INVESTMENT PLANNING FOR ELECTRIC POWER SYSTEMS TO
MITIGATE EXTREME EVENTS

A Dissertation
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Doctor of Philosophy

by
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INVESTMENT PLANNING FOR ELECTRIC POWER SYSTEMS TO MITIGATE EXTREME EVENTS

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Cornell University 2012

My research has focused on two inter-related questions. First, how do we model the impacts of terrorism and earthquake events on electric power systems? Second, how might we optimize investments in these systems when there are limited resources? For intentional attack we model the interaction between the offender and the operator of the network where both parties have limited budgets and behave in their own self-interest. The problem was formulated as a multi-level mixed-integer programming problem and we implemented a Tabu search with an embedded greedy algorithm to find the optimum defense strategy. We model the regional earthquake hazard using a four step process that included an optimization problem to select a small collection of events from a candidate set, including a probability of occurrence for each event that matches the hazard. Since electric power systems are spatially distributed, their performance is driven by the joint distribution for damage of the components. Hence we estimated this distribution by constructing a collection of consequence scenarios for each earthquake scenario, where each consequence scenario identifies the level of damage to each component. For each consequence scenario, we used an economic dispatch model to predict the load shed and repair costs throughout the repair process. We expanded the analysis of the power network under the seismic risk by modeling
the additional impact of cascading outages and the consequences on the air passenger transportation system due to the interdependency of both networks. We formulated the problem of selecting seismic mitigation strategies to increase resilience of electric power system to earthquake hazards as a two-stage stochastic program. We develop a custom solution procedure which we show to be computationally effective for extremely large problem instances.
Natalia was born and raised in Cali, Colombia. After graduating from Liceo Benalcazar, she took six months of English as a Second Language at Algonquin College, Nepean, Canada, and six months of French as a Second Language at University of The West, Angers, France. In July 2001 she started Law at Javeriana University, Bogota, Colombia, and in August 2002 she transferred to University of Los Andes, Bogota, Colombia, where she graduated with a Bachelor of Science in Civil Engineering and Minor in Law. Natalia worked for a year at University of Los Andes, Center in Water Supply Systems and Sewers, as a Research Engineer in the “Condition Assessment and Damage Management of the Water Supply System of Bucaramanga”. Natalia moved to Ithaca in August 2007 to pursue her master and doctoral degrees in the School of Civil and Environmental Engineering at Cornell.
Le dedico esta tesis doctoral a mi familia; especialmente a mi amigo, amor y esposo, Alex quien ha estado a mi lado en tantos buenos y malos momentos de mi investigación. Agradezco a Caro, Valle, Carolyn y Ed por su apoyo y cariño incondicional. Finalmente, quiero dedicar este trabajo a mis padres por inculcarme a siempre ser lo mejor que puedo ser, y ofrecerme las bases y el amor para lograrlo.
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CHAPTER 1

INVESTMENT PLANNING FOR ELECTRIC POWER SYSTEMS UNDER TERRORIST THREAT

Nomenclature

A. Model Parameters

\( T \) set of transmission lines and transformers (assumed to be directed to facilitate formulation).

\( B \) set of buses.

\( L \) set of transformers.

\( S \) set of substations.

\( E \) set of generators.

\( a_{ij}^D \) cost of adding \( c_{ij} \) units to transmission line \((i, j)\).

\( b_g^D \) cost of adding \( q_g \) units to generator \( g \).

\( M^A \) terrorist’s budget.

\( M^D \) defender’s budget.

\( c_g^E \) per-unit cost for generation using generator \( g \).

\( c_{ij} \) size of capacity increments that can be added to transmission line \((i, j)\).

\( c_l^L \) cost to replace interdicted transformer \( l \).

\( D_i \) demand at bus \( i \).

\( e_{ij}^A \) cost of attacking transmission line \((i, j)\).

\( f_s^A \) cost of attacking substation \( s \).

\( f_s^D \) cost of protecting substation \( s \).

\( G_g^m \) capacity of generator \( g \).

\( h_g^D \) cost of protecting generator \( g \).
\( h_g^A \) cost of attacking generator \( g \).

\( h_l^L \) unit purchasing cost of transformer \( l \).

\( k \) stage of recovery, \( k \in \{1,2,3,4\} \).

\( m_{ij} \) reactance for transmission line \((i,j)\).

\( p_{ij}^m \) capacity of line \((i,j)\).

\( q_g \) increments of capacity that can be added to generator \( g \).

\( r_s^S \) cost of repairing substation \( s \).

\( r_{ij}^T \) cost of repairing transmission line \((i,j)\).

\( r_g^E \) cost of repairing generator \( g \).

\( s_i \) the substation where bus \( i \) is located.

\( \mu_i \) unit load shedding cost at bus \( i \).

\( t_k \) time to complete stage \( k \).

\( T_{ijkl} \) binary parameter, 1 indicates a transformer; otherwise indicates a transmission line.

\( \delta^+(i) \) set of lines that start in bus \( i \).

\( \delta^-(i) \) set of lines that end in bus \( i \).

\( I_i \) set of generators that are connected to bus \( i \).

\( O_s \) set of buses in substation \( s \).

\( (k,l) || (i,j) \) transmission line \((k,l)\) is the line sharing right of way with \((i,j)\).

### B. Defender’s Decision Variables

\( x^D = (x_s^D) \) vector of binary decision variables, \( x_s^D = 1 \) if substation \( s \) is protected and 0 otherwise.

\( z^D = (z_g^D) \) vector of binary decision variables, \( z_g^D = 1 \) if generator \( g \) is protected and 0 otherwise.

\( w^D = (w_g^D) \) vector of integer decision variables, \( w_g^D \) indicates that \( q_g \) units of capacity are added to generator \( g \).
\( v^D = (v^D_{ij}) \) vector of integer decision variables, \( v^D_{ij} \) indicates that \( c_{ij} \) units of capacity are added to transmission line \((i,j)\).

\( H^D = (H^D_l) \) vector of integer decision variables, \( H^D_l \) indicates the number of transformers of type \( l \) purchased.

\section*{C. Terrorist’s Decision Variables}

\( x^A = (x^A_s) \) vector of binary decision variables, \( x^A_s = 1 \) if substation \( s \) is attacked and 0 otherwise.

\( y^A = (y^A_{ij}) \) vector of binary decision variables, \( y^A_{ij} = 1 \) if transmission line \((i,j)\) is attacked and otherwise.

\( z^A = (z^A_g) \) vector of binary decision variables, \( z^A_g = 1 \) if generator \( g \) is attacked and 0 otherwise.

\section*{D. Power-link Network Decision Variables}

\( G = (G^k_g) \) vector of nonnegative continuous decision variables, \( G^k_g \) indicates the power generated by generator \( g \) in period \( k \).

\( P = (P^k_{ij}) \) vector of continuous decision variables, \( P^k_{ij} \) indicates the electric power flows in line \((i,j)\) in period \( k \).

\( U = (U^k_i) \) vector of nonnegative continuous decision variables, \( U^k_i \) indicates the load shed at bus \( i \) in stage \( k \).

\( \theta = (\theta^k_i) \) vector of continuous decision variables, \( \theta^k_i \) indicates the phase angle at bus \( i \) in period \( k \).

\( R = (R^k_{ijl}) \) vector of binary decision variables, \( R^k_{ijl} = 1 \) if spare transformers \( l \) is used in line \((i,j)\) and 0 otherwise.
E. Tabu Search Parameters

$\alpha$ indicator of iteration.
$\bar{\alpha}$ iteration limit
$\tau$ indicator of neighbor.
$\bar{\tau}$ number of neighbors to explore
$\bar{\tau}^2$ maximum number of neighbors to construct from previous attacks.
$\varphi_\tau$ objective value for neighbor $\tau$.
$\varphi^\alpha$ objective value for initial solution in iteration $\alpha$.
$\varphi^*$ objective value for best neighbor.
$\varphi^{**}$ objective value of best solution for the search.
$\Delta_\tau$ defense strategy for neighbor $\tau$.
$\Delta^\alpha$ defense strategy for initial solution in iteration $\alpha$.
$\Delta^*$ defense strategy for best neighbor.
$\Delta^{**}$ defense strategy of best solution for the search.
$\mu_\tau$ defense strategy cost neighbor $\tau$.
$\mu^*$ defense strategy cost for best neighbor.
$\pi$ previous number of attack strategies to use when identifying components for investment.

Introduction

Deliberate attacks do not occur frequently but when they do they can be disastrous. From 1999 to 2002 there were over 150 attacks on electric power systems across the world [Zimmerman et al. 2005]. In the United States there is awareness that it is important to make these systems more resilient to terrorist attacks. For example, the National Academies suggests that it is important to consider protection of key equipment and whether there should be additional reserve capacity for generation,
transmission and distribution to promote resiliency to address these threats [National Research Council [2002].

This paper presents a formulation for the problem of a strategic defender to a carefully crafted attack. We develop a leader-follower model representation of this strategic interaction. The leader determines the protection measures to be adopted including specific investments to increase operating margin and the acquisition of spare transformers. The follower is the terrorist that selects an attack with full knowledge of the defender investments and the understanding that the operator will optimize the use of the system after the attack in order to minimize the consequences of the attack.

National Research Council [2008] describes a range of related models across a variety of infrastructures focused on a bioterrorist threat. Bienstock and Mattia [2007] develop an investment planning model using a linear dc power flow representation of an electric power transmission network. They include a range of scenarios for the disruption and solve for investments in line capacities that minimize the investment needed to honor all demands under all scenarios. Smith et al. [2007] analyze the fortification problem for networks. They assume different profiles for the attacker behavior: destruction based on capacity of arcs, destruction based on flow, or optimal behavior to minimize or maximize the flow in the system. Scaparra and Church [2006] use a bilevel mixed-integer program to identify the critical components in a network under terrorist threats. They use an implicit enumeration algorithm to solve the fortification problem or upper level. The lower level is formulated as an r-interdiction median problem. Smith and Lim [2008] study different models and algorithms to solve network interdiction games. They also explore the problem from a three level perspective including the fortification of the network.
Salmeron et al. [2004] and [2009] develop an interdiction model for electric power systems, using a set of linear dc power flow models, with the goal of identifying the attack that would result in the maximum disruption. Salmeron et al. [2004] develop a heuristic iterative scheme to identify prices which represent the value of each component in an attack. They then select those components which result in the largest estimated damage, given the prices developed and that are consistent with the terrorist’s budget. Salmeron et al. [2009] focus on the same formulation but use generalized Benders decomposition to create a heuristic solution procedure. Arroyo & Galiana [2005] and Motto et al. [2005] develop a bilevel formulation of the interdiction problem. In Motto et al. [2005] the terrorist’s objective is to cause a maximal amount of load shed with the removal of as few lines as possible and the defender wants to minimize load shed. Arroyo & Galiana [2005] and Motto et al. [2005] uses the Karush–Kuhn–Tucker conditions to convert the bilevel formulation into a single level optimization problem. Bier et al. [2007] employs a simple procedure to identify “near-optimal” interdiction strategies for power transmission systems. They consider attacks to lines only and implement a greedy algorithm. At each iteration of that algorithm the line with maximum flow is identified. This process continues until the budget of the terrorist is exhausted. Additionally, it explores the benefits from hardening (protecting) a number of lines that the terrorist would most likely choose.

Carrion et al. [2007] uses a stochastic programming formulation to reinforce and expand a power system with the objective of reducing the impact of a deliberate attack. They use a previously defined set of possible attacks and possible expansion strategies. The attacks are generated using results from Motto et al. [2005]. Alguacil et al. [2009] analyzes the expansion plans proposed in Carrion et al. [2007] in relation to economic and vulnerability issues. Arroyo & Fernandez [2009] evaluates the benefit of line switching as a mitigation strategy. They use a genetic algorithm to solve the
interdiction problem. Arroyo [2009] analyzes the vulnerability of a power grid to unintentional and deliberate outages. The interdiction bilevel mixed-integer problem is transformed into a single level problem using two different methods: its Karush–Kuhn–Tucker conditions, and duality theory. Chen et al. [in press] expands on the defender-attacker problem by focusing on the amount of information available to the attacker when they form the attack as a mechanism to determine the defender's optimal strategy.

We formulate a multi-level mixed-integer programming problem. The defender’s optimization is solved using a Tabu Search similar to that presented by Wen and Huang [1996]. The attack is found with a greedy algorithm. It has three different variations that correspond to three assumptions as to how the terrorist will craft the attack. These assumptions are identified as: capacity based attack (CBA), maximum flow based attack (MFA), and greedy attack (GA).

The defender can allocate a limited budget to protect generators and substations, to increase generation and line capacity, or to purchase spare transformers. The terrorist can attack any combination of lines, substations, and/or generation units subject to their budget constraint. This paper compares the results for three different assumptions of how the terrorist will craft the attack and four budgets each for the defender and the terrorist using the one area IEEE Reliability Test System (RTS) – 1996 [Grigg, 1999].

This paper can be viewed as extensions of Bier et al. [2007] in that it extends their interdiction problem to consider the performance of the systems during the entire repair process, not just in the period right after the attack. It also explores the limitations of a CBA attack in comparison to GA and MFA crafted attacks. Finally we significantly broaden the approach to the investment aspect of the problems to include opportunities to add operating margin to the systems as well as to stockpile
spares. This paper can also be viewed as an extension of [Salmeron et al. 2004] and [Salmeron et al. 2009] in that their focus is on interdiction under the assumption that it is known how spare transformers will be used. This is a key element of this analysis however; we also focus on the investment elements of the problem in addition to how to optimally use spares after an attack.

**Approach**

**A. Formulation**

We formulate the defender-attacker-operator problem as a three-level optimization problem.

The upper level corresponds to the defender’s decision to minimize the costs after an attack to the power grid through investments (pre-attack). The intermediate and lower levels are the interdiction problem. In the interdiction problem, the attacker damages a set of components in the system in order to optimize her/his interest. However, we assumed that the attacker knows what the operating strategy would be to minimize the costs after the attack. The third level corresponds to operator’s minimization of operating costs after the attack.

The repair process consists of four stages. During the first stage, all attacked components and those in proximity to attacked components can not have any flow. During the second stage, the only lines that are not repaired are those connected to attacked substations. During the third stage, all substations components have been repaired except damaged transformers. In the fourth and final stage, spare transformers are assumed to be in place.

The linear dc power flow network model is used to estimate the power flows in the network at each stage during the repair process and the demands not satisfied. The defender’s objective is to minimize the sum of the power generation costs, load shed
costs and the repair costs by implementing an effective pre-attack investment strategy. The objective of the terrorist is to maximize the sum of the power generating costs, load shed costs, and repair costs. Both players have limited resources. The objective of the operator is to minimize the sum of the generation costs, the load shed costs and the repair costs for transformers which are attacked and for which there is not a spare.

B. Power-Flow Optimization Model

The linear dc power flow model formulated below represents the optimal response of the operator during the recovery period. In this model, the investment strategy of the defender is assumed known to the terrorist. We assume that the operator observes the defender’s and attacker’s decisions and makes his decision during these four time periods by minimizing the total operational cost during the four-stage repair process. Mathematically, for the given \((x^A, y^A, z^A, w^D, v^D, H^D)\), this network optimization problem is to choose \((\theta, G, P, U, R)\) that minimizes

\[
\sum_{k=1,2,3,4} \left( \sum_{l \in B} \mu_i U^k_l + \sum_{g \in E} c_g^E G^k_g \right) (t_k - t_{k-1}) + \sum_{s \in S} \sum_{l \in O_s} \sum_{i \in A} c_i^T (1 - R_{ij}) T_{ij} x^A_{s} \tag{1}
\]

subject to

\[
(\theta_1^i - \theta_1^j)(1 - y_{ij}^A)(1 - x_{si}^A)(1 - x_{sj}^A) \prod_{(k,l)|(i,j)} (1 - y_{kl}^A) = m_{ij} p_{ij}^1 * P_{ij}^m / (1 + c_{ij} v_{ij}^D), \quad (i,j) \in T \tag{2}
\]

\[
(\theta_2^i - \theta_2^j)(1 - x_{si}^A)(1 - x_{sj}^A) = m_{ij} p_{ij}^2 * P_{ij}^m / (1 + c_{ij} v_{ij}^D), \quad (i,j) \in T \tag{3}
\]

\[
(\theta_3^i - \theta_3^j)(1 - x_{si}^A \sum_{l \in L} T_{iji}) = m_{ij} p_{ij}^3 * P_{ij}^m / (1 + c_{ij} v_{ij}^D), \quad (i,j) \in T \tag{4}
\]
(\theta_i^+ - \theta_j^+)(1 - x_{si}^A \sum_{l \in L} (1 - R_{ijl})T_{ijl}) = m_{ij} \frac{P^m_{ij}}{1 + c_{ij}v_{ij}^D}, \quad (i,j) \in T \quad (5)

\sum_{g \in l(i)} G_g^k - \sum_{(i,j) \in \delta^+(i)} P_{ij}^k + \sum_{(j,i) \in \delta^-(i)} P_{ji}^k = D_i - U_i^k, \quad i \in B, k = 1,2,3,4 \quad (6)

0 \leq U_i^k \leq D_i, \quad i \in B, k = 1,2,3,4 \quad (7)

0 \leq G_g^k \leq (G_g^m + q_g w_g^D)(1 - z_g^A), \quad g \in E, k = 1,2,3,4 \quad (8)

|P_{ij}^1| \leq (P^m_{ij} + c_{ij}v_{ij}^D)(1 - y_{ij}^A)(1 - x_{si}^A)(1 - x_{sj}^A) \prod_{(k,j) \in (i,j)} (1 - y_{ik}^A), \quad (i,j) \in T, \quad (9)

|P_{ij}^2| \leq (P^m_{ij} + c_{ij}v_{ij}^D)(1 - x_{si}^A)(1 - x_{sj}^A), \quad (i,j) \in T, \quad (10)

|P_{ij}^3| \leq (P^m_{ij} + c_{ij}v_{ij}^D)(1 - x_{si}^A \sum_{l \in L} T_{ijl}), \quad (i,j) \in T, \quad (11)

|P_{ij}^4| \leq (P^m_{ij} + c_{ij}v_{ij}^D)(1 - x_{si}^A \sum_{l \in L} (1 - R_{ijl})T_{ijl}), \quad (i,j) \in T, \quad (12)

\sum_{(i,j) \in T} R_{ijl} \leq H_l, \quad l \in L \quad (13)

R_{ijl} = 0,1 \quad (i,j) \in T, l \in L \quad (14)

The objective function given in equation (1) is the sum of the power generation, load shed, and replacement costs (for spare transformers for which there are no spares). Constraints (2), (3), (4), and (5) approximate the active power flows on the transmission lines in the four stages of the repair process. It is important to notice that some forms of line capacity upgrades, such as reconductoring or adding parallel lines may reduce line reactance. We model this by reducing the line's reactance in proportion to the capacity increase due to the enhancement. Constraints (6) preserve the power balance at the buses in the four-stage repair process. Constraints (7) state that the load shedding at a bus cannot exceed the demand at the bus. Constraints (8)
bounds the power produced at each generator. If a generator is not attacked, the
generator has the full capacity, which is its original capacity plus any capacity added
by the defender. Otherwise the generator does not function during the four time
periods. Note that the power flow in each transmission line can go in either direction
therefore the flow load on each transmission line can take either a negative or positive
value. Constraints (9), (10), (11), and (12) set the maximum absolute values of the
flows for transmission lines in each stage. A line is not available in the first stage if it
is attacked, if a substation to which is connected is attacked, and/or if a line in close
proximity is attacked. Otherwise, the line has full transmission capacity. For the
second stage, lines that are connected to attacked substations are the only ones with no
transmission capacity. Substations without transformers can be repaired by the third
stage; thus, during the third stage only lines that represent transformers damaged
during a substation attacked are not available. In the final stage, the interdicted
transformers, for which there is a spare, are back in operation. Constraints (13) state
that the number of transformers of each type used to replace the interdicted
transformers of that same type cannot exceed the number available. Constraints (14)
impose binary restrictions. It is important to notice that this formulation includes the
length of time each component attacked will be out of service. It is these durations
that drive the definition of the stages, indexed by k, in the model. We assume, as in
[Gomberg and Schweig 2007], that lines, transformers for which there are available
spares and substations are repaired within 72h, 360h, 768h, respectively.

Recall that mathematical program (1) – (14) depends on the defender and
terrorist’s decisions \((x^2, y^2, z^2, w^1, v^1, H^1)\). Let \(F_L(x^2, y^2, z^2, w^1, v^1, H^1)\) be the
minimum value in (1).
C. Terrorist’s Optimization Problem

The terrorist is assumed to have perfect information on network protection, network capacity, and number of stored spare transformers. The terrorist also understands that the operator will strive to mitigate the impact of the attack to the extent possible including identifying how to use the spare transformers effectively. Based all information, the terrorist chooses its attack strategy under a budget constraint. Mathematically, for the given \((x^D, z^D, v^D, w^D, H^D)\), the terrorist’s optimization problem is to choose \((x^A, y^A, z^A)\) that maximizes

\[
F_L(x^A, y^A, z^A, w^D, v^D, H^D) + \sum_{(i,j) \in T} y^T_{ij} y^A_{ij} + \sum_{g \in E} r^g z^g_A + \sum_{s \in S} r^S x^A_s
\]

subject to

\[
x^A_s \leq 1 - x^D_s, \quad s \in S,
\]

\[
z^A_g \leq 1 - z^D_g, \quad g \in E,
\]

\[
y^A_{ij} \leq 1 - \sum_{l \in L} T_{ili}, \quad (i,j) \in T,
\]

\[
\sum_{s \in S} f^A_s x^A_s + \sum_{(i,j) \in T} e^A_{ij} y^A_{ij} + \sum_{g \in E} h^A_g z^A_g \leq M^A,
\]

\[
x^A_s, z^A_g, y^A_{ij} = 0,1, \quad s \in S, g \in E, (i,j) \in T
\]

The objective function (15) is the sum of the power generation costs and load shed costs during the four stages, and the repair costs. Constraints (16) and (17) enforce the rule that only the unprotected generators and substations can be attacked. Constraints (18) prohibit attacks on single transformers; these can only be damaged through attacks to substations. Constraint (19) states that the total cost of attacking the
generators, lines, and substations cannot exceed the available budget. Constraints (20) require the decision variables are binary.

Recall that mathematical program (15) – (20) depends on the defender’s decisions$(x^D, z^S, v^D, w^D, H^D)$. Then let $F_A(x^D, z^S, v^D, w^D, H^D)$ be the maximum value.

**D. Defender’s Optimization Problem.**

The defender understands that the terrorist has perfect information and will optimize his attack to further his objectives. Therefore the defender chooses a protection strategy to minimize their objective function subject to a limited budget. Mathematically, we formulate this optimization problem as a static transmission and generation planning problem [Latorre et al. 2003] to choose $(x^D, z^S, v^D, w^D, H^D)$ that minimizes

$$F_A(x^D, z^S, v^D, w^D, H^D)$$

subject to

$$\sum_{s \in S} f_s^D x_s^D + \sum_{g \in E} h_g^D z_g^D + \sum_{(i,j) \in T} a_{ij}^D v_{ij}^D + \sum_{g \in E} b_g^D w_g^D + \sum_{l \in L} h_l^D H_l^D \leq M^D,$$

$$x_s^D, z_g^D, v_{ij}^D, w_g^D = 0,1, \ s \in S, g \in E, (i,j) \in T, H_l^D = \text{non-negative integer } l \in L$$

The treatment of investment costs is parallel to [Alguacil et al. 2003] and [Bienstock and Mattia 2007].

**E. Solution Procedure**

It is known that this three-level optimization problem is NP-hard [Bard, 1988]. We used a Tabu Search to solve the defender’s problem and greedy algorithms to address the attacker’s problem. For the Tabu Search, the first neighbors are generated based on attacks generated from past neighborhoods. All the substations and generators attacked
on these previous iterations are likely critical components; thus, in these first iterations, we explore the benefit from protecting them. However, if these protection strategies are infeasible, the neighbors are generated randomly. The other neighbors are randomly generated. The attack is estimated using a greedy algorithm, which represent different assumptions about how the terrorist will craft their attack. Those algorithms are: CBA, MFA, or GA. CBA is an algorithm which iteratively removes the component with the largest capacity until the attacker’s budget is exhausted [Smith et al., 2007]. MFA iteratively removes components with the largest weighted flow across the four repair periods as computed by the dc power flow model. This algorithm is an extension to that given in Bier et al. [2007]. In each iteration, the GA algorithm removes the component which causes the greatest increase in the terrorist’s objective function until the terrorist’s budget is exhausted.

For the case study developed based on the One Area IEEE RTS-96 [Grigg 1999]. The system described in the next section, the algorithm was implemented and executed using IBM ILOG OPL 6.3 in a Dell Optiplex 755, Intel® Core™ 2 Quad, with 2.83 GHz and 3.25 GB of RAM memory (though the code was not parallelized). For 20 iterations, 10 neighbors per iteration, keeping solutions as tabu for 5 iterations, an attack budget of 6 units, and a defense budget of US$ 25 million, the execution times was approximately: 3 and a half minutes for CBA, 22 minutes for MFA, and about 12 hours for GA. For a dc model of the Eastern Interconnect which has about 23,000 links and 15,000 nodes for a defense budget of US$ 100 million, an attack budget of 10, and an attack strategy of CBA, about 14 hours was required. In this case, CPLEX 12.1 C++ concert technology was used. Computation time can be reduced if the problem is parallelized using CPLEX 12.2.

Case Study
We applied our method to the IEEE One Area RTS - 1996. It has 24 nodes and 38 links that correspond to 24 buses and 38 transmission lines. We included generation units and substations as independent sets of components that can be related to the buses. Substations were defined as combinations of one or more buses. With the exception of buses 3, 9, 10, 11, 12, 13, and 24, each bus represents a substation. Buses 3 and 24 are connected by a transmission line identified as a transformer; thus, these two buses are part of the same substation. Likewise, buses 9, 10, 11, and 12 are part of a single substation. With these considerations, the model has 20 substations. The IEEE RTS document has a comprehensive description of the location of generation units among the buses [Grigg 1999]. The 5 transformers on the one area RTS-96 were modeled as links for the load-flow model.

Salmeron et al. [2004] identify the average outage time when certain components of a power system are disrupted. Overhead lines, transformers for which there are available spares, single busses and substations are assumed to take 72 h, 360 h, and 768 h, respectively. In addition to this data, we assume that replacing transformers and generation units can take on the order of 4320 h (6 months). These outage lengths are the basis for the four time period considered in this model.

The costs to attack a component can vary based on a variety of factors including the type of attack and the location of the component. Therefore, we assigned a relative cost to attack each type of element. Attacking a line costs one unit, attacking a substation, three units, and attacking a generator four units. An attack to a line damages the line and any line with common right of way (See ovals identified with letters from A to G in One-area IEEE RTS-96 [Grigg 1999]). An attack to a substation damages the buses and transformers within the substation and all the lines coming connected to the substation. The attack to a generation unit exclusively damages the generation plant; it does not affect the bus where it is located. The costs to protect
substations and generators, increase capacity of lines and generation units, and replace components were obtained from a variety of sources. Billington et al. [1989] provides the generation unit costs for hydro and coal plants; they include capital costs and operational costs. Balducci et al. [2005] classifies costs for several power system capacity enhancement projects; it was a source for repair costs and lines capacity enhancement strategies. Vaziri et al. [2004] presents data for line upgrades and new transformers costs. New generation unit capital costs and generating cost were obtained from Wald [2007], the Energy Information Administration [2009], and Secretary-General of the OECD [2005]. All the costs were converted to 2002 U.S. dollars and adjusted to make them consistent among sources.

Table 1 shows how the defender’s budget is allocated under each scenario. Protection (Pro.) corresponds to protecting substations and generators. Protecting the nuclear generators and the coal/steam generator (U350) is very expensive but these generators are the most attractive for the terrorist. Since it is not possible for the defender to protect these components with any of the explored budgets, protection of generators is not considered. Therefore, protection, in these experiments is focused on substation security. Enhancement (Enh.) refers to investments in operating capacity for transmission lines and generators. In this analysis, the largest benefits are the result of investments in generation capacity. It is useful to notice that the model tends to generally recommend a relatively equal investment in protection and enhancement and, because of the cost of transformers, no investment in spares.
Table 1. Defender’s resources allocation

<table>
<thead>
<tr>
<th>Attack</th>
<th>Defender's Budget [Millions of U.S. dollars]</th>
<th>Defense resource allocation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>25.0</td>
<td>50.0</td>
</tr>
<tr>
<td>Type</td>
<td>Budget</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>21.6</td>
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<tr>
<td></td>
<td>6</td>
<td>21.6</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>21.6</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.0</td>
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<td></td>
<td>6</td>
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<td>7</td>
<td>21.6</td>
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<td></td>
<td>6</td>
<td>10.8</td>
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<tr>
<td></td>
<td>7</td>
<td>10.8</td>
</tr>
</tbody>
</table>

Table 2 shows the components that the defender decides to protect or enhance. As in Table 1, this table has an entry for each attack rule and defender and attacker budgets. Substations are identified with an $S$ and with the number of the bus ($B$) within them. Enhancements to generators, $G$ are referenced using the corresponding bus and the total generating MW added to generators at the bus. Notice how protecting substations 3 and 9 is a key element of many defense strategies. In addition, notice the recurrence of enhancements to the capacity of generators in bus 1, 2, and 15. Increasing generation capacity in these three buses is cheaper than in most of the other buses. In addition, buses 1 and 2 are located in an area of the system with less generating capacity and there is a risk of isolation from other sources of generation depending on the character of the attack.
Table 2. Defense strategy critical components

<table>
<thead>
<tr>
<th>Attack</th>
<th>Type</th>
<th>Budget</th>
<th>Defender's Budget [Millions of U.S. dollars]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>25.0</td>
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<td></td>
<td></td>
<td>Protection</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Protection</td>
</tr>
<tr>
<td>2</td>
<td>GA</td>
<td>S.3**</td>
<td>G(B:1(4MW), B:2(12MW), B:15(4.8MW))</td>
</tr>
<tr>
<td>3</td>
<td>GA</td>
<td>S.3**</td>
<td>G(B:1(8MW), B:2(12MW), B:15(4.8MW))</td>
</tr>
<tr>
<td>6</td>
<td>GA</td>
<td>S.3**</td>
<td>G(B:1(8MW), B:2(12MW), B:15(4.8MW))</td>
</tr>
<tr>
<td>7</td>
<td>GA</td>
<td>S.3**</td>
<td>G(B:1(8MW), B:2(12MW), B:15(4.8MW))</td>
</tr>
<tr>
<td>2</td>
<td>MFA</td>
<td>S.3**</td>
<td>L:(28),G(B:1(4MW), B:2(4MW))</td>
</tr>
<tr>
<td>3</td>
<td>MFA</td>
<td>S.3**</td>
<td>L:(28),G(B:1(4MW), B:2(4MW))</td>
</tr>
<tr>
<td>6</td>
<td>MFA</td>
<td>S.3**</td>
<td>G(B:1(8MW), B:2(8MW), B:15(4.8MW))</td>
</tr>
<tr>
<td>7</td>
<td>MFA</td>
<td>S.3**</td>
<td>G(B:1(8MW), B:2(4MW), B:15(7.2MW))</td>
</tr>
<tr>
<td>2</td>
<td>CBA</td>
<td>S.3**</td>
<td>L:(28),G(B:1(4MW), B:2(4MW))</td>
</tr>
<tr>
<td>3</td>
<td>CBA</td>
<td>S.3**</td>
<td>S.3**</td>
</tr>
<tr>
<td>6</td>
<td>CBA</td>
<td>S.9</td>
<td>G(B:2(4MW), B:15(4.8MW))</td>
</tr>
<tr>
<td>7</td>
<td>CBA</td>
<td>S.3**</td>
<td>L:(1),G(B:1(4MW), B:2(8MW))</td>
</tr>
</tbody>
</table>

* S:9 Corresponds to bus 9, 10, 11, and 12
** S:3 corresponds to bus 3, and 24
<table>
<thead>
<tr>
<th>Attack</th>
<th>Type</th>
<th>Budget</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GA</td>
<td>2</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.37</td>
</tr>
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<td>0.55</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>1.59</td>
</tr>
<tr>
<td>MFA</td>
<td>2</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.37</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>1.22</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>1.59</td>
</tr>
<tr>
<td>CBA</td>
<td>2</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.37</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>0.55</td>
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<tr>
<td></td>
<td>7</td>
<td>1.59</td>
</tr>
</tbody>
</table>
For the different attack budgets and strategies and different defense budgets, the attack decision follows a simple behavior. When the terrorist’s budget is low, lines are the target of attacks. As their budget grows, substations become attractive and finally, when there are sufficient funds, generators. The nuclear power plant is frequently targeted when the terrorist has sufficient resources, regardless of the rule the terrorist uses in crafting the attack.

Table 3 shows the incurred costs after an attack. Each row corresponds to a unique combination of attack rule and budget. There are three types of costs: repair, generation (Gen.), and load-shed (L.shed). Note that the after-attack generation costs vary based on costs of available generation units, and connectivity between power sources and demand. When there is no defense budget and the terrorist has 2 units, each decision rule for the terrorist selects different lines to attack. All the selected lines are among lines with greater transmission capacity. For 3 units, the terrorist can attack substations and lines; the best attack will include damaging substation 9. Substation 9 has 4 transformers and corresponds to 80% of the connectivity between high voltage transmission and low voltage transmission areas. The low voltage area does not have enough generation capacity to cope with the local demand. The logic behind MFA leads the attacker to target substation 3. This situation highlights a weakness in the MFA rule. MFA depends on the solution to the dc power flow model which is often not unique. Therefore, it may be possible for the operator to effectively compensate for the loss as understood through this model.

When the attacker has 6 units, there are sufficient funds to attack two substations. It is important to remember that attacks to substations with transformers and to any generator last for all four periods. It is also important to notice that example network has a total of 3405 MW of generation capacity and 2850 MW of demand. Therefore, capacity exceeds demand by about 20%. The three larger generators are the two
nuclear plants, that can produce 400 MW, and one coal/steam plant that can produce 350 MW. If one of these generators were attacked, the system would still have 5% excess capacity. On the other hand, attacking substations 3 and 9 would split the system into two areas. The lower voltage area includes the first 10 buses of the RTS; it has a total demand of 1332 MW and a local generation capacity of 684 MW. In addition to the limitations in local generation capacity in this area, it is important to highlight that 370 MW of the demand are located in buses 9 and 10 which are part of substation 9. If attacked, the lines connected to this substation would remain disrupted for the first two time periods cutting off any energy supply to these two buses. The second area has enough local generation capacity; it only has load-shed problems due to connectivity. Consequently, the system would have a total load-shed of 35% during the first two time periods and 25% in the third and fourth periods. The costs of the load-shed are more significant than the difference between the repair costs of the nuclear plant versus the 5 transformers. Therefore, the optimal attack with a budget of 6 units corresponds to damaging substations 3 and 9. The system has enough redundancy to cope with losing 400 MW and with part of the new limitations of connectivity. CBA and GA decision rules choose to attack substations 9 and 3; but the MFA rule results in an attack on one of the nuclear plants and two lines and therefore a much lower post-event consequence for the operator.

When the terrorist has 7 units and the defender zero budget, effective attacks include a high capacity generator and substation 9. All three rules identify the same generator and substation. Investing in protection reduces losses in connectivity and investing in generation capacity provides redundancy. This explains the slight increment in generation costs with larger defense budgets. Notice the reductions in load-shed costs, for GA and CBA, when the attacker has 3 or more units and the defense budget increases to US$25 million. In the later case, the defender protects
substations 3 and 9. With only 3 units a secondary substation is attacked. With 6 units the attack focuses on a nuclear plant and two lines. With 7 units, the nuclear plant and a secondary substation are targeted.

It is useful to understand what the impacts might be if a defensive strategy is crafted based on an incorrect estimate of how the terrorist will craft their strategy. We explored this question when the investment budget is US$25 million and the terrorist’s budget is 6 units. If the defender assumes that the terrorist will use an MFA based strategy but they use GA or CBA instead, the post-event costs would be about US$4.7 billion in contrast the US$1.5 billion the defender might have anticipated. In this example, planning for a GA attack is the most conservative. However, examples can be constructed for which GA does not lead to favorable post-event consequences when the original defense was based on a different attack strategy.

**Conclusions**

This paper presents a formulation for the problem of a strategic defender of an electric power system to a carefully crafted attack. We develop a leader-follower model representation of this strategic interaction. We then apply this model to the One Area RTS-96 [Grigg 1999] with several different budgets for the terrorist and defender and three different rules for how the terrorist might craft the attack. Based on those experiments, we identify four important ideas for investment in these types of networks. First, MFA as a decision rule for the attacker is often inferior to GA and CBA. A key reason for this is that there are often alternative optimums to the network flow problem. This makes the connection between high flow and criticality fragile. Second, when protecting electric power systems against these types of attacks, investments in operating margin are important to consider in addition to more traditional protection measures. Operating margin can make the systems more
resilient to attacks and alleviate the need for some types of traditional protection measures. Third, as the defender’s budget increases many of the investments recommended by smaller budgets remain useful. This is important because these types of investments are often done incrementally. Fourth, if the defensive strategy is developed assuming a different strategy will be used to craft the attack than that which is actually used, the post-event costs may be significantly higher than anticipated. This implies that it is important to look for investment strategies that are robust against different methods which might be used to craft the attack.

This research points to several different avenues for future research. First, the probability of an attack is not considered as part of this analysis hence extensions which would include this element is important. This analysis assumes that the user of the model’s results subjectively incorporates what “soft” information about the likelihood of an attack might be available to support decision-making. In some countries, terrorism has been a long-term problem that persists on a time scale of decades. A country that has already suffered from persistent attacks would have data to construct an analysis estimating the probability of attack and could weigh the costs of an attack accordingly. A country with rare attacks or no previous attacks might need to develop a more subjective approach and integrate this approach with the estimation of a trade-off frontier of the reduction in attack consequences vs. costs. This trade-off frontier might be somewhat similar to the results given in Tables 1-3 in the case study.

Second, we formulate a static defender-attacker-operator model. There is significant value in the creation of a dynamic representation of this strategic interaction over a longer time scale. For example, we consider the initial investment costs for capacity expansion and protection as well as the costs incurred during a 6 month recovery period. A dynamic representation would allow explicit treatment of operating and maintenance costs, return on investment constraints, environmental
constraints, etc. It would also allow for the simultaneous consideration of traditional issues like changes in demand over time. Further, improvements in generation and transmission capacity would provide economic and reliability benefits (in addition to benefits of protection from attack) and quantifying these is useful. Third, the attack strategy actually used is difficult to determine a priori therefore developing a method which identifies robust investments against a range of rules is important. Fourth, infrastructure networks are interdependent; therefore it is possible that investments in other networks on which the electric power system relies could be as important as investments in the electric power systems itself. Therefore, research to extend the modeling to consider interdependencies is important. Fifth, there are a range of targets available, should an entity be interested in engaging in terrorist activities. In order to defend infrastructures it is useful to consider the broad range of actions that can be taken to reduce the attractiveness of targets or to increase the difficulty of mounting a successful attack. It is also important that the costs and benefits of these measures be fully assessed. For example, [Murray-Tuite 2007] focuses on the costs of pre-event and post-event security measures in transportation systems. [Prentice 2008] focuses on estimating tangible and intangible benefits of security measures. Therefore, research to look across the range of targets available and the range of mitigation measures, including costs incurred and benefits accrued, is critical. Sixth, and finally, electric power systems are subject to a range of hazards therefore it is important to think about investment strategies in a multi-hazard context. Some investments, which could be made, improve post-event performance after multiple types of events (e.g. earthquakes, hurricanes, cascading from a small initial failure etc.) while others are only helpful for one type of event. A multi-hazard approach is likely to create the most effective investment strategy for the total funds expended.
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CHAPTER 2

MODELING VULNERABILITY OF ELECTRIC POWER SYSTEMS TO EARTHQUAKE EVENTS

Introduction

Our nation’s security as well as the quality of life of its citizenry depends on the continuous reliable operation of a collection of complicated interdependent infrastructures including electric power and more generally energy supply, water, transportation and emergency services. As demonstrated in the 2003 Northeast blackout, a disruption in energy infrastructure can quickly and significantly disrupt others, causing ripples across the nation.

Natural hazards including earthquakes pose a significant hazard to electric power systems. On January 17, 1994 the Northridge earthquake struck the city of Los Angeles and surrounding areas. 2.5 million customers lost power [Dong et al. 2004]. The Great Hanshin earthquake occurred a year later affecting the city of Kobe, Japan. Twenty fossil-fire power generation units, six 275 kV substations, and two 154 kV substations were damaged. Approximately, 2.6 million customers were affected by outages [Noda 2001]. On May 18, 2008, the Wenchuan earthquake caused extensive damage to the local power transmission, and distributions systems in the Sinchuan province, China. Approximately 900 substations and 270 transmission lines of the State Power Grid were damaged. At least 90% of the damages could have been avoided by adopting new guidelines of seismic design [Eidinger 2009]. 90% of Chileans did not have electricity immediately following the February 27, 2010 8.8 Mw earthquake. The largest power transmission company in the country had direct losses of about US $ 6.5 billion [Long 2010]. The devastating Tohoku Chiho – Taiheiyo-Oki
earthquake on March 11, 2011 and its aftershocks damaged 14 power plants, 70 transformers, and 42 transmission towers, among other failures. Outage stemming from the event affected 4.6 million residences and the April 7 aftershock affected an additional 4 million [Shumuta 2011].

Seismic risk in the Western region of the United States has been extensively studied. However, there has been relatively less attention expended on seismic risk in the Central and Eastern United States. These regions do contain seismic zones that have been the source of strong earthquakes. In 1886 a 6.6 to 7.1 m\text{b} earthquake occurred in Charleston, South Carolina [Obermeier et al. 1985]. In the winter of 1811-1812 a 3 intraplate earthquakes, including one with an estimated 7.8 M\text{W}, centered in the New Madrid Seismic Zone (NMSZ) struck the central United States inducing liquefaction and permanent ground motion [Tuttle 2002]. For the purpose of this study, we focus on the NMSZ.

There are three key characteristics which make understanding and representing the hazard in the NMSZ important. First, studies of historic records of large earthquakes in the area suggest that the time between large events is somewhere between 200 to 800 years with an average of about 500 years. Considering that the last recorded high magnitude earthquake was in 1811-1812, we might be close to an event based on the low estimate of the recurrence relationship [Tuttle 2002]. Gomberg and Schweig estimate that the probability of a 7.5 – 8.0 M\text{W} earthquake occurring in the next 45 – years is between 7% to 10%, and the probability of a 6.0 M\text{W} or larger is between 25 and 40% [2007]. Second, due to soil conditions, ground shaking in this area is expected to affect a greater area than would be expected in California for a similar magnitude earthquake [Gomberg and Schweig 2007]. Third, since earthquakes in the NMSZ are not as frequent as those on other faults, like San Andreas, there is an
insufficient understanding of the earthquake risk in the area and therefore mitigation strategies in use may not be adequate.

This paper presents a seismic risk analysis of the performance of a U.S. Eastern Interconnect power grid (EI). The representation of the EI was developed in 1998 by the Multi-Area Modeling Working Group. It is a representation of the system as of 1998 with demands reflective of a prediction of summer 2003. This case includes direct representation of every region in the EI. Detailed representation is only provided for voltages greater than 100 kV. It includes information for 23,416 transmission lines and 14,957 buses. These buses are grouped in 2,765 substations with two or more buses and 6,448 single buses. Table 4 lists the voltages of the buses.

<table>
<thead>
<tr>
<th>Total Number of Buses</th>
<th>14,957</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt;= 765 kV</td>
<td>82</td>
</tr>
<tr>
<td>&gt;= 500 kV</td>
<td>192</td>
</tr>
<tr>
<td>&gt;= 345 kV</td>
<td>544</td>
</tr>
<tr>
<td>&gt;= 230 kV</td>
<td>1126</td>
</tr>
<tr>
<td>&gt;= 115 kV</td>
<td>6956</td>
</tr>
<tr>
<td>&gt;= 69 kV</td>
<td>2386</td>
</tr>
<tr>
<td>&gt;= 34.5 kV</td>
<td>742</td>
</tr>
<tr>
<td>&lt; 34.5 kV</td>
<td>2929</td>
</tr>
<tr>
<td>Active Generation Buses</td>
<td>2,316</td>
</tr>
</tbody>
</table>

Similar studies of power grid performance have been previously published. Shinozuka et al. [2003] examine the performance of the LADWP power system under 47 earthquake scenarios that collectively represent the seismic hazard of the Los Angeles area. Shinozuka et al. [2007] identify the sequential failure of components in
Los Angeles electric transmission system under a severe earthquake. They use Monte Carlo simulation to create the consequence scenarios. They run IPFLOW for each of the consequence scenarios; all the components with too high or too low voltages are eliminated. The procedure iterates between IPFLOW and the elimination of components with extreme voltages until reaching convergence. Elnashai et al. [2008] use seismic events developed by USGS to estimate the overall seismic risk in NMSZ and Wabash Valley Seismic Zone. They use HAZUS fragility curves to evaluate the impact on the power grid as well as other vulnerable infrastructure systems. The damage of components is determined using a threshold value. A similar analysis is presented in Elnashai et al. [2009]; the seismic hazard is defined by a deterministic scenario of magnitude 7.7 $M_W$ over an artificial fault created by USGS. Portante et al. [2012] analyze the response of the EI to a strong earthquake scenario; they use a magnitude 7.7 $M_W$ earthquake scenario in the north arm of the USGS artificial fault. Similar to Shinozuka et al. [2007], they not only analyze the performance of the grid from the initial damage, but they also consider the downstream impacts due to extreme voltages. Portante et al. [2010] consider component’s repair and replacement times.

The analysis in this paper innovates beyond the previous studies in four main aspects. First, as in Shinozuka et al. [2003], the natural hazard is modeled with a set of earthquake scenarios that collectively represents the seismic hazard of the region of interest; however, we use an optimization method that enables modeling the seismic hazard by a smaller subset of scenarios with adjusted occurrence probabilities. We also extend that optimization method to include a constraint on the number of events that can be used to describe the hazard. Second, the consequence scenarios are not generated using Monte Carlo Simulation, but an optimization algorithm. With this algorithm the number of consequence scenarios can be reduced considerably and we can demonstrate how well those scenarios match the marginal distribution of damage.
for each component. Third, we extend the optimization to construct consequence scenarios to include damage state information for each component and scenario based on mitigation performed. This allows us to consider mitigation opportunities. Finally, the risk of cascading failure is considered with a new statistical method that is based on observed data and simple assumptions about the blackout area and does not depend on modeling and simulating a selected subset of cascading mechanisms.

The next section provides an overview of each of the four modeling steps needed to understand the seismic vulnerability of an electric power system and to provide an estimate of the benefit of a set of specific mitigation actions across the entire restoration period. The third section provides a detailed discussion of modeling the seismic hazard of a geographic area in a manner that is consistent with understanding the seismic vulnerability of an electric power system. These ideas are then applied to an analysis of the EI. The fourth section focuses on providing an understanding of how to estimate the distribution of damage from seismic events on an electric power system in general and the EI in specific. The fifth section focuses on quantifying the impact of cascading outages that can spread the blackout beyond the initial seismic damage to an electric power system. The sixth section focuses on estimating the load shed and costs incurred over the restoration period. The seventh section discusses conclusions.

**Method Overview**

This section develops each of the four modeling steps needed to understand the seismic vulnerability of an electric power system and to provide an estimate of the benefit of a set of specific mitigation actions. Those steps are illustrated in Figure 1.
Figure 1. Modeling Earthquake Impacts on Electric Power Networks.
Figure 2. Locations at which ground motion is analyzed to select earthquake scenarios.
The first step is to develop a suite of earthquake scenarios (location and magnitude) that replicate important measures of the seismic hazard. For electric power systems the key measure is the exceedance curves for peak ground acceleration (PGA) where seismically sensitive components are located. Figure 2 illustrates the 81 locations at which the hazard is measured in the NMSZ. Figure 3 illustrates the PGA exceedance curve for control point 35, which is located near Pinckneyville, Illinois.

Under each scenario the probability that each component sustains specific levels of damage is then computed using a regional loss estimation methodology such as, HAZUS [FEMA 2003]. It can be used to provide estimates of damage to buildings and individual elements of utility and transportation systems based on a single event. HAZUS accomplishes this task by bringing together two key pieces of data for each element in the built infrastructure or lifeline system: fragility curves that are dependent on the type of structure/lifeline component and a probability distribution for the ground shaking at that location based on the event. A fragility curve specifies the probability that a structure of a specific type experiences a certain level of damage or greater for a given amount of ground shaking. Figure 4 gives an example of a set of

![Exceedance curve for PGA at location P35 (Near Pinckneyville, IL).](image-url)

Figure 3. Exceedance curve for PGA at location P35 (Near Pinckneyville, IL).
fragility curves for a low voltage substation with seismic components. HAZUS categorizes damage to substations into five classes: no damage, slight, moderate, extensive and complete. The curves are interpreted as follows. If the peak ground acceleration is 0.4 g, the probability that the substation experiences at least Slight damage is 90%. This fact implies that the probability that No Damage occurs is 10%. Furthermore, the probability it experiences at least moderate, extensive or complete damage is 70%, 40% and 5%, respectively.

Figure 4. Example Fragility Curves for Low Voltage Substations with Seismic Components [FEMA 2003]

The second element that is needed is a probability distribution for the ground shaking at the location of interest. This ground shaking is often estimated using empirical relationships called ground motion prediction equations (empirical equations are often referred to as attenuation equations), which predict the distribution of ground motion amplitudes using a logarithmic mean that represents the distance, magnitude,
type of rupture, soil classification, etc., and a standard deviation. It is commonly assumed that the residuals, as estimated by the standard deviation, are normally distributed about the mean.

Since the performance of lifelines is governed by the joint distribution of damage over all components, each earthquake scenario must be translated into a set of consequence scenarios and their hazard-adjusted occurrence probabilities. In each consequence scenario, the level of damage of each component is specified. The development of these consequence scenarios is the second step in the process. More generally, we develop these consequence scenarios specifying the condition for each component with and without seismic mitigation.

Electric power systems are vulnerable to cascading failures. That is, when an initial set of components fail, in this case as a result of an earthquake, those failures can lead to current and voltage fluctuations in other components and outages in those components. "Outage" means that the component is removed from service and is therefore unavailable to transmit electric power. Cascading outages are recursive in that the lines that outage as a result of cascade can lead to the outage of additional lines. Dobson and Carreras [2010] develop a branching process model to represent the propagation of transmission line outages from an initiating event through successive generations of cascading line outages. We use their branching process model in the analysis to determine a probability distribution of the additional line outages due to cascading and hence the additional load shed in the resultant blackout.

The final step is the use of an economic dispatch model over the repair duration to understand the distribution in the costs to be incurred. We also explore how those costs would be affected through mitigation. The duration of component outages stemming from a cascade are significantly different than the length of outages of seismically damaged components. Hence the final two steps can be envisioned as
parallel analyses. Understanding the scope of a cascade in addition to the impact of outages stemming from damage is important because cascade induced damage does produce additional societal costs and complicates restoration activities after natural hazard events. The following four sections describe each of these steps in turn.

**Characterizing the Hazard**

A key element of modeling the vulnerability of a region, which is a necessary precursor to understanding the impact of these events on infrastructure, is developing a mechanism to express the hazards. We express the hazard by identifying a small set of earthquake scenarios and their hazard-adjusted probability of occurrence. We use the scenario approach because electric power systems are spatially distributed therefore it is critical to describe the hazard in a manner that preserves the spatial impacts on the system. We focus on limiting the number of scenarios because the computations needed to predict the system impacts for each scenario are extensive.

We use PGA as the key measure of the hazard. We extend the mathematical optimization method to describe the hazard developed by Vaziri et al. [in press] to find the best subset of earthquake events and their associated probabilities which cause the least discrepancy with the seismic behavior as represented in the exceedance curves for PGA for each of the control points. The exceedance curves were obtained from the USGS website [2008b]. We choose to focus on the exceedance curves rather than the recurrence relationship as a description of the hazard because (24) the connection between earthquake events and damage to the infrastructure is more effectively summarized in the exceedance curves since they contain highly resolved spatial information; (25) our goal is to identify a very small subset of events that represent the hazard and a significantly larger number is generally needed to fit a frequency magnitude relationship well; and (26) for NMSZ, there is considerable debate about
the nature of the frequency-magnitude relationship which has not been resolved [Newman et al. 1999].

The mathematical formulation developed in Vaziri et al. [in press] requires a set of earthquake scenarios, $S$, associated with a single seismic zone where each earthquake is characterized by a magnitude, $M_s$, and the probability distribution of ground shaking at control points, $i$. Let $F_{i,s}(g)$ represent the probability that earthquake $s$ leads to a PGA greater than or equal to $g$ in location $i$. $F_{i,s}(g_i,r)$ is defined in (24). $g_{i,r}$ (in units of $[g]$) is the PGA which has a $1/r$ annual probability of exceedance where $r$ is the return period or estimated time interval between events of same or greater intensity. Notice that it is a weighted sum of the probability of exceeding acceleration $g_{i,r}$. $w_a$, is the weight associated with attenuation relation $a$ as given in Petersen et al [2008]. More specifically, this probability corresponds to the weighted sum of the probabilities that the lognormal random variable with mean, $ln (\mu_{i,s,a})$ and standard deviation $\sigma_{i,s,a}$, is greater than the acceleration $g_{i,r}$, associated to point $i$ and return period $r$.

$$F_{i,s}(g_{i,r}) = \sum_{a=1}^{A} w_a \times \left[ 1 - \Phi \left( \frac{ln g_{i,r} - ln \mu_{i,s,a}}{\sigma_{i,s,a}} \right) \right] \quad (24)$$

This problem can be formulated as a linear problem (LP) with the following decision variables: the adjusted occurrence probability of each scenario, $P_s$, and the errors from over and under estimating the probability of exceeding PGA in point $i$ and for return period $r$, and $e_{i,r}^+$ and $e_{i,r}^−$ respectively. The objective function given in (25) corresponds to the minimization of the sum of all the errors. Equation (26) requires that the weighted sum of the probabilities of exceeding PGA value in point $i$ and with return period $r$ for all the earthquake scenarios to be as close as possible to the return period provided in USGS [2008b]. Equation (27) establishes the bounds for the occurrence probability of each earthquake scenario. The lower bound is zero and the
upper bound a defined $P_{max}$; for the purpose of this study we defined this value as 0.05. Equation (28) defines the errors as non-negative variables.

$$\text{Min } \sum_{i=1}^{I} \sum_{r=1}^{R} (e_{ir}^+ + e_{ir}^-)$$

$$\sum_{s=1}^{S} [P_s * F_i,s(g_{i,r})] - e_{ir}^+ + e_{ir}^- = 1/r \quad \forall i, r$$

$$0 \leq P_s \leq P_{max} \quad \forall s$$

$$e_{ir}^+, e_{ir}^- \geq 0 \quad \forall i, r$$

The LP will likely assign nonzero probabilities to many earthquake scenarios. Each scenario increases considerably the computation required for the downstream analyses. Therefore, we extend this formulation to create a mixed-integer linear programming problem by requiring that no more than $N$ scenarios have a non-zero adjusted occurrence probability. This is done by augmenting the LP with a set of binary variables, $\bar{P}_s$, each of which takes on a value of 1 if scenario $s$ is part of the set and zero otherwise. Constraint (27) is then replaced with constraint (29). Constraint (30) limits the number of chosen earthquakes to $N$, and (31) defines the new set of binary variables.

$$0 \leq P_s \leq P_{max} * \bar{P}_s \quad \forall s$$

$$\sum_{s=1}^{S} \bar{P}_s \leq N$$

$$\bar{P}_s \in \{0,1\} \quad \forall s$$

We use two sources of candidate earthquakes scenarios. The first source is the earthquake catalog from the USGS website [2008a]. This catalog includes 433 earthquakes that occurred within the NMSZ. The magnitudes were converted from $m_{blg}$ to $M_W$ using Atkinson and Boore [1995] and Johnston [1996] equally weighted as given in Petersen et al [2008]. The mean PGA for each control point was estimated using Toro et al [1997], Frankel et al [1996], Campbell [2003], Atkinson and Boore
[2006], Tavakoli and Pezeshk [2005], and Silva et al [2002] assuming BC site conditions (shear-wave velocity, 760 m/s), the relative weights given in Petersen et al [2008]. In addition to the 433 earthquakes identified in the Central-East Unites States earthquake scenarios catalog, we use 20 synthetic events on 5 synthetic faults created by USGS to represent the hazard in New Madrid. The 20 scenarios correspond to each of the 4 possible magnitudes (7.3, 7.5, 7.7 and 8) for ruptures in the 5 different branches described in Petersen et al [2008]. USGS [2008c] provides computer code that can be compiled and run to generate each of these deterministic scenarios in New Madrid. For this additional set of events, the attenuation relationship from Somerville et al [2001] is also included in the analysis.

Table 5. List of 8 earthquake scenarios with adjusted probability of occurrence.

<table>
<thead>
<tr>
<th>Location</th>
<th>Fault Info.</th>
<th>Depth [km]</th>
<th>Magnitude m_bgL</th>
<th>Magnitude M_w</th>
<th>Date</th>
<th>Source</th>
<th>Adjusted occurrence probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>36.7</td>
<td>-90.3</td>
<td>0.0</td>
<td>4.3</td>
<td></td>
<td>2/2/1954</td>
<td>NCEER</td>
<td>0.0500</td>
</tr>
<tr>
<td>38.2</td>
<td>-89.8</td>
<td>11.0</td>
<td>4.3</td>
<td></td>
<td>4/9/1955</td>
<td>NCEER</td>
<td>0.0500</td>
</tr>
<tr>
<td>37.9</td>
<td>-88.4</td>
<td>21.0</td>
<td>5.5</td>
<td></td>
<td>11/9/1968</td>
<td>NCEER</td>
<td>0.0078</td>
</tr>
<tr>
<td>38.7</td>
<td>-88.0</td>
<td>10.0</td>
<td>5.2</td>
<td></td>
<td>6/10/1987</td>
<td>USHIS</td>
<td>5.20mn</td>
</tr>
<tr>
<td>36.8</td>
<td>-89.2</td>
<td>5.0</td>
<td>4.5</td>
<td></td>
<td>9/29/1987</td>
<td>USHIS</td>
<td>4.50mn</td>
</tr>
<tr>
<td>35.8</td>
<td>-90.2</td>
<td>9.0</td>
<td>4.2</td>
<td>7.7</td>
<td>5/1/2005</td>
<td>PDE</td>
<td>4.20mw</td>
</tr>
</tbody>
</table>

Table 5 gives the 8 earthquake scenarios selected (from the 433 earthquake events in the historic catalogue and the 20 synthetic events developed by USGS) with their adjusted occurrence probability. Figure 5 illustrates the “location” of the 8 earthquake scenarios. Scenarios from the USGS catalog are located with a star with a size relative to the magnitude of the scenario; there are 6 of these scenarios. There are 2 scenarios from the fault sources; the complete synthetic rupture is also illustrated in the map.
Figure 5. New Madrid Seismic Zone and location of the 8 earthquake scenarios (2 from fault ruptures and 6 point locations).
As illustrated in Figure 2, there are 81 locations at which the optimization needs to match the exceedance curves within the limitations of the earthquake events provided and the maximum number of scenarios from that set that may be used. The solution illustrated in Table 5 and Figure 5 does result in some departures from the exceedance curves at a small number of locations. The largest discrepancy between the exceedance curve and the exceedance curve implied by the scenarios selected and their hazard-adjusted probability of occurrence occurs at location P84 (border between Tennessee and Arkansas at the Mississippi River near Osceola, Arkansas) as illustrated in Figure 6.

![Figure 6. Exceedance curves for location P84.](image)

Figure 6 illustrates the exceedance curve from USGS, the implied exceedance curve obtained from the scenarios (and their probabilities of occurrence) based on the formulation given in Vaziri et al. [in press], and based on that model with the extension to limit the number of events. All, curves correspond to location P84. When the number of events, which have a positive probability of occurrence is limited to ten, the number of events to include based on the optimization drops from 149 to 8.
Figure 7. Histogram of PGA optimization error for every combination of location and return period.

Notice that there is very little difference in the implied exceedance curves. Figure 7 gives a histogram of the difference between the PGA for each of the return periods based on the USGS data at each location and the probability based on the earthquakes selected for that same value of PGA. It is useful to notice that limiting the number of earthquakes selected has a minimal impact on the quality of the estimation and that the magnitudes of these differences in probability are quite small. It is also useful to notice that the distribution of errors is symmetric indicating that the approximation of the exceedance curves is not biased.

**Vulnerability Modeling**

For each scenario selected to represent the underlying hazard, the probability that each component falls into each damage state can be computed using an earthquake regional loss methodology such as HAZUS. However, the performance of a lifeline system is dependent on the joint distribution of damage to the components. Each element in the infrastructure impacts the connectivity of the network, therefore, the
marginal damage distribution of damage to each element in isolation is insufficient for analysis. The common mechanism to represent this joint distribution is the development of consequence scenarios where each consequence scenario has a realized damage state for each component of the infrastructure. Also, associated with each consequence scenario is a probability of occurrence. For each consequence scenario, system performance is then computed.

Consequence scenarios to represent seismic risk are generally obtained using Monte Carlo Simulation. Since performing computationally intensive calculations on a large number of samples is impractical, in practice small sample sizes are used or the computations performed on each sample observation is simplified. Chang, et al. [2000] randomly generates 10 consequence scenarios for each single event using the probabilistic information derived for each bridge to analyze the impacts of earthquakes on the Los Angeles highway system. Shiraki et al. [2007] extend the analysis in Chang et al. [2000] to consider 47 earthquake events using 10 Monte Carlo sample realizations each of each. Çağnan, Davidson, and Guikema [2006] use a mixture of Monte Carlo simulation and expert opinion to model the Los Angeles electrical system with 10 or 20 Monte Carlo samples to assess the impacts of an event. Jayaram and Baker [2010] apply Monte Carlo sampling to construct consequence scenarios for the San Francisco highway system. They use a large number of ground-motion intensity maps to represent the hazard (which can be viewed as ground shaking realizations from the scenarios identified in the previous section). For each ground-motion map they use Monte Carlo simulation to draw consequence scenarios to assess system performance. Since the downstream analysis is a static user equilibrium computation, which is somewhat computationally intensive, they reduce the complexity of the San Francisco highway system by focusing on major facilities thereby reducing the computational burden of the analysis.
We use and extend the optimization method introduced by Brown et al. [2011] to develop consequence scenarios and their hazard-consistent probability of occurrence. Each consequence scenario has a realized damage state for each component of the infrastructure and an associated probability. For a given component the damage state and probability from each scenario can be used to determine the implied vulnerability. The objective of the optimization is to select the consequence scenarios and associated probabilities so the implied vulnerabilities of each component match the “true” (input) vulnerability as closely as possible. We describe this formulation in terms of components of the electric power system for clarity and brevity with the understanding that this formulation could be applied to any spatially distributed lifeline systems by replacing the notions of generation units, substations and distribution circuits with the appropriate seismically vulnerable components in that lifeline system.

The power grid components’ vulnerability is represented as described in the HAZUS regional loss estimation methodology [FEMA 2003]. HAZUS classifies transmission substations voltages that are less than 150 kV, 150 to 350 kV, and above 350 kV as low, medium and high voltage, respectively. Additionally, all the substations, generation units and distribution circuits are classified as seismically designed and not seismically designed. HAZUS divides damage to all components in 5 mutually exclusive and exhaustive damage states: no-damage, slight, moderate, extensive, and complete. For each damage state and component group, HAZUS includes the median and dispersion of the damage probability function. As illustrated in Figure 5, these fragility curves depend on the PGA at the component’s location. As mentioned previously, for each scenario which has a positive probability based on the method described in the previous subsection, there is a probability distribution for the PGA at each component’s location.
In this analysis, we do not consider damage to generation units. According to Schiff [1998] power plants have good overall seismic performance in the United States and in countries such as Chile and Japan. For substation, only four of the five damage states are included: no-damage, moderate, extensive, and complete. Similarly, for transmission lines, the damage states are no-damage, extensive and complete.

Brown et al. [2011] did not consider the possibility of investing in seismic reinforcement for components. For the purposes of this research their optimization model is expanded to include the performance of the components prior to reinforcement and after reinforcement. (32) is the objective function for our optimization. It represents the minimization of the errors from overestimating or underestimating the probability that each component is in each damage state. $k$ identifies each component, $d$ represents each of the damage states, $e_{kd}^+$ and $e_{kd}^-$ are the overestimation and underestimation errors when component $k$ is not seismically designed, and $\varepsilon_{kd}^+$ and $\varepsilon_{kd}^-$ are the errors when the component is seismically designed.

$$\min \sum_{kd} (e_{kd}^+ + e_{kd}^- + \varepsilon_{kd}^+ + \varepsilon_{kd}^-)$$

The errors correspond to the difference between the probability of component $k$ being in damage state $d$, and the sum of the product of the consequence scenario’s occurrence probability for those consequence scenarios in which the component is in damage state $d$ (See (33) and (34)). The probability that component $k$ is in damage state $d$ when it is not seismically design is $m_{kd}$; when it is seismically designed, this parameter is $\mu_{kd}$. $s_j$ is the likelihood of occurrence of consequence scenario $j$. $b_{jkd}$ and $\beta_{jkd}$ are binary variables that take value one when component $k$ is in damage state $d$ in
consequence scenario \( j \) in the absence of mitigation (non-seismic) and after mitigation, respectively (seismic).

\[
\sum_j s_j b_{jkd} - e^+_{kd} + e^-_{kd} = m_{kd} \quad \forall k, d \quad (33)
\]

\[
\sum_j s_j \beta_{jkd} - e^+_{kd} + e^-_{kd} = \mu_{kd} \quad \forall k, d \quad (34)
\]

(35) and (36) guarantee that component \( k \) can only take one damage state per consequence scenario for each design level (seismic and non-seismic).

\[
\sum_d b_{jkd} = 1 \quad \forall k, j \quad (35)
\]

\[
\sum_d \beta_{jkd} = 1 \quad \forall k, j \quad (36)
\]

(37) establish the lower and upper bound for the consequence scenarios occurrence likelihood. For the current problem \( s_{\text{min}} \) is zero and \( s_{\text{max}} \) one. Equation (38) requires that the sum of the probability of all consequence scenarios must equal one.

\[
s_{\text{min}} < s_j < s_{\text{max}} \quad \forall j \quad (37)
\]

\[
\sum_j s_j = 1 \quad (38)
\]

(39) defines all error variables as positive, and (40) define \( b_{jkd} \) and \( \beta_{jkd} \) as binary variables.

\[
e^+_{kd}, e^-_{kd}, \varepsilon^+_{kd}, \varepsilon^-_{kd} \geq 0 \quad \forall k, d \quad (39)
\]

\[
b_{jkd}, \beta_{jkd} \in \{0,1\} \quad \forall j, k, d \quad (40)
\]

The optimization problem is a nonlinear integer problem. To solve it, we used the same heuristic proposed and implemented in Brown et.al. [2011]. The extension to
address the representation of seismic mitigation does not change the underlying problem structure that yielded that solution procedure. It is important to notice that the notation we use to develop this formulation does overlap with the notation used to describe the optimization to select the scenarios to match the exceedance curves at each of the 81 locations. Our notation for each of these separate optimizations is consistent with the original publications that described these core ideas. Therefore, it is important to understand the different meaning for similar notation in the two subsections. We have elected to do this to make the related publications more accessible.

Based on the analysis in the previous section, 8 events effectively captured to hazard (Table 5). The formulation developed in this section provides a mechanism (through the use of consequence scenarios) to construct a collection of consequence scenarios to represent the joint distribution of damage to the components in the electric power system under each of those scenarios. The six events with \( m_{blg} \) less than 6, result in effectively no physical damage to the electric power grid based on the fragility curves given in HAZUS and the probability distribution for PGA for each event.
Figure 8. PGA from earthquake scenario in West fault of magnitude 8.0 MW, and one of the consequence scenarios.

The earthquake scenario on the Mid-East fault of magnitude 7.7 MW, and the earthquake scenario on the West fault of magnitude 8.0 MW do result in considerable damage, however. For each of these events, we generate 6 consequence scenarios using the formulation described above. The average error across all six scenarios (based on their probabilities of occurrence) and the probability each component is in each of the damage states with a positive probability is about 3.5%. Table 7 gives the probability of each consequence scenario as well as the number of transmission lines and substations in each relevant damage state, with and without seismic improvements.

Figure 8 illustrates the damage state for each component under the first consequence scenario listed in Table 6 associated with the earthquake scenario of
magnitude 8.0 $M_w$ on the West fault. It is useful to notice the extensive damage to the
Memphis, Tennessee and Paducah, Kentucky, area as well as Southeastern, Missouri.

Table 6. Consequence scenarios general information.

| Source                  | Adjusted occurrence probability | Basic design | | | | | Seismically reinforced components |
|-------------------------|---------------------------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
|                         |                                 | Transmission Lines damage | Substations damage | Transmission Lines damage | Substations damage |
| West branch, 8.0 $M_w$  | 0.00016                         | 5 | 24 | 18 | 8 | 15 | 2 | 24 | 15 | 3 | 7 |
|                         | 0.0002                          | 5 | 25 | 23 | 19 | 8 | 6 | 20 | 13 | 15 | 7 |
|                         | 0.00008                         | 15 | 24 | 68 | 24 | 10 | 25 | 5 | 66 | 26 | 9 |
|                         | 0.00018                         | 4 | 25 | 13 | 14 | 10 | 5 | 20 | 7 | 12 | 7 |
|                         | 0.00024                         | 6 | 24 | 12 | 16 | 9 | 4 | 22 | 8 | 20 | 6 |
|                         | 0.00014                         | 6 | 22 | 26 | 22 | 12 | 4 | 21 | 26 | 12 | 6 |
| Mid-East branch, 7.7 $M_w$ | 0.000126                        | 27 | 11 | 35 | 16 | 14 | 21 | 9 | 29 | 13 | 14 |
|                         | 0.000396                        | 28 | 14 | 4 | 13 | 11 | 16 | 6 | 3 | 13 | 9 |
|                         | 0.000288                        | 30 | 13 | 17 | 12 | 12 | 19 | 8 | 9 | 8 | 11 |
|                         | 0.000342                        | 28 | 13 | 6 | 7 | 19 | 24 | 8 | 9 | 10 | 11 |
|                         | 0.000432                        | 24 | 14 | 5 | 13 | 13 | 18 | 8 | 3 | 19 | 6 |
|                         | 0.000216                        | 28 | 26 | 25 | 10 | 13 | 15 | 9 | 13 | 16 | 5 |

Mod.=Moderate, Ext.=Extensive, Com.=Complete

Estimating Cost during Restoration Process

An economic dispatch model of electric power can be used to estimate the impact
generated by each consequence scenario. These impacts can then be combined using
the estimated probability of occurrence of the earthquake scenario they stems from
and the probability of the consequence scenario itself. It is important to remember that
each consequence scenario specified the damage state of each component. Based on
the HAZUS methodology [FEMA 2003], again, we assume that substations have three
possible levels of damage: moderate, extensive, and complete. Moderate damage
causes repair costs of 40% of general substation cost and does not affect any of the
transformers in the substation. Extensive damage in average, affects 70 % of the substation and 50% of transformers in the facility. Complete damage, causes complete loss of the substation including all the transformers. When substations sustain moderate damage, the estimate time for repair is 3 days, for extensive damage, a week, and for complete damage, repairs can vary depending on transformers. High voltage and/or customized transformers can have very large lead-times. Therefore, for modeling purposes, we assume that for Low voltage transformers the operator would have access to spares within a month. All the components in substations under complete damage are back to normal within a month with exception of Medium and High voltage transformers. We assume an average lead-time for these transformers of 6 months. Thus, the complete system can get back to normal conditions after 6 months. For transmission lines we only model two levels of damage, extensive and complete. Extensive damage for a transmission line corresponds to a damage ratio of 50% of total cost and complete, 100%. Transmission lines under extensive damage can be repaired within 3 days and under complete damage within a week.

Based on the different repair periods for substations and transmission lines, we model 4 recovery stages. The stages extend from right after the event to 3 days, from 3 days to a week, a week to a month, and the final stage extends from the end of the first month to 6 months after the earthquake. Each time period is modeled by a dc flow model with the objective of minimizing operating costs. Operating costs include power generation and load shed costs. Average power generation costs are estimated based on the data collected by the U.S. Department of Energy [2009]. We model two types of generators: nuclear, and a weighted average of the rest of energy sources that reflected national energy consumption. Based on data from 2002, the relative shares of energy consumed by source fuel is 39%, 24%, 22.5%, 8.5%, and 6% for oil, gas, coal, nuclear, hydro and renewable constitute, respectively. Finally, Billington et al. [1989]
presents business interruption costs (which are indirect costs) per kW and kWh for different sectors.

The linear dc power flow model formulated below represents the optimal operation during the recovery period after the seismic event. Mathematically, the objective is to minimize equation (41). It represents the total generation and load shed costs for the system in the four recovery periods.

\[
\min \sum_{k=1,2,3,4} \left( \sum_{i \in B} c^B U_i^k + \sum_{g \in G} c_g^G G_g^k \right) (t_k - t_{k-1}) \tag{41}
\]

subject to

\[
(\theta_i^1 - \theta_j^1) (1 - y_{ij}^{E,C}) (1 - x_{si}^{M,E,C}) (1 - x_{sj}^{M,E,C}) = m_{ij} P_{ij}^1, \quad \forall (i,j) \tag{42}
\]

\[
(\theta_i^2 - \theta_j^2) (1 - y_{ij}^{C}) (1 - x_{si}^{E,C}) (1 - x_{sj}^{E,C}) = m_{ij} P_{ij}^2, \quad \forall (i,j) \tag{43}
\]

\[
(\theta_i^3 - \theta_j^3) (1 - x_{si}^{C}) (1 - x_{sj}^{C}) = m_{ij} P_{ij}^3, \quad \forall (i,j) \tag{44}
\]

\[
(\theta_i^4 - \theta_j^4) (1 - x_{si}^{C} T_{ij}) = m_{ij} P_{ij}^4, \quad \forall (i,j) \tag{45}
\]

\[
\sum_{g \in I(i)} G_g^k - \sum_{(i,j) \in \delta^+(i)} P_{ij}^k + \sum_{(j,i) \in \delta^-(i)} P_{ji}^k = D_i - U_i^k, \quad \forall i, k \tag{46}
\]

\[
0 \leq U_i^k \leq D_i, \quad \forall i, k \tag{47}
\]

\[
0 \leq G_g^k \leq G_g^m, \quad \forall g, k \tag{48}
\]

\[
|P_{ij}^1| \leq P_{ij}^m (1 - y_{ij}^{E,C}) (1 - x_{si}^{M,E,C}) (1 - x_{sj}^{M,E,C}), \quad \forall (i,j) \tag{49}
\]

\[
|P_{ij}^2| \leq P_{ij}^m (1 - y_{ij}^{C}) (1 - x_{si}^{E,C}) (1 - x_{sj}^{E,C}), \quad \forall (i,j) \tag{50}
\]

\[
|P_{ij}^3| \leq P_{ij}^m (1 - x_{si}^{C}) (1 - x_{sj}^{C}), \quad \forall (i,j) \tag{51}
\]

\[
|P_{ij}^4| \leq P_{ij}^m (1 - x_{si}^{C} T_{ij}), \quad \forall (i,j) \tag{52}
\]

The objective function given in equation (41) is the sum of the power generation, and load shed. \(c_g^G\) represent the unitary cost of power generation associated with generator \(g\), \(G_g^k\) the generation output in generator \(g\) and time period \(k\), \(c^B\) the unitary
cost per load shed, and $U_i^k$ the load shed in bus $i$ and time period $k$. Notice that the length of the time periods is represented with $(t_k - t_{k-1})$. Constraints (42), (43), (44), and (45) approximate the active power flows on the transmission lines in the four stages of the repair process. In these equations, $m_{ij}$ is the reactance in line $(i,j)$, $y_{ij}^{E,C}$ is an indicator that takes the value of one when a line is either in extensive or complete damage states, otherwise is zero (similarly takes the value of one when a line is in damage state complete), and $x_{s_i}^{M,E,C}$ is an indicator that is one when a substation is under moderate or greater damage (notice that superscript “$E,C$”, stands for extensive or greater damage, and “$C$” exclusively for complete damage). $\theta_i^k$ is a variable that represents the phase angle in bus $i$ and time period $k$, and $P_{ij}^k$ represents the power flow in transmission line $(i,j)$ for time period $k$. Constraints (46) preserve the power balance at the buses in the four-stage recovery process. Constraints (47) state that the load shedding at a bus cannot exceed the demand at the bus. Constraints (48) bounds the power produced at each generator; $G_{g}^m$ is the maximum generation output in generator $g$. Note that the power flow in each transmission line can go in either direction therefore the flow load on each transmission line can take either a negative or positive value. Constraints (49), (50), (51), and (52) set the maximum absolute values of the flows for transmission lines in each stage. A line is not available in the first stage if it is in damage states extensive or complete, or if a substation to which is connected is in moderate or more intense damage. In the final stage, high voltage and/or customized transformers in substations under complete damage are still out of operations. These transformers are identified by $T_{ij}$, an indicator that takes the value of one for high voltage and/or customized transformers and zero in all the other cases.

Figure 9 gives exceedance relationships for load shed and repair costs for the EI with and without complete seismic mitigation. For example, if an earthquake occurs that is sufficiently strong to affect the electric power infrastructure, that event will
cause damages in excess of US$2 billion and result in load shed costs in excess of US$10 billion.

Figure 9. Exceedance relationship for repair and load shed costs.

**Modeling Cascades**

The initial failures directly caused by the earthquake can often propagate to cause a more widespread blackout by a process of cascading outages. The power system weakens as each outage occurs and it is then more likely that further outages occur. The equipment outaged by cascading is usually not damaged; it is typically removed from service by automatic protection devices or by operator actions. The outaged equipment can be restored to service without repair, but if the blackout is large and widespread, full restoration can take hours to days. The direct cost of the blackout associated with the cascading is primarily due to cost of the interruption of the supply. However, it should be noted that indirect blackout costs such as adverse interactions with other infrastructures, and particularly the impediments caused by a lack of electrical power to support the efforts to rescue people affected by the earthquake.
damage and executing an effective infrastructure repair and restoration process can be very significant.

Our objective is to estimate the order of magnitude of the increase in the load shed due to cascading. Since the systematic study of cascading failure is recent, to perform this first, rough estimate we apply new methods to describe the probability distribution of the additional lines that will trip out and implicitly identify a joint probability distribution for where those lines will be in the network.

Here the assumed number of initial outages is the number of lines damaged by the earthquake. It should be noted that not all damaged lines are outaged (they can sometimes remain operational even if they have to be repaired later) and that lines may be outaged by the quake without damage (for example, swaying lines flash over and trip out). However, in the absence of data on the exact relation of outaged lines to damaged lines, we make the simple assumption that the number of lines initially outaged by the earthquake is equal to the number of damaged lines.

Modeling cascades has two parts. The first part probabilistically estimates how many lines are outaged after cascading given the number of initial line outages, and is based on standard utility data and a branching process model of cascading. The second part samples a possible blackout area with the estimated total number of lines outaged. The blackout load shed can then be estimated from the blackout area.

A. Estimating the Number of Lines in the Cascade

Branching process models of cascading are high-level probabilistic models that can track the cascading in terms of the number of lines outaged. The use of branching process models to estimate the probability distribution of the number of lines outaged after cascading is supported by comparisons with observations and simulations of transmission line outages (Dobson et al. [2010], Ren and Dobson [2008], Dobson and
Carreras [2010]) and by the approximation of other high-level probabilistic models of cascading by branching processes [Kim and Dobson 2010]. Given the number of initial failures and the average amount by which the cascade propagates, the branching process gives formulas that can be evaluated to estimate the probability distribution of the number of lines outaged by cascading. Here we obtain the initial number of line failures from the earthquake scenarios and use propagation estimates obtained from observed data for the propagation of line outages. The details of the formulas and data used are given in Dobson and Carreras [2010]. For example, Figure 10 shows the probability distribution of the total number of lines outaged after cascading for assumed numbers of initial line failures. As an illustration of the probability distributions given in Figure 10, suppose the earthquake damages 29 lines. There is about a 5% chance that more than 57 lines will be outaged when the opportunity for cascades is considered.

![Figure 10. Probability of line outages for 4 different values of initial failures.](image-url)
The mechanisms of cascading failure in electric power systems are complicated and diverse. One advantage of using an observed data set instead of simulated transmission line outages is that there are no modeling assumptions restricting or approximating the interactions between the line outages. The observed data set used to estimate the propagation is 12.4 years of transmission line outages in the Northwest USA that is publicly available in [Bonneville Power Administration 2009].

B. Estimating the Enlarged Blackout Area

Given a sample of the total number of lines outaged after cascading, we approximate a sample blackout area that includes the initial lines outaged. To do this, we add outaged lines at random to grow the area of the blackout until the area includes the total number of lines outaged. The lines are randomly added so that they are next to lines that have already outaged, so that the final blackout area resembles a connected region, as are quite commonly observed after a blackout. This process approximates the final outcome of a blackout and is not intended to represent the sequence in which lines outage in the blackout, since some mechanisms of blackout propagation do not spread via neighboring lines. The process amounts to a random sample of a possible blackout area including the initial line failures that regards spreading of the blackout in any direction as equally likely. Additionally, we assume that any substation connected to one or more outaged transmission lines would also be out of service. A sample of the outcome of a cascade is then produced and then the load shed is identified. Figure 11 shows an example of a blackout extent with and without the cascade. In the original scenario 29 lines are outaged; this number increases to 57 in the cascade scenario. The probability of this blackout or larger is about 5%. Notice that the cascade causes the impacts on the electric power system to
be felt as far away as Southern Louisiana. The original blackout interrupts 37,600 MW, and the cascade scenario interrupts 53,300 MW, an increase of 40%.

Figure 11. Load shed stemming from the first consequence scenario in Table 7 including a potential cascade.
We focus on modeling cascades exclusively for the case when components are not mitigated to achieve additional seismic resilience. For each consequence scenario we estimate the load shed after the earthquake without considering cascading failure. We also estimate the total load shed for each possible number of line outages due to cascading until we reach 99\% of the cumulative distribution. For each number of initial lines out, we generate 10 cascade scenarios, each with a probability of 0.1.

We generate more than 10,000 damage scenarios to evaluate the risk due to cascading effects right after the earthquake. The adjusted occurrence probability for each consequence scenario can be multiplied by the probability of cascade to obtain a new occurrence probability for each of the 10,000 scenarios. We use a standard dc load flow model to estimate the total load shed per hour in the occurrence of each one of the events. In this dc load flow model we assume that the operator does not minimizes the operation costs and only minimizes the load shed. The objective function in (41) becomes equation (53). Notice that equation (54) and (58) include an indicator for line outages due to cascading, $y_{ij}^{Casc}$.

\[
\min \sum_{i \in B} U_i \tag{53}
\]

subject to

\[
(\theta_i - \theta_j)(1 - y_{ij}^{E,C})(1 - y_{ij}^{Casc})(1 - x_{si}^{M,E,C})(1 - x_{sj}^{M,E,C}) = m_{ij}P_{ij}, \forall (i,j) \tag{54}
\]

\[
\sum_{g \in I(i)} G_g - \sum_{(i,j) \in \delta^+(i)} P_{ij} + \sum_{(j,i) \in \delta^-(i)} P_{ij} = D_i - U_i, \quad \forall i \tag{55}
\]

\[
0 \leq U_i \leq D_i, \quad \forall i \tag{56}
\]

\[
0 \leq G_g \leq G_g^m, \quad \forall g \tag{57}
\]
\[ |P_{ij}| \leq P_{ij}^m (1 - y_{ij}^{E,C})(1 - y_{ij}^{\text{casc.}}) (1 - x_{si}^{M,E,C}) (1 - x_{sj}^{M,E,C}), \quad \forall (i,j) \] (58)

Figure 12. Exceedance probability of initial load shed when analysis considers cascading failure and when it does not.

Figure 12 shows the exceedance curve for load shed immediately after a seismic event with and without the impacts of cascades. Line cascading was modeled following the procedure described above. Substation performance under cascade conditions has not been previously studied. Therefore, we included three different cascade analyses that differ in the risk of substation tripping. In the first analysis, any substation connected to one or more affected lines will trip; the other two analyses correspond to the cases when the tripping is caused by two or more, and three or more tripped lines, respectively. It is useful to notice that cascade consequences have greater impact on the relative size of blackouts for scenarios with smaller amounts of earthquake damage. It is also useful to notice that the number of outaged lines that are
assumed to cause a substation to trip has a significant impact on the magnitude of the cascade that stems from the original earthquake damage; hence it is important to deepen our understanding of substation tripping during the cascading process.

Conclusions
This paper has augmented existing methods in the literature to develop a comprehensive modeling process for estimating the impacts of earthquakes on electric power systems including explicit representation of cascading. It has also demonstrated that the methods can be applied to large-scale electric power systems.

The modeling process is composed of four steps. The first step is a robust mechanism, using optimization, to represent the earthquake hazard through the selection of specific events from a candidate set, including the hazard adjusted probabilities of occurrence for each event, that is consistent with the exceedance curves for the hazard across the region. The second step is the translation of each of those earthquake scenarios (location and magnitude) into a collection of consequence scenarios (and their probabilities of occurrence) where each consequence scenario identifies the level of damage for each component in the electric power system. In the case study developed in this paper, the regional loss estimation methodology HAZUS was used to compute the probability that each component fell into each of a set of mutually exclusive and exhaustive damage states for each consequence scenario. The third step uses an economic dispatch model of the electric power system to compute the repair costs and the load shed under each consequence scenario. Then, using the probability of occurrence of each consequence scenario and the probability of occurrence of the earthquake event that lead to that consequence scenario the distribution for load shed and repair costs are computed. The fourth and final step is
the computation of the additional load shed that stems from each consequence
scenario via cascading failure of the electrical grid.

There is a range of opportunities for further research. First, while it is important to
understand the risks posed by earthquakes on the electric power system it is even more
important to identify how those risks might be mitigated. This modeling process has
illustrated how to construct consequence scenarios which include how damage would
change with mitigation. This leads to the opportunity to construct an optimization
model to understand how fixed funds might be expended to mitigate earthquake
hazard for electric power systems. Second, this modeling has assumed that the
components damaged in the earthquake are also the components that are outaged.
Understanding and refining this approximation would be useful. Third, modeling
cascades in electric power systems stemming from earthquakes is complicated. They
occur when the system is fragile and in a damaged state. Unusual events and
interactions will occur and operators are busy trying to understand what has been
damaged and how to organize a repair effort. There are some actions that can be taken
to lessen the load shed from the cascade, including re-assessing the feasible
dispatching. The modeling assumed that these measures were taken and then assumes
a probabilistic representation of blackout area. Understanding what strategies are
possible and how to represent them accurately is important. Fourth, the electric power
system is critical to the operation of many other infrastructures. Conversely, the
electric power system is dependent on other infrastructures. Understanding the impact
of earthquakes on interdependent infrastructures (including electric power networks) is
very important. There are opportunities to apply these methods to other infrastructures
and to link the resultant infrastructure models together. In the context of mitigation,
this avenue for additional research becomes even more compelling.
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CHAPTER 3

INVESTMENT PLANNING FOR ELECTRIC POWER SYSTEMS TO MITIGATE EARTHQUAKE HAZARDS

Introduction

Earthquakes pose a significant risk to electric power systems as illustrated by a range of recent events. For example, on January 17, 1994 the Northridge earthquake struck the city of Los Angeles and surrounding areas. Two and a half million customers lost power [Dong et al. 2004]. The Great Hanshin earthquake occurred a year later affecting the city of Kobe, Japan. Twenty fossil-fire power generation units, six 275 kV substations, and two 154 kV substations were damaged. Approximately, 2.6 million customers were affected by outages [Noda 2001]. On May 18, 2008, the Wenchuan earthquake caused extensive damage to the local power transmission and distributions systems in the Sinchuan province, China. Approximately 900 substations and 270 transmission lines of the State Power Grid were damaged. It has been estimated that at least 90% of the damage could have been avoided by adopting new guidelines for seismic design [Eidinger 2009]. 90% of Chileans did not have electricity immediately following the February 27, 2010 8.8 M_W earthquake. The event caused the largest power transmission company in Chile to have direct losses of approximately US $ 6.5 billion [Long 2010]. The devastating Tohoku Chiho – Taiheiyo-Oki earthquake on March 11, 2011 and its aftershocks damaged 14 power plants (including Fukushima nuclear power plant), 70 transformers, and 42 transmission towers, among other failures. Outage stemming from the event affected 4.6 million residences and the April 7 aftershock affected an additional 4 million [Shumuta 2011].
Further, damage in the electric power system often causes impacts on other infrastructures including transportation, electric power, oil, gas, telecommunications and emergency services. The 2003 Northeast blackout provides an excellent illustration of how a disruption in the power network can quickly and comprehensively affect other infrastructures across a wide area. The 2003 Northeast blackout affected Ohio, Michigan, Pennsylvania, New York, Vermont, Massachusetts, Connecticut, New Jersey and the Canadian province of Ontario with a combined population of over 50 million and 61,800 megawatts of electric load [U.S.-Canada Power System Outage Task Force 2004]. At least 70 auto and parts plants were shutdown idling more than 100,000 workers in the Motor Vehicle & Automotive Parts Industries. The blackout affected at least 8 oil refineries in the U.S. and Canada. It also triggered emergency shutdown procedures at the Marathon Oil Corporation’s Marathon Ashland refinery about 10 miles south of Detroit that ended up in a small explosion and the release of a mixture of hydrocarbons and steam. The steel, chemical, aluminum, paper, and food were among affected industries. Local telephone service was operational but jeopardized by the energy emergency generated by the blackout. Airports were closed in Toronto, Newark, New York, Detroit, Cleveland, Montreal, Ottawa, Islip, Syracuse, Buffalo, Rochester, Erie, and Hamilton [ELCON 2004].

“[During the 2003 Northeast blackout] except for ferries and feet, nearly all other forms of transportation around the [New York] metropolitan region failed miserably [on August 14, 2003]” [Kennedy 2003]. Seven hours after the beginning of the blackout airlines with flights to and from the three major airports in the region (Newark Liberty International, JFK International, and New York La Guardia) began to reestablish limited operations. Air traffic controllers in the three airports lost contact with all airplanes during the 30-45 seconds that the backup emergency generation took to switch on. The night of August 14th all three airports instituted their “snow plan”
for stranded passengers spending the night in the airports and agreed to accept some flights throughout the night. Newark Liberty Airport operations were almost fully restored within 22 hours Operations in La Guardia and JFK Airports were restored in about 30 hours [U.S. Department of Transportation 2003].

On January 4, 2010, Reagan National Airport in Washington, DC was paralyzed for one hour due to a power outage. Flights were grounded and only two of the four security screening points worked during that period. The two screening points that were working were on backup power generation [KTLA 2010]. On January 20, 2010, a seven and a half hour blackout affected thousands of passengers traveling to and from Cleveland Hopkins International Airport. Backup generation powered the control tower, airfield, and operations center, which accounts for 20% of the airport’s power needs. The airport can’t operate without electricity to power ticket counters, security checkpoints, baggage carousels and jetways. Full operations were restored on the afternoon of January 21st, more than 24 hours after the beginning of the blackout. “Cleveland Hopkins International Airport is willing to risk another power outage like the one that grounded hundreds of flights Sunday [January 20, 2010] rather than spend millions of dollars in backup generators to keep terminals operating” [Gillispie 2010]. Furthermore airlines are responsible for having backup systems at their ticket counters, but most do not have them [Miller 2010].

The objective of this paper is to develop a model to optimize the selection of mitigation strategies to stem the consequences of earthquake events. These mitigation strategies include investments in anchoring as well as the addition of operating margin to the system. It is important to realize that anchoring of electric power components is only of value against earthquake hazards, whereas investments in operating margin are useful across a range of situations, including spikes in demand stemming from
prolonged periods of intense hot or cold spells, equipment failures, hurricanes, and ice storms. Hence it is important to consider both types of mitigation strategies.

In addition to the development of this optimization model, we include an analysis of the potential benefits of these investments on the reliable delivery of electric power to airports. It is clear that blackouts can have serious impacts on many other important infrastructures and this study gives one example of estimating the impact on the air travel infrastructure. The decisions to invest or do not invest in hardening infrastructure have profound impacts. Our purpose is to explore these impacts with the understanding that it is possible and important to integrate these impacts into the investment modeling directly.

The model and solution procedure is illustrated through its application to the Eastern Interconnect Power Grid (EI) to investigate opportunities to stem the hazards generated by the New Madrid Seismic Zone (NMSZ). These mitigation plans are then evaluated to assess their benefits to the national passenger air transportation system. The power network used is a 1998 representation of the EI and the air model is developed based on the top 80 airports as measured by enplanements in 2010. We model the NMSZ risk on EI using HAZUS’ regional loss estimation method. The performance of the power grid is evaluated using a dc flow economic dispatch model, which is used to estimate the load shed in all the nodes of the grid and particularly in the substations providing energy to airports included in the air system model.

We implement a knapsack based heuristic to solve the non-linear integer programming problem (NLIP) to optimize the selection of mitigation strategies for electric power system components. To model the seismic risk, we use a suite of earthquake scenarios that nearly replicates the exceedance curves for peak ground acceleration (PGA) as measured at 81 control locations across the NMSZ. Since the electric power system is a spatially distributed system, we create a suite of
consequence scenarios for each earthquake scenario where each consequence scenario identifies the resulting damage state of each component. Once the damage state of a component is known, the expected time required for the component to be operational again and the cost of the repair can be estimated. The construction of these consequence scenarios provides an implicit representation of the joint distribution of damage for each earthquake scenario. The damage to the power grid considered is limited to transmission lines and substations. As mentioned previously, the operation of the power grid is modeled using an economic dispatch model and it is assumed that the operator of the network has a limited budget to invest in mitigating the risk. A simpler version of the NLIP is solved using Lingo 13. The results using Lingo and our proposed heuristic are compared for different mitigation budgets to gain a sense of the performance of the heuristic.

The contribution of this paper is three fold. First, this is the formulation to simultaneously optimize structural mitigation decisions and capacity enhancement opportunities. Second, this is the first solution procedure for the optimization of transmission system capacity expansion that can be applied to very large problem instances. We focus on the EI in its entirety; hence the problem instances have about 15,000 nodes and 23,000 arcs. This is about two orders of magnitude larger than the biggest network found in the literature. Third, we explore the impacts on the air transportation system of investments in the electric power system. There are many papers focused on modeling interdependencies, however, they use graph theory based abstractions of each of the underlying system models rather than modeling the “physics” of the operation of each system.

The next section reviews the relevant literature. The third section develops the formulation. The fourth section presents the solution procedure. The fifth section describes the key elements of the case study. The sixth section describes the results of
the application of the tools developed in sections two and three to the case study described in section four. It also includes a comparison of the performance of the solution procedure developed in the third section to LINGO 13, a commercial solver, for a simplified problem instance. The seventh section summarizes the key elements of the paper and next steps for future research.

**Literature Review**

This work represents an extension of three related areas in the literature: (1) optimization of structural reinforcement of components in electric power systems, (2) the broader transmission capacity expansion literature, and (3) modeling the interdependencies in infrastructure systems.

Vanzio found optimized structural upgrading strategies for electric power networks using a new index to choose among critical nodes in the network [2000]. The method was tested using a representation of the Sicily, Italy power network, which includes 181 nodes and 220 lines. Shumuta focused on upgrading substation equipment [2007]. He evaluated the criticality of components with 4 indexes; two indexes were used to represent the earthquake resistant capacity of each component, the third index represents the seismic performance of the component, and the fourth index is related to the upgrading costs. This method was tested on a hypothetical electric power system with 16 substations, located in the Nagoya region, Japan.

Transmission expansion planning for power networks includes a variety of approaches with exact or heuristic solution procedures, and for static or dynamic planning horizons. An extensive literature review can be found in Latorre et al. [2003]. Work in this area does not specifically consider the benefits under seismic risk and normally applies to small scale networks. Samarakoon et al. used a mixed integer linear programming model to solve the transmission and generation expansion
problem [2001]. Prior to the model application, they identified all possible single circuit outages associated with all possible expansion investments. The critical contingencies (possible single circuit outages) are included in the model as constraints. The method is tested on the Sri Lankan power network, a 38 node model, and 8 new nodes and 19 new branches are considered among the expansion options. Alguacil et al. used a revised mixed integer linear formulation of the static transmission expansion problem that is computationally efficient using conventional solvers [2003]. Both Samarakoon et al. [2001] and Alguacil et al. [2003] use a “disjunctive parameter to allow enough degrees of freedom to the voltage angle difference between every disconnected nodes” [Alguacil et a. 2003]. Alguacil et al. [2003] tested their model in the One Area IEEE reliability test system [Grigg 1999] which has 24 nodes and 38 links. With the same problem formulation, Carrion et al. presented a methodology to expand the transmission network in order to reduce its vulnerability to intentional attacks [2007]. This formulation includes not only the enhancement of current lines but the development of new lines, which is achieved by including other transmission lines with initial capacity zero. Carrion et al. [2007] tested their methodology on the Two Area IEEE reliability test system [Grigg 1999]; the test system has 48 nodes and 79 lines, and the model considered 20 prospective new lines. Georgilakis presented an improved differential evolution solution to the transmission expansion problem [2010]. The methodology uses a reference network subproblem which is topologically identical to the expanded network, and with generation and load unchanged. The reference network subproblem is used to find the optimal capacities of transmission lines; this subproblem includes a set of contingency scenarios as constraints of the individual load flow problem. The optimum network problem is the same as the subproblem with the additional difficulty of having to choose the lines to add to the network. Georgilakis [2010] tested the methodology in a 30-bus IEEE reliability and 9
prospect transmission lines. Aguado et al. recently presented a transmission expansion model that explicitly considers a multi-year planning horizon [2012]. The model is formulated as a mixed-integer linear problem and the method is tested on a 6 bus system and applied on the transmission system of mainland Spain, which includes 86 buses and 168 circuits.

We build on the literature for modeling electric power systems by integrating the optimization of structural hardening with capacity expansion. This is important because investments in hardening are only valuable under earthquake conditions where as operating margin is valuable under a range of events. Further, our solution strategy can be used for very large problem instances. There are related problems solved in the literature but those problems reach a maximum of a couple of hundred nodes and arcs. Our algorithm has been successfully applied to the EI which is on the order of tens of thousands of nodes and arcs.

Infrastructure models that incorporate interdependencies between systems are important to address the infrastructure planning problem holistically. Previous approaches that address the interdependency problem use input-output analysis, graph theory, network flow and Markov models. Rinaldi et al. [2001] presented a framework to understand and analyze infrastructure interdependencies. They explicitly discuss the centrality of the electric power system to the operation of a range of infrastructures. Haines and Jiang [2001] and Crowther and Haines [2005] develop a input-output models to analyze the interdependencies between infrastructures. Crowther and Haines provide a methodological framework to model multisectoral and multiregional economic interdependencies [2010]. This model is based on the work developed by Haines and Jiang [2001]. Dueñas-Osorio et al. use graph theory methods to analyze the response of the water and power networks of Shelby Count, Tennessee [2007]. Dueñas-Osorio et al. introduces geographical proximity as a rule to establish
interdependencies among infrastructure systems; furthermore it analyzed the response in one network based on the degree of coupling between interdependent networks [2007]. Poljansek et al. assessed the performance of two interdependent infrastructure systems using graph theory: the European gas network and the electric network [2011]. Xu et al. [2007] developed a method based on network flow and Markov models to estimate the recovery time in interdependent infrastructure after a disruption. Xu et al. [2007] use transition matrices in the Markov model to represent the changes in the capacity of links overtime. The method is illustrated in a small gas-electric infrastructure network. Lee II et al. [2007] uses a network flow approach to model restoration of service after disruption of the interdependent power, telecommunications and subway system of the lower Manhattan region of New York.

We build on the research in modeling interdependencies by representing the physical links that exist between those networks as well as the behavior of the material that flows across each infrastructure. In the electric power network, that is real power flow. In the airline network that is aircraft and travelers. Our motivation for including the estimation of the impacts of the air system from investments in the electric power system is to illustrate the need to move towards coupled infrastructure models that differentiate between the different end uses for electric power and quantify the wider impacts of losing the power.

**Formulation**

The key question addressed by the formulation is how to optimally invest in seismic mitigation strategies for the power network. The strategies include reinforcing the lines and substations, and adding capacity to the lines and generators given the budget limitations. We measure the performance of the power network as the sum of the power generating cost, load shed and repair costs under a set of consequence
scenarios that model the seismic hazard in the region and vulnerability of the network to that hazard.

We evaluate the performance of a passenger air transportation system for the existing and improved power system assuming the lowest cost operation of the electric power system after the event. The performance of the air transportation system is measured by the number of passengers that are able to be accommodated after the event. We use an optimization model to estimate the number of passengers that can be accommodated after the event. The electric power and air transport models are discussed in turn in the next two subsections.

A. Electric Power Transmission System

The electric power transmission system investment planning model is formulated as a two-stage stochastic program. A two-stage stochastic program is an optimization model formulation that incorporates uncertainty in the parameters of the model. The two-stage structure assumes that all decisions are made at one time instance prior to the resolution of all uncertainty. In this case, the uncertainty revolves around what damage will occur to each component in the electric power system. This uncertainty is expressed through the use of a number of consequence scenarios, where each consequence scenario gives the damage to each component. The decisions are made in what is termed the “first-stage” of the model. In our formulation, the first-stage is the identification of what components in the electric power system should be reinforced and what components should be enhanced. The consequences of those decisions, under each consequence scenario, occur in the “second-stage” of the model. In this problem formulation, the second-stage is the power flow across each component including what demands for power are not satisfied under each consequence scenario.
We first introduce the topology of the power network. Let $\Pi$ be the set of transmission lines. Let $S$ be the set of substations. Let $G$ be the set of generators. Let $B$ be the set of buses. Let $I(i)$ be the set of the generators connected to bus $i$. We define the first-stage binary decision variables as follows. Let $x_s = 1$ if substation $s$ is reinforced and $x_s = 0$ otherwise. The cost to reinforce substation $s$ is $b_s$. Let $y_{ij} = 1$ if transmission line $(i,j)$ is reinforced and $y_{ij} = 0$ otherwise. The cost to reinforce transmission line $(i,j)$ is $f_{ij}$. Let $z_g$ take integer values 0, 1 or 2 representing the respective amount of discrete capacity increments in the capacity of power generator $g$. The cost to add a discrete increment $\mu$ to the existing capacity of generator $g$ is $o_g$. Let $w_{ij}$ take integer values 0 to 4 representing the amount of discrete increments in transmission capacity for line $(i,j)$. The cost to add a discrete capacity increment $\rho$ to the existing capacity of transmission line $(i,j)$ is $h_{ij}$. We assume that the total available budget for seismic reinforcement is $M^D$, and that the total budget available for transmission capacity and power generation enhancement is $M^C$. Since the total investments to reinforce the components and to adding capacities to the components cannot exceed the available respective budgets, then equations (59) and (60) must hold.

\[
\sum_{s \in S} b_s x_s + \sum_{(i,j) \in \Pi} f_{ij} y_{ij} \leq M^D, \tag{59}
\]

\[
\sum_{(i,j) \in \Pi} h_{ij} w_{ij} + \sum_{g \in G} o_g z_g \leq M^C. \tag{60}
\]

In this application, the seismic reinforcement of transmission lines is assumed to be a percentage of the total replacement cost of the line obtained from Balducci et. al. [2006]. Seismic reinforcement of substations entails anchoring the transformers in the substations. Therefore the cost to reinforce a substation is estimated by multiplying the
cost of anchoring a transformer [Shinozuka 2003] by the number of transformers in
the substation. Transmission line enhancement is modeled as discrete increments of a
quarter of total original capacity of the line. The maximum capacity expansion for
lines is a doubling of current capacity. Generation enhancement is modeled as discrete
increments of a fifth of initial capacity. The cost of the enhancement is modeled as a
percentage of the total cost of the line or generator.

Based on the HAZUS seismic risk assessment methodology [FEMA 2003], five
damage states are defined for electric power components: none, minor, moderate,
extensive and complete. We choose to focus on the damage states none, moderate,
extensive and complete because the damage associated with minor is not substantial in
this context. Moderate damage generates a repair cost of 40% of substation cost and
does not affect any of the transformers in the substation. Extensive damage is assumed
to imply damages costing 70% of the value of the substation including impacting 50%
of the transformers in the substation. Complete damage causes the complete loss of the
substation including all the transformers. The estimated time for repair is 3 days for
the moderate damage and a week for extensive damage. For complete damage, repairs
can vary depending on the ease with which the transformers can be replaced. High
voltage and/or customized transformers can have very large lead-times. Thus, for
modeling purposes, we assume that for low voltage transformers, the operator would
have access to spares within a month. We assume that the average lead-time for
medium and high voltage transformers is 6 months. Therefore, all the components in
substations under complete damage are back to normal within a month with the
exception of medium and high voltage transformers which is 6 months. For
transmission lines we only model two levels of damage: extensive and complete.
Extensive damage for a transmission line corresponds to a damage ratio of 50% of the
total cost of the line and complete damage results in costs totaling the full cost of the
line. Transmission lines under extensive damage can be repaired within 3 days and under complete damage within a week. This implies that by the end of 6 months, in the worst case, the system is back to normal.

From a modeling perspective, this implies that the repair process is composed of 4 time periods. The first period extends from the quake event to the end of the third day. By then transmission lines that have experienced extensive damage have been restored. Also, substations under moderate damage have been repaired. The second time period extends from the beginning of day four to the end of the first week. By then transmission lines that have experienced complete damage have been repaired as well as substations under extensive damage. The third time period extends from the end of the first week to the end of the first month. By the end of this time period, low voltage transformers will have been replaced. The final time period extends from one month to six months. Six months after the event, medium and large voltage transformers will have been repaired. The following notation encapsulates these time period definitions. Let $t_0 = 0$, $t_1 = 3$ days, $t_2 = 1$ week, $t_3 = 1$ month, and $t_4 = 6$ months. Then $t_k - t_{k-1}$ is the time length in days of period $k$ for $k = 1, 2, 3, 4$.

We assume that there are $N$ earthquake consequence scenarios, i.e., $n = 1, ..., N$. The associated annual probability of scenario $n$ is $Pr(n)$. Let $c^B$ be the per unit load shed cost. Let $c^G_g$ be the per unit power generation cost of generator $g$. Note that the first-stage reinforcement decisions and the earthquake scenario determine the level of damage in the component; hence, the length of time from the earthquake that the component is unavailable and the repair cost is known. Let $\Psi^n_s(x_s)$ be the repair cost for substation $s$ under the first-stage decision $x_s$ for the earthquake scenario $n$. Let $\Omega^n_{ij}(y_{ij})$ be the repair cost of damaged transmission line $(i, j)$ under the first-stage decision $y_{ij}$ and given earthquake scenario $n$. Let $\Delta^{nk}_s(x_s) = 1$ if substation $s$ is not functional in period $k$ under earthquake scenario $n$ for the given first-stage decision.
\(x_s^D\) and \(\Delta_{sk}^n(x_s) = 0\) otherwise. Similarly, let \(\Lambda_{ij}^{nk}(y_{ij}) = 1\) if transmission line \((i,j)\) is not functional in period \(k\) under scenario \(n\) for the given first-stage decision \(y_{ij}\) and \(\Lambda_{ij}^{nk}(y_{ij}) = 0\) otherwise. Note that in this application, functions \(\Psi_s^D(x_s)\) and \(\Delta_{sk}^n(x_s)\) are nonlinear functions in \(x_s\) and functions \(\Omega_{ij}^n(y_{ij})\) and \(\Lambda_{ij}^{nk}(y_{ij})\) are nonlinear functions in \(y_{ij}^D\). Let \(\rho\) be the percentage of total line’s capacity corresponding to the discrete capacity increments, and let \(\mu\) be the same for generators.

Now we define the second-stage decision variables. Let \(\theta_{i}^{nk}\) be the voltage phase angle in bus \(i\) and period \(k\) under scenario \(n\). Let \(P_{ij}^{nk}\) be the real power flow in transmission line \((i,j)\) in period \(k\) under scenario \(n\). Since the electric flows can go in both directions, \(P_{ij}^{nk}\) can be positive or negative. Let \(G_{g}^{nk}\) be the nonnegative generation output from generator \(g\) in period \(k\) under scenario \(n\). Let \(U_{i}^{nk}\) be the nonnegative load shed in bus \(i\) in period \(k\) under scenario \(n\).

Let \(m_{ij}\) be the reactance of transmission line \((i,j)\) and let \(T_{ij}\) be an indicator parameter with value 1 when the operator has a spare transformer for transmission line \((i,j)\) and 0 otherwise.

\[
\begin{align*}
(\theta_i^{n1} - \theta_j^{n1})(1 - \Lambda_{ij}^{n1}(y_{ij}))(1 - \Delta_{si}^{n1}(x_{si}))(1 - \Delta_{sj}^{n1}(x_{sj}))(1 + \rho w_{ij}) &= m_{ij}P_{ij}^{n1}, \quad \forall (i,j), n \quad (61) \\
(\theta_i^{n2} - \theta_j^{n2})(1 - \Lambda_{ij}^{n2}(y_{ij}))(1 - \Delta_{si}^{n2}(x_{si}))(1 - \Delta_{sj}^{n2}(x_{sj}))(1 + \rho w_{ij}) &= m_{ij}P_{ij}^{n2}, \quad \forall (i,j), n \quad (62) \\
(\theta_i^{n3} - \theta_j^{n3})(1 - \Delta_{si}^{n3}(x_{si}))(1 - \Delta_{sj}^{n3}(x_{sj}))(1 + \rho w_{ij}) &= m_{ij}P_{ij}^{n3}, \quad \forall (i,j), n \quad (63) \\
(\theta_i^{n4} - \theta_j^{n4})(1 - \Delta_{si}^{n4}(x_{si})(1 - T_{ij}))(1 + \rho w_{ij}) &= m_{ij}P_{ij}^{n4}, \quad \forall (i,j), n \quad (64)
\end{align*}
\]

Constraints (61), (62), (63), and (64) approximate the active power flows on the transmission lines in the four periods of the repair process. If the per day demand at
bus $i$ is $D_i$, then constraints (65) state flow conservation at each bus under each earthquake scenario.

$$\sum_{g \in I(i)} G_{g}^{nk} - \sum_{(i,j) \in \delta^+(i)} P_{ij}^{nk} + \sum_{(j,i) \in \delta^-(i)} P_{ji}^{nk} = D_i - U_i^{nk}, \quad \forall i, k, n (65)$$

where $\delta^+(i)$ is the set of the transmission lines such that $(i,j) \in \Pi$ and $\delta^-(i)$ is the set of transmission lines such that $(j,i) \in \Pi$. Since the load shed at a bus cannot exceed the demand at the bus.

$$0 \leq U_i^{nk} \leq D_i, \quad \forall i, k, n (66)$$

We assume that generator $g$ has capacity $G_g^m$ and transmission line $(i,j)$ has capacity $P_{ij}^m$. Equations (67)-(71) reflect the capacity constraints in each generator and each transmission line in each time period under each earthquake scenario.

$$0 \leq G_{g}^{nk} \leq G_{g}^m (1 + \mu z_g), \quad \forall g, k, n (67)$$

$$|P_{ij}^{n1}| \leq P_{ij}^m (1 + \rho w_{ij}) \left(1 - \Lambda_{ij}^{n1}(y_{ij}) \right) \left(1 - \Delta_{s_i}^{n1}(x_{s_i}) \right) \left(1 - \Delta_{s_j}^{n1}(x_{s_j}) \right), \quad \forall (i,j), n (68)$$

$$|P_{ij}^{n2}| \leq P_{ij}^m (1 + \rho w_{ij}) \left(1 - \Lambda_{ij}^{n2}(y_{ij}) \right) \left(1 - \Delta_{s_i}^{n2}(x_{s_i}) \right) \left(1 - \Delta_{s_j}^{n2}(x_{s_j}) \right), \quad \forall (i,j), n (69)$$

$$|P_{ij}^{n3}| \leq P_{ij}^m (1 + \rho w_{ij}) \left(1 - \Delta_{s_i}^{n3}(x_{s_i}) \right) \left(1 - \Delta_{s_j}^{n3}(x_{s_j}) \right), \quad \forall (i,j), n (70)$$

$$|P_{ij}^{n4}| \leq P_{ij}^m (1 + \rho w_{ij}) \left(1 - \Delta_{s_i}^{n4}(x_{s_i}) \right) \left(1 - T_{ij} \right), \quad \forall (i,j), n (71)$$

where $s_i$ is the substation, to which bus $i$ belongs. The objective function of the two-stage stochastic program is to minimize the expected generation, load shed and repair costs in the four recovery periods as given in Equation (72).
where \( \Psi_s(x_s^D) = \sum_{n=1}^{N} Pr(n) \Psi_s^n(x_s) \) and \( \Omega_{ij}(y_{ij}) = \sum_{n=1}^{N} Pr(n) \Omega_{ij}^n(y_{ij}) \).

Note that the two-stage stochastic program (59) – (72) is a nonlinear mixed integer stochastic program. To better understand the structure of the two-stage nonlinear mixed integer stochastic program, we rewrite the program as follows. Let \( x = (x_s: s \in S) \), \( y = (y_{ij}: (i,j) \in \Pi) \), \( w = (w_{ij}: (i,j) \in \Pi) \), and \( z = (z_g: g \in E) \). For the given first-stage decision variables \((x,y,w,z)\), the second-stage optimization problem is to choose \((G_g^{nk}, P_{ij}^{nk}, U_1^{nk})\) to minimize

\[
\sum_{n=1}^{N} Pr(n) \sum_{k=1,2,3,4} \left( \sum_{i \in B} c_B U_1^{nk} + \sum_{g \in G} c_g^G G_g^{nk} \right) (t_k - t_{k-1})
\]

subject to constraints (61) – (71). Note that the second-stage optimization problem is a linear program (LP) and the program can be solved by scenario when the first stage decisions are known. Let \( \Phi(x,y,w,z) \) be the objective function value associated with Equation (73) subject to constraints (61)-(71) for the given first stage variables \((x,y,w,z)\). Then the first-stage of the two-stage nonlinear mixed integer stochastic program is to choose the binary variables \((x,y,w,z)\) to minimize

\[
\Phi(x,y,w,z) + \sum_{s \in S} \Psi_s(x_s) + \sum_{(i,j) \in \Pi} \Omega_{ij}(y_{ij})
\]

subject to (59) and (60). Notice that this is a knapsack problem. This motivates our solution procedure described in the next section.
B. Passenger Air Transportation System

As discussed previously, one goal of this paper is to understand how outages in the electric power network affect the performance of a passenger air transportation system and how those impacts might be mitigated through investments in the transmission system. Again, we do not suggest investments in the electric power system to directly address impacts of outages in electric because there is no dataset that specifically identifies the purpose of each load in a transmission model.

Disruption in the air transportation system is modeled for the first three time periods of the power outage. It is difficult for airlines to change their flight schedules quickly due to a myriad of issues including work rules for flight crews, basing of crews, aircraft maintenance support and gate availabilities. However, if their access to an airport for operations is restricted for a long period of time (greater than about a month), changes to the flight plan are likely to occur. Hence we focus on impacts in the first month.

The objective assumed for the passenger air transportation model is to maximize the number of passengers that can be accommodated given a fixed number of passengers that wish to travel, each with known origins and destinations. Let $U^* = (U^*_{i,n,k})$ be the optimal load shed at each bus in all periods and under all scenarios from the previous stochastic program. We suppress the dependence of $U^*$ on the optimal improvement of the power network $(x^*, y^*, w^*, z^*)$ from the previous stochastic program to simplify the notation.

We first define the topology of the passage air transportation system as follows. Let $R_{od}$ be the set of all the possible routes between origin-destination pair $(o, d)$, let $L$ be the set of all the links representing individual segments of trips, and let $R^l_{od}$ be the set of all the possible routes between origin-destination pair $(o, d)$ that include link $l$. Let $A$ be the set of all the origin-destination pairs for the possible trips in the system.
We assume that $\overline{D}_{od}$ is the daily average number of passengers traveling between origin-destination pair $(o, d)$ and $\overline{q}_l$ is the daily maximum number of passengers using link $l$ as a segment of their trip.

The decision variables are defined as follows. Let $p_{rk}^{nk}$ be the daily average passenger flow in route $r$ under the power outage during time period $k$ and scenario $n$, and let $q_l^{nk}$ be the daily average passenger flow in link $l$ under the power outage during time period $k$ and scenario $n$.

$$\sum_{(o,d) \in A} \sum_{r \in R_{od}} p_{rk}^{nk} = q_l^{nk}, \quad \forall l, n, k = 1,2,3 \tag{75}$$

$$\sum_{r \in R_{od}} p_{rk}^{nk} \leq \overline{D}_{od}, \quad \forall (o, d), n, k = 1,2,3 \tag{76}$$

$$p_{rk}^{nk} \geq 0, \quad \forall r, n, k = 1,2,3 \tag{77}$$

Equations (75) computes the total number of passengers using link $l$ based on the route flows across all $(o,d)$ pairs. Equation (76) ensures that the route flows do not exceed the demand for travel. Equations (77) require that the passenger flow variables are nonnegative.

For each scenario in the electric power network, either with or without investment, the locations where insufficient power is supplied can be identified using the economic dispatch model given in the previous subsection. Hence, we assume that if either the origin and/or destination airport of a link do not receive sufficient power, the capacity of the link is zero. To model this dependence, let $o_l$ and $d_l$ be the respectively origin and destination of link $l$. Let $B_o$ be the set of all buses supplying energy to airport $o$. Also let $M^p$ be the load shed that would cause an airport to be unable to support aircraft arrivals and departures (which includes insufficient power for security, baggage handling, etc.). We define the function $\delta_{o}^{nk}(U^*)$ to be one if $\sum_{l \in B_o} U_{l}^{nk} \geq$
\( M^p \) and to be zero, otherwise. Then equation (78) gives the dependence of the air passenger transportation system on the power network.

\[
q_{nk}^{i_l} \leq \bar{q}_i \left( 1 - \vartheta_{o_l}^{nk}(U^*) \right) \left( 1 - \vartheta_{d_l}^{nk}(U^*) \right), \quad \forall l, n, k = 1, 2, 3 \tag{78}
\]

The objective of the air flow system is to find \((p_{nk}^{i_l}, q_{nk}^{i_l})\) to maximize the expected daily average passengers accommodated. This leads to the maximization of the following equation:

\[
\sum_{n=1}^{N} \sum_{k=1,2,3} \sum_{(o,d) \in A} \sum_{r \in Rod} p_{nk}^{i_l} (t_k - t_{k-1}) / 24. \tag{79}
\]

Recall that \(U^*\) is known in the optimization model (75)-(79) and then both \(\vartheta_{o_l}^{nk}(U^*)\) and \(\vartheta_{d_l}^{nk}(U^*)\) are known. Thus the mathematical program (75)-(79) is a linear program that can be decomposed by scenario and time period.

**Solution Procedure**

Once the load shed in each of the power network buses is known the operability of each of the airports, the passenger air traffic problem is a linear programming network flow problem which can be solved using a solver such as IBM ILOG Optimization Studio CPLEX 12.2. Therefore this section we focus on the solution to the power network investment planning problem. As mentioned previously, this is a two-stage mixed integer nonlinear stochastic program for which realistic instances will be very large (on the order of many hundreds to thousands of integer variables, each with relatively small values) therefore; we develop a heuristic solution procedure.

Since the costs for seismic reinforcement are considerably smaller than those for investments in additional capacity, we first estimate the optimal reinforcement
investments and then estimate the optimal capacity expansion decisions. The key idea that underlies each heuristic is to construct a knapsack problem with a linear objective function so that the solution of the knapsack problem is also a good solution to either the optimization for reinforcement or for capacity expansion.

A. Seismic Reinforcement

This heuristic has four steps. The first step is to run the dc load flow economic dispatch model assuming all components are available. The second step is to run the economic dispatch assuming that a single component \( \in \text{set}(R) \) (set of all components that can be reinforced) is not functional. The third step is done for each component that can be reinforced. This step involves computing the relative benefit for reinforcing each component. The fourth step identifies the subset of reinforcement strategies that maximizes the benefit (as approximated using the weights developed in step 3) subject to the budget constraint given in equation (59). To simplify the notations, let us consider the following parametric dc load flow dispatch problem where we determine \((\theta_i, G_g, P_{ij}, U_i)\) that minimizes

\[
\sum_{i \in B} c^B U_i + \sum_{g \in G} c^G G_g
\]

subject to

\[
(\theta_i - \theta_j)(1 - \tau_{ij})(1 - \zeta_{s_i})(1 - \zeta_{s_j}) = m_{ij} P_{ij} \quad \forall (i, j)
\]

\[
\sum_{g \in I(i)} G_g - \sum_{(i, j) \in \delta^+(i)} P_{ij} + \sum_{(j, i) \in \delta^-(i)} P_{ji} = D_i - U_i \quad \forall i
\]

\[
|P_{ij}| \leq P_{ij}^m (1 - \tau_{ij}) \quad \forall (i, j)
\]

\[
0 \leq U_i \leq D_i \quad \forall i
\]

\[
0 \leq G_g \leq G_g^m \quad \forall g
\]
where $\tau_{ij}$ and $\zeta_{si}$ are input parameters for all $(i,j)$. The solution procedure is then as follows.

**Step i:** Run the dc flow economic dispatch problem (80) – (85) with $\tau_{ij} = \zeta_{si} = \zeta_{sj} = 0$ for all $(i,j)$ to determine $(\theta_i, G_g, P_{ij}, U_i)$. Note that the load shed at each bus $U_i = 0$ since we assume that all components in the network are functional. Let $\lambda$ be the optimal objective value.

**Step ii:** Let the set($R$) be comprised of the collection of components $r$ for which there is at least one consequence scenario under which the component is not operational but with mitigation it becomes operational, for at least one time period. Run the dc flow economic dispatch problem (80) – (85) for each component $r \in \text{set}(R)$ assuming that component $r$ is not functional. To do this, the parameters in the program are set as follows. If $r = (i,j)$, we let $\tau_{ij} = 1$ for $(i,j) = r$ and let $\tau_{ij} = 0$ for $(i,j) \neq r$. We also let $\zeta_{si} = \zeta_{sj} = 0$ for all $(i,j)$. If $r = s$, we let $\zeta_{si} = 1$ for $s_i = r$ and let $\zeta_{si} = 0$ for $s_i \neq r$. We also let $\tau_{ij} = 0$ for $(i,j)$. We determine the solution $(\theta_i, G_g, P_{ij}, U_i)$ and let $\lambda_r$ be the optimal objective value.

**Step iii:** Estimate the benefit of reinforcement for all components for which it is a consideration. Let $r \in \text{set}(R)$ be the members of that set.

If $r = (i,j)$, let
\[
\beta_{ij}^D = \sum_{n=1}^{N} Pr(n) \sum_{k=1,2} (\lambda^{ij} - \bar{\lambda}) \left( \Lambda_{ij}^{nk}(0) - \Lambda_{ij}^{nk}(1) \right) (t_k - t_{k-1}) + \Omega_{ij}(0) - \Omega_{ij}
\]

If $r = s$, let
\[
\beta_{s}^D = \sum_{n=1}^{N} Pr(n) \left( \sum_{k=1,2,3} (\lambda^{s} - \bar{\lambda}) \left( \Delta_{s}^{nk}(0) - \Delta_{s}^{nk}(1) \right) (t_k - t_{k-1}) \right) + \sum_{n=1}^{N} Pr(n) \left( (\lambda^{s} - \bar{\lambda}) \bar{S}(s) \left( \Delta_{s}^{nk+4}(0) - \Delta_{s}^{nk+4}(1) \right) (t_4 - t_3) \right) + \Psi_{s}(0) - \Psi_{s}(1)
\]

(86)
It is important to notice that in the fourth time period some large transformers may not
be operational in some substations. The rest of the substation may still be used if there
are other functioning transformers, but its performance has not been fully restored. To
reflect this we define $\bar{S}(s)$ be the serviceability of a substation. The definition of this
quantity is the fraction of lines and transformers that are functional within the
substation to the total number of lines and transformers (both functional and non-
functional) within the substation. Notice that the values $\lambda_{ij}$ and $\lambda_s$ are the result of step (ii). Also, $\Lambda_{ij}^nk(0)$ ($\Delta_s^n(0)$) are binary input data that reflect whether a line
(substation) is operational during period $k$ under scenario $n$ if no mitigation is
performed. Similarly, $\Lambda_{ij}^nk(1)$ ($\Delta_s^n(1)$) are binary input data that reflect whether a line
(substation) is operational during period $k$ under scenario $n$ if mitigation is
performed. Finally, $\Omega_{ij}(0) - \Omega_{ij}(1)$ reflects the repair costs savings if mitigation is
performed on a line. Similarly, $\Psi_s(0) - \Psi_s(1)$ is the same quantity, except it is
associated with substation repair cost savings.

**Step iv:** Determine the reinforcement strategy for the network. Run the integer
problem to determine the reinforcement strategy $(x,y)$ that maximize

$$\sum_{s \in S} \beta_s^D x_s + \sum_{(i,j) \in \Pi} \beta_{ij}^D y_{ij}$$  \hspace{1cm} (87)

subject to

$$\sum_{s \in S} b_s^D x_s + \sum_{(i,j) \in \Pi} f_{ij}^D y_{ij} \leq M^D$$  \hspace{1cm} (88)

$$x_s = 0, y_{ij} = 0 \text{ for } s \notin \text{set}(R), (i,j) \notin \text{set}(R)$$  \hspace{1cm} (89)

**B. Capacity Enhancement**

For the capacity enhancement problem we used a heuristic that iterates over three
main steps to gradually add capacities to the selected components. The first step is to
run a linearized version of the enhancement problem given in equations (60)-(71) and (73). The second step is to estimate the relative benefit from investment in each of the components. The third step is to identify a subset of $\mathcal{E}$ enhancements that maximizes the benefit subject to the budget constraint given in equation (2). To reduce computational time and memory requirements, we split the problem by scenario $n$ and time period $k$. Let $w_{ij}^{nk}$ be a continuous variable representing the capacity enhancement of transmission line $(i,j)$ under scenario $n$ and during time period $k$. In the same way, let $z_g^{nk}$ be a continuous variable representing the capacity enhancement of generator $g$ under scenario $n$ and during time period $k$. Notice there is no requirement that these variables be the same across scenarios and time periods. Of course these recommendations cannot be implemented directly. The heuristic will integrate these decisions together and draw conclusions that will then be implementable.

This is an iterative procedure, therefore we define $M^{C\ast}$ budget that has been allocated in previous iterations. At the beginning of the procedure, this value is zero. Also, let $l$ be the iteration number.

**Step v.** Initialize parameters. Let $l = 0$, $w_{ij}^l = 0$ for all transmission lines $(i,j)$, and $z_g^l = 0$ for all generators $g$. Also $M^{C\ast} = 0$. Select the maximum number, $\tilde{C}$, of enhancement units to be added per iteration. Also select the $\tilde{Z}$ and $\tilde{W}$, the upper bounds for $w$ and $z$. Notice there is single maximum level of capacity augmentation for generators and lines, respectively.

**Step vi.** Run a linearized version of the dc economic dispatch defined by constraints (60) – (71) and objective function defined in equation (73) but decomposed by scenario and time period and as given in equations (90)-(46). The linearization of the model is achieved through the following two modifications. First, equations (61) – (64) are replaced by equations (91) to (94) by replacing the term $(1 + \rho w_{ij})$ with
\((1 + \rho w^l_{ij})\). Effectively this assumes that the change in the reactance is proportional to the additional capacity added in previous iterations. Second, the sets of integer variables \(z^{nk}_g\) and \(w^{nk}_{ij}\) become positive continuous variables that can take values from \(z^l_g\) and \(w^l_{ij}\) (the value already obtained in previous iterations) up to \(\bar{Z}\) and \(\bar{W}\) respectively as given in equations (98)-(103). Notice that constraints (95) to (97), (102) to (104) are common to the \(n\) by \(k\) problems; however, equations (91) to (94) define only the \(k\)th problem as indicated in the superscript of the variables. Equations (98) to (101) are also different definitions of the same constraint for each of the four time periods. We solve \(n\) by \(k\) linear problems choosing variables \((G^{nk}_g, p^{nk}_{ij}, U^{nk}_i, w^{nk}_{ij}, z^{nk}_g)\) that minimize

\[
\sum_{i \in B} c_B U^{nk}_i + \sum_{g \in G} c_G G^{nk}_g
\]

subject to

\[
(\theta^{n1}_i - \theta^{n1}_j) \left(1 - \Lambda^{n1}_{ij}(y_{ij})\right) \left(1 - \Delta^{n1}_{si}(x_{si})\right) \left(1 - \Delta^{n1}_{sj}(x_{sj})\right)(1 + \rho w^l_{ij}) = m_{ij} p^{n1}_{ij} \quad \forall (i, j) \quad (91)
\]

\[
(\theta^{n2}_i - \theta^{n2}_j) \left(1 - \Lambda^{n2}_{ij}(y_{ij})\right) \left(1 - \Delta^{n2}_{si}(x_{si})\right) \left(1 - \Delta^{n2}_{sj}(x_{sj})\right)(1 + \rho w^l_{ij}) = m_{ij} p^{n2}_{ij} \quad \forall (i, j) \quad (92)
\]

\[
(\theta^{n3}_i - \theta^{n3}_j) \left(1 - \Delta^{n3}_{si}(x_{si})\right) \left(1 - \Delta^{n3}_{sj}(x_{sj})\right)(1 + \rho w^l_{ij}) = m_{ij} p^{n3}_{ij} \quad \forall (i, j) \quad (93)
\]

\[
(\theta^{n4}_i - \theta^{n4}_j) \left(1 - \Delta^{n4}_{si}(x_{si})(1 - T_{ij})\right)(1 + \rho w^l_{ij}) = m_{ij} p^{n4}_{ij} \quad \forall (i, j) \quad (94)
\]

\[
\sum_{g \in I(i)} G^{nk}_g - \sum_{(i,j) \in \delta^+(i)} p^{nk}_{ij} + \sum_{(j,i) \in \delta^-(i)} p^{nk}_{ji} = D_i - U^{nk}_i \quad \forall i \quad (95)
\]
\[ 0 \leq U_{i}^{nk} \leq D_{i}, \quad \forall i \quad (96) \]
\[ 0 \leq G_{g}^{nk} \leq G_{g}^{m}(1 + \mu z_{g}^{nk}), \quad \forall g \quad (97) \]
\[ |P_{ij}^{n1}| \leq P_{ij}^{m}(1 + \rho w_{ij}^{nk}) \left( 1 - \Lambda_{ij}^{n1}(y_{ij}) \right) \left( 1 - \Delta_{s_{i}}^{n1}(x_{s_{i}}) \right) \left( 1 - \Delta_{s_{j}}^{n1}(x_{s_{j}}) \right) \forall (i, j) \quad (98) \]
\[ |P_{ij}^{n2}| \leq P_{ij}^{m}(1 + \rho w_{ij}^{nk}) \left( 1 - \Lambda_{ij}^{n2}(y_{ij}) \right) \left( 1 - \Delta_{s_{i}}^{n2}(x_{s_{i}}) \right) \left( 1 - \Delta_{s_{j}}^{n2}(x_{s_{j}}) \right) \forall (i, j) \quad (99) \]
\[ |P_{ij}^{n3}| \leq P_{ij}^{m}(1 + \rho w_{ij}^{nk}) \left( 1 - \Delta_{s_{i}}^{n3}(x_{s_{i}}) \right) \left( 1 - \Delta_{s_{j}}^{n3}(x_{s_{j}}) \right) \forall (i, j) \quad (100) \]
\[ |P_{ij}^{n4}| \leq P_{ij}^{m}(1 + \rho w_{ij}^{nk}) \left( 1 - \Delta_{s_{i}}^{n4}(x_{s_{i}}) \right) \left( 1 - T_{ij} \right) \quad \forall (i, j) \quad (101) \]
\[ w_{ij}^{l} \leq w_{ij}^{nk} \leq \bar{W} \quad \forall (i, j) \quad (102) \]
\[ z_{g}^{l} \leq z_{g}^{nk} \leq \bar{Z} \quad \forall g \quad (103) \]
\[ \sum_{(i,j)\in \Pi} h_{ij}^{c} w_{ij}^{nk} + \sum_{g \in E} o_{g}^{c} z_{g}^{nk} \leq M^{c}, \quad (104) \]

Note that \( \Lambda_{ij}^{n1}(y_{ij}) \) for all \( (i, j) \) and \( \Delta_{s}^{nk}(x_{s}) \) for all \( s \), are known parameters from steps (i) to (iv). Then mathematical program (90) – (104) is a linear program. Let \((G_{g}^{nk}, P_{ij}^{nk}, U_{i}^{nk}, w_{ij}^{nk}, z_{g}^{nk})\) be the solution.

**Step vii.** Estimate the benefit from individual transmission and generation enhancements. These correspond to the sum of the component’s marginal capacity enhancements in iteration \( l \), for each scenario and time period. Notice that for equation
(106) we include the ratio between the maximum enhancement units for lines, $\overline{W}$ to the maximum enhancement units for generators, $\overline{Z}$. This ratio warranties that the benefits are within the same proportion to ensure that generator and transmission enhancement are coordinated. The benefit from the enhancement in transmission line $(i, j)$ is defined as follows:

$$\beta_{ij}^C = \sum_{n=1}^N Pr(n) \sum_{k=1,2,3,4} (w_{ij}^{nk} - w_{ij}^l) (t_k - t_{k-1})$$  

(105)

The benefit from the enhancement in generator $g$ is defined as follows:

$$\beta_g^C = \left( \frac{\overline{W}}{\overline{Z}} \right) \sum_{n=1}^N Pr(n) \sum_{k=1,2,3,4} (z_g^{nk} - z_g^l)(t_k - t_{k-1})$$  

(106)

**Step viii.** Select a subset of the capacity enhancement strategy of the network. Run the integer program to determine binary variables $(\overline{w}, \overline{z})$ that maximize

$$\sum_{(i,j)\in \Pi} \beta_{ij}^C \overline{w}_{ij} + \sum_{g\in E} \beta_g^C \overline{z}_g$$  

(107)

subject to

$$\sum_{(i,j)\in \Pi} h_{ij}^C \overline{w}_{ij} + \sum_{g\in E} a_g^C \overline{z}_g \leq M^C - M^{C*}$$  

(108)

$$\sum_{(i,j)\in \Pi} \overline{w}_{ij} + \sum_{g\in E} \overline{z}_g \leq \overline{C}$$  

(109)

Let $(\overline{w}, \overline{z})$ be the solution.

Notice that we only add one discrete unit of capacity per component per iteration; therefore, it is important that the number $\overline{C}$, which limits the number of selected
enhancements per iteration, is small enough in relation to the number of enhancements we can choose from the total possible for the available budget.

**Step ix.** Update the solutions and the budget as follows:

\[ w_{ij}^{l+1} = w_{ij}^l + \bar{w}_{ij} \quad (110) \]

\[ z_{g}^{l+1} = z_{g}^l + \bar{z}_{g} \quad (111) \]

\[ M^{C*} = \sum_{(i,j) \in \Pi} h_{ij}^C w_{ij}^{l+1} + \sum_{g \in E} o_g^C z_{g}^{l+1} \quad (112) \]

**Step x.** Check stopping conditions: If \( M^C - M^{C*} < \epsilon \), let \( l = l + 1 \) and go back to step vi. Otherwise, report the enhancement solutions \( (w, z) = (w^l, z^l) \).

**Case Study**

We focus on questions of seismic mitigation of the EI under limited budget, and the repercussions on passenger air transportation between the top 80 airports as measured by the number of enplanements. The representation of the EI was developed in 1998 by the Multi-Area Modeling Working Group. It is a representation of the system as of 1998 with demands reflective of a prediction of the summer of 2003. This case includes direct representation of every region in the EI, which extends approximately from the Rocky Mountains to the East Coast excluding Texas. Detailed representation is for voltages greater than 100 kV. It includes information for 23,416 transmission lines and 14,957 buses. These buses are grouped in 2,765 substations with two or more buses and 6,448 single buses. Load shed, generation output, repair, and mitigation costs were estimated in 2002 U.S. dollars.

We only consider the seismic risk from the NMSZ. The hazard is modeled by a set of earthquake scenarios selected using the mathematical optimization method developed by Vaziri et al.[in press]. The method also assigns an adjusted occurrence
probability to each scenario minimizing the discrepancy with the seismic behavior as represented in the exceedance curves for PGA. We located 81 control points in the NMSZ area and obtained the PGA exceedance curves for each point from the U.S. Geological Survey (USGS) Seismic Hazard Maps [2008b]. A key input to that optimization is the identification of the candidate set of earthquake events. We used two sources to create the candidate set: the earthquake catalog from the USGS website [2008a], and 20 synthetic scenarios created using code provided by USGS [2008c]. The earthquake catalog includes 433 earthquakes that occurred within the NMSZ. The magnitudes were converted from mbLg to MW as described in Petersen et al [2008]. The mean PGA for each control point was estimated using Toro et al [1997], Frankel et al [1996], Campbell [2003], Atkinson and Boore [2006], Tavakoli and Pezeshk [2005], and Silva et al [2002] assuming soil type BC (Firm Rock), the relative weights given in Petersen et al [2008]. In addition to the 433 earthquakes identified in the Central-East Unites States earthquake scenarios catalog, we use 20 synthetic events on 5 synthetic faults created by USGS to represent the hazard in New Madrid. The 20 scenarios correspond to each of the 4 possible magnitudes (7.3, 7.5, 7.7 and 8) for ruptures in the 5 different branches described in Petersen et al [2008]. USGS provides computer code that can be compiled and run to generate each of these deterministic scenarios in New Madrid [2008c].

For electric power systems, the key measure is the exceedance curves for PGA where seismically sensitive components are located. Figure 13 illustrates the exceedance curve from USGS and the implied exceedance curve obtained from the selected scenarios (and their probabilities of occurrence) based on the formulation given in Vaziri et al. [in press] for a single control point located near the border between Tennessee and Arkansas at the Mississippi River near Osceola, Arkansas.
Table 7 presents the 8 earthquake scenarios selected with their adjusted occurrence probability.

<table>
<thead>
<tr>
<th>Location</th>
<th>Fault Info.</th>
<th>Depth [km]</th>
<th>Magnitude m\textsubscript{bgl}</th>
<th>Date</th>
<th>Source</th>
<th>Adjusted occurrence probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>36.7</td>
<td>-90.3</td>
<td>0.0</td>
<td>4.3</td>
<td>2/2/1954</td>
<td>NCEER</td>
<td>0.0500</td>
</tr>
<tr>
<td>38.2</td>
<td>-89.8</td>
<td>11.0</td>
<td>4.3</td>
<td>4/9/1955</td>
<td>NCEER</td>
<td>0.0500</td>
</tr>
<tr>
<td>37.9</td>
<td>-88.4</td>
<td>21.0</td>
<td>5.5</td>
<td>11/9/1968</td>
<td>NCEER</td>
<td>0.0078</td>
</tr>
<tr>
<td>38.7</td>
<td>-88.0</td>
<td>10.0</td>
<td>5.2</td>
<td>6/10/1987</td>
<td>USHIS</td>
<td>5.20mn</td>
</tr>
<tr>
<td>36.8</td>
<td>-89.2</td>
<td>5.0</td>
<td>4.5</td>
<td>9/29/1987</td>
<td>USHIS</td>
<td>4.50mn</td>
</tr>
<tr>
<td>35.8</td>
<td>-90.2</td>
<td>9.0</td>
<td>4.2</td>
<td>5/1/2005</td>
<td>PDE</td>
<td>4.20mw</td>
</tr>
<tr>
<td>Mid-East West</td>
<td></td>
<td></td>
<td>7.7</td>
<td></td>
<td>USGS faults</td>
<td>0.0018</td>
</tr>
<tr>
<td>West</td>
<td></td>
<td></td>
<td>8.0</td>
<td></td>
<td>USGS faults</td>
<td>0.0010</td>
</tr>
</tbody>
</table>

Using a HAZUS [FEMA 2003] we estimate the probability that each component sustains specific levels of damage under each scenario. HAZUS is regional loss estimation methodology that categorizes damage to substations and transmission lines into five classes: no damage, slight, moderate, extensive and complete. As mentioned before we only modeled moderate, extensive and complete damage for substations and extensive and complete for transmission lines. The probability that each component sustains the different levels of damages is modeled as a set of consequence scenarios, each one with an adjusted occurrence probability. We use the optimization method introduced by Brown et al. [2011] to develop the consequence scenarios and their hazard-consistent probability of occurrence. The objective of the optimization is to select the consequence scenarios and associated probabilities so the implied vulnerabilities of each component match the “true” (input) vulnerability as closely as
possible. [Brown et al. 2011] optimization is expanded to include the no-reinforcement and reinforcement scenarios.

![Exceedance curves for control point](image)

Figure 13. Exceedance curves for control point located at the border between Tennessee and Arkansas at the Mississippi River near Osceola, Arkansas.

The earthquake scenario on the Mid-East fault of magnitude 7.7 $M_W$, and the earthquake scenario on the West fault of magnitude 8.0 $M_W$ are the only two scenarios resulting in considerable physical damage to the electric grid. For each of these events, we generate 6 consequence scenarios. Table 8 presents the 12 consequence scenarios and the number of transmission lines and substations that fall into each of the possible damage states. Figure 14 shows the spatial distribution of PGA for the earthquake scenario in the West branch with a magnitude of 8.0 $M_W$ and the consequence scenario 3 (See ID column in Table 8).

We use a static network flow model to represent the passenger air traffic system. The model focuses on the largest 80 airports (as measured by enplanements) in the U.S for the year 2010 [FAA 2010]. The number of seats flown in each direction for pairs of airports was obtained from the T100 dataset for 2010 [BTS 2010]. Trips between the selected airports correspond to more than 80% of total domestic
passenger trips. To obtain the routes between each pair of considered airports, we use the Airline Origin and Destination Survey, DB1B from the BTS website, for the year 2010 [BTS 2010]. We only included routes with a maximum of three flight segments; this reduces the total number of passengers considered by less than 1%. With the DB1B we estimated the number of seats taken by passengers not considered in our model. This information combined with the total seats per segments were used to estimate the capacity available in each link, $q_i$. The average daily number of passengers traveling between the 80 largest airports on routes with 3 or less segments was a little over 916,000 for the year 2010. The daily number of passengers between origin-destination pairs, $D_{d_i}$, is the demand considered in the model.

Table 8. Consequence scenarios use to represent the NMSZ hazard on the EI.

<table>
<thead>
<tr>
<th>Source ID</th>
<th>Adjusted occurrence probability</th>
<th>Basic design</th>
<th>Seismically reinforced components</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line damage</td>
<td>Substations damage</td>
<td>Lines damage</td>
<td>Substations damage</td>
</tr>
<tr>
<td>West branch, 8.0 MW</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.000160</td>
<td>5</td>
<td>24</td>
</tr>
<tr>
<td>2</td>
<td>0.000200</td>
<td>5</td>
<td>25</td>
</tr>
<tr>
<td>3</td>
<td>0.000080</td>
<td>15</td>
<td>24</td>
</tr>
<tr>
<td>4</td>
<td>0.000180</td>
<td>4</td>
<td>25</td>
</tr>
<tr>
<td>5</td>
<td>0.000240</td>
<td>6</td>
<td>24</td>
</tr>
<tr>
<td>6</td>
<td>0.000140</td>
<td>6</td>
<td>22</td>
</tr>
<tr>
<td>Mid-East branch, 7.7 MW</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.000126</td>
<td>27</td>
<td>11</td>
</tr>
<tr>
<td>8</td>
<td>0.000396</td>
<td>28</td>
<td>14</td>
</tr>
<tr>
<td>9</td>
<td>0.000288</td>
<td>30</td>
<td>13</td>
</tr>
<tr>
<td>10</td>
<td>0.000342</td>
<td>28</td>
<td>13</td>
</tr>
<tr>
<td>11</td>
<td>0.000432</td>
<td>24</td>
<td>14</td>
</tr>
<tr>
<td>12</td>
<td>0.000216</td>
<td>28</td>
<td>26</td>
</tr>
</tbody>
</table>

Mod.=Moderate, Ext.=Extensive, Com.=Complete
Figure 14. Distribution of PGA and damage for an 8.0 Mw earthquake on the West Branch under consequence scenario 3.

We approximately identify the substations in the EI that provide electricity to each airport based on GIS information. In addition, we used the energy cost per enplanement for large airports from [Salamone 2007] and energy costs per load in the transportation sector from the U.S. Energy Information Administration website to estimate the airport’s average energy consumptions [EIA 2012]. With this information and the list of substations in the vicinity of the airports, we connected each airport to a substation or a group of substations such as the total demand of the substations was at least twice the airport’s energy demand.
**Results**

For reinforcement, the heuristic was tested using a simplified version of the EI model which considers only the first time period and two of the twelve earthquake scenarios. We solved the NLIP for 3 different mitigation budgets: US$100 million, US$20 million, and US$10 million using two methods: the proposed heuristic coded in C++ with IBM ILOG Optimization Studio CPLEX 12.2 serving as an LP solver, and the full NLIP in Lingo 13 (which has an non-linear integer solver) in a Dell Precision T5500, Intel® Xeon® X5650 with 2 processors of 2.66 GHz., and 6.00 GB of total RAM memory. Lingo found a 0.5% better solution for a mitigation budget of US$100 million; for the other two problem instances the heuristic method found solutions that were 20% better in quality. Lingo took over 8 hours to solve and the heuristic took 8 minutes. Given the computational burden, Lingo cannot be used to address the full problem formulation for the EI.

We used the heuristic to find the solution to the problem of estimating the optimal seismic reinforcement strategy for the EI full problem formulation. The analysis included the 12 consequence scenarios identified to represent the hazard in the NMSZ. For each consequence scenario we modeled the repair process using the 4 time periods described above. The total running time varies depending on the budget; the average computation time is 1 hour using the machine described above Figure 15 shows the exceedance probability of load shed costs for 4 different investment scenarios: no-mitigation, US$ 5 million, US$ 10 million, and US$ 100 million.

We assume about 50 lines and 110 substations are viable candidates for reinforcement. Depending on the budget available, some lines may be too expensive hence the exact number of candidates varies between the experiments. For a budget of US$ 5 million, the model suggests no seismic reinforcement for lines, and that about 63% of substations should be anchored. For a budget of US$ 10 million, 6% of
transmission lines and 87% of substations are selected for reinforcement. Finally, for US$ 100 million, 70% transmission lines and 100% of substations should be reinforced. Investing US$ 10 million can reduce the load shed costs by almost 40% of the load shed costs in no reinforcement situation, and the repair costs by almost 20%. In expectation this reduction translates to a saving of about US$ 12 million annually stemming from an upfront investment of US$ 10 million. Investing US$ 100 million provides and additional US$ 2 million benefit.

Figure 15. Exceedance probability for load shed costs during repair for different reinforcement mitigation investment budgets.

The second part of the problem, finding an optimum capacity enhancement solution, is more computationally demanding. Therefore, it could not be tested with a simplified version of the EI seismic risk model. We used the one area IEEE Reliability Test System (RTS) – 1996 [Grigg 1999]. We formulated the problem for a hypothetical case in which demands in the test system were doubled. The nonlinear integer programming problem (NLIP) was evaluated for 4 different mitigation budgets: US$50 million, US$100 million, US$500 million, and US$1 billion using
two methods: the proposed heuristic coded in C++ with IBM ILOG Optimization Studio CPLEX 12.2 serving as an LP solver, and the full NLIP in Lingo 13 (using the global solver). The four solutions are within a 0.1% error from the optimal solution found using Lingo. Lingo requires between 20 seconds to 3 minutes depending on the investment budget, and the heuristic, less than 20 seconds.

We used the heuristic to find optimum enhancement strategies that help to mitigate the consequences on the EI of a seismic event in the NMSZ. The problem included the 12 consequence scenarios, and the 4 time repair periods and over 110,000 potential investments over 30,000 components. We found solutions for three enhancement budgets under two situations, no previous reinforcement, and US$ 10 million investment. The 3 enhancement budgets are: US$ 100 million, US$ 1 billion, and US$ 10 billion. Computational time varies significantly depending on the relation between the total budget and parameter $\tilde{C}$, with the smallest computation times on the order 2 hours and the longest on the order of 12 hours.

Figure 16 shows the exceedance probability of load shed costs during repair time for the case of no previous investment in reinforcement and for 4 different investment scenarios: no-mitigation, US$ 100 million, US$ 1 billion, and US$ 10 billion. Figure 17 shows the same results for the case of investment in reinforcing for a budget of US$ 10 million.

When there is no previous reinforcement investment, the enhancement mitigation strategies include enhancement of 57 lines for a budget of US$ 100 million, 204 lines and 1 generator for a budget of US$ 1 billion, and 444 lines and 52 generators for a budget of US$ 10 billion. When there is previous reinforcement the solutions include 54 lines, 226 lines and 1 generator, and 448 lines and 52 generators, respectively. An investment of US$ 100 million exclusively in capacity enhancement represents a 25% savings in load-shed over the 6 months repair period. For a budget of US$ 1 billion
and US$ 10 billion, the load shed savings are 37% and 48% respectively. If US$ 10 million were invested in reinforcement and US$ 100 million in capacity enhancement there is a 51% savings in load shed costs. The saving goes as high as 64% when the capacity enhancement investment is US$ 10 billion. As an exercise, we estimated the benefit from investing US$ 110 million exclusively in reinforcement. We found that the expected annual saving in load shed costs is US$ 1 million smaller in this case than in the case when US$ 10 million are invested in reinforcement and US$ 100 million in capacity expansion. The combined investment is better for the mitigation of seismic risk; furthermore, investing US$ 100 million in capacity enhancement can benefit the power network not only in a seismic event but also against other hazardous events and spikes in demand.

Figure 16. Exceedance probability for load shed costs during repair when there is no investment in reinforcement and some investment in capacity enhancement.
We evaluated the performance of the U.S. domestic air traffic system under power outages due to the NMSZ. We did not model the seismic vulnerabilities of the air traffic infrastructure and we only considered the first three time periods of the power disruption (because for long term outages, the airlines may choose to change the flight schedule). The solution varies among the consequence scenarios depending on whether Memphis International Airport (MEM), Nashville international Airport (BNA), and Lambert-St. Louis international Airport (STL) are part of the blackout area or not. Under no mitigation investment, MEM is always affected by the power outage. Problems in BNA and STL vary from one scenario to another.

First, we focus on the first time period (first three days after an earthquake) for the first three consequence scenarios. The first scenario affects MEM and STL and it causes the disruption of more than 30,000 daily trips, and affects 315 origin-destination pairs out of the total 6,222. All the disruptions occur to flights departing or arriving at the affected airports. The second scenario only affects MEM, and causes no
consequences on passengers travelling between airports different than MEM. The third scenario affects MEM, BNA, and STL. It affects more than 7% of the origin-destination pairs causing no with origin and or destination at the affected airports.

Table 9 shows the number of daily passengers whose trip is cancelled under each of the earthquake scenarios and during the three repair time periods within the first month after the seismic event. Results are presented for a scenario of US$ 10 million and US$ 100 million for reinforcement and capacity enhancement, respectively.

Table 9. Average daily cancelled trips.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>No mitigation</th>
<th>Seismic reinf.: US$ 10 million, enhancement: US$ 100 million</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>First 72 hours</td>
<td>3rd – 7th day</td>
</tr>
<tr>
<td>1</td>
<td>30,532</td>
<td>7,374</td>
</tr>
<tr>
<td>2</td>
<td>73,74</td>
<td>7,374</td>
</tr>
<tr>
<td>3</td>
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Conclusions

This paper develops a computational effective procedure, using stochastic programming, to optimize seismic mitigation (both anchoring of components as well as capacity expansion in transmission and generation) in electric power systems for large-scale application. The solution procedure was tested using smaller problem instances. It was then applied to perform seismic mitigation planning for EI, which
has almost 15,000 nodes and 23,000 links. As mentioned previously, this is the first paper to model both the opportunity to strengthen system components as well as to add operating margin. Further this is the first attempt to address capacity planning for very large transmission systems.

Future work is valuable in at least three related areas. First, the interdependency between the electric power system and the air system is only in one direction. That is, lack of power affects the air transportation system. Lack of air travel capabilities does not impact the electric power system. For this particular infrastructure interaction, this is reasonable. However, in other systems, the one way impact is too simplistic. For example, consider the relationship between water and electric power. Substantial amounts of water are used in generation for electric power, and electric power is used for pumping in the water system. Explicit representation of these “two way” linkages is important. Therefore, extending the modeling to incorporate these complex linkages with explicit representation of the physical operation of each of the systems is important. Second, we simply compute the impacts on the air transportation system of investments in the electric power system, rather than use that computation as part of the optimization to dispatch the remaining power after an event. Currently there is very little data available on the end uses for the electric power demanded at the high voltage busses represented in the transmission network model. That is, there is very limited information about how the lower voltage distribution system allocates end uses to the transmission network busses. However, if that data were available, and there were methods available to exploit that data, decision making for investment planning would be much improved. Third, the focus of this modeling has been earthquake hazards. In reality many kinds of events impact infrastructure systems including hurricanes, ice storms and intentional disruption (though this has occurred primarily outside the U.S). Extending the modeling to address multiple hazards is important.
REFERENCES


