ESSAYS ON INTERNATIONAL COMOVEMENTS OF FINANCIAL MARKETS

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International portfolio diversification is beneficial only if asset returns are not significantly correlated across countries. Therefore, it is essential for investors who want to make an appropriate portfolio selection to understand the nature of asset return correlations. This thesis consists of three essays on international comovements of financial markets.

The first essay analyzes the effects of heterogeneous beliefs and learning on international comovements of equity returns and portfolio rebalancing mechanism. This essay develops a continuous-time general equilibrium model in a two-asset and two-good economy with two representative agents, who differ in perceived rates of output growth and accuracy of beliefs. The equilibrium correlations of equity returns across counties and optimal portfolios are expressed in terms of the differences in beliefs. The main findings are: (1) the differences in perceived rates of output growth generate equity home or foreign bias, resulting in lower cross-country equity return correlations; and (2) the volatilities of optimal portfolios and capital flows increase with the differences in perceived output growth and with the differences in accuracy of beliefs.

The second essay studies the effects of trade costs in goods market on international comovements of equity markets and those on equity home bias. This essay develops a continuous-time general equilibrium model in a two-country, two-asset, and two-good setting where international trade of goods is costly. I solve for the

optimal portfolios and the equilibrium correlations of cross-country equity returns and analyze how they change depending on the size of trade costs, the coefficient of risk aversion, and the elasticity of substitution between domestic and foreign goods. It is found that the cross-country equity return correlations decrease with the size of trade costs. This result is robust to different sizes of trade costs and asymmetry related to potential growth and consumer preferences. It is also found that the size of the trade costs and other parameter values determine whether trade costs would generate equity home bias or foreign bias.

The third essay is devoted to an empirical analysis of the effects of financial integration on international comovements of financial markets. The essay provides a characterization of synchronization among 24 countries over the period 1980-2003. A country-pair panel instrumental variables framework is employed to explain time-varying bilateral correlations among national stock returns, by utilizing the dataset on trade costs in Fitzgerald (2008). It is found that financial integration driven by reduction of trade costs leads to a higher degree of synchromization across stock markets.

BIOGRAPHICAL SKETCH

Yusuke Tateno was born on July 28, 1983 in Chiba, Japan. After receiving his high school diploma in 2002 from Kaisei Academy in Tokyo, Japan, he moved to the US and attended Beloit College, a small liberal arts college in Wisconsin. He received a BA in economics in 2005, earning Summa Cum Laude and Phi Beta Kappa.

In August 2005, he enrolled as a graduate student in the economics department of Cornell University in Ithaca, New York, majoring in international economics and macroeconomics. His doctoral research, under the supervision of Professors Assaf Razin, Eswar Prasad, and Viktor Tsyrennikov, focused on international comovements of financial markets. During his student career, he served as a consultant to graduate students needing assistance with teaching methods and activities. He also held short-term research positions at the International Monetary Fund and at a private consulting firm.

After graduation, he will pursue his career as a public official of the United Nations where he was recruited through the National Competitive Recruitment Exam in economics.

This thesis is dedicated to my parents, Fujio Tateno and Toshimi Tateno, who taught me discipline, honor, and respect.

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Finally, I would like to thank my parents, brothers and sister for all their incessant support during all the years of my life. They are the reason for my existence.

Yusuke Tateno, August 2011

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CHAPTER 1

INTRODUCTION

International portfolio diversification is beneficial because it enables us to smooth consumption by hedging against country-specific risks. However, cross-boarder investments may come at a cost in the presence of market frictions such as trading costs of assets and heterogeneity in beliefs as they would change the nature of the joint dynamics of asset returns across countries. If these market frictions and their changes make equity returns significantly correlated across countries, international portfolios diversification would not be as beneficial as it is supposed to be. Thus, it is essential for investors who want to make an appropriate portfolio selection to understand how market frictions affect asset prices and dynamics. This thesis consists of three essays on international comovements of financial markets. Chapter 2 analyzes the effects of heterogeneous beliefs and learning on international comovements of equity returns and portfolio rebalancing mechanism. Chapter 3 examines the mechanism of joint dynamics of asset returns across imperfectly integrated goods market. Finally, Chapter 4 contains an empirical analysis of the effects of financial integration on international comovements of financial markets.

Chapter 2 analyzes the effects of heterogeneous beliefs and learning on the equilibrium correlations of equity returns and portfolio rebalancing mechanism. Asset prices and dynamics reflect investors' beliefs and preferences. Thus, heterogeneity in beliefs and preferences plays an important role in the formation of portfolio structure and capital flows across markets. This chapter develops a continuous-time general equilibrium model in a two-asset and two-good economy with two representative agents, who differ perceived rates of output growth and accuracy of their beliefs. The equilibrium correlations of equity returns across countries and optimal portfolios are expressed in terms of the differences in beliefs. The main findings are: (1) the differences in perceived rates of output growth generate equity home or foreign bias, resulting in lower cross-country equity return correlations; and (2) the volatilities of optimal portfolios and capital flows increase with the differences in perceived output growth and with the differences in accuracy of beliefs.

Chapter 3 examines the effects of trade costs in goods market on international comovements of equity markets and those on equity home bias. Trade costs in goods market are not a negligible market friction as pointed out by Anderson and van Wincoop (2004): international trade costs such as transportation costs and tariffs are as high as 74 percent of production costs. Roughly speaking, since trade costs in goods market affect good prices and consumers behavior, they should also affect equity prices and their dynamics. This chapter analyzes the effects of trade costs in goods market on international comovements of equity markets and those on equity home bias. The chapter develops a continuous-time general equilibrium model in a two-country, two-asset, and two-good setting where international trade of goods is costly. I solve for the optimal portfolios and the equilibrium correlations of cross-country equity returns and analyze how they change depending on the size of trade costs, the coefficient of risk aversion, and the elasticity of substitution between domestic and foreign goods. It is found that the cross-country equity return correlations decrease with the size of trade costs. This result is robust to different sizes of trade costs and asymmetry related to potential growth and consumer preferences. It is also found that the size of the trade costs and other parameter values determine whether trade costs would generate equity home bias or foreign bias.

Chapter 4 is devoted to an empirical analysis of the effects of financial integration on international comovements of financial markets. In the last three decades, we have observed a rapid increase in international financial integration, not only among industrial countries but also among most emerging market economics. We have also witnessed evidence of rising comovements of cross-country equity returns. This chapter examines how these two events are linked together, that is, how financial integration affect international comovements of stock returns. The chapter provides a characterization of synchronization among 24 countries over the period 1980-2003. A country-pair panel instrumental variables framework is employed to explain time-varying bilateral correlations among national stock returns, by utilizing the dataset on trade costs in Fitzgerald (2008). It is found that financial integration driven by reduction of trade costs leads to higher degree of synchronization across stock markets.

CHAPTER 2

INTERNATIONAL COMOVEMENTS OF EQUITY MARKETS: THE EFFECTS OF HETEROGENEOUS BELIEFS AND LEARNING

This chapter analyzes the effects of heterogeneous beliefs and learning on international comovements of equity returns and portfolio rebalancing mechanism. This chapter develops a continuous-time general equilibrium model in a two-asset and two-good economy with two representative agents, who differ in perceived rates of output growth and accuracy of beliefs. The equilibrium correlations of equity returns across countries and optimal portfolios are expressed in terms of the differences in beliefs. The main findings are: (1) the differences in perceived rates of output growth generate equity home or foreign bias, resulting in lower cross-country equity return correlations; and (2) the volatilities of optimal portfolios and capital flows increase with the differences in perceived output growth and with the differences in accuracy of beliefs.

2.1 Introduction

Asset prices and dynamics reflect investors' beliefs and preferences. Thus, heterogeneity in beliefs and preferences should be important factors to be considered in portfolio selections and cross boarder investments. This chapter analyzes the effects of heterogeneous beliefs and Bayesian learning on stock return comovements and portfolio rebalancing mechanism. To this aim, the chapter develops a continuous-time general equilibrium model in a two-asset and two-good economy with two representative agents, who differ in preferences, perceived output growth, and accuracy of their beliefs.

Heterogeneous beliefs can capture the phenomenon such that local investors are better informed about the local production than cross-border investors. The literature has been modeling it in such a way that investors receive common information but interpret it in different ways. The early works of the literature include Varian (1985, 1986), Abel (1990), and Harris and Raviv (1993), where a static discrete-time setting is employed. The development of the continuous-time models allow the different subjective beliefs to be arose from a Bayesian learning of the investors perceived output growth potential as in Williams (1997), Detemple and Murthy (1994), Basak (2000, 2005), Gallmeyer and Hollifield (2008).

The paper builds up on these continuous-time models by adding an asymmetry related to accuracy of beliefs. It is typically assumed that accuracy of beliefs are the same across investors for simplicity, and therefore the investors' behavior is simply affected by their initial perceived levels of output growth. In this chapter, investors also differ in accuracy of perceived output growth (i.e. one has more accurate perception than the other), which may further affect their investment behavior. This assumption is to model more precisely the difference between domestic and cross-border investment. Typically, domestic investors have accurate perception about the economic fundamentals about their own countries than the foreign investors, though they may sometime become over-confident about the potential of their own production. The model also includes demand uncertainty and consumption home bias, so that investor behavior is not purely derived from output risk sharing. The reason for adapting a two-good setting is to allow the terms of trade to be deviate from unity as existing works show that the terms of trade volatility is one of the important factors which influence the degree of stock return comovements (Fratzscher, 2002).

Another contribution of the chapter is to allow output shocks to play a role in portfolio rebalancing mechanism, apart from wealth effects. In a conventional case where investors have homogeneous beliefs and preferences, investors rebalance portfolio purely due to the change in wealth, keeping portfolio structure unchanged. However, with heterogeneous beliefs, investors adjust their portfolio not only in response the changes in wealth but also by changing portfolio structure through updating beliefs. Portfolio rebalancing mechanism is now decomposed into three parts: wealth effects, portfolio adjustment due to the realization of demand shocks, and portfolio adjustment due to updating their beliefs in response to the realization of supply shocks.

The model is based on the literature of good and asset trades under uncertainty in an international context (Helpman and Razin, 1978; Cole and Obstfeld, 1991). It is also closed related to the works on international comovements of equity markets such as Zapatero (1995), Pavlova and Rigobon (2007), and Soumare (2007). Zapatero (1995) shows that equity markets are perfectly correlated internationally under the assumption of logarithmic preferences. Pavlova and Rigobon (2007) extends the work by Zapatero (1995) and show that, with existence of demand shocks, equity markets are not perfectly correlated internationally even under the assumption of the logarithmic preferences as investors hedge against their own demand risks. Soumare (2007) uses CRRA preferences and concludes that the perfect correlation between stock markets does not hold under more general specifications of utility. This chapter extends the works by Pavlova and Rigobon (2007) by taking into account the effects of heterogeneous beliefs and learning and provides full characterization of return comovements and portfolio rebalancing.

The rest of the chapter is organized as follows: Section 2.2 presents an international asset pricing model where representative agents have time-additively separable logarithmic preferences, and characterize the equilibrium. Section 2.3 show that stock market correlations decrease in the differences in perceived output growth. The dynamics of stock prices and optimal portfolios are derived in closed-form. Section 2.4 discusses an alternative source of market frictions and their effects on stock market correlations. Section 2.5 contains my conclusions.

2.2 The Model

This section presents a two-country, two-asset, two-good asset pricing model under incomplete information.

2.2.1 Production

The chapter considers a continuous-time Lucas-type pure-exchange economy with finite horizon T. The economy consists of two countries: Home and Foreign, respectively denoted by H and F. The uncertainty in this economy is represented by a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, P)$, on which is defined a standard three-dimensional Brownian motion $\mathbf{w}(t) = \left(w^H(t), w^F(t), w^\delta(t)\right)^T$. The Brownian motions $w^H(t)$, $w^F(t)$, and $w^\gamma(t)$ capture country-specific production risk in H, country-specific production risk in F, and relative demand risk, respectively.

Each country specializes in producing its own good. The amount of goods produced in country $i \in \{H, F\}$ follows a geometric Brownian process:

$$\begin{aligned} dY^H(t) &= Y^H(t)\left[\mu^{Y^H} dt + \sigma^{Y^H} dw^H(t)\right], \\ dY^F(t) &= Y^F(t)\left[\mu^{Y^F} dt + \sigma^{Y^F} dw^F(t)\right] \end{aligned} \tag{2.1}$$

where μ^{Y^H} and μ^{Y^F} denote rates of output growth, and σ^{Y^H} and σ^{Y^F} denote diffusion parameters of the output growth. These four parameters are constant and exogenous. The terms of trade are defined as the price of the Foreign goods relative to the price of the Home goods. By normalizing the price of the Home goods ($p^H(t) = 1$), the terms of trade can be written as $p(t) \equiv p^F(t)/p^H(t)$, and follows a geometric Brownian process:

$$dp(t) = p(t)\left[\mu^p(t)\,dt + \boldsymbol{\sigma}^p(t)^T d\mathbf{w}(t)\right] \tag{2.2}$$

where $\mu^p(t)$ and $\boldsymbol{\sigma}^p(t)$ are the drift and diffusion parameters of the process and determined at the equilibrium.

2.2.2 Heterogeneous Beliefs

Even though the investors commonly observe the realized output $Y^H(t)$ and $Y^F(t)$, they do not know true growth rates of output μ_{Y^H} and μ_{Y^F}. This assumption has been widely made in the literature such as Basak (2000, 2005), and Gallmeyer and Hollifield (2008). Since outputs are observed continuously, all investors perfectly estimate its volatility σ_{Y^H} and σ_{Y^F} by computing the output process's quadratic variation (Gallmeyer and Hollifield, 2008). As discussed in the introduction, the initial levels of precision are typically assumed to be the same across investors for simplicity, and therefore the investors' behavior is simply affected by their initial levels of perceived output growth. In this model, investors differ in the initial levels of accuracy of beliefs (i.e. one is better informed than the other), which may further affect the investment behavior. To be precise, the Home investor has a higher (lower) precision on the perceived growth rates of the Home (Foreign) production than the Foreign investor. This assumption is to model more accurately the difference between domestic and cross-border investment.

Investor i's prior is normally distributed with mean $\mu_i^j(0)$ and variance $v_i^j(0)$ to the rate of production growth in country

$$k \in \{H, F\} : \mu^{Y^k}(0) \sim N\left(\mu_i^{Y^k}(0), v_i^{Y^k}(0)\right). \tag{2.3}$$

$v_i^{Y^k}(0)$ can be interpreted as the level of precision or confidence. Both $\mu_i^{Y^k}(0)$ and $v_i^{Y^k}(0)$ are private information. It can be assumed without loss of generality that $v_i^{Y^i}(0) > v_k^{Y^i}(t), i \neq k$. This assumption is realistic since the domestic investors tend to know better about the production in their own country than the foreign investors. Investors continuously update their beliefs in a Bayesian fashion as they observe the level of production $Y^H(t)$ and $Y^F(t)$. Following Chernoff (1968) and Liptser and Shiryayev (2001), the investor's posterior distribution can be expressed as:

$$\mu^{Y^k}(t) \sim N\left(\mu_i^{Y^k}(t), v_i^{Y^k}(t)\right) \tag{2.4}$$

where

$$\begin{aligned} \mu_i^{Y^k}(t) &= v_i^{Y^k}(t)\left[\mu_i^{Y^k}(0)\left(v_i^{Y^k}(0)\right)^{-1} + \left(\sigma^{Y^k}\right)^{-2}\int_0^t \frac{dY^k(u)}{Y^k(u)}\right], \\ v_i^{Y^k}(t) &= \left[\left(v_i^{Y^k}(0)\right)^{-1} + \left(\sigma^{Y^k}\right)^{-2} t\right]^{-1}. \end{aligned} \tag{2.5}$$

Define the data generating process which is conditional on the information available to investor i as $w_i^k(t)$, $k \in \{H, F\}$. For investor i, each production process is viewed as

$$dY^k(t) = Y^k(t)\left[\mu_i^{Y^k}(t)\,dt + \sigma^{Y^k} dw_i^k(t)\right]$$

By rearranging the terms, we get:

$$\begin{aligned} dw_i^k(t) &= \frac{1}{\sigma^{Y^k}}\left[\frac{dY^k(t)}{Y^k(t)} - \mu_i^{Y^k}(t)\,dt\right] \\ &= \frac{1}{\sigma^{Y^k}}\left[\mu^{Y^k} - \mu_i^{Y^k}(t)\right]dt + dw^k(t) \end{aligned}$$

Therefore, the data generating process which is conditional on the information available to investor i is

$$d\mathbf{w}_i(t) = \begin{pmatrix} \frac{1}{\sigma^{Y^H}}\left[\mu^{Y^H} - \mu_i^{Y^H}(t)\right]dt \\ \frac{1}{\sigma^{Y^F}}\left[\mu^{Y^F} - \mu_i^{Y^F}(t)\right]dt \\ 0 \end{pmatrix} + d\mathbf{w}(t). \tag{2.6}$$

2.2.3 Investment Opportunities

There are two financial securities in each country: a risky stock and a locally risk-free bond. Denote the prices of the Home stock, Foreign stock, Home bond, and Foreign bond by S^H, S^F, B^H, and B^F, respectively. All of them are defined in units of the numeraire good, i.e., the Home goods. The returns and volatilities of investable assets under the investor i's beliefs are:

$$\begin{aligned} dB^H(t) &= B^H(t)\,\mu_i^{B^H}(t)\,dt, && (2.7) \\ d\left[p(t)\,B^F(t)\right] &= p(t)\,B^F(t)\left[\mu_i^{pB^F}(t)\,dt + \boldsymbol{\sigma}_i^{pB^F}(t)^T d\mathbf{w}_i(t)\right], \\ dS^H(t) + Y^H(t)\,dt &= S^H\left[\mu_i^{S^H}(t)\,dt + \boldsymbol{\sigma}_i^{S^H}(t)^T d\mathbf{w}_i(t)\right], \\ dS^F(t) + p(t)\,Y^F(t)\,dt &= S^F\left[\mu_i^{S^F}(t)\,dt + \boldsymbol{\sigma}_i^{S^F}(t)^T d\mathbf{w}_i(t)\right] \end{aligned}$$

where $\mu_i^{B^H}(t)$ and $\mu_i^{B^F}(t)$ are the rates of returns in bond markets, $\mu_i^{S^H}(t)$ and $\mu_i^{S^F}(t)$ are perceived asset returns in equity markets, and $\boldsymbol{\sigma}_i^{S^H}(t)$ and $\boldsymbol{\sigma}_i^{S^F}(t)$ are volatility of asset returns. All variables for prices, returns, and volatilities are to be determined at the equilibrium. The diffusion parameter of the Foreign bond returns is not necessarily zero since the Foreign bonds are not riskfree in units of the Home goods. They are riskfree only in terms of the Foreign goods. Because there are four securities and three sources of uncertainty, the market is potentially dynamically complete.

Since the Home and Foreign investors agree to the asset prices, no arbitrage condition holds and implies

$$\begin{aligned}
\boldsymbol{\sigma}_H^{pB^F}(t) &= \boldsymbol{\sigma}_F^{pB^F}(t) = \boldsymbol{\sigma}^p(t) \\
\boldsymbol{\sigma}_H^{S^H}(t) &= \boldsymbol{\sigma}_F^{S^H}(t) = \boldsymbol{\sigma}^{S^H}(t) \\
\boldsymbol{\sigma}_H^{S^F}(t) &= \boldsymbol{\sigma}_F^{S^F}(t) = \boldsymbol{\sigma}^{S^F}(t) \\
\mu_H^{B^H}(t) &= \mu_F^{B^H}(t) = r(t) \\
\overrightarrow{\boldsymbol{\mu}}_F(t) - \overrightarrow{\boldsymbol{\mu}}_H(t) &= \boldsymbol{\sigma}(t)^T \begin{pmatrix} \frac{\mu_F^{Y^H}(t)-\mu_H^{Y^H}(t)}{\sigma^{Y^H}} \\ \frac{\mu_F^{Y^F}(t)-\mu_H^{Y^F}(t)}{\sigma^{Y^F}} \\ 0 \end{pmatrix} \qquad (2.8)
\end{aligned}$$

where $\boldsymbol{\mu}_i(t) = \begin{pmatrix}\mu_i^{S^H}(t) & \mu_i^{S^F}(t) & \mu_i^{pB^F}(t)\end{pmatrix}^T$ is a vector of i's perceived asset returns, and $\boldsymbol{\sigma}(t) \equiv \begin{bmatrix}\boldsymbol{\sigma}^{S^H}(t) & \boldsymbol{\sigma}^{S^F}(t) & \boldsymbol{\sigma}^p(t)\end{bmatrix}$ is a volatility matrix for the investable asset returns. $r(t)$ can be interpreted as a riskfree rate. Investors agree to the asset volatilities, but agree to disagree to the perceived asset returns. The difference in the perceived asset returns is expressed in terms of the difference in the perceived production growth.

Consider the case where heterogeneous beliefs are asymmetry related to the center of perceived growth. Following the literature such as Gallmeyer and Hollifield (2008), I assume that local investors are more optimistic than cross-border investors, that is, $\mu_H^{Y^H}(0) > \mu_H^{Y^F}(0)$ and $\mu_F^{Y^F}(0) > \mu_H^{Y^F}(0)$. For simplicity, I also assume that the initial levels of precision are the same across investors, that is, $\upsilon_H^{Y^k}(0) = \upsilon_F^{Y^k}(0) = \upsilon^{Y^k}(0), k \in \{H, F\}$. The prior and posterior distributions are, for each production $k \in \{H, F\}$,

$$
\begin{aligned}
\mu^{Y^k}(0) &\sim N\left(\mu_i^{Y^k}(0), \upsilon^{Y^k}(0)\right), \\
\mu^{Y^k}(t) &\sim N\left(\mu_i^{Y^k}(t), \upsilon_i^{Y^k}(t)\right)
\end{aligned}
$$

where

$$
\begin{aligned}
\mu_i^{Y^k}(t) &= \upsilon^{Y^k}(t)\left[\mu_i^{Y^k}(0)\left(\upsilon^{Y^k}(0)\right)^{-1} + \left(\sigma^{Y^k}\right)^{-2}\int_0^t \frac{dY^k(u)}{Y^k(u)}\right], \\
\upsilon^{Y^k}(t) &= \left[\left(\upsilon^{Y^k}(0)\right)^{-1} + \left(\sigma^{Y^k}\right)^{-2} t\right]^{-1}.
\end{aligned}
$$

Given that the initial levels of precision are the same across investors, the difference in the levels of precision stay the same over time. The difference in perceived asset returns depends on the difference in the initial perceived output growth and is given by

$$
\overrightarrow{\boldsymbol{\mu}}_F(t) - \overrightarrow{\boldsymbol{\mu}}_H(t) = \boldsymbol{\sigma}(t)^T \begin{pmatrix} \frac{\sigma^{Y^H}}{\left(\sigma^{Y^H}\right)^2 + \upsilon^H(0)t}\left(\mu_F^{Y^H}(0) - \mu_H^{Y^H}(0)\right) \\ \frac{\sigma^{Y^F}}{\left(\sigma^{Y^F}\right)^2 + \upsilon^F(0)t}\left(\mu_F^{Y^F}(0) - \mu_H^{Y^F}(0)\right) \\ 0 \end{pmatrix} \tag{2.9}
$$

Now I describe the case where heterogeneity in beliefs is asymmetry related to the levels of precision. In particular, I assume that $\upsilon_H^{Y^k}(0) \neq \upsilon_F^{Y^k}(0), k \in \{H, F\}$, whereas the initial perceived output growth rates are the same: $\mu_H^{Y^H}(0) = \mu_F^{Y^H}(0) = \mu^{Y^H}(0)$ and $\mu_F^{Y^F}(0) = \mu_H^{Y^F}(0) = \mu^{Y^F}(0)$. As before, investors do not know other investor's belief nor the accuracy of her/his own beliefs. The posterior and prior distribution of investor i's beliefs are:

$$
\begin{aligned}
\mu^{Y^k}(0) &\sim N\left(\mu^{Y^k}(0), v_i^{Y^k}(0)\right), i \in \{H, F\}, k \in \{H, F\}, \\
\mu^{Y^k}(t) &\sim N\left(\mu_i^{Y^k}(t), v_i^{Y^k}(t)\right), i \in \{H, F\}, k \in \{H, F\}
\end{aligned}
$$

where

$$
\begin{aligned}
\mu_i^{Y^k}(t) &= v_i^{Y^k}(t)\left[\mu^{Y^k}(0)\left(v_i^{Y^k}(0)\right)^{-1} + \left(\sigma^{Y^k}\right)^{-2}\int_0^t \frac{dY^k(u)}{Y^k(u)}\right], \\
v_i^{Y^k}(t) &= \left[\left(v_i^{Y^k}(0)\right)^{-1} + \left(\sigma^{Y^k}\right)^{-2} t\right]^{-1}.
\end{aligned}
$$

Although the initial perceived output growth rates are the same, the perceived output growth rates can be different over time. The difference in perceived asset returns are given by

$$
\begin{aligned}
& \overrightarrow{\boldsymbol{\mu}}_F(t) - \overrightarrow{\boldsymbol{\mu}}_H(t) \\
= \; & \boldsymbol{\sigma}(t)^T \begin{pmatrix} \frac{\left(v_H^H(0) - v_F^H(0)\right)}{\left[\left(\sigma^{Y^H}\right)^2 + v_F^H(0)t\right]\left[\left(\sigma^{Y^H}\right)^2 + v_H^H(0)t\right]}\left[\mu^{Y^H}(0)\,t - \int_0^t \frac{dY^H(u)}{Y^H(u)}\right] \\ \frac{\left(v_H^F(0) - v_F^F(0)\right)}{\left[\left(\sigma^{Y^F}\right)^2 + v_F^F(0)t\right]\left[\left(\sigma^{Y^F}\right)^2 + v_H^F(0)t\right]}\left[\mu^{Y^F}(0)\,t - \int_0^t \frac{dY^F(u)}{Y^F(u)}\right] \\ 0 \end{pmatrix} \quad (2.10)
\end{aligned}
$$

2.2.4 Preferences

Each country is populated by a representative agent who has time-additively separable logarithmic preferences and they can be represented as follows:

$$
U_i(0) = E_{i,0}\left[\int_0^T e^{-\rho t}\delta_i(t)\left(a_i \ln c_i^H(t) + (1 - a_i)\ln c_i^F(t)\right)dt\right], i \in \{H, F\}. \quad (2.11)
$$

where $a_i \in (0, 1)$ measures the degree of home bias and satisfies $a_H + a_F = 1$. The assumption of consumption home bias is equivalent to $a_H > a_F$. $\delta_i(t)$ represents relative preference shocks, and satisfies $\delta_i(0) = 1$, $\frac{d\delta_i(t)}{\delta_i(t)} = \sigma^{\delta^i} dw^\gamma$ and $\sigma^{\delta^H} \neq \sigma^{\delta^F}$. Without a loss of generality, we can assume $\sigma^{\delta^H} > \sigma^{\delta^F}$.

The wealth process of agent i expressed in units of the Home good is

$$\begin{aligned} \frac{dW_i(t)}{W_i(t)} &= x_i^{S^H}(t)\left(\frac{dS^H(t) + p^H(t)\,Y^H(t)\,dt}{S^H(t)}\right) \\ &\quad + x_i^{S^F}(t)\left(\frac{dS^F(t) + p^F(t)\,Y^F(t)\,dt}{S^F(t)}\right) + \\ &\quad x_i^{B^H}(t)\left(\frac{dB^H(t)}{B^H(t)}\right) + x_i^{B^F}(t)\left(\frac{d\left[p(t)\,B^F(t)\right]}{p(t)\,B^F(t)}\right) \\ &\quad - \left(c_i^H(t) + p(t)\,c_i^F(t)\right)dt \end{aligned} \tag{2.12}$$

where $c_i^H(t)$ and $c_i^F(t)$ are investor i's consumption. At $t = 0$, each investor owns the entire stock in her/his own country (i.e., $W_H(0) = S^H(0)$, and $W_F(0) = S^F(0)$).

Matching the diffusion terms gives us the condition that the optimal portfolio satisfies

$$\mathbf{x}_i(t) = \begin{pmatrix} x_i^{S^H}(t) \\ x_i^{S^F}(t) \\ x_i^{B^F}(t) \end{pmatrix} = \boldsymbol{\sigma}(t)^{-1}\boldsymbol{\sigma}^{W_i}(t) \tag{2.13}$$

where $\mathbf{x}_i(t) = \begin{pmatrix} x_i^{S^H}(t) & x_i^{S^F}(t) & x_i^{B^F}(t) \end{pmatrix}^T$ is a vector of shares for i's wealth invested on each asset, and $\boldsymbol{\sigma}(t) = \begin{bmatrix} \boldsymbol{\sigma}^{S^H}(t) & \boldsymbol{\sigma}^{S^F}(t) & \boldsymbol{\sigma}^{p}(t) \end{bmatrix}$ is a volatility matrix of asset returns. This implies that the optimal portfolio can be derived in closed-form if the volatility of investors' wealth can be expressed in closed-form.

2.2.5 Equilibrium

Definition 1 *Given beliefs, preferences and initial endowments, a competitive equilibrium is a collection of adapted processes for asset prices, consumption* $\left(c_i^H(t), c_i^F(t)\right)$*, and asset allocations* $\left(x_i^{S^H}(t), x_i^{S^F}(t), x_i^{B^F}(t)\right)$*,* $i \in \{H, F\}$*, such that*

$\left(c_i^H(t), c_i^F(t), x_i^{S^H}(t), x_i^{S^F}(t), x_i^{B^F}(t)\right)$ *is a solution to agent* i*'s optimization problem, and all markets clear for all* $t \in [0, T]$*,*

$$\begin{aligned}
c_H^H(t) + c_F^H(t) &= Y^H(t) && \text{(Home good)} \\
c_H^F(t) + c_F^F(t) &= Y^F(t) && \text{(Foreign good)} \\
W_H(t)\, x_H^{S^H}(t) + W_F(t)\, x_F^{S^H} &= S^H(t) && \text{(Home stock)} \\
W_H(t)\, x_H^{S^F}(t) + W_F(t)\, x_F^{S^F}(t) &= S^F(t) && \text{(Foreign stock)} \\
W_H(t)\, x_H^{B^H}(t) + W_F(t)\, x_F^{B^H}(t) &= 0 && \text{(Home bond)} \\
W_H(t)\, x_H^{B^F}(t) + W_F(t)\, x_F^{B^F}(t) &= 0 && \text{(Foreign bond)}
\end{aligned} \tag{2.14}$$

Each agent's dynamic consumption-portfolio problem can be converted into the static problem using martingale techniques (Cox and Huang, 1989; Karatzas et al., 1987).

$$\max_{\left\{c_i^H, c_i^F\right\}} E_{i,0}\left[\int_0^T e^{-\rho t}\delta_i(t)\left(a_i \ln c_i^H(t) + (1-a_i)\ln c_i^F(t)\right)dt\right] \tag{2.15}$$
$$\text{subject to } E_0\left[\int_0^T \xi_i(t)\left(c_i^H(t) + p(t)\, c_i^F(t)\right)dt\right] \leqq W_i(0),$$

where ξ_i is the state price density process. ξ_i can be understood as the price faced by agent i of a security paying dt at time t, and it follows the dynamics $d\xi_i(t) = \xi_i(t)\left[-r(t)\,dt - \boldsymbol{\theta}_i(t)^T d\mathbf{w}_i(t)\right]$ with $\xi_i(0) = 1$. These state price density processes are investor-specific since they face heterogeneous equilibrium price dynamics. The market price of risk $\boldsymbol{\theta}_i(t)$ is defined as

$$\begin{aligned} \boldsymbol{\theta}_i(t) &= \left(\boldsymbol{\sigma}(t)^T\right)^{-1}\left(\overrightarrow{\boldsymbol{\mu}}_i(t) - r(t)\,\overrightarrow{\mathbf{1}}\right) \\ \text{where } \boldsymbol{\sigma}(t) &= \left[\boldsymbol{\sigma}^{S^H}(t) \quad \boldsymbol{\sigma}^{S^F}(t) \quad \boldsymbol{\sigma}^{p}(t)\right] \end{aligned} \tag{2.16}$$

where $\overrightarrow{\mathbf{1}} = (1 \quad 1 \quad 1)^T$ is a three-dimensional vector of ones. In Section 2.3, I find that the difference in the perceived asset returns is expressed in terms of the difference in the perceived production growth. The difference in the market price of risk can be expressed in terms of the difference in the perceived production growth.

$$\boldsymbol{\theta}_F(t) - \boldsymbol{\theta}_H(t) = \left(\boldsymbol{\sigma}(t)^T\right)^{-1}\left(\overrightarrow{\boldsymbol{\mu}}_F(t) - \overrightarrow{\boldsymbol{\mu}}_H(t)\right) \tag{2.17}$$

The necessary and sufficient conditions for optimal consumption are

$$\begin{aligned} e^{-\rho t}\delta_i(t)\,a_i c_i^H(t) &= y_i \xi_i(t) \\ e^{-\rho t}\delta_i(t)(1-a_i)\,c_i^F(t) &= y_i \xi_i(t)\,p(t) \end{aligned} \tag{2.18}$$

where y_i is the Lagrange multiplier such that the budget constraint holds with equality at the optimum.

Since financial markets are potentially dynamically complete, the equilibrium prices of the financial securities are identical to those in an aggregated representative agent economy where the representative agent has the following aggregate intratemporal utility function:

$$\begin{aligned} &U\left(Y^H(t), Y^F(t)\right) = \delta_H(t)\left(a_H \ln c_H^H(t) + (1-a_H)\ln c_H^F(t)\right) \\ &+\lambda(t)\,\delta_F(t)\left(a_F \ln c_F^H(t) + (1-a_F)\ln c_F^F(t)\right) \\ &\text{ subject to } c_H^H(t) + c_F^H(t) = Y^H(t), \text{ and } c_H^F(t) + c_F^F(t) = Y^F(t). \end{aligned} \tag{2.19}$$

The stochastic weighting process $\lambda(t)$ represents the relative welfare weight of the Foreign agent and is identified as

$$\lambda(t) = \frac{y_H}{y_F} \frac{\xi_H(t)}{\xi_F(t)}. \tag{2.20}$$

Ito's Lemma gives us the drift and diffusion terms of the stochastic weighting process:

$$\begin{aligned} \boldsymbol{\sigma}^{\lambda}(t) &= \boldsymbol{\theta}_F(t) - \boldsymbol{\theta}_H(t) = \left(\boldsymbol{\sigma}(t)^T\right)^{-1} \left(\overrightarrow{\boldsymbol{\mu}}_F(t) - \overrightarrow{\boldsymbol{\mu}}_H(t)\right) \\ \boldsymbol{\mu}^{\lambda}(t) &= \boldsymbol{\sigma}^{\lambda}(t) \cdot \boldsymbol{\theta}_F(t) \end{aligned} \tag{2.21}$$

The equilibrium is characterized as follows. The agents' optimal consumption allocation rules are

$$\begin{aligned} c_H^H(t) &= \frac{\delta_H(t) a_H}{\delta_H(t) a_H + \delta_F(t) a_F \lambda(t)} Y^H(t), \\ c_F^H(t) &= \frac{\delta_F(t) a_F \lambda(t)}{\delta_H(t) a_H + \delta_F(t) a_F \lambda(t)} Y^H(t), \\ c_H^F(t) &= \frac{\delta_H(t)(1-a_H)}{\delta_H(t)(1-a_H) + \delta_F(t)(1-a_F)\lambda(t)} Y^F(t), \\ c_F^F(t) &= \frac{\delta_F(t)(1-a_F)\lambda(t)}{\delta_H(t)(1-a_H) + \delta_F(t)(1-a_F)\lambda(t)} Y^F(t). \end{aligned} \tag{2.22}$$

The equilibrium investor-specific state price density is given by

$$\begin{aligned} \xi_H(t) &= \frac{e^{-\rho t}}{y_H} \frac{\delta_H(t) a_H + \lambda(t)\delta_F(t) a_F}{Y^H(t)}, \\ \xi_F(t) &= \frac{e^{-\rho t}}{y_F} \frac{\delta_H(t) a_H + \lambda(t)\delta_F(t) a_F}{\lambda(t) Y^H(t)}. \end{aligned} \tag{2.23}$$

The terms of trade and volatility are given by

$$\begin{aligned} p(t) &= \frac{\delta_H(t)(1-a_H) + \lambda(t)\delta_F(t)(1-a_F)}{\delta_H(t) a_H + \lambda(t)\delta_F(t) a_F} \frac{Y^H(t)}{Y^F(t)} \\ \boldsymbol{\sigma}^p(t) &= A(t)\left(\boldsymbol{\sigma}^{\delta_H} - \boldsymbol{\sigma}^{\delta_F} - \boldsymbol{\sigma}^{\lambda}(t)\right) + \boldsymbol{\sigma}^{Y^H} - \boldsymbol{\sigma}^{Y^F} \end{aligned} \tag{2.24}$$

where $A(t) = -\frac{\lambda(t)\delta_H(t)\delta_F(t)(a_H - a_F)}{[\delta_H(t)a_H + \delta_F(t)a_F\lambda(t)][\delta_H(t)(1-a_H)+\delta_F(t)(1-a_F)\lambda(t)]}$. (2.25)

This $A(t)$ is strictly negative if consumption home bias exists and strictly increasing with the degree of consumption home bias.

The market prices of risks are given by

$$\begin{aligned}
\boldsymbol{\theta}_H(t) &= \boldsymbol{\sigma}^{Y^H} - \frac{\delta_H(t)a_H}{\delta_H(t)a_H + \lambda(t)\delta_F(t)a_F}\boldsymbol{\sigma}^{\delta_H} \\
&\quad - \frac{\lambda(t)\delta_F(t)a_F}{\delta_H(t)a_H + \lambda(t)\delta_F(t)a_F}\left(\boldsymbol{\sigma}^{\delta_F} + \boldsymbol{\sigma}^{\lambda}(t)\right) \\
\boldsymbol{\theta}_F(t) &= \boldsymbol{\sigma}^{Y^H} - \frac{a_H\delta_H(t)}{\delta_H(t)a_H + \lambda(t)\delta_F(t)a_F}\left(\boldsymbol{\sigma}^{\delta_H} - \boldsymbol{\sigma}^{\lambda}(t)\right) \\
&\quad - \frac{\lambda(t)\delta_F(t)a_F}{\delta_H(t)a_H + \lambda(t)\delta_F(t)a_F}\boldsymbol{\sigma}^{\delta_F}
\end{aligned} \tag{2.26}$$

The formal derivation of the market prices of risks are given in the appendix.

Finally, the stock prices and dynamics are

$$\begin{aligned}
S^H(t) &= E_{H,t}\left[\int_t^T \frac{\xi_H(s)}{\xi_H(t)}Y^H(s)\,ds\right] \\
&= Y^H(t)\frac{1-e^{-\rho(T-t)}}{\rho} \\
S^F(t) &= E_{H,t}\left[\int_t^T \frac{\xi_H(s)}{\xi_H(t)}p(s)Y^F(s)\,ds\right] \\
&= p(t)Y^F(t)\frac{1-e^{-\rho(T-t)}}{\rho} \\
\boldsymbol{\sigma}^{S^H}(t) &= \boldsymbol{\sigma}^{Y^H} \\
\boldsymbol{\sigma}^{S^F}(t) &= \boldsymbol{\sigma}^{Y^F} + \boldsymbol{\sigma}^{p}(t)
\end{aligned} \tag{2.27}$$

The formal derivation of the stock prices and volatilities are given in Appendix A.

2.3 Correlations, Portfolios, and Capital Flows

2.3.1 Correlations

Understanding correlations of asset returns is an essential component of portfolio selection process. The Modern Portfolio Theory (MPT) tells us that investors can reduce their exposure to risk by holding a combination of weakly correlated assets. When two stock returns are uncorrelated, for instance, agents can reduce the volatility of portfolio returns by investing in both stocks. On the other hand, when two stock returns are highly and positively correlated, it becomes more difficult to diversify their portfolio risks. Furthermore, the extent of stock market synchronization has an important policy implication since it indicates to what degree external shocks could be transmitted into the domestic capital market. Before implementing new government policies (e.g. lowering foreign income tax), one should assess how such policy changes affect vulnerability of the domestic capital market to external shocks.

This subsection considers the covariances and correlations between the Home and Foreign stock returns. The covariance between two stock returns is given by

$$\sigma_{S^H S^F}(t) = \sigma_{Y^H Y^F} + \sigma_{Y^H p}(t) \tag{2.28}$$

The first term represents the fundamental comovements; and the second term represents the excess comovements through the terms of trade.

The correlation between two stock returns is given by

$$\rho_{S^H S^F}(t) = \frac{\sigma_{S^H S^F}(t)}{\left\|\boldsymbol{\sigma}_{S^H}(t)\right\| \left\|\boldsymbol{\sigma}_{S^F}(t)\right\|} \tag{2.29}$$

$$= \frac{\boldsymbol{\sigma}_{Y_H} \cdot \left(-A(t)\,\boldsymbol{\sigma}^{\lambda}(t) + \boldsymbol{\sigma}_{Y^H}\right)}{\left\|\boldsymbol{\sigma}_{Y^H}\right\| \left\|A(t)\left(\boldsymbol{\sigma}^{\delta_H} - \boldsymbol{\sigma}^{\delta_F} - \boldsymbol{\sigma}^{\lambda}(t)\right) + \boldsymbol{\sigma}_{Y^H}\right\|}$$

The derivation of the correlation between two stock returns is given in Appendix A.

In equilibrium in a frictionless economy (i.e., when $\lambda(t)$ is deterministic) and no effective demand uncertainty ($\boldsymbol{\sigma}^{\delta_H} = \boldsymbol{\sigma}^{\delta_F}$), the two stock prices are perfectly correlated. For example, a positive output shock in the Home output increases the Home stock price. However, due to relative abundance of the Home goods, the terms of trade move against the Home output, which in turn move in favor of the Foreign output. Hence, the value of the Foreign goods rises, thereby providing a boost to the Foreign stock price. It can be viewed as shock transmission through the perfect terms-of-trade adjustment. This result is consistent with Zapatero (1995) and Pavlova and Rigobon (2007). Perfect correlation does not hold when the demand uncertainty is ineffective ($\boldsymbol{\sigma}^{\delta_H} \neq \boldsymbol{\sigma}^{\delta_F}$) or when any heterogeneity in beliefs is present (i.e., when $\lambda(t)$ is stochastic).

The derivative of the correlations between two stock returns $\rho_{S^H S^F}$ with respect to the degree of consumption home bias a at $t = 0$ is

$$\frac{d\rho_{S^H S^F}(0)}{da} < 0, \tag{2.30}$$

which implies that stock market correlations decrease with the degree of consumption home bias. The detailed derivation of this is given in Appendix A.

The derivative of the correlations between two stock returns $\rho_{S^H S^F}$ with respect to the difference in the initial levels of perceived output growth τ at $t = 0$ is

$$\frac{d\rho_{S^H S^F}(0)}{d\tau} < 0, \tag{2.31}$$

which again implies that stock market correlations decrease with the differences in the initial levels of perceived output growth rates.

The following proposition summarizes the findings from this subsection.

Proposition 2 *1. Stock market correlations decrease in the degree of consumption home bias. 2. Stock market correlations decrease in the differences in initial perceived output growth. 3. Perfect correlation between two stock returns hold when there is no information friction ($\boldsymbol{\sigma}^{\lambda}(t) = 0$) and no effective demand uncertainty ($\boldsymbol{\sigma}^{\delta_H} = \boldsymbol{\sigma}^{\delta_F}$).*

2.3.2 Portfolio Choice

This subsection derives agents' optimal portfolio in closed-form under the assumption of time-additively separable logarithmic preferences. Stock market correlations discussed in the previous subsection can be also explained by the portfolio rebalancing channel. A positive supply shock in the Home country increases wealth of investors who own the Home stock. This wealth expansion increases the demand of the Foreign stock, which results in the increase in the Foreign stock price.

The diffusion terms of the dynamic budget constraint implies

$$\boldsymbol{\sigma}^{W_H}(t) = \boldsymbol{\sigma}(t)\,\mathbf{x}_H(t) \tag{2.32}$$

where $\mathbf{x}_H(t) = \begin{bmatrix} x_H^{S^H}(t) & x_H^{S^F}(t) & x_H^{B^F}(t) \end{bmatrix}^T$ and
$\boldsymbol{\sigma}(t) = \begin{bmatrix} \boldsymbol{\sigma}^{S^H}(t) & \boldsymbol{\sigma}^{S^F}(t) & \boldsymbol{\sigma}^{p}(t) \end{bmatrix}$.

The wealth dynamics can be derived by applying Ito's Lemma to the wealth equation derived earlier.

$$\begin{aligned}
\boldsymbol{\sigma}^{W_H}(t) &= \boldsymbol{\sigma}^{Y^H} + \frac{\lambda(t)\,\delta_F(t)\,a_F}{\delta_H(t)\,a_H + \lambda(t)\,\delta_F(t)\,a_F}\left(\boldsymbol{\sigma}^{\delta_H} - \boldsymbol{\sigma}^{\delta_F} - \boldsymbol{\sigma}^{\lambda}(t)\right) \quad (2.33)\\
\boldsymbol{\sigma}^{W_F}(t) &= \boldsymbol{\sigma}^{Y^H} - \frac{a_H\delta_H(t)}{\delta_H(t)\,a_H + \delta_F(t)\,a_F\lambda(t)}\left(\boldsymbol{\sigma}^{\delta_H} - \boldsymbol{\sigma}^{\delta_F} - \boldsymbol{\sigma}^{\lambda}(t)\right)
\end{aligned}$$

The optimal portfolio is given by matching the right hand sides of equations 2.32 and 2.33

$$\begin{aligned}
\mathbf{x}_H(t) &= \boldsymbol{\sigma}(t)^{-1}\boldsymbol{\sigma}^{Y^H} \quad (2.34)\\
&+\frac{\lambda(t)\,\delta_F(t)\,a_F}{\delta_H(t)\,a_H + \lambda(t)\,\delta_F(t)\,a_F}\boldsymbol{\sigma}(t)^{-1}\left(\boldsymbol{\sigma}^{\delta_H} - \boldsymbol{\sigma}^{\delta_F}\right) \quad (2.35)\\
&-\frac{\lambda(t)\,\delta_F(t)\,a_F}{\delta_H(t)\,a_H + \lambda(t)\,\delta_F(t)\,a_F}\boldsymbol{\sigma}(t)^{-1}\left(\overrightarrow{\boldsymbol{\mu}}_F(t) - \overrightarrow{\boldsymbol{\mu}}_H(t)\right)
\end{aligned}$$

$$\begin{aligned}
\mathbf{x}_F(t) &= \boldsymbol{\sigma}(t)^{-1}\boldsymbol{\sigma}^{Y^H}(t) \quad (2.36)\\
&-\frac{a_H\delta_H(t)}{\delta_H(t)\,a_H + \delta_F(t)\,a_F\lambda(t)}\boldsymbol{\sigma}(t)^{-1}\left(\boldsymbol{\sigma}^{\delta_H} - \boldsymbol{\sigma}^{\delta_F}\right) \quad (2.37)\\
&+\frac{a_H\delta_H(t)}{\delta_H(t)\,a_H + \delta_F(t)\,a_F\lambda(t)}\boldsymbol{\sigma}(t)^{-1}\left(\overrightarrow{\boldsymbol{\mu}}_F(t) - \overrightarrow{\boldsymbol{\mu}}_H(t)\right)
\end{aligned}$$

Since the first two terms also appear in optimal portfolios from the frictionless case, the third term is a component of optimal portfolios generated by heterogeneity in beliefs. Define these additional terms as follows:

$$\begin{aligned}
\widetilde{\mathbf{x}}_H(t) &= -\frac{\lambda(t)\,\delta_F(t)\,a_F}{\delta_H(t)\,a_H + \lambda(t)\,\delta_F(t)\,a_F}\boldsymbol{\sigma}(t)^{-1}\left(\overrightarrow{\boldsymbol{\mu}}_F(t) - \overrightarrow{\boldsymbol{\mu}}_H(t)\right) \quad (2.38)\\
\widetilde{\mathbf{x}}_F(t) &= \frac{a_H\delta_H(t)}{\delta_H(t)\,a_H + \delta_F(t)\,a_F\lambda(t)}\boldsymbol{\sigma}(t)^{-1}\left(\overrightarrow{\boldsymbol{\mu}}_F(t) - \overrightarrow{\boldsymbol{\mu}}_H(t)\right)
\end{aligned}$$

It is implied that the deviation from the optimal portfolios in a frictionless economy is linear in the deference between the excepted asset returns across agents.

Asymmetry related to the center of perceived output growth

Now I assume that the initial levels of precision are the same: $v_H^{Y^k}(0) = v_F^{Y^k}(0) = v^{Y^k}(0), k \in \{H, F\}$. Following the literature such as Gallmeyer and Hollifield (2008), we also assume that local investors are more optimistic than cross-border investors, that is, $\mu_H^{Y^H}(0) > \mu_H^{Y^F}(0)$ and $\mu_F^{Y^F}(0) > \mu_H^{Y^F}(0)$. The posterior and prior distribution of investor i's beliefs are:

$$\begin{aligned} \mu^{Y^k} &\sim N\left(\mu_i^{Y^k}(0), v^{Y^k}(0)\right), i \in \{H, F\}, k \in \{H, F\} \qquad (2.39) \\ \mu^{Y^k} &\sim N\left(\mu_i^{Y^k}(t), v^{Y^k}(t)\right), i \in \{H, F\}, k \in \{H, F\} \end{aligned}$$

where

$$\begin{aligned} \mu_i^{Y^k}(t) &= v^{Y^k}(t)\left[\frac{\mu_i^{Y^k}(0)}{v^{Y^k}(0)} + \frac{\int_0^t \frac{dY^k(u)}{Y^k(u)}}{\left(\sigma^{Y^k}\right)^2}\right], \qquad (2.40) \\ v^{Y^k}(t) &= \frac{v_i^{Y^k}(0)\left(\sigma^{Y^k}\right)^2}{v_i^{Y^k}(0)\, t + \left(\sigma^{Y^k}\right)^2}. \end{aligned}$$

Given that the initial levels of precision are the same, the levels of precision stay the same over time. The difference in perceived assets returns are given by

$$\overrightarrow{\boldsymbol{\mu}}_F(t) - \overrightarrow{\boldsymbol{\mu}}_H(t) = \boldsymbol{\sigma}(t)^T \begin{pmatrix} \frac{\sigma^{Y^H}}{\left(\sigma^{Y^H}\right)^2 + v^H(0)t}\left(\mu_F^{Y^H}(0) - \mu_H^{Y^H}(0)\right) \\ \frac{\sigma^{Y^F}}{\left(\sigma^{Y^F}\right)^2 + v^F(0)t}\left(\mu_F^{Y^F}(0) - \mu_H^{Y^F}(0)\right) \\ 0 \end{pmatrix} \qquad (2.41)$$

The deviation term of each optimal portfolio is given by

$$\widetilde{\mathbf{x}}_H(t) = -\frac{\lambda(t)\,\delta_F(t)\,a_F}{\delta_H(t)\,a_H + \lambda(t)\,\delta_F(t)\,a_F} \times \tag{2.42}$$

$$\begin{pmatrix} \frac{\sigma^{Y^H}}{\left(\sigma^{Y^H}\right)^2 + v^H(0)t}\left(\mu_F^{Y^H}(0) - \mu_H^{Y^H}(0)\right) \\ \frac{\sigma^{Y^F}}{\left(\sigma^{Y^F}\right)^2 + v^F(0)t}\left(\mu_F^{Y^F}(0) - \mu_H^{Y^F}(0)\right) \\ 0 \end{pmatrix} \tag{2.43}$$

$$\widetilde{\mathbf{x}}_F(t) = \frac{a_H\delta_H(t)}{\delta_H(t)\,a_H + \delta_F(t)\,a_F\lambda(t)} \times$$

$$\begin{pmatrix} \frac{\sigma^{Y^H}}{\left(\sigma^{Y^H}\right)^2 + v^H(0)t}\left(\mu_F^{Y^H}(0) - \mu_H^{Y^H}(0)\right) \\ \frac{\sigma^{Y^F}}{\left(\sigma^{Y^F}\right)^2 + v^F(0)t}\left(\mu_F^{Y^F}(0) - \mu_H^{Y^F}(0)\right) \\ 0 \end{pmatrix} \tag{2.44}$$

The deviation from the benchmark case is characterized in closed-form, in particular, in terms of the differences in the initial perceived output growth rates. It implies that as agents are more optimistic about the domestic output and more pessimistic about the foreign output, the degree of portfolio home bias increases. The following proposition summarize the findings from this subsection.

Proposition 3 *1. The degree of portfolio home bias decreases with the difference in the initial perceived rates of output growth. 2. The degree of portfolio home bias decreases as t increases, i.e., as agents learn about the outputs.*

Asymmetry related to the levels of precision

Now I describe the situation where the initial levels of precision are different: $v_H^{Y^k}(0) \neq v_F^{Y^k}(0), k \in \{H, F\}$, whereas the initial perceived output growth rates are the same: $\mu_H^{Y^H}(0) = \mu_H^{Y^F}(0) = \mu^{Y^H}(0)$ and $\mu_F^{Y^F}(0) = \mu_H^{Y^F}(0) = \mu^{Y^F}(0)$. As before, investors do not know other investor's belief nor the accuracy of her/his own beliefs.

The posterior and prior distribution of investor i's beliefs are:

$$\begin{aligned}
\mu^{Y^k} &\sim N\left(\mu^{Y^k}(0), v_i^{Y^k}(0)\right), i \in \{H,F\}, k \in \{H,F\} \\
\mu^{Y^k} &\sim N\left(\mu_i^{Y^k}(t), v_i^{Y^k}(t)\right), i \in \{H,F\}, k \in \{H,F\}
\end{aligned}$$

where

$$\begin{aligned}
\mu_i^{Y^k}(t) &= v_i^{Y^k}(t)\left[\frac{\mu^{Y^k}(0)}{v_i^{Y^k}(0)} + \frac{\int_0^t \frac{dY^k(u)}{Y^k(u)}}{\left(\sigma^{Y^k}\right)^2}\right], \\
v_i^{Y^k}(t) &= \frac{v_i^{Y^k}(0)\left(\sigma^{Y^k}\right)^2}{v_i^{Y^k}(0)\,t + \left(\sigma^{Y^k}\right)^2}.
\end{aligned}$$

Although the initial perceived output growth rates are the same, the perceived output growth rates can be different over time. The levels of precision are different initially and over time.

The deviation term from the benchmark portfolios are given by

$$\begin{aligned}
\widetilde{\mathbf{x}}_H(t) &= -\frac{\lambda(t)\,\delta_F(t)\,a_F}{\delta_H(t)\,a_H + \lambda(t)\,\delta_F(t)\,a_F} \times \\
&\left(\begin{array}{c}
\frac{\sigma^{Y^H}\left(v_H^H(0)-v_F^H(0)\right)}{\left[\left(\sigma^{Y^H}\right)^2+v_F^H(0)t\right]\left[\left(\sigma^{Y^H}\right)^2+v_H^H(0)t\right]}\left[\mu^{Y^H}(0)\,t - \int_0^t \frac{dY^H(u)}{Y^H(u)}\right] \\
\frac{\sigma^{Y^F}\left(v_H^F(0)-v_F^F(0)\right)}{\left[\left(\sigma^{Y^F}\right)^2+v_F^F(0)t\right]\left[\left(\sigma^{Y^F}\right)^2+v_H^F(0)t\right]}\left[\mu^{Y^F}(0)\,t - \int_0^t \frac{dY^F(u)}{Y^F(u)}\right] \\
0
\end{array}\right) \quad (2.45) \\
\widetilde{\mathbf{x}}_F(t) &= \frac{a_H\delta_H(t)}{\delta_H(t)\,a_H + \delta_F(t)\,a_F\lambda(t)} \times \\
&\left(\begin{array}{c}
\frac{\sigma^{Y^H}\left(v_H^H(0)-v_F^H(0)\right)}{\left[\left(\sigma^{Y^H}\right)^2+v_F^H(0)t\right]\left[\left(\sigma^{Y^H}\right)^2+v_H^H(0)t\right]}\left[\mu^{Y^H}(0)\,t - \int_0^t \frac{dY^H(u)}{Y^H(u)}\right] \\
\frac{\sigma^{Y^F}\left(v_H^F(0)-v_F^F(0)\right)}{\left[\left(\sigma^{Y^F}\right)^2+v_F^F(0)t\right]\left[\left(\sigma^{Y^F}\right)^2+v_H^F(0)t\right]}\left[\mu^{Y^F}(0)\,t - \int_0^t \frac{dY^F(u)}{Y^F(u)}\right] \\
0
\end{array}\right) \quad (2.46)
\end{aligned}$$

Further assume that $v_H^H(0) = v_F^F(0) = 0$, and $v_H^F(0) = v_F^H(0) = v(0)$, then the deviations become

$$\widetilde{\mathbf{x}}_H(t) = -\frac{\lambda(t)\,\delta_F(t)\,a_F}{\delta_H(t)\,a_H + \lambda(t)\,\delta_F(t)\,a_F}\begin{pmatrix} \frac{v(0)w^H(t)}{\left(\sigma^{YH}\right)^2 + v(0)t} \\ \frac{-v(0)w^F(s)}{\left(\sigma^{YF}\right)^2 + v(0)t} \\ 0 \end{pmatrix} \tag{2.47}$$

$$\widetilde{\mathbf{x}}_F(t) = \frac{a_H\delta_H(t)}{\delta_H(t)\,a_H + \delta_F(t)\,a_F\lambda(t)}\begin{pmatrix} \frac{v(0)w^H(t)}{\left(\sigma^{YH}\right)^2 + v(0)t} \\ \frac{-v(0)w^F(t)}{\left(\sigma^{YF}\right)^2 + v(0)t} \\ 0 \end{pmatrix} \tag{2.48}$$

The deviation from the benchmark case now depends on the levels and differences in the initial levels of precision. It implies that portfolio home bias is generated by agents' over-confidence about the domestic output and under-confidence about the foreign output. If cross-border investors have the lower level of precision on expectation of output growth than local investors, then the optimal portfolio structure fluctuates around the frictionless case. This may increase or decrease the stock return correlations.

The deviation of optimal portfolio depends on the current realization of output shocks. For example, in the case of a positive Home shock, the Foreign investor increases the share of the Home stock, while the Home investor decreases it. This is because the Foreign investor knows less about the Home production than the Home investor, so that after observing this positive shock, she/he adjusts portfolio structure by updating her/his beliefs, while the Home investor keeps the same portfolio structure. The optimal portfolio approaches to the benchmark portfolio as time passes by as investors learn about the output growth.

2.3.3 Portfolio Rebalancing

After the realization of demand and supply shocks, investors adjust their asset allocation. In the benchmark case where there is no heterogeneity in beliefs, the Home investor's optimal share of the Foreign equity holding is

$$x_{H0}^{S^F}(t) = \frac{a_F \delta_H(t)(1-a_H) + a_F \delta_F(t)(1-a_F)\overline{\lambda}}{(a_H - a_F)\delta_H(t)}. \tag{2.49}$$

Ito's Lemma gives

$$dx_{H0}^{S^F}(t) = [\text{Ito term}]\, dt - \frac{a_F \delta_F(t)(1-a_F)\overline{\lambda}}{(a_H - a_F)\delta_H(t)}(\sigma_{\delta_H} - \sigma_{\delta_F})\, dw^{\delta}(t) \tag{2.50}$$

This tell us in the economy with no demand uncertainty and homogeneous beliefs, optimal portfolios are deterministic as the second term becomes zero.

Since net capital flows can be defined as changes in the amount of wealth invested aboard:

$$\begin{aligned} CF_{HF0}(t) &= d\left[W_H(t)\, x_{H0}^{S^F}(t)\right] \\ &= \underbrace{x_{H0}^{S^F}(t)\, dW_H(t)}_{\substack{\text{capital flows due to} \\ \text{the change in wealth}}} + \underbrace{W_H(t)\, dx_{H0}^{S^F}(t)}_{\substack{\text{capital flows due to} \\ \text{the change in optimal portfolio}}}. \end{aligned} \tag{2.51}$$

The volatility of capital flows is solely due to the volatility of wealth generated from demand shock realization.

In the economy with no heterogeneity in beliefs but with demand uncertainty, optimal portfolio follows a stochastic path. Now the volatility of capital flows comes from both the volatility of wealth and the volatility of optimal portfolio generated from demand shocks.

In the case where investors differ in the initial perceived output growth, volatility of the Home investor's optimal share of assets invested abroad is

$$
\begin{aligned}
dx_H^{S^F}(t) &= [Ito]\,dt \qquad (2.52) \\
&+\frac{a_F(1-a_F)\lambda(t)\delta_F(t)}{(a_H-a_F)\delta_H(t)}(\sigma_{\delta_H}-\sigma_{\delta_F})\,dw^{\delta}(t) \\
&+\frac{\lambda(t)\delta_H(t)\delta_F(t)a_Ha_F}{[\delta_H(t)a_H+\lambda(t)\delta_F(t)a_F]^2}\frac{\sigma^{Y^F}}{\left(\sigma^{Y^F}\right)^2+v^F(0)t}\times \\
&\left(\mu_F^{Y^F}(0)-\mu_H^{Y^F}(0)\right)(\sigma_{\delta_H}-\sigma_{\delta_F})\,dw^{\delta}(t) \\
&-\frac{\lambda(t)\delta_H(t)\delta_F(t)a_Ha_F}{[\delta_H(t)a_H+\lambda(t)\delta_F(t)a_F]^2}\frac{\sigma^{Y^F}}{\left(\sigma^{Y^F}\right)^2+v^F(0)t}\times \\
&\left(\mu_F^{Y^F}(0)-\mu_H^{Y^F}(0)\right)\begin{pmatrix} \frac{\sigma^{Y^H}}{\left(\sigma^{Y^H}\right)^2+v^H(0)t}\left(\mu_F^{Y^H}(0)-\mu_H^{Y^H}(0)\right) \\ \frac{\sigma^{Y^F}}{\left(\sigma^{Y^F}\right)^2+v^F(0)t}\left(\mu_F^{Y^F}(0)-\mu_H^{Y^F}(0)\right) \\ 0 \end{pmatrix}^T d\mathbf{w}(t)
\end{aligned}
$$

It can be shown that the volatility of optimal portfolio is higher than that in the frictionless case. The second line is the diffusion term which comes from demand uncertainty. This part is equal to the diffusion in the homogeneous beliefs case. The third line is also the diffusion related to demand shocks, and this is generated from the initial heterogeneity in beliefs. The effect of this term gets weaker at t increases. Finally, the fourth line contains the diffusion term related to supply shocks, and this is generated from the initial heterogeneity in beliefs. Since the direction of uncertainty in this term is independent of that in the second and the third lines, the volatility of optimal portfolio is higher than that in the frictionless case.

Capital flows from H to F is given

$$CF_{HF}(t) = d\left[W_H(t)\, x_H^{S^F}(t)\right] = \underbrace{x_H^{S^F}(t)\, dW_H(t)}_{\substack{\text{capital flows due to} \\ \text{the change in wealth}}} + \underbrace{W_H(t)\, dx_H^{S^F}(t)}_{\substack{\text{capital flows due to} \\ \text{the change in optimal portfolio}}} \tag{2.53}$$

Now the volatility of capital flows can be decomposed into three parts: capital flows due to the change in wealth, capital flows due to the change in optimal portfolio as a result of demand shock realization, and capital flows due to the change in optimal portfolio as a result of supply shock realization.

In the case where investors differ in the levels of precision of initial beliefs, volatility of the Home investor's optimal share of assets invested abroad is given

$$\begin{aligned} dx_H^{S^F}(t) = {} & [Ito]\, dt \\ & + \frac{a_F (1-a_F)\,\lambda(t)\,\delta_F(t)}{(a_H - a_F)\,\delta_H(t)} \left(\sigma_{\delta_H} - \sigma_{\delta_F}\right) dw^{\delta}(t) \\ & + \frac{\lambda(t)\,\delta_H(t)\,\delta_F(t)\, a_H a_F}{\left[\delta_H(t)\, a_H + \lambda(t)\,\delta_F(t)\, a_F\right]^2} \frac{v(0)\, w^F(s)}{\left(\sigma^{Y^F}\right)^2 + v(0)\, t} \left(\sigma_{\delta_H} - \sigma_{\delta_F}\right) dw^{\delta}(t) \\ & - \frac{\lambda(t)\,\delta_H(t)\,\delta_F(t)\, a_H a_F}{\left[\delta_H(t)\, a_H + \lambda(t)\,\delta_F(t)\, a_F\right]^2} \frac{v(0)\, w^F(s)}{\left(\sigma^{Y^F}\right)^2 + v(0)\, t} \begin{pmatrix} \frac{v(0) w^H(t)}{\left(\sigma^{Y^H}\right)^2 + v(0)t} \\ \frac{-v(0) w^F(s)}{\left(\sigma^{Y^F}\right)^2 + v(0)t} \\ 0 \end{pmatrix}^T d\mathbf{w}(t). \end{aligned} \tag{2.54}$$

The interpretation of this equation is similar to the one for equation 2.52.

2.4 Equity Trade Costs

Section 2.4 considers equity trade costs as an alternative source of market frictions and examines how the change in the size of market frictions affect stock market correlations. The market friction I consider is a proportional equity trade cost. Agent in each country incurs a proportional cost $\tau \in (0,1)$ on the dividends received from abroad. The equity trade costs in this model essentially act as a withholding tax, which can be interpreted metaphorically as a reduced form for informational costs, transaction costs, differential taxation, etc. This withholding tax story is realistic because, under the current US tax code, dividend income earned by foreign corporation is subject to a 30 percent withholding tax. This withholding tax is not required on dividends paid to domestic shareholders. The tax withheld from a payment may be recovered by a refund claim, depending upon a double taxation treaty. For simplicity, a withholding tax is redistributed in a lump-sum fashion so that there is no cost associated with this tax system. Coeurdacier and Guibaud (2009) study the effect of a proportional withholding tax on stock prices in a one-good, two-asset setting. The rest of the model setting is as described in Section 2. No arbitrage condition implies that the difference between investor-specific perceived asset returns is linear in the proportional equity trade costs:

$$\vec{\boldsymbol{\mu}}_F(t) - \vec{\boldsymbol{\mu}}_H(t) = \tau \begin{pmatrix} -\frac{Y^H(t)}{S^H(t)} \\ \frac{Y^F(t)}{S^F(t)} \\ 0 \end{pmatrix} \tag{2.55}$$

Thus, the following findings hold: 1. Stock market correlations decrease with the equity trade costs. 2. Stock market correlations decrease in the degree of consumption home bias. Assuming the logarithmic preferences, the deviation term of the optimal portfolio from the frictionless case is given by

$$\widetilde{\mathbf{x}}_H(t) = -\tau \frac{\lambda(t)\,\delta_F(t)\,a_F}{\delta_H(t)\,a_H + \lambda(t)\,\delta_F(t)\,a_F} \begin{pmatrix} -\frac{Y^H(t)}{S^H(t)} \\ \frac{Y^F(t)}{S^F(t)} \\ 0 \end{pmatrix}, \tag{2.56}$$

which implies that the degree of home bias increases with the equity trade costs. Unlike heterogeneous beliefs, it does not disappear even if t increases, and this portfolio home bias stays over time.

2.5 Conclusions

This paper analyzes the effects of heterogeneous beliefs and learning on stock return comovements and portfolio rebalancing mechanism. The paper develops a continuous-time general equilibrium model in a two-country, two-asset, two-good setting, and in the presence of heterogeneous beliefs on output growth. The equilibrium correlations of stock market returns and optimal portfolios are expressed in terms of the differences in beliefs. The main findings are: 1. Stock market correlations decrease with the differences in perceived output growth; and 2. Volatilities of optimal portfolios and capital flows increase with the differences in perceived output growth and with the differences in levels of precision of beliefs.

There are two factors that affects the degree of stock return comovements: demand uncertainty together with consumption home bias; and the presence of heterogeneity in beliefs.

In a frictionless economy with no demand uncertainty, stock prices across markets perfectly comove each other under logarithmic preferences (Zapatero, 1995; Pavlova and Rigobon, 2007). Suppose that there are two countries, Home and Foreign, in an economy, and each specializes in producing its own goods. A positive supply shock in the Home country increases the Home stock price. However, due to relative abundance of the Home goods, the terms of trade deteriorate the Home currency, which in turn move in favor of the Foreign currency. Hence, the value of the Foreign goods rise, thereby providing a boost to the Foreign stock price. It can be viewed as shock transmission through perfect terms-of-trade adjustment. Perfect comovements of two stock prices can be also explained by the portfolio rebalancing channel. A positive supply shock in the Home country increases wealth of investors who own the Home stock. This wealth expansion increases the demand of the Foreign stock, which results in the increase in the Foreign stock price. Any deviation from the logarithmic preferences lowers the degree of comovements because income effects and substitution effects no longer cancel each other.

Why do stock market correlations decrease in the degree of consumption home bias? With demand uncertainty and consumption home bias, stock price correlation is not equal to one. The Home supply shocks still impact the Foreign stock market through the terms of trade adjustment but not perfectly as more shocks are absorbed domestically. Demand uncertainty together with consumption home bias generates portfolio home bias. When a positive supply shock hits the Home production, investors' wealth of stock holder of the Home stock in the Home country increases more than those in the Foreign country. This generates relative "wealth transfer" from the Foreign country to the Home country, and therefore the demand for the Home stock rises more than that for the Foreign stock, thereby providing a surge in the Home stock price.

Why do stock market correlations decrease in the size of market friction? The way financial friction impacts the mechanism of the stock comovements is somewhat similar to the demand uncertainty. Financial friction generates portfolio home bias, which prevents the terms of trade from adjusting to the changes in relative outputs. It also leads to different portfolio rebalancing behavior across markets, resulting in the lower stock price correlation. Any deviation from the logarithmic preferences even lowers the correlation.

In addition, heterogeneous beliefs in the model allows us to analyze investors' portfolio rebalancing behavior. Under the situation where the Foreign investors are less informed about the Home investor, a supply shock in the Home country have direct wealth effect and indirect portfolio rebalancing effect, and these effects are in the opposite directions. Wealth effects work just like in a frictionless case but have more impact on the Home investor due to the portfolio home bias, while the Foreign investor rebalances their portfolio to a higher extent than the Home investor. Since the Foreign investors know less about the Home production, they learn relatively more by observing the realized output and therefore rebalance more. This helps explaining why cross-border investments tend to be taken out from the local markets during the crises and why domestic investments are more stable.

CHAPTER 3

TRADE COSTS IN GOODS MARKET AND INTERNATIONAL COMOVEMENTS OF EQUITY MARKETS

This chapter analyzes the effects of trade costs in goods market on international comovements of equity markets and those on equity home bias. The chapter develops a continuous-time general equilibrium model in a two-country, two-asset, and two-good setting where international trade of goods is costly. I solve for the optimal portfolios and the equilibrium correlations of cross-country equity returns and analyze how they change depending on the size of trade costs, the coefficient of risk aversion, and the elasticity of substitution between domestic and foreign goods. It is found that the cross-country equity return correlations decrease with the size of trade costs. This result is robust to different sizes of trade costs and asymmetry related to potential growth and consumer preferences. It is also found that the size of the trade costs and other parameter values determine whether trade costs would generate equity home bias or foreign bias.

3.1 Introduction

Trade costs in goods market are not a negligible market friction as pointed out by Anderson and van Wincoop (2004): international trade costs such as transportation costs and tariffs are as high as 74 percent of production costs. Roughly speaking, since trade costs in goods market affect good prices and consumers behavior, they should also affect equity prices and their dynamics. However, there is a broad consensus such that the effects of trade costs on financial variables would depend on the size of trade costs, the elasticity of substitution between goods, and the coefficient of risk aversion. For instance, trade costs generate equity *home* bias if the elasticity of substitution between goods is set to an inverse of the coefficient of risk aversion, while equity *foreign* bias is generated if the former exceeds the latter and if the size of trade costs is less than a certain threshold level. In spite of these mixed results regarding the effects of trade costs, my finding about the joint dynamics of equity returns is monotonically decreasing with the size of trade costs for plausible sets of parameter values.

This chapter fits in two streams of the literature. First, it can be interpreted as a continuous-time / dynamic extension to the literature that studies the effects of trade costs on optimal portfolios. There are several important papers that study the effects of trade costs to explain the equity home bias puzzle: people hold a disproportionate share of domestic assets. Obstfeld and Rogoff (2000, 2007) use a simple two-period, two-country, two-good model with iceberg-type trade costs and find that trade costs in goods markets can explain the equity home bias puzzle. On the other hand, using a two-period model with plausible sets of parameter values, Coeurdacier (2009) argues that introducing trade costs in goods markets is not sufficient to explain this equity home bias puzzle. This paper analyzes the similar topic in a more general set-up in the sense that the model employs a dynamic continuous-time setting. Trade costs considered in the model are of iceberg-type and exogenous, following the above papers. In a broader sense, this chapter can be also seen as an extension to the standard international portfolio choice literature such as Lucas (1982), Dumas (1992), Uppal (1993), Zapatero (1995) and Kollmann (2006) in that it considers a wide range of parameter choices for elasticity of substitution between goods and coefficient of risk aversion.

The second stream of literature is the international asset pricing models that study the effect of market frictions on dynamics of asset returns and correlations. Basak and Gallmeyer (2003) study differential dividend taxation in a standard international asset pricing model and find that the stock return volatility is increased. Coeurdacier and Guibaud (2008) use a similar continuous-time model with trade costs in equity markets and show that the stock return correlations decrease with the size of trade costs. Pavlova and Rigobon (2007) introduce demand shocks into an otherwise standard model. They find that the stock return correlations are reduced after introduction of effective demand uncertainty. Luo and Visaltanachoti (2010) extend the model of Pavlova and Rigobon (2007) and analyze the role of non-tradable. Chapter 2 of this thesis also extends Pavlova and Rigobon (2007) with an addition of heterogeneous beliefs and learning.

Methodologically, this chapter is related to Devereux and Sutherland (2007) and Tille and van Wincoop (2010), which provide methods to solve for international equity portfolios. They take approximations around the non-stochastic steady-state, whereas Coeurdacier and Guibaud (2008) takes around the size of market friction. I will take his technique and apply it to my model by taking approximations around the case with no trade cost.

The rest of the paper is organized as follows: In Section 3.2, I present a continuous-time international asset pricing model with a proportional trade costs in goods market. In Section 3.3, the equilibrium terms of trade, asset return correlations, and optimal portfolios are characterized. Section 3.4 contains simulation results and interpretation of them. Section 3.5 gives some sensitivity analyses. Finally, Section 3.6 contains the discussion and concludes.

3.2 The Model

The paper considers a continuous-time Lucas-type pure-exchange economy with finite horizon T. The economy consists of two countries: Home and Foreign. Home variables are denoted with H, and Foreign variables are denoted with F. The uncertainty in this economy is represented by a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, P)$, on which is defined a standard two-dimensional Brownian motion $\mathbf{w}(t) = \left(w^H(t), w^F(t)\right)^T$. The Brownian motions $w^H(t)$ and $w^F(t)$ capture country-specific production risk in H and F, respectively.

3.2.1 Production

Each country specializes in producing one tradable good. The amount of goods produced in country $i \in \{H, F\}$ follows a geometric Brownian process:

$$\begin{aligned} dY^H(t) &= Y^H(t)\left[\mu^{Y^H} dt + \sigma^{Y^H} dw^H(t)\right], \text{ and} \\ dY^F(t) &= Y^F(t)\left[\mu^{Y^F} dt + \sigma^{Y^F} dw^F(t)\right], \end{aligned} \tag{3.1}$$

where μ^{Y^H} and μ^{Y^F} denote rates of output growth, and σ^{Y^H} and σ^{Y^F} denote diffusion parameters of the output growth. These four parameters are constant and exogenous. The terms of trade are defined as the price of Foreign goods relative to the price of the Home goods. By normalizing the price of Home goods ($p^H(t) = 1$), the terms of trade can be written as $p(t) \equiv p^F(t)/p^H(t)$, and follows a geometric Brownian process:

$$dp(t) = p(t)\left[\mu^p(t)\, dt + \boldsymbol{\sigma}^p(t)^T d\mathbf{w}(t)\right] \tag{3.2}$$

where $\mu^p(t)$ and $\boldsymbol{\sigma}^p(t)$ are the drift and diffusion terms of the process and determined at the equilibrium.

3.2.2 Investment Opportunities

There are three financial securities available in this economy: Home equity, Foreign equity, and risk-free bond. Each equity is a claim on the future endowment of the country. The prices of Home equity, Foreign equity, and bond are denoted by S^H, S^F, and B, respectively, and all of them are defined in units of the numeraire goods, i.e., Home goods. The dynamics of asset returns are:

$$\begin{aligned} dS^H(t) + Y^H(t)\,dt &= S^H\left[\mu^{S^H}(t)\,dt + \boldsymbol{\sigma}^{S^H}(t)^T d\mathbf{w}(t)\right] && \text{(Home equity)} \\ dS^F(t) + p(t)\,Y^F(t)\,dt &= S^F\left[\mu^{S^F}(t)\,dt + \boldsymbol{\sigma}^{S^F}(t)^T d\mathbf{w}(t)\right] && \text{(Foreign equity)} \\ dB(t) &= B(t)\,\mu^B(t)\,dt && \text{(Risk-free bond)} \end{aligned} \tag{3.3}$$

where μ^{S^H} and μ^{S^F} are drift terms of asset returns in equity markets, $\boldsymbol{\sigma}^{S^H}$ and $\boldsymbol{\sigma}^{S^F}$ are diffusion terms of asset returns, and μ^B is a risk-free rate. All variables for prices, drifts, and diffusions are to be determined at the equilibrium. Since bonds are riskfree, there is no diffusion associated with the bond price. Because there are three securities and two sources of uncertainty in a continuous-time dynamic economy, the market is potentially dynamically complete.

3.2.3 Trade Costs

I assume that goods can be shipped from one country to the other, but this shipment is subject to exogenous iceberg-type costs of trade: a fraction $\frac{1}{1+\tau}$ of a unit of good shipped abroad arrives at its destination. So the price of Home goods faced by Foreign agent is $1+\tau$, and the price of Foreign goods faced by Home agent is $(1+\tau)\,p$.

3.2.4 Preferences

Each country is populated by a representative agent who has the standard relative risk aversion preferences as follows:

$$U_i(0) = E_0 \left[\int_0^T e^{-\rho t} \left(\frac{C_i(t)^{1-\gamma}}{1-\gamma} \right) dt \right], i \in \{H, F\} \tag{3.4}$$

where γ denotes coefficient of relative risk aversion, and C_i is the aggregate consumption index for an agent i. When $\gamma = 1$, the utility function becomes $E_0 \left[\int_0^T e^{-\rho t} \ln C_i(t)\, dt \right]$.

Each representative agent consume both goods, and her/his aggregate consumption index C_i is:

$$C_i(t) = \left[(a)^{\frac{1}{\psi}} c_i^H(t)^{\frac{\psi-1}{\psi}} + (1-a)^{\frac{1}{\psi}} c_i^F(t)^{\frac{\psi-1}{\psi}} \right]^{\frac{\psi}{\psi-1}} \tag{3.5}$$

where c_i^H is the total consumption of Home goods by a representative agent i, c_i^F is the total consumption of Foreign goods by a representative agent i, $a \in (0,1)$ represents the degree of preference toward Home goods, and the parameter ψ is the elasticity of (intratemporal) substitution between Home and Foreign goods. I will assume for now that ψ is constant and strictly greater than one. Two goods are perfect substitutes when ψ approaches infinity and perfect complements when $\psi = 0$. When $\psi = 1$, this aggregate consumption index C_i becomes $\left(c_i^H\right)^a \left(c_i^F\right)^{1-a}$.

The dynamic budget constraint is given by:

$$\begin{aligned}
&\frac{dW_H(t)}{W_H(t)} = x_H^{S^H}(t)\left(\frac{dS^H(t)+Y^H(t)dt}{S^H(t)}\right) + x_H^{S^F}(t)\left(\frac{dS^F(t)+p(t)Y^F(t)dt}{S^F(t)}\right) \\
&+\left[1 - x_H^{S^H}(t) - x_H^{S^F}(t)\right]\mu^B(t)\,dt - \left[c_H^H(t) + (1+\tau)\,p(t)\,c_H^F(t)\right]dt \\
&\frac{dW_F(t)}{W_F(t)} = x_F^{S^H}(t)\left(\frac{dS^H(t)+Y^H(t)dt}{S^H(t)}\right) + x_F^{S^F}(t)\left(\frac{dS^F(t)+p(t)Y^F(t)dt}{S^F(t)}\right) \\
&+\left[1 - x_F^{S^H}(t) - x_F^{S^F}(t)\right]\mu^B(t)\,dt - \left[(1+\tau)\,c_F^H(t) + p(t)\,c_F^F(t)\right]dt
\end{aligned} \tag{3.6}$$

where W_i is the size of i's wealth in units of Home goods, and x_i^j is a share of wealth invested on a security j by an agent i. I assume that at $t = 0$, each agent owns the entire stock in her/his own country (i.e., $W_H(0) = S^H(0)$, and $W_F(0) = S^F(0)$).

By matching the diffusion terms of the dynamic budget constraint, the optimal portfolio for each agent is given:

$$\mathbf{x}_i(t) = \boldsymbol{\sigma}(t)^{-1}\,\boldsymbol{\sigma}^{W_i}(t) \tag{3.7}$$

where $\mathbf{x}_i(t) = \begin{bmatrix} x_i^{S^H}(t) & x_i^{S^F}(t) \end{bmatrix}^T$ is a vector of shares for i's wealth invested on equities, $\boldsymbol{\sigma}(t) = \begin{bmatrix} \boldsymbol{\sigma}^{S^H}(t) & \boldsymbol{\sigma}^{S^F}(t) \end{bmatrix}$ is a volatility matrix of asset returns, and $\boldsymbol{\sigma}^{W_i}$ is the diffusion of i's wealth.

3.2.5 Equilibrium

The definition of an equilibrium is given as follows:

Definition 4 *Given preferences and initial endowments, a competitive equilibrium is a collection of adapted processes for asset prices, consumption* $\left(c_i^H(t), c_i^F(t)\right)$*, and asset allocations* $\left(x_i^{S^H}(t), x_i^{S^F}(t)\right)$*,* $i \in \{H, F\}$*, such that* $\left(c_i^H(t), c_i^F(t), x_i^{S^H}(t), x_i^{S^F}(t)\right)$ *is a solution to agent i's optimization problem, and all markets clear for all* $t \in [0, T]$*.*

The market clearing conditions for goods and assets are as follows

$$
\begin{aligned}
c_H^H(t) + (1+\tau)\, c_F^H(t) &= Y^H(t) && \text{(Home good)} \\
(1+\tau)\, c_H^F(t) + c_F^F(t) &= Y^F(t) && \text{(Foreign good)} \\
W_H(t)\, x_H^{S^H}(t) + W_F(t)\, x_F^{S^H} &= S^H(t) && \text{(Home stock)} \\
W_H(t)\, x_H^{S^F}(t) + W_F(t)\, x_F^{S^F}(t) &= S^F(t) && \text{(Foreign stock)} \\
W_H(t)\left(1 - x_H^{S^H}(t) - x_H^{S^F}(t)\right) + W_F(t)\left(1 - x_F^{S^H} - x_F^{S^F}(t)\right) &= 0 && \text{(Bond)}
\end{aligned}
\tag{3.8}
$$

To solve for the equilibrium, the following three-step procedures are employed. First, the individual intratemporal utility maximization gives the intratemporal allocation of consumption expenditure across goods for each agent. Next, the individual dynamic utility maximization for each agent gives the intertemporal allocation of wealth for each agent. On the third step, the intratemporal allocation of consumption expenditure across agents is given by solving the intratemporal utility maximization of the world representative agent, appealing to a dynamically complete market setting.

Each representative agent maximizes the aforementioned dynamic utility function 3.4 by solving the following intratemporal utility maximization problem:

$$
\max_{c_i^H(t), c_i^F(t)} u_i(\kappa_i(t), p(t)) = \max_{c_i^H(t), c_i^F(t)} \frac{C_i(t)^{1-\gamma}}{1-\gamma} \tag{3.9}
$$

$$
\begin{aligned}
\text{subject to } \kappa_H(t) &= c_H^H(t) + (1+\tau)\, p(t)\, c_H^F(t) \text{ for Home agent, and} \\
\kappa_F(t) &= (1+\tau)\, c_F^H(t) + p(t)\, c_F^F(t) \text{ for Foreign agent.}
\end{aligned}
$$

where κ_i is the consumption expenditure for an agent i.

The first order conditions give the intratemporal allocation rules across goods:

$$\begin{aligned} c_H^H(t) &= \frac{a}{a+(1+\tau)^{1-\psi} p(t)^{1-\psi}(1-a)} \kappa_H(t) \qquad (3.10)\\ c_H^F(t) &= \frac{(1-a)(1+\tau)^{1-\psi} p(t)^{1-\psi}}{a+(1+\tau)^{1-\psi} p(t)^{1-\psi}(1-a)} \frac{1}{1+\tau} \frac{1}{p(t)} \kappa_H(t) \end{aligned}$$

for Home agent, and

$$\begin{aligned} c_F^H(t) &= \frac{a(1+\tau)^{1-\psi}}{a(1+\tau)^{1-\psi}+(1-a)p(t)^{1-\psi}} \frac{1}{1+\tau} \kappa_F(t) \qquad (3.11)\\ c_F^F(t) &= \frac{(1-a)p(t)^{1-\psi}}{a(1+\tau)^{1-\psi}+(1-a)p(t)^{1-\psi}} \frac{1}{p(t)} \kappa_F(t) \end{aligned}$$

for Foreign agent.

Now the indirect utility function can be expressed in terms of the consumption expenditure κ_i:

$$\begin{aligned} u_H^*(\kappa_H(t), p(t)) &= \left[a+(1-a)(1+\tau)^{1-\psi} p(t)^{1-\psi}\right]^{\frac{1-\gamma}{\psi-1}} \frac{\kappa_H(t)^{1-\gamma}}{1-\gamma} \quad \text{(Home agent)}\\ u_F^*(\kappa_F(t), p(t)) &= \left[a(1+\tau)^{1-\psi}+(1-a)p(t)^{1-\psi}\right]^{\frac{1-\gamma}{\psi-1}} \frac{\kappa_F(t)^{1-\gamma}}{1-\gamma} \quad \text{(Foreign agent)} \end{aligned} \qquad (3.12)$$

Using the martingale techniques (Cox and Huang, 1989; Karatzas et al., 1987), each agent's dynamic consumption-portfolio problem can be converted into the static problem:

$$\begin{aligned} &\max_{\{\kappa_i(t)\}_{t=0}^T} E_0\left[\int_0^T e^{-\rho t} u_i^*(\kappa_i(t), p(t))\, dt\right] \qquad (3.13)\\ &\text{subject to } E_0\left[\int_0^T \xi(t)\kappa_i(t)\, dt\right] \le W_i(0) \end{aligned}$$

where ξ is the state price density process. ξ can be understood as the price of a security paying dt at time t, and it follows the dynamics $d\xi(t) = \xi(t)\left[-\mu^B(t)\,dt - \boldsymbol{\theta}(t)^T d\mathbf{w}(t)\right]$ with $\xi(0)=1$, where $\boldsymbol{\theta}$ denotes the market price of risk which is defined as

$$\boldsymbol{\theta}(t) \equiv \left(\boldsymbol{\sigma}(t)^T\right)^{-1}\left(\vec{\boldsymbol{\mu}}(t) - \mu^B(t)\,\vec{\mathbf{1}}\right) \qquad (3.14)$$

where $\overrightarrow{\mathbf{1}}$ is a two-dimensional vector of ones.

The necessary and sufficient conditions for optimality of the consumption expenditure stream are:

$$\begin{aligned} \xi(t) &= \frac{e^{-\rho t}}{y_H}\left[a+(1-a)(1+\tau)^{1-\psi}p(t)^{1-\psi}\right]^{\frac{1-\gamma}{\psi-1}}\kappa_H(t)^{-\gamma} \\ \xi(t) &= \frac{e^{-\rho t}}{y_F}\left[a(1+\tau)^{1-\psi}+(1-a)p(t)^{1-\psi}\right]^{\frac{1-\gamma}{\psi-1}}\kappa_F(t)^{-\gamma} \end{aligned} \tag{3.15}$$

where $y_i > 0$ is the Lagrange multiplier such that the budget constraint holds with equality at the optimum.

Since financial markets are potentially dynamically complete, the equilibrium prices of the financial securities are identical to those in an aggregated representative agent economy where the representative agent has the following aggregate intratemporal utility function:

$$\begin{aligned} & U(\kappa_H(t), \kappa_F(t), p(t)) \\ = & \max_{\kappa_H(t), \kappa_F(t)} \left[u_H^*(\kappa_H(t), p(t)) + \lambda u_F^*(\kappa_F(t), p(t))\right] \\ & \text{subject to } \kappa_H(t) + \kappa_F(t) = Y^H(t) + p(t) Y^F(t). \end{aligned} \tag{3.16}$$

where the weight $\lambda > 0$ represents the relative welfare weight of Foreign agent and is identified as

$$\frac{\partial u_H^*(t)}{\partial \kappa_H(t)} = \lambda \frac{\partial u_F^*(t)}{\partial \kappa_F(t)} \tag{3.17}$$

In this economy, since all agents face the same state price density, λ is a constant.

The intratemporal allocation of resources is given by:

$$
\begin{aligned}
\kappa_H(t) &= h(p(t))\left[Y^H(t) + p(t)\,Y^F(t)\right] && (3.18)\\
\kappa_F(t) &= \left[1 - h(p(t))\right]\left[Y^H(t) + p(t)\,Y^F(t)\right]\\
\text{where } h(p(t)) &= \frac{\left[a + (1-a)(1+\tau)^{1-\psi}\,p(t)^{1-\psi}\right]^{\frac{1}{\psi-1}\frac{1-\gamma}{\gamma}}}{\left(\begin{array}{c}\left[a + (1-a)(1+\tau)^{1-\psi}\,p(t)^{1-\psi}\right]^{\frac{1}{\psi-1}\frac{1-\gamma}{\gamma}}\\ +\lambda^{\frac{1}{\gamma}}\left[a(1+\tau)^{1-\psi} + (1-a)\,p(t)^{1-\psi}\right]^{\frac{1}{\psi-1}\frac{1-\gamma}{\gamma}}\end{array}\right)}
\end{aligned}
$$

This $h(p(t))$ can be interpreted as the wealth weight for the Home agent relative to the Foreign agent.

3.3 Characterization of the Equilibrium

In this section, I give some description of the equilibrium in the neighborhood of the zero-cost case. Section 3.3.1 gives analytical results for the consumption allocation rules. Section 3.3.2 examines analytically the terms of trade and dynamics. And finally, Section 3.3.3 gives asset price dynamics and optimal portfolios.

3.3.1 Consumption

The following proposition is given by combining 3.18 with 3.10.

Proposition 5 *The equilibrium consumption allocations are*

$$
\begin{aligned}
c_H^H(t) &= \overbrace{\frac{a}{a+(1-a)(1+\tau)^{1-\psi}p(t)^{1-\psi}}}^{\textit{Home agent's allocation across goods}} \overbrace{\overbrace{h(p(t))}^{\textit{allocation across countries}} \overbrace{\left[Y^H(t)+p(t)Y^F(t)\right]}^{\textit{total output}}}^{\textit{Home agent's consumption expenditure}}, \\
c_H^F(t) &= \tfrac{(1-a)(1+\tau)^{-\psi}p(t)^{-\psi}}{a+(1-a)(1+\tau)^{1-\psi}p(t)^{1-\psi}} h(p(t))\left[Y^H(t)+p(t)Y^F(t)\right], \\
c_F^H(t) &= \tfrac{a(1+\tau)^{-\psi}}{a(1+\tau)^{1-\psi}+(1-a)p(t)^{1-\psi}}\left[1-h(p(t))\right]\left[Y^H(t)+p(t)Y^F(t)\right], \\
c_F^F(t) &= \tfrac{(1-a)p(t)^{-\psi}}{a(1+\tau)^{1-\psi}+(1-a)p(t)^{1-\psi}}\left[1-h(p(t))\right]\left[Y^H(t)+p(t)Y^F(t)\right].
\end{aligned}
\tag{3.19}
$$

Each of these allocation rules has three components. The first component represents how each agent allocates her/his consumption expenditure across goods; the second is for resource allocation across countries; and the third component corresponds to the total output. When $\tau > 0$, the amounts of consumption of imported goods, namely c_H^F and c_F^H, are reduced, resulting in the consumption home bias.

3.3.2 Terms of Trade

Following Coeurdacier and Guibaud (2008), I take approximations in the size of trade costs around the no-cost case. The terms of trade p can be derived from the market clearing condition for goods (3.8) and the equilibrium consumption allocations (3.19). The detailed derivation of the terms of trade is given in the Appendix B.

Proposition 6 *To the first order, the terms of trade p is given by:*

$$
p(t) = \left(1+\tau\frac{\psi-1}{\psi}\frac{1-\lambda^{\frac{1}{\gamma}}}{1+\lambda^{\frac{1}{\gamma}}}\right)\left(\frac{1-a}{a}\frac{Y^H(t)}{Y^F(t)}\right)^{\frac{1}{\psi}} + o(\tau). \tag{3.20}
$$

When there is no cost associated with international trade of goods ($\tau = 0$), the terms of trade reflect the relative output ratio regardless of the relative wealth size λ or the risk aversion coefficient γ. If Y^H increases relative to Y^F, the terms of trade p, i.e., the relative prices of Foreign goods goes up because the Home goods become more abundant than the Foreign goods. With a greater ψ, the terms of trade reflects the change in the relative output ratio to a less extent because agents will adjust their consumption bundles more easily in response to observed shocks.

The first-order effect of trade costs depends on the values of ψ, λ, and γ. At the symmetric equilibrium, there is no first-order effect of τ on the terms of trade as $\lambda = 1$. There is also no first-order effect on the terms of trade if the elasticity of substitution between goods is one ($\psi = 1$). Otherwise, τ affects p, but the direction of the effects varies. In the case where the elasticity of substitution between goods is greater than unity (that is, $\psi > 1$, where goods are more substitutable than $\psi = 1$) and the weight is more on the Home agent ($\lambda < 1$), the effect of τ on p is positive, which means that the relative price of Foreign goods increases with the size of trade costs. On the other hand, where the weight is more on the Foreign agent ($\lambda > 1$), the effect of τ on p is positive, which means that the relative price of Foreign goods decreases with the size of trade costs. That is, τ increases the price of goods imported from a smaller country to a larger country, while τ decreases the price of goods imported from a larger country to a smaller country. From a viewpoint of a larger country, imports become more expensive with τ, while they become less expensive for a smaller country.

The intuition is the following: Consider the case where Home is greater than Foreign in wealth ($\lambda < 1$) and two goods are more substitutable than unity ($\psi > 1$). When trade costs are reduced, both countries start to consume more of imported goods. But due to the difference in size, an increase in Home agent's consumption of Foreign goods is greater than that in Foreign agent's consumption of Home goods. This asymmetric reaction pushes up the price of Foreign goods relative to Home goods. On the other hand, when two goods are less substitutable than unity, then the increased wealth due to reduced trade costs will be spent more on domestic goods and less on imported goods. Since the Home country is larger in size, the impact of the change in her/his behavior exceeds that for the Foreign agent. Therefore, the price of Foreign goods relative to Home goods down.

Next, I consider the second-order approximation of the terms of trade as in the following proposition.

Proposition 7 *Second-order approximation for p is:*

$$\begin{aligned} p(t) &= \left(1+\tau\frac{\psi-1}{\psi}\frac{1-\lambda^{\frac{1}{\gamma}}}{1+\lambda^{\frac{1}{\gamma}}}\right)\left(\frac{1-a}{a}\frac{Y^H(t)}{Y^F(t)}\right)^{\frac{1}{\psi}} \\ &+\frac{\tau^2}{2}(\psi-1)\frac{1-\lambda^{\frac{1}{\gamma}}}{1+\lambda^{\frac{1}{\gamma}}}\left[1-\frac{1-\lambda^{\frac{1}{\gamma}}}{1+\lambda^{\frac{1}{\gamma}}}\left(\frac{\psi-1}{\psi}\right)^2\right]\left(\frac{1-a}{a}\frac{Y^H(t)}{Y^F(t)}\right)^{\frac{1}{\psi}} \\ &+\tau^2\left(\frac{1}{\gamma}-\psi\right)\left(\frac{\psi-1}{\psi}\frac{1-\lambda^{\frac{1}{\gamma}}}{1+\lambda^{\frac{1}{\gamma}}}\right)^2\frac{\left(\frac{1-a}{a}\right)^{\frac{2}{\psi}}\left(\frac{Y^H(t)}{Y^F(t)}\right)^{\frac{2-\psi}{\psi}}}{1+\left(\frac{1-a}{a}\right)^{\frac{1}{\psi}}\left(\frac{Y^H(t)}{Y^F(t)}\right)^{\frac{1-\psi}{\psi}}} \\ &+\tau^2\left(\frac{1}{\gamma}-\psi\right)\frac{\psi-1}{\psi}\left[\begin{array}{c}\frac{1}{1+\lambda^{\frac{1}{\gamma}}}\frac{\left(\frac{1-a}{a}\right)^{\frac{2}{\psi}}\left(\frac{Y^H(t)}{Y^F(t)}\right)^{\frac{2-\psi}{\psi}}}{1+\left(\frac{1-a}{a}\right)^{\frac{1}{\psi}}\left(\frac{Y^H(t)}{Y^F(t)}\right)^{\frac{1-\psi}{\psi}}} \\ -\frac{\lambda^{\frac{1}{\gamma}}}{1+\lambda^{\frac{1}{\gamma}}}\frac{\left(\frac{1-a}{a}\right)^{\frac{1}{\psi}}\left(\frac{Y^H(t)}{Y^F(t)}\right)^{\frac{1}{\psi}}}{1+\left(\frac{1-a}{a}\right)^{\frac{1}{\psi}}\left(\frac{Y^H(t)}{Y^F(t)}\right)^{\frac{1-\psi}{\psi}}}\end{array}\right] \\ &+o\left(\tau^2\right). \end{aligned} \tag{3.21}$$

The first line is the same as the first-order approximation for p. The signs and impacts of the second-order effects of trade costs (lines 2, 3 and 4) are time-variant and depend on the current relative output ratio as well as the values of ψ, λ, and

γ. With $\psi = 1$, there is no effect of τ on p just like in the first-order effect. When $\lambda = 1$, the second and third lines become zero, while the fourth line remains. When $\frac{1}{\gamma} = \psi$, the expression of p becomes a simple reflection of the relative output ratio. Finally, at the symmetric equilibrium, the terms of trade are reduced to the one in the no-cost case.

Proposition 8 *First- and second-order approximation of terms of trade diffusion coefficient* $\boldsymbol{\sigma}^p(t)$ *are:*

$$\boldsymbol{\sigma}^p(t) = \frac{\boldsymbol{\sigma}^{Y^H} - \boldsymbol{\sigma}^{Y^F}}{\psi} + o(\tau) \tag{3.22}$$

$$\boldsymbol{\sigma}^p(t) = f(t)\, p(t)^{-1} \left(\frac{1-a}{a} \frac{Y^H(t)}{Y^F(t)} \right)^{\frac{1}{\psi}} \frac{\boldsymbol{\sigma}^{Y^H} - \boldsymbol{\sigma}^{Y^F}}{\psi} + o\left(\tau^2\right) \tag{3.23}$$

$$\text{where } f(t) = \begin{pmatrix} 1 + \tau \frac{\psi-1}{\psi} \frac{1-\lambda^{\frac{1}{\gamma}}}{1+\lambda^{\frac{1}{\gamma}}} \\ -\frac{\tau^2}{2}(\psi-1) \frac{1-\lambda^{\frac{1}{\gamma}}}{1+\lambda^{\frac{1}{\gamma}}} + \frac{\tau^2}{2}(\psi-1)\left(\frac{\psi-1}{\psi} \frac{1-\lambda^{\frac{1}{\gamma}}}{1+\lambda^{\frac{1}{\gamma}}} \right)^2 \\ -\tau^2 \left(\frac{1}{\gamma} - \psi\right)\left(\frac{\psi-1}{\psi}\right)^2 \frac{2-\psi+p_0(t)b(t)^{-1}}{\left[1+p_0(t)b(t)^{-1}\right]^2} p_0(t)\, b(t)^{-1} \left(\frac{1-\lambda^{\frac{1}{\gamma}}}{1+\lambda^{\frac{1}{\gamma}}} \right)^2 \\ +\tau^2 \left(\frac{1}{\gamma} - \psi\right) \frac{\psi-1}{\psi} \begin{bmatrix} \frac{\lambda^{\frac{1}{\gamma}}}{1+\lambda^{\frac{1}{\gamma}}} \frac{1+\psi p_0(t)b(t)^{-1}}{\left[1+p_0(t)b(t)^{-1}\right]^2} \\ -\frac{1}{1+\lambda^{\frac{1}{\gamma}}} \frac{2-\psi+p_0(t)b(t)^{-1}}{\left[1+p_0(t)b(t)^{-1}\right]^2} p_0(t)\, b(t)^{-1} \end{bmatrix} \end{pmatrix}$$

The diffusions of the terms of trade are time-invariant up to first order, while they depend on the relative output ratio in the second order. The second-order will be ineffective if $\psi = 1$. When $\lambda = 1$, the second and third lines of f disappear, while the fourth line remains. At the symmetric equilibrium, the diffusion becomes

$$\boldsymbol{\sigma}^p(t) = \left[1 + \tau^2 \frac{1}{4} \left(\frac{1}{\gamma} - \psi \right)(1+\psi) \frac{\psi - 1}{\psi} \right] \frac{\boldsymbol{\sigma}^{Y^H} - \boldsymbol{\sigma}^{Y^F}}{\psi} + o\left(\tau^2\right), \tag{3.24}$$

Given the terms of trade diffusion, the terms of trade volatilities are given by $\|\boldsymbol{\sigma}^p(t)\| = \left[\boldsymbol{\sigma}^p(t)^T \boldsymbol{\sigma}^p(t)\right]^{\frac{1}{2}}$. With $\psi = 1$, the volatility is $\|\boldsymbol{\sigma}^p(t)\| = \left\|\boldsymbol{\sigma}^{Y^H} - \boldsymbol{\sigma}^{Y^F}\right\|$, which means that the volatility of terms of trade purely comes from the volatilities of outputs, and there is no effect of trade costs whatsoever as $\frac{d\|\boldsymbol{\sigma}^p(t)\|}{d\tau} = 0$. When $\psi > 1$, τ affects the terms of trade volatility. For example, at the symmetric equilibrium, the terms of trade volatility becomes

$$\|\boldsymbol{\sigma}^p (t)\| = \left[1 - \frac{\tau^2}{4}\left(\psi - \frac{1}{\gamma}\right)(1+\psi)\frac{\psi - 1}{\psi}\right]\left\|\frac{\boldsymbol{\sigma}^{Y^H} - \boldsymbol{\sigma}^{Y^F}}{\psi}\right\|. \tag{3.25}$$

The effect of τ on the volatility is

$$\frac{d\|\boldsymbol{\sigma}^p (t)\|}{d\tau} = \frac{\tau}{2}\left(\frac{1}{\gamma} - \psi\right)(1+\psi)\frac{\psi - 1}{\psi}\left\|\frac{\boldsymbol{\sigma}^{Y^H} - \boldsymbol{\sigma}^{Y^F}}{\psi}\right\|. \tag{3.26}$$

This means that τ makes the terms of trade less volatile if $\psi > \frac{1}{\gamma}$, which is assumed widely in the related literature. The intuition is the following: With no trade costs, the terms of trade purely reflect the change in the relative output ratio. On the other hand, in present of trade costs, the terms of trade does not reflect the change in outputs as much as the benchmark no-cost case. This happens when consumers can easily substitute one type of goods to the other, and "easily" means $\psi > \frac{1}{\gamma}$.

3.3.3 Asset Returns, Wealth Processes, and Optimal Portfolios

The two equity prices $S^H$and S^F and their dynamics can be derived by the following lemmas. The derivations of these lemma are given in Appendix C.

Lemma 9 *Home and Foreign equity prices can be written as*

$$\begin{aligned} S^H(t) &= E_t\left[\int_t^T \frac{\xi(s)}{\xi(t)} Y^H(s)\, ds\right], \\ S^F(t) &= E_t\left[\int_t^T \frac{\xi(s)}{\xi(t)} p(s)\, Y^F(s)\, ds\right]. \end{aligned} \tag{3.27}$$

Lemma 10 *Home and Foreign equity prices diffusions are given*

$$\begin{aligned} \boldsymbol{\sigma}^{S^H}(t) &= \boldsymbol{\theta}(t) + \boldsymbol{\sigma}^{Y^H} + \frac{E_t\left[\int_t^T \xi(s)\, Y^H(s)\, \frac{D_t\xi(s)}{\xi(s)} ds\right]}{E_t\left[\int_t^T \xi(s)\, Y^H(s)\, ds\right]}, \\ \boldsymbol{\sigma}^{S^H}(t) &= \boldsymbol{\theta}(t) + \boldsymbol{\sigma}^{Y^F} + \frac{E_t\left[\int_t^T \xi(s)\, Y^F(s)\, p(s) \left(\frac{D_t\xi(s)}{\xi(s)} + \frac{D_t p(s)}{p(s)}\right) ds\right]}{E_t\left[\int_t^T \xi(s)\, Y^F(s)\, p(s)\, ds\right]}. \end{aligned} \tag{3.28}$$

Lemma 11 *The correlation of equity returns is*

$$\rho_{S^H S^F}(t) = \frac{\boldsymbol{\sigma}^{S^H S^F}(t)}{\left\|\boldsymbol{\sigma}^{S^H}(t)\right\| \left\|\boldsymbol{\sigma}^{S^F}(t)\right\|} \tag{3.29}$$

The equity prices are derived in a typical way: the sum of discounted future dividend flows. The diffusion coefficients of equity prices consist of two parts: one represents the current market price of risks and the diffusion of their production, while the other is the same for the future. Assuming that there is systematic asymmetry between the Home and Foreign, the diffusion term that comes from the future production would be the same for the Home and Foreign. Therefore, the degree of international comovements of stock prices depends on the degree of comovements between the current levels of Y^H and pY^F.

The two wealth processes W_H and W_F and their dynamics are characterized by the following lemmas. The derivations are given in Appendix C.

Lemma 12 *Home and Foreign wealth processes can be written as*

$$\begin{aligned} W_H(t) &= E_t\left[\int_t^T \frac{\xi(s)}{\xi(t)}\kappa_H(s)\,ds\right] \\ W_F(t) &= E_t\left[\int_t^T \frac{\xi(s)}{\xi(t)}\kappa_F(s)\,ds\right] \end{aligned} \tag{3.30}$$

Lemma 13 *Home and Foreign wealth process diffusions are given*

$$\begin{aligned} \boldsymbol{\sigma}^{W_H}(t) &= \boldsymbol{\theta}(t) + \frac{E_t\left[\int_t^T \xi(s)\,\kappa_H(s)\left(\frac{D_t\kappa_H(s)}{\kappa_H(s)} + \frac{D_t\xi(s)}{\xi(s)}\right)ds\right]}{E_t\left[\int_t^T \xi(s)\,\kappa_H(s)\,ds\right]} \\ \boldsymbol{\sigma}^{W_F}(t) &= \boldsymbol{\theta}(t) + \frac{E_t\left[\int_t^T \xi(s)\,\kappa_F(s)\left(\frac{D_t\kappa_F(s)}{\kappa_F(s)} + \frac{D_t\xi(s)}{\xi(s)}\right)ds\right]}{E_t\left[\int_t^T \xi(s)\,\kappa_F(s)\,ds\right]} \end{aligned} \tag{3.31}$$

Finally, the optimal portfolios are given as in 3.7

$$\begin{aligned} \mathbf{x}_H(t) &= \boldsymbol{\sigma}(t)^{-1}\boldsymbol{\sigma}^{W_H}(t), \\ \mathbf{x}_F(t) &= \boldsymbol{\sigma}(t)^{-1}\boldsymbol{\sigma}^{W_F}(t), \end{aligned} \tag{3.32}$$

and, given $Y^H(t), Y^F(t), \mu^{Y^H}, \mu^{Y^F}, \boldsymbol{\sigma}^{Y^H}, \boldsymbol{\sigma}^{Y^F}, a$ and ρ, the degree of equity home bias is defined as

$$\begin{aligned} Bias &= \mathbf{x}_H(t) - \mathbf{x}_F(t) \\ &= \boldsymbol{\sigma}(t)^{-1}\left(\boldsymbol{\sigma}^{W_H}(t) - \boldsymbol{\sigma}^{W_F}(t)\right) \\ &= \boldsymbol{\sigma}(t)^{-1}\left(\begin{array}{c} \frac{E_t\left[\int_t^T \xi(s)\kappa_H(s)\left(\frac{D_t\kappa_H(s)}{\kappa_H(s)} + \frac{D_t\xi(s)}{\xi(s)}\right)ds\right]}{E_t\left[\int_t^T \xi(s)\kappa_H(s)ds\right]} \\ -\frac{E_t\left[\int_t^T \xi(s)\kappa_F(s)\left(\frac{D_t\kappa_F(s)}{\kappa_F(s)} + \frac{D_t\xi(s)}{\xi(s)}\right)ds\right]}{E_t\left[\int_t^T \xi(s)\kappa_F(s)ds\right]} \end{array}\right) \end{aligned} \tag{3.33}$$

If the first component of this measure is positive (or equivalently, the second component is negative), equity home bias exists. Similarly, if the first component is negative, equity foreign bias exists.

3.4 Simulation Results

Analytical results are not available for equity price dynamics or optimal portfolios as they cannot be expressed in closed-form except for some special cases. This section provides Monte Carlo simulation results for the effects of trade costs in goods market on equity return volatilities, terms of trade volatilities, equity return correlations, and degree of equity biases.

Some obvious cases are when the elasticity of substitution between goods ψ is set to unity. With $\psi = 1$, the individual utility functions stand for a simple logarithmic-type preference, so that income and substitution effects cancel out each other in response to supply shocks. In this case, the terms of trade perfectly reflect the relative output ratio, and thus, two equity prices comove perfectly, which makes the asset return volatility matrix invertible. The optimal portfolio is of course indeterminate, and furthermore, trade costs in goods market have no effect on financial variables. The asset return correlations are one for any levels of trade costs.

Once the elasticity of substitution between goods ψ deviates from unity, trade costs become effective on the asset price dynamics. The parameter values used in these numerical examples are as follows: $a = 0.5$, $\rho = 0.04$, $\mu^{Y^H} = \mu^{Y^F} = 0.03$, $\boldsymbol{\sigma}^{Y^H} = [0.15 \quad 0.05]^T$, $\boldsymbol{\sigma}^{Y^F} = [0.05 \quad 0.15]^T$, $T = 5$, $n = 12$. The number of iterations are 10,000.

I consider the case when the elasticity of substitution between goods ψ is greater than the inverse of the risk aversion coefficient. Following Coeurdacier (2009), the elasticity of substitution between goods is set to $\psi = 5$, and the coefficient of risk aversion is set to $\gamma = 2$. The basic results of simulation is shown on Figures 3.1-

3.10. The Foreign stock return volatility is increasing with the size of trade costs as in Figure 3.1. The reason for this increasing volatility is explained in Figure 3.2, which shows how the Foreign equity returns are sensitive to output shocks in each country. The diffusion values are decreasing with the size of trade costs with respect to Home shocks and increasing with respect to Foreign shocks. This implies that the larger the trade barriers are, the more sensitive to domestic shocks the equity returns become.

Now, the question is: why do equity returns become more sensitive to domestic shocks in presence of trade barriers? The key is to understand how the terms of trade react to these shocks. Figure 3.3 tells us that the terms of trade volatility decreases with the size of trade costs up to a certain threshold level ϕ, becomes zero at ϕ, and increases with the size of trade costs for any trade costs higher than ϕ. The diffusions of the terms of trade are shown in Figure 3.4. According to this figure, the terms of trade is positively associated with Home shocks and negatively with Foreign shocks when trade costs are small, while all associations are reversed when trade costs are above the threshold level.

The equilibrium covariance between equity returns and the terms of trade is shown in Figure 3.5 for various levels of trade costs. The covariance is negative when the trade costs are below the threshold level ϕ and positive if the trade costs are above ϕ. This implies that in presence of small trade barriers, equities provide less returns when the price of goods is high. Similarly, in presence of large trade barriers, equities provide more returns when the price of goods is high.

Such patterns of equity returns and the terms of trade generate equity biases as in Figures 3.6 and 3.7. Figure 3.6 shows the shares of foreign equity in domestic wealth and in foreign wealth. When the size of trade costs is below the threshold level ϕ, the share of foreign equity in domestic wealth is more than the share in foreign wealth, meaning that investors prefer equities abroad (i.e., equity foreign bias). Foreign agent prefers Home equity to Foreign equity because the Foreign equity provides less returns when the price of Foreign goods (which she/he prefers to consume) is high. Similarly, Home agent prefers Foreign equity because Home equity provides less returns when the price of Home goods is high. When the size of trade costs is above ϕ, the share of foreign equity in foreign wealth is more than the share in domestic wealth. Foreign agent prefers Foreign equity to Home equity because now the Foreign equity provides more returns when the price of Foreign goods is high. Since the degree of equity biases is defined as the difference in shares of Home equity in Home wealth and in Foreign wealth, the degree of equity bias is negative when trade barriers are large and positive when trade barriers are small (Figure 3.7).

The overall story goes as follows: When there is no trade cost, the terms of trade act as a close refection of the relative output as discussed in the previous section. Consumers can substitute goods one from the other without any costs associated with international trade of goods. When a permanent positive shock hits the Home production, the relative price of Foreign goods increase due to abundance of Home goods (Figure 3.8). Each agent's portfolio is the same as the market portfolio, so there is no equity bias. Both agents shift their consumption bundle towards Home goods because Home goods are now relatively cheaper. Agents also shift their portfolio towards Home equity because the permanent shock increases the expected returns of Home equity (Figure 3.9). Since both agents change their portfolio in the exact same way, no equity bias is generated after the shock.

In presence of small trade costs, the terms of trade reflect the output ratio to a less extent, i.e., the degree of reflection goes down with the size of trade costs as they alleviate the interaction between the terms of trade and the output ratio. Consumers would still substitute from one goods to the other goods, but they are subject to trade costs for consumption of imported goods. In terms of optimal portfolios, equity foreign bias is generated because the equilibrium covariance between the Foreign asset returns and the relative prices of Foreign goods are negative. Suppose that both agents held the same market portfolio as in the zero-cost case, the Home agent was to be more subjective to the Home shock than the Foreign agent since the relative price of Foreign goods would not be affected as much to reflect the output ratio. This means that the Home agent could improve her/his risk sharing by make herself/himself more subjective to Foreign shocks, that is to say, by holding more Foreign equity. Similarly, the Foreign agent would better off in terms of risk sharing by holding more Home equity than the market portfolio. As a results, equity foreign bias is generated, and this result holds as long as the size of trade costs is less than a certain threshold level ϕ.

When the size of trade costs is exactly at the threshold level ϕ, the terms of trade do not at all reflect the output ratio. A positive supply shock in Home equity puts upward pressure on the terms of trade, but at the same time, the substitution effects towards Home goods put the opposite pressure. At this threshold level of trade costs, these two pressures exactly cancel out each other. Since the equilibrium covariances between the terms of trade and either equity prices are zero, no equity bias is generated, that is, each agent holds the market portfolio.

When the size of trade costs is above the threshold level ϕ, the terms of trade move in a way such that the more abundant the goods are, the more expensive they become. For example, a positive supply shock in Home equity increases the wealth of both Home and Foreign agents. Since the trade costs are so high, consumers do not shift their consumption towards imports, and rather consume more of domestically produced goods. Since this particular shock is a positive shock on Home equity, the terms of trade move in favor of Home goods. The equilibrium covariances between the Foreign equity returns and the relative prices of Foreign goods are now positive. Suppose that both agents held the market portfolio, the Home agent was to be more subjective to Foreign shocks than the Foreign agents as a negative supply shock on Foreign production increase the relative price of Home goods, which the Home agent can consume without incurring high trade costs. This means that the Home agent could improve her/his risk sharing by holding more of the Home equity. As a results, equity home bias is generated, and this result holds as long as the size of trade costs is above the threshold level ϕ. The equilibrium correlations decreases with the size of trade costs.

Finally, the equilibrium correlations between equity returns decrease with the size of trade costs, in spite of the mixed effects of trade costs on the terms of trade volatility and optimal portfolios (Figure 3.10). This is because the equity return volatility is increasing in the size of trade costs (Figure 3.1) as trade costs make equity returns more sensitive to domestic shocks (Figure 3.2). Another reason is that the equilibrium covariances between equity returns are decreasing in the size of trade costs due to lowered adjustability of the terms of trade (Figure 3.11).

3.5 Sensitivity Analysis

These findings are robust to different levels of the elasticity of substitution between goods and the coefficient of risk aversion. The simulation results are shown in

Figures 3.12-3.19.

First, Figure 3.12 gives the correlations between equity returns as a function of trade costs and the elasticity of substitution between goods. When the elasticity of substitution between goods is one, as aforementioned before, the equilibrium correlations between equity returns are one no matter what the size of the trade costs is. As soon as the elasticity deviates from one, the correlations become a decreasing function of the size of the trade costs, which is consistent with our results. It can be also seen on the figure that the effects of trade costs on the correlations increase with the elasticity of substitutions. In other words, at some fixed level of trade costs, the correlations are decreasing in the elasticity of substitution. This is because trade costs prevents consumers from substituting their consumption of one goods from the other goods. With high trade costs and a large elasticity, the incentive to shift consumption bundles are high, but the shift will not be fully taken place due to the trade costs, which lower the equilibrium correlations of equity returns.

Figure 3.13 shows the correlations between equity returns as a function of trade costs and the coefficient of risk aversion. It is found that at any levels of the coefficient of risk aversion, the correlations are decreasing with the size of trade costs, which is again consistent with our results.

Do parameter choices matter in the formation of portfolio structures? Figures 3.14 and 3.16 contain the answer to this question. Figure 3.14 shows the shares of foreign equity in domestic wealth as a function of trade costs and the elasticity of substitution between goods, holding the risk aversion coefficient constant. When the elasticity is equal to one, the optimal portfolio is indeterminate because as seen earlier two equity returns perfectly comove each other. When the elasticity is greater than one but less than the threshold level ϕ, the Home agent holds more foreign equity and less domestic equity, i.e., equity foreign bias. Finally when the elasticity is greater than the threshold level ϕ, her/his portfolio is biased toward domestic equity. These findings are the same as the one we discussed earlier. It can be also seen on this figure that the threshold level ϕ of trade costs becomes lower as the elasticity of substitution between goods are higher. The share of foreign equity in domestic wealth is decreasing in the elasticity at any levels of trade costs. The effects of the elasticity of substitution seem to be larger with lager trade costs, which also supports one of the earlier findings: agents want to hold more of foreign equity when the price of foreign goods is high. The intuition is as follows: for a low elasticity of substitution, an agent cares a lot about which goods to consume, and thus, she/he holds portfolio whose returns are high when the foreign goods price is high. (The desire for not loosing consumption of foreign goods is higher for a lower elasticity of substitution.) On the other hand, for a high elasticity of substitution, an agent cares little about which goods to consume since her/his consumption bundle can be relatively easily shift towards either goods in response to realization of goods. This allows equity returns and prices to move to the opposite directions, resulting in equity home bias.

The opposite side of the foreign equity market can be seen on Figure 3.15 which show the shares of foreign equity in foreign wealth as a function of trade costs and the elasticity of substitution between goods. Finally, the degree of equity home bias is shown on Figure 3.16 as a function of trade costs and the elasticity of substitution. Equity biases are generated toward domestic or foreign equities depending on the size of trade costs and the choice of elasticity of substitution.

The conditions for a set of parameter values that gives a negative association between the equilibrium correlations and the size of trade costs are summarized in Figure 3.20. If a set of parameter values of the elasticity of substitution and the risk aversion coefficient falls into the areas denoted by "*negative*", located at top-right and bottom-left, then the equilibrium correlations between equity returns decrease with the size of trade costs. If a set of parameters are right on the boarder, i.e., $\psi = 1$ or $\psi = \frac{1}{\gamma}$, then trade costs have no impact on the equilibrium correlations. Otherwise, the equilibrium correlations increase with the size of trade costs.

The relationships between parameter values and the degree of equity biases can be summarized as follows: If $\psi = 1$, optimal portfolios are indeterminate as the equilibrium correlations between equity prices is one. If $\psi \neq 1$, then the value of γ determines whether equity biases are generated. If $\gamma > 1$, then trade costs generate equity foreign bias; if $\gamma = 1$, then trade costs has no impacts on optimal portfolios; and otherwise, trade costs generate equity home bias. These relationships hold only for small trade barriers. For large trade barriers, the association could be reversed depending on the threshold levels of trade costs.

Another robustness checks conducted here is regarding an asymmetric setting. Figures 3.21-3.24 show how the related variables are affected by the relative wealth size. In Section 3.3.2, it was suggested that the equal weights for the Home and Foreign agents ($\lambda = 1$) simplifies the terms of trade dramatically. It might be the case that any asymmetry could change my results about the equilibrium correlations and equity home biases. However, all of these variables do not change qualitatively, and thus, it can be suggested that our findings are robust to asymmetric settings.

3.6 Conclusions

This paper analyzes the mechanism of international comovements across imperfectly integrated markets. The chapter develops a continuous-time general equilibrium model in a two-country, two-asset, and two-good setting where international trade of goods are costly. The equilibrium correlations of two equity returns are expressed in terms of the size of trade costs.

It is found that the degree of equity bias depends on the size of trade costs, the elasticity of substitution between goods, and the coefficient of risk aversion. When the equilibrium covariances between equity returns and the terms of trade is negative, equity foreign bias exists, while when it is positive, equity home bias exists. This results is consistent with the findings by Coeurdacier (2009), and this chapter shows that his findings hold in a continuous-time dynamic setting. In spite of these parameter-dependent results in portfolio structures, the equilibrium correlations between equity returns are monotonically decreasing in the size of trade costs for a plausible set of parameter values. This is because: 1. Trade costs make equity returns sensitive to domestic shocks, and 2. The equilibrium covariance between equity returns decreases with the size of trade costs due to lowered adjustability of the terms of trade.

All the results derived in this chapter are qualitative, and any potential empirical implications are left for future research.

Figures and Tables

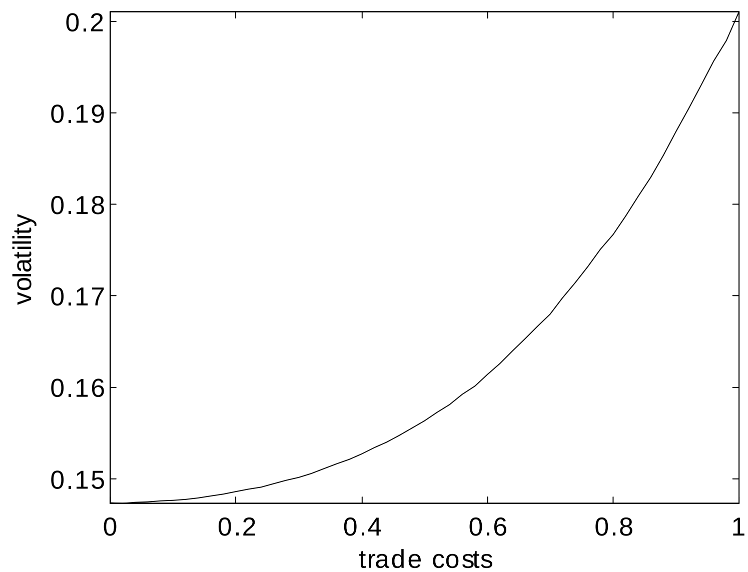

Figure 3.1: Equity return volatility as a function of trade costs.

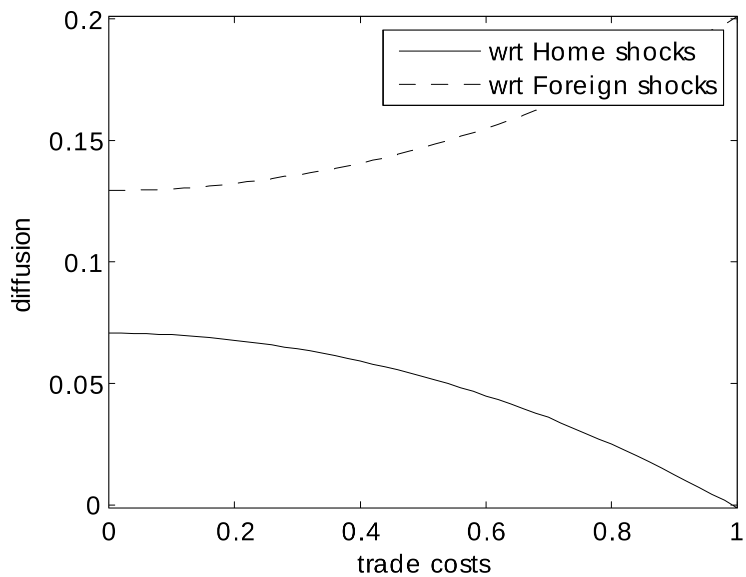

Figure 3.2: Diffusion coefficients of equity returns as a function of trade costs

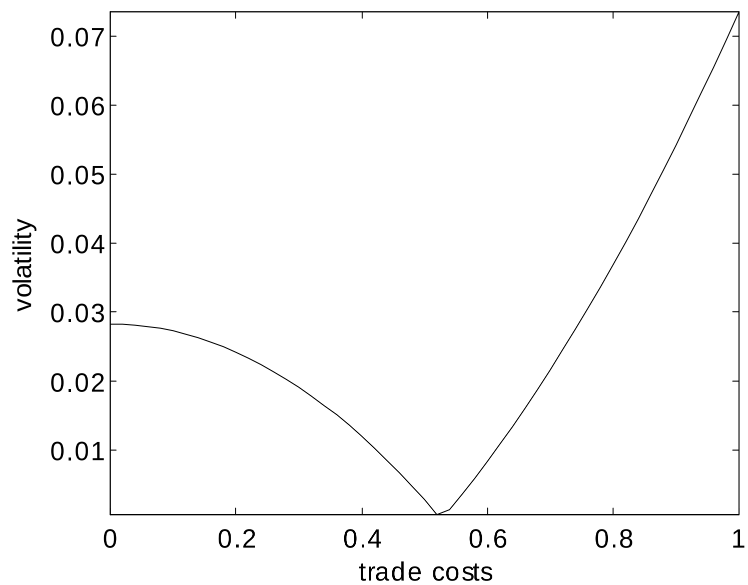

Figure 3.3: Terms of trade volatility as a function of trade costs

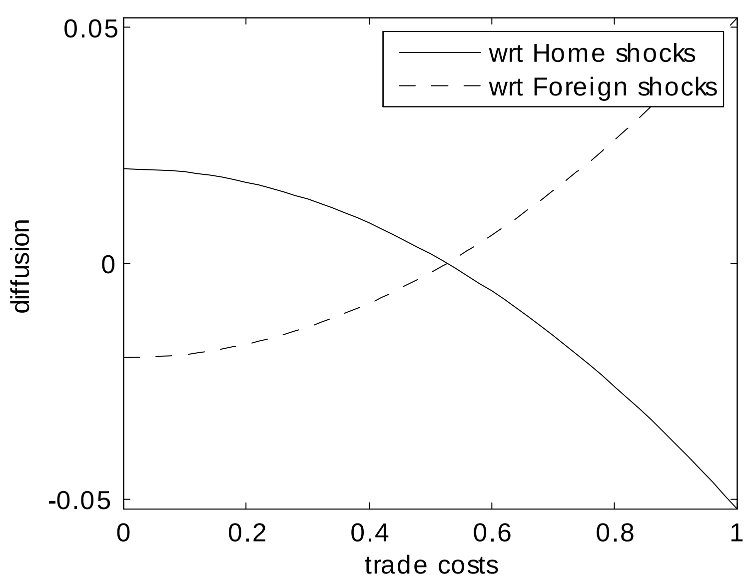

Figure 3.4: Diffusion coefficients of the terms of trade as a function of trade costs

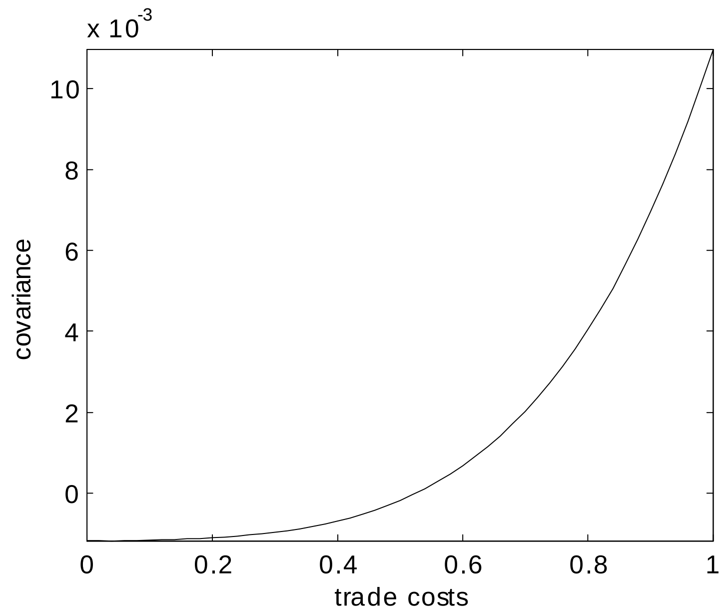

Figure 3.5: Equilibrium covariance between equity returns and the terms of trade as a function of trade costs.

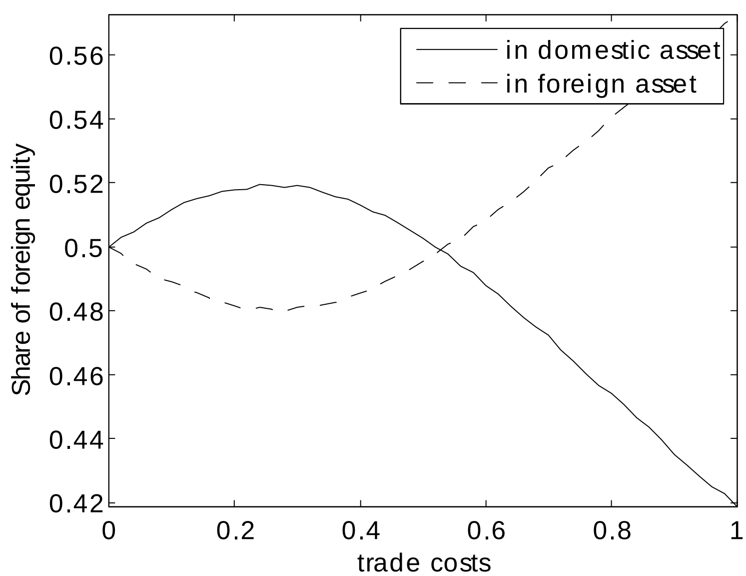

Figure 3.6: Shares of foreign equity

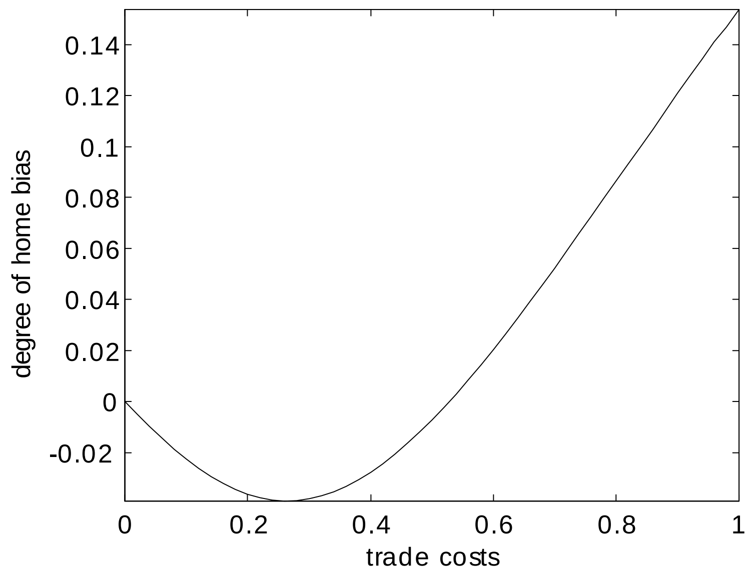

Figure 3.7: Degree of equity home bias as a function of trade costs.

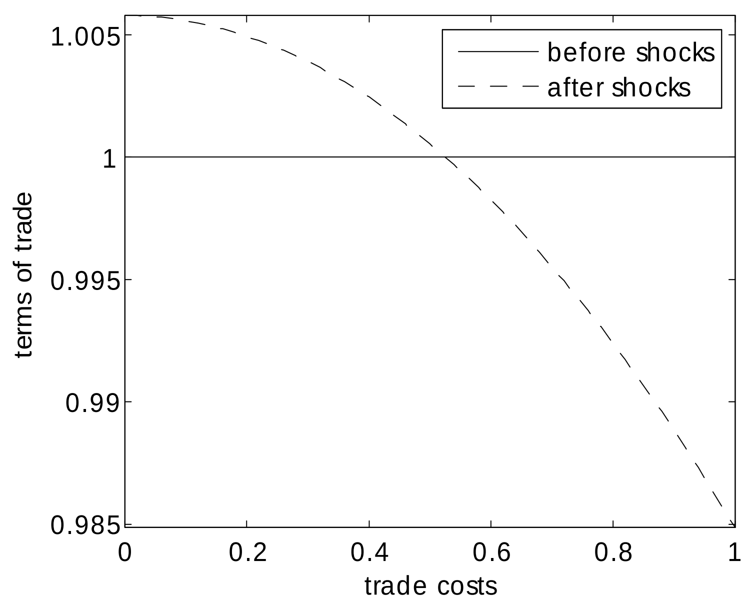

Figure 3.8: Change in the terms of trade after a positive shock to Home production.

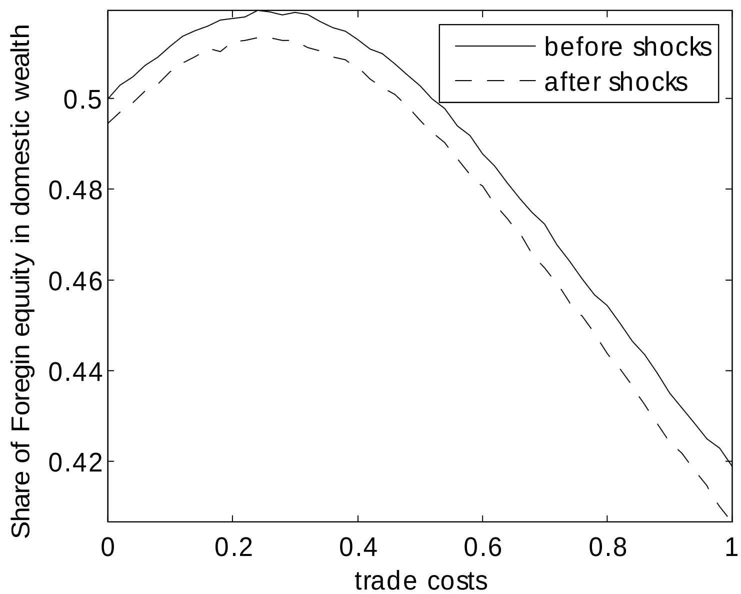

Figure 3.9: Change in shares of foreign equity in domestic asset after a positive shock to Home production.

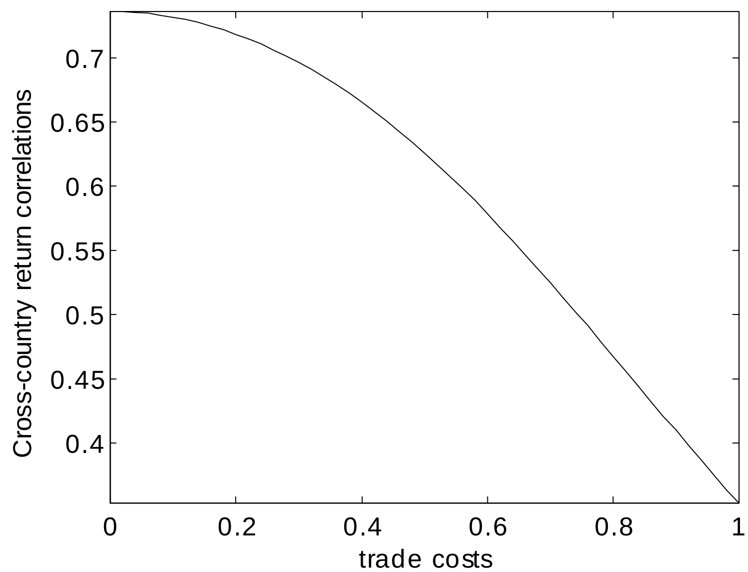

Figure 3.10: Cross-country equity return correlations as a function of trade costs.

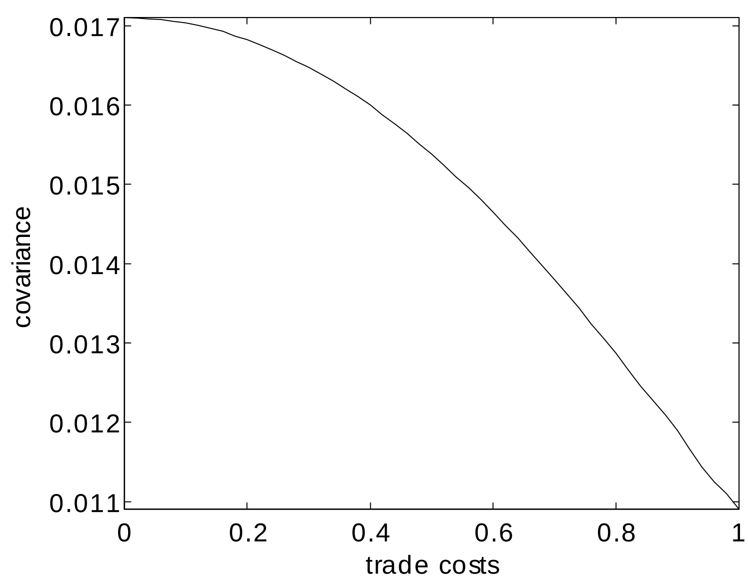

Figure 3.11: Equilibrium covariance between domestic and foreign equity returns as a funciton of trade costs.

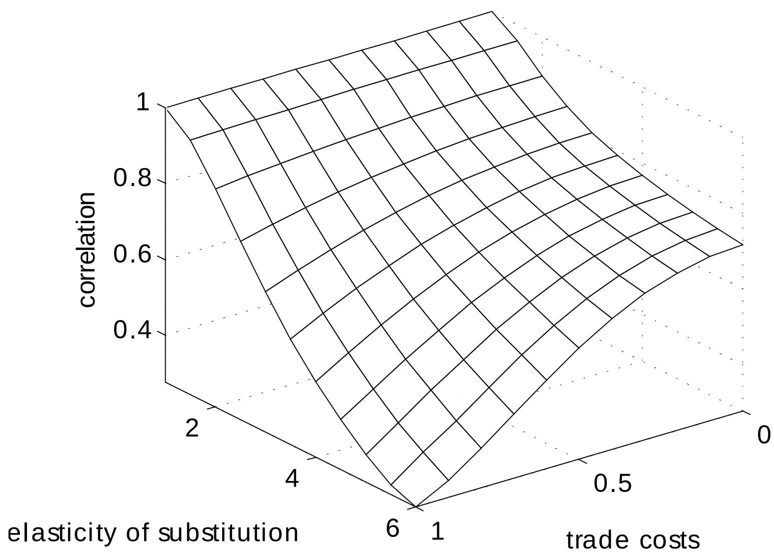

Figure 3.12: Cross-country equity return correlations as a function of τ and ψ when $\gamma = 2$.

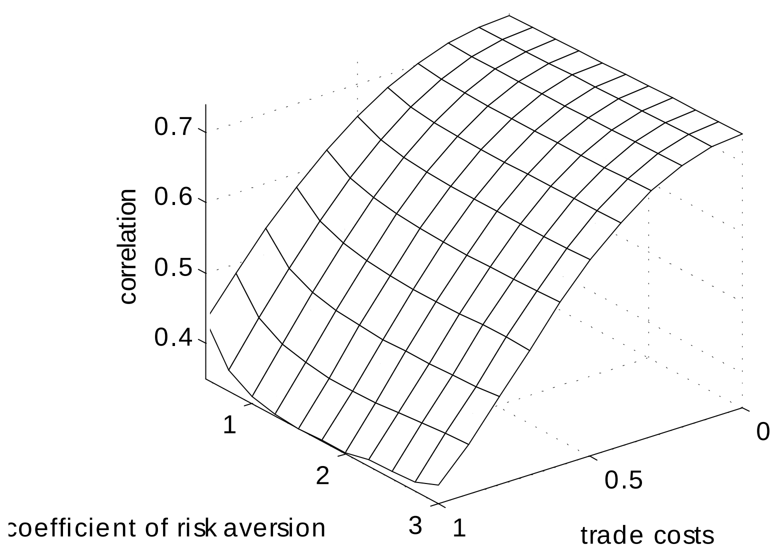

Figure 3.13: Cross-country return correlations as a function of τ and γ when $\psi = 5$.

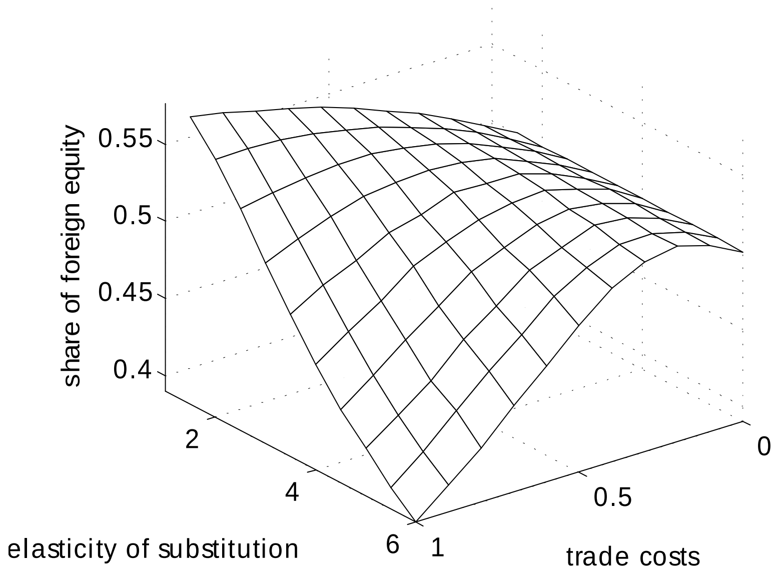

Figure 3.14: Share of foreign equity in domestic wealth as a function of τ and ψ when $\gamma = 2$.

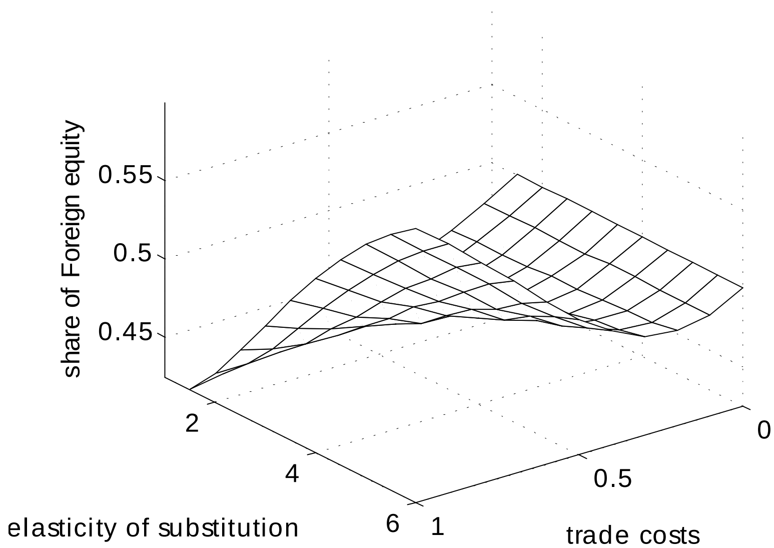

Figure 3.15: Share of foreign equity in foreign wealth as a function of τ and ψ when $\gamma = 2$.

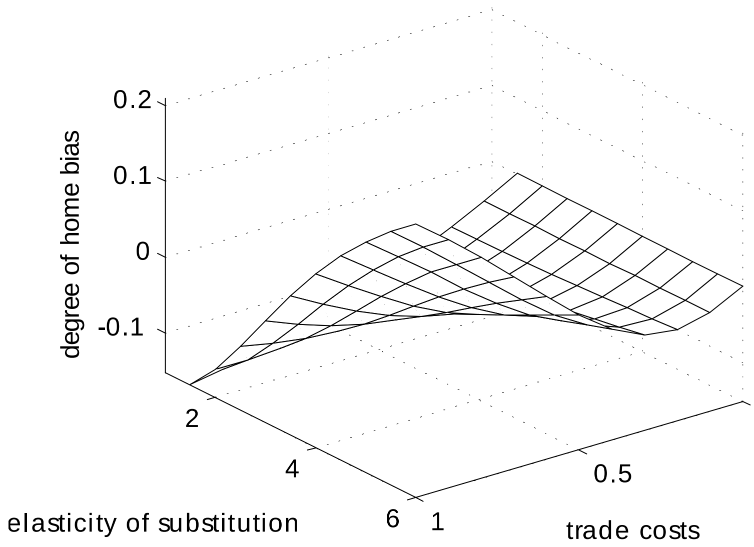

Figure 3.16: Degree of equity home bias as a function of τ and ψ when $\gamma = 2$.

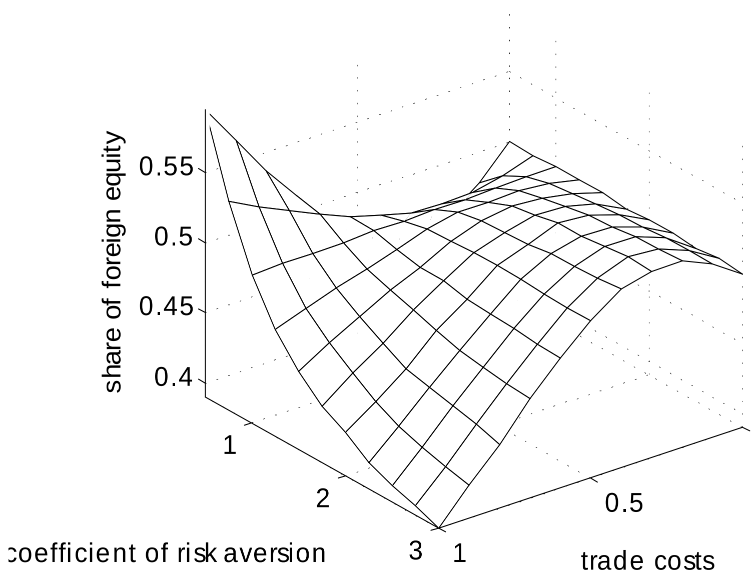

Figure 3.17: Share of foreign equity in domestic wealth as a function of τ and γ when $\psi = 5$.

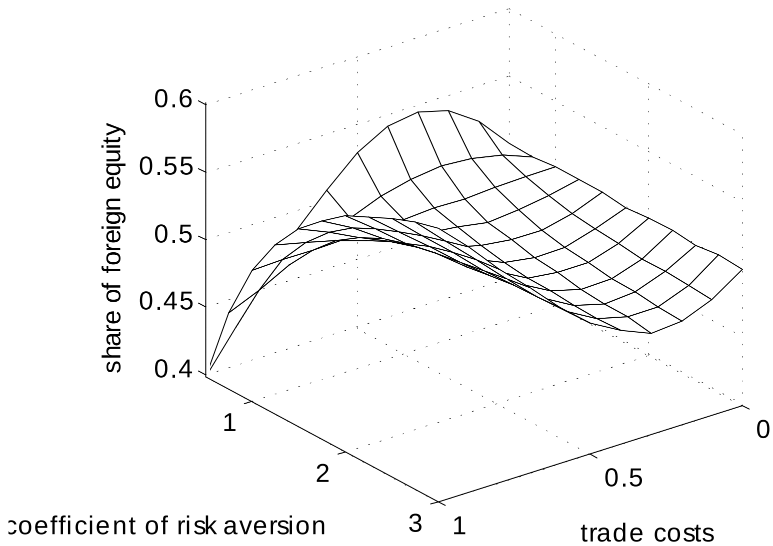

Figure 3.18: Share of foreign equity in foreign wealth as a function of τ and γ when $\psi = 5$.

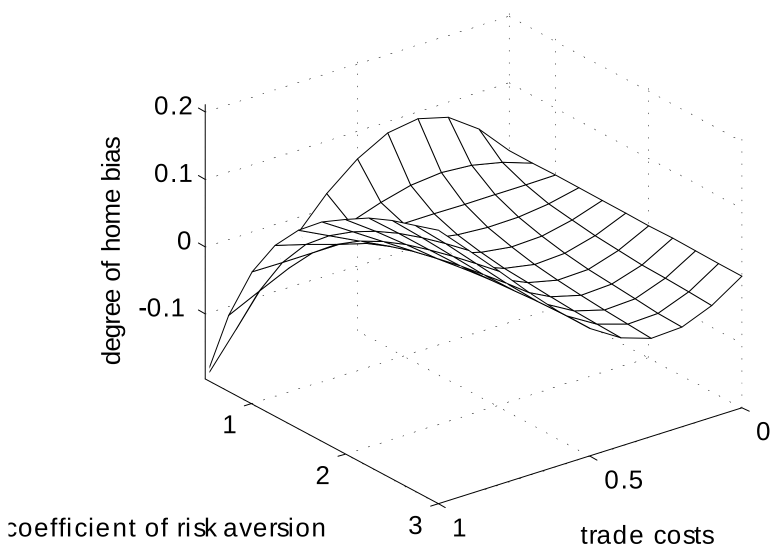

Figure 3.19: Degree of equity home bias as a function of τ and γ when $\psi = 5$.

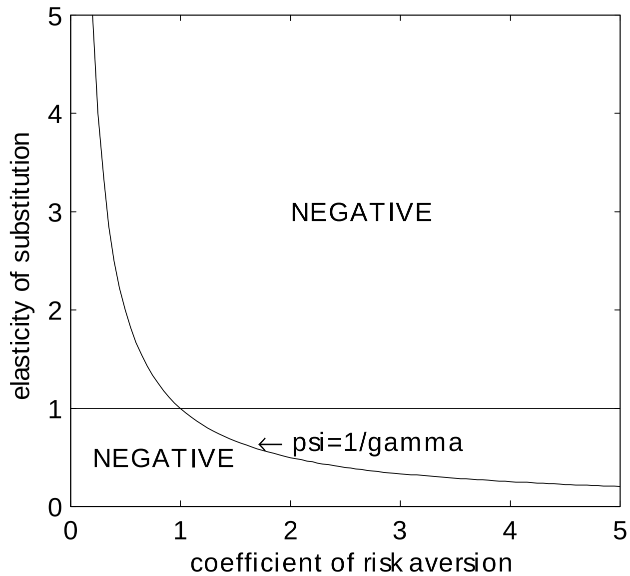

Figure 3.20: Conditions for a negative association between the equilibrium correlations and the size of trade costs.

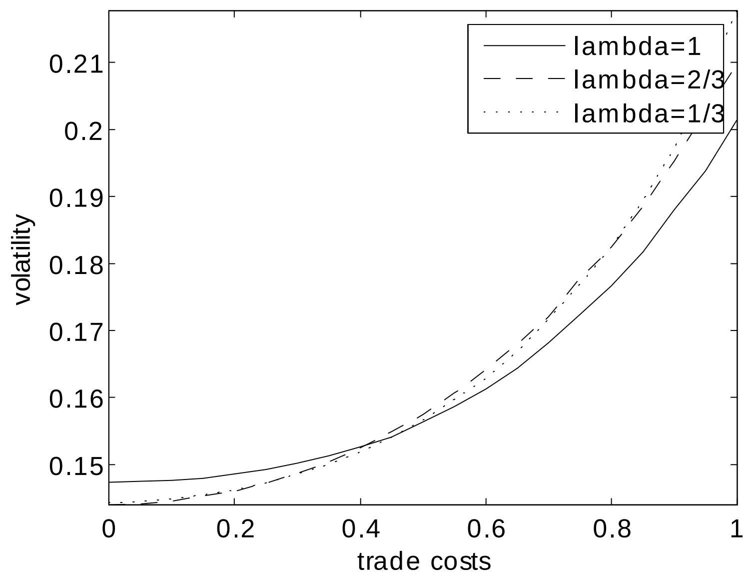

Figure 3.21: Equity return volatility as a function of trade costs.

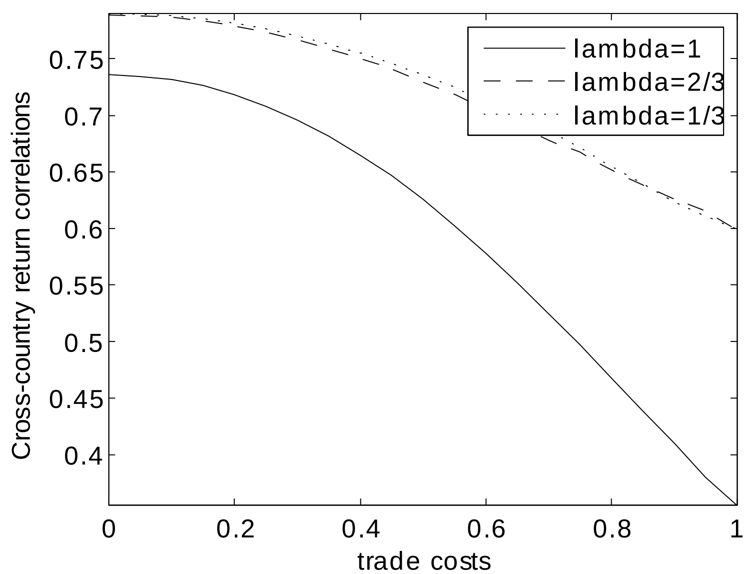

Figure 3.22: Cross-country equity return correlations as a function of trade costs.

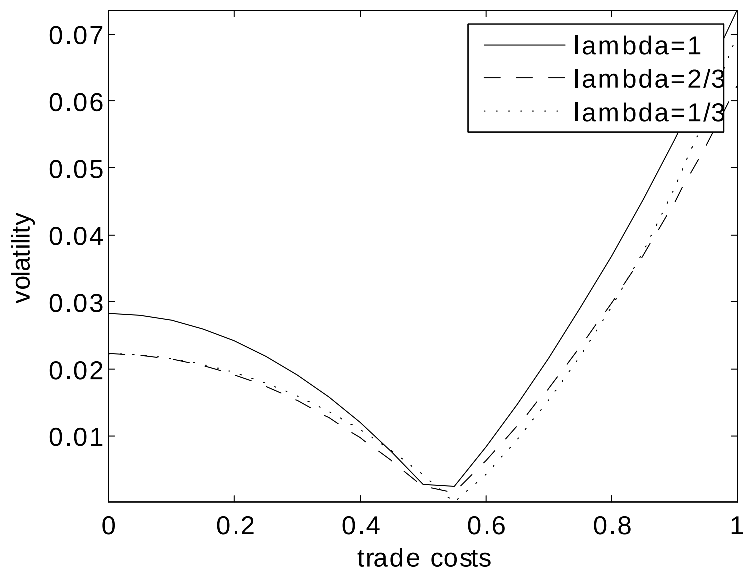

Figure 3.23: Terms of trade volatility as a function of trade costs.

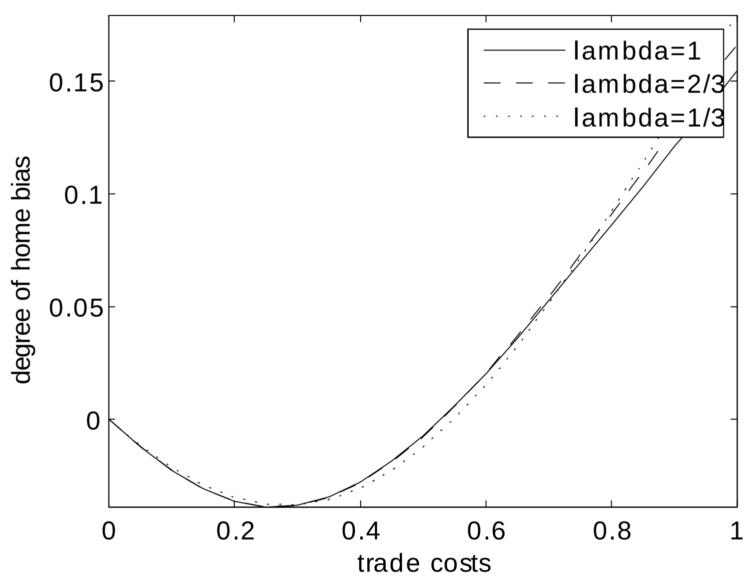

Figure 3.24: Shares of foreign equity in domestic asset.

CHAPTER 4

EMPIRICAL ANALYSIS ON THE EFFECTS OF FINANCIAL INTEGRATION ON INTERNATIONAL COMOVEMENTS OF STOCK MARKETS

This chapter analyzes the effects of financial integration on the extent of stock market synchronization. At a theoretical level, financial integration has an ambiguous impact on stock market synchronization. I characterize synchronization among 24 countries over the period 1980-2003. A country-pair panel instrumental variables framework is employed to explain time-varying correlations among national stock returns, by utilizing the Fitzgerald (2008) dataset on trade costs. It is found that financial integration driven by reduction of trade costs leads to a higher degree of synchronization across stock markets.

4.1 Introduction

In the last three decades, we have observed a rapid increase in international financial integration. Figure **4.1** depicts a recent surge in the measure of financial integration since 1980 for 24 advanced and emerging market economies. Financial integration is measured by the amounts of assets and liabilities as a ratio to GDP, and the figure shows the median value for each year. We have also witnessed evidence of rising comovements of stock returns across markets. The trend of increasing correlations between stock returns has been pointed out by many related works such as Yang et al., 2006.

At a theoretical level, financial integration has both positive and negative impacts on stock market synchronization. Financial integration could increase the degree of asset return comovements through the transmission of country-specific shocks ("contagion"). Under completely segmented financial markets, country-specific shocks affect only domestic investors since they do not hold foreign stocks. On the other hand, when financial markets are somewhat integrated, country-specific shocks will be transmitted through wealth effects. For example, a positive productivity shock in one country increases relative wealth of stock holders in the world. Increasing wealth in turn shifts up their demand for stocks in other countries, thereby inducing return comovements, regardless of the evolution of market fundamentals. Financial integration may also lead to sectoral specialization of production through the reallocation of capital. It alleviates country-specific shock transmission and simultaneous impacts of sector-specific shocks that affect one particular industry in all countries. Overall impact of increasing financial integration is therefore ambiguous.

Understanding stock return comovements is an essential component of asset management. The modern portfolio theory tells us that investors can reduce their exposure to risk by holding a combination of weakly correlated assets. When two stock returns are uncorrelated, for instance, agents can reduce the volatility of portfolio by investing in both stocks. On the other hand, when two stock returns are highly and positively correlated, it becomes more difficult to diversify their portfolio risks. Furthermore, the extent of stock market synchronization has an important policy implication since it indicates to what degree external shocks could be transmitted into the domestic financial market. Before implementing new government policies (e.g. lowering foreign income tax), one should assess how such policy changes affect vulnerability of the domestic financial market to external shocks.

Integration reduces the cost of capital and attracts foreign investors to local markets appealing to their better diversification benefits. However, if the degree of synchronization goes up as a result of increasing foreign participation, the diversification benefits would become smaller. This paper examines how integration and synchronization are linked together, that is, how financial integration affect comovements of stock returns. To this aim, I look at how financial integration is associated with the degree of comovements. The empirical investigation is conducted on 24 advanced and emerging market economies 1980 to 2003. The degree of synchronization is measured by the correlations between weekly national stock returns, computed over one-year period. The dynamic panel instrumental variables framework is used to control for a potential endogeneity problem associated with financial integration by utilizing the dataset on trade costs in Fitzgerald (2008). It is found that financial integration stemming from reduction of trade costs is followed by a higher degree of synchronization across stock markets.

This chapter adds to the literature in the following ways. First, my analysis is more comprehensive than the previous studies as my dataset covers both advanced and emerging market economies. Most existing works use bilateral financial flow data to measure strength of financial linkages between two countries. However, such data is available only for a small number of countries with a short time dimension. I get around this issue by following Walti (2008) in constructing an alternative measure of financial integration between two countries. Walti (2008) uses the logarithm of the product of two countries' measures of financial openness taken from the Lane and Milesi-Ferretti (2007) dataset of financial assets and liabilities, and therefore the coverage is wide in both cross-sectional and time-series dimensions. Second, I employ a country-pair time-varying instrumental variables method by utilizing the dataset of Fitzgerald (2008). Fitzgeral (2008) reports exogenous and time-varying bilateral trade costs over the period 1980-2003 for 24 advanced and emerging market economies. The use of this trade cost dataset as instruments allows the models to reflect a causal effect of financial integration on stock market synchronization.

The rest of the paper is organized as follows: Section 4.2 reviews related literature on the impacts of globalization on comovements; Section 4.3 provides a brief description of the data and the econometric specifications; Section 4.4 presents regression results showing the relationship between the stock return correlations and the measures of financial integration, and checks whether the results are robust; and finally, Section 4.5 concludes.

4.2 Related Literature

Several aspects of financial integration have been identified as drivers of stock market synchronization. Bekaert and Harvey (2000) find that liberalized equity markets show a higher degree of comovements with world indices. They find that stock return correlations and market betas increase after liberalization of capital account by comparing them for pre- and post-liberalization. Along the same line, Goetzman et al. (2005) and Quinn and Voth (2008) use a long-run dataset on capital account regulations over 100 years and find evidence of a positive relationship between the level of capital account openness and stock return correlations.

Bilateral financial and trade intensity are also found to be key drivers of stock market synchronization. Forbes and Chinn (2004) compare the importance of direct trade linkages with that of bilateral financial flows and conclude that trade flows are significant determinants of the effect of large stock markets on other financial markets. Beine and Candelon (2007) provide evidence for a positive impact of both financial and trade integration on the degree of cross-country stock market correlations among 25 developing countries. Morana (2008) focus on G-7 and show that economic integration increases international stock market comovements through the common response of stock markets to global economic shocks, while financial integration would operate through financial shocks spillover.

In addition to equity market liberalization and financial and trade intensity, a number of factors are identified as robust determinants of international synchronization of equity returns. Dellas and Hess (2005) examine a cross section of both industrial and emerging market economies and show that stock market synchronization increases with the liquidity of equity markets and greater financial depth. Fratzscher (2002) shows that exchange rate volatility negatively impacts stock market comovements. Walti (2008) studies the impact of monetary integration on stock market synchronization and find that the adoption of a single currency increases correlation. Tavares (2009) documents that bilateral trade intensity increases the correlation of returns, while real exchange rate volatility and the asymmetry of output growth decrease it. Roll (1992) and Dutt and Mihov (2005) document the role of industrial structure on stock market comovements and conclude that the similarity in industry composition lead stock markets to comove more than stock markets with a different industry composition. Finally, Bartoram, Griffin, and Ng (2009) conclude that the stock return correlations depend on the international ownership.

Beine and Candelon (2007) suggest that these divergent results may be due to the high degree of heterogeneity when industrial countries and developing countries are analyzed together. I challenge this issue by properly controlling for country-specific characteristics and international trends that affect all sample countries.

In a broader sense, this chapter is related to the literature that has explored the costs and benefits of integration. For instance, Sachs and Warner (1995) and Wacziarg and Welch (2008) find that trade openness promotes economic growth. Although empirical evidence is less clear for the effect of financial integration (see for instance the survey paper by Kose et al., 2009), capital account liberalization is identified as an important source of economic growth (Bekaert and Harvey, 2003).

4.3 Data and Methodology

There are various approaches to measure international stock market comovements. One simple way is to look at the pairwise correlations between stock returns. This approach is widely used to assess the joint behavior of the two stock markets (Bekaert and Harvey, 2000; Dellas and Hess, 2005; Beine and Candelon, 2007; Walti, 2008). However, Forbes and Rigobon (2002) point out that the correlation coefficients are biased upwards when markets become more volatile and conclude that there was virtually no increase in unconditional correlation coefficients. They suggest that the correlations should be adjusted by the relative increase in the variance of returns:

$$CORR_{i,j,t}^{FR} = \frac{CORR_{i,j,t}^{realized}}{\sqrt{1 + v_{i,j,t}\left[1 - \left(CORR_{i,j,t}^{realized}\right)^2\right]}} \tag{4.1}$$

where $CORR_{i,j,t}^{FR}$ is the volatility-corrected correlation coefficient for countries i and j at time t, $CORR_{i,j,t}^{realized}$ is the realized correlations, and $v_{i,j,t}$ is the relative increase in the variance of the returns defined by

$$v_{i,j,t} = \frac{max(\sigma_{i,t}, \sigma_{j,t})}{min(\sigma_{i,t}, \sigma_{j,t})} - 1$$

where $\sigma_{i,t}$ denotes the variance of the returns for country i at time t. Since these correlation coefficients cannot be normally distributed, I adapt a Fisher-Z transformation of the dependent variable following Otto et al. (2001), Beine and Candelon (2007), and Walti (2008):

$$CORR_{i,j,t} = \ln\left(\frac{1 + CORR^{FR}_{i,j,t}}{1 - CORR^{FR}_{i,j,t}}\right) \tag{4.2}$$

This $CORR_{i,j,t}$ is the variable used in my regression analysis. In the sensitivity analysis, I also use the uncorrected corrections to make sure that the results do not solely depend on measures of stock return correlations.

Our sample covers 24 advanced and emerging market economies from 1980 to 2003. These countries are: Australia, Austria, Belgium, Canada, Switzerland, Germany, Denmark, Spain, Finland, France, United Kingdom, Greece, Ireland, Italy, Japan, Korea, Mexico, Netherlands, Norway, New Zealand, Portugal, Sweden, Turkey, and the United States. Stock return data is computed using the MSCI country indexes, compiled from DataStream. Observations are weekly and converted into US dollars using nominal exchange rates. Correlations between national stock returns are taken over one-year period. The sample statistics of the data is described on Table 4.1.

Since our main interest lies in identifying the impacts of financial integration on stock market synchronization, measuring financial and economic integration properly is a key issue. Kose, Prasad, Rogoff and Wei (2009) discuss financial openness measures in detail. De jure measures of capital account openness rely on the dating of financial market liberalization and/or government policies on capital account restriction, whereas de facto measures of financial integration focus on the outcomes of such liberalizations. Bekaert and Harvey (2003) emphasize the distinction between market liberalization and market integration and conclude that announcement of market liberalization does not always coincide with the completion of de facto integration. Kose, Prasad and Terrones (2009) suggest that the use of the de facto measures is more appropriate than the de jure measures when the effects of an outcome-based measure of financial integration are of interest. They also point out that the de facto measures allow us to obtain a finer characterization of financial openness by looking closely at the effects of different types of capital flows.

A measure of financial integration used in this chapter is constructed based on the usual de facto measures of financial openness, that is, the gross stock of external assets and liabilities as a ratio to GDP taken from the External Wealth of Nations Database complied by Lane and Milesi-Ferretti (2007). The bilateral measure of financial integration ($FININT$) is calculated as the logarithm of the product of two countries' measures of de facto financial openness, following Walti (2008).

A panel data framework with time fixed effects allows us to capture unobserved country specific effects and global shocks. Our baseline specification is:

$$CORR_{i,j,t} = \alpha_t + \alpha_{i,j} + \beta FININT_{i,j,t-1} + \mathbf{X}'_{i,j,t-1}\mathbf{\Psi} + \varepsilon_{i,j,t} \tag{4.3}$$

where $CORR_{i,j,t}$ is the volatility-corrected correlations between national stock returns for countries i and j at time t, $FININT_{i,j,t-1}$ is the measure of financial integration, $\mathbf{X}_{i,j,t-1}$ is a matrix that includes exogenous regressors, α_t and $\alpha_{i,j}$ represent period dummies and time-invariant country-pair specific effects, respectively. The independent variables are lagged by one period in order to account for the difference in timing between two events.

Given this panel data approach, the inclusion of period dummies allows us to capture the roles of common international shocks on the comovements. Likewise, by the inclusion of cross-sectional effects we are able to account for unobserved country pair specific heterogeneity. The Hausman tests are conducted to decide on the use of fixed or random effects. The results tend to support the use of cross-sectional random effects.

In addition to the core independent variables that measure the degree of financial integration, we include in regressions several control variables. Inclusion of these control variables ensures that the estimated β is not influenced by the omitted variables. Bilateral trade intensity ($TRAINT$) is included to capture the effects of trade integration on stock market synchronization, which is measured in a standard way: the logarithm of bilateral imports and exports as a percent of the two countries' GDP. Bilateral exchange rate volatility ($FXVOL$) is used to control for differences in currency risk. Fratzcher (2002) find that exchange rate volatility negatively impacts stock market comovements as the costs of portfolio rebalancing are low.

Even with the inclusion of country-pair effects, time dummies, and some proper control variables, it is still concerned that the OLS estimates may be subject to potential country-pair time-varying omitted variable biases. To account for the biases, panel instrumental variable models are used by utilizing the Fitzgeral (2008) dataset of trade costs. Fitzgerald (2008) made public her dataset, which contains time-varying bilateral trade costs among 24 countries from 1970-2003 and is treated as exogenous to country pairs. I make use of this trade costs data ($TCOST$) to instrument for financial integration as well as trade integration. The first-stage relationship between financial integration ($FININT$) and trade costs ($TCOST$) are:

$$FININT_{i,j,t} = \delta_t + \delta_{i,j} + \gamma TCOST_{i,j,t} + \mathbf{Z}'_{i,j,t}\Phi + v_{i,j,t} \tag{4.4}$$

where $\mathbf{Z}_{i,j,t}$ is a matrix that includes exogenous regressions, and δ_t and $\delta_{i,j}$ represent period dummies and time-invariant country-pair specific effects, respectively. The exogenous instruments other than trade costs include dummy variables for common languages and for the joint EU members. The validity of the instruments are discussed in the next section.

4.4 Results

I start my analysis by obtaining cross-sectional OLS estimates. Columns (1) to (3) on Table 4.2 reports the OLS estimates using the volatility-corrected correlations of stock returns as the dependent variable. The average values of variables are taken over the latest 12 years for each country pair. As there is one observation for each country pair, the number of observations is equal to the number of country pairs. The cross-sectional coefficient on the financial integration measure is positive and significant at standard confidence levels. This suggests that the return correlations are higher for pairs with higher financial integration.

In columns (4) and (5) on Table 4.2, the 2SLS regression results are reported. The first column of (4) shows the reduced-form relationship between return correlations and the size of trade costs. All independent variables in the reduced-form regression are assumed to be exogenous to the economy. The regression yields a negative and highly significant estimate on $TCOST$. This suggests that reduction of trade costs leads to a higher degree of stock market synchronization. The next two columns report the estimates from the first stage regressions. The coefficients on $TCOST$ are negative and highly significant, which suggests that country pairs with low trade costs are more integrated in terms of trade and finance. As the first-stage F-scores are significantly high, I would not worry about the weak instrument problems. Finally, the last column of (4) reports the second-stage coefficients. The 2SLS estimate of financial integration is positive and significant at the 95% confidence level, suggesting that increases in financial integration stemming from reduction of trade costs are followed by higher comovements of financial markets. The 2SLS estimate of financial integration remains positive and significant even after controlling for exchange rate volatilities (Column (5) of Table 4.2).

In Table 4.3, I report the panel OLS and panel 2SLS with and without country-pair and/or period-specific effects. The average values of variables are taken for each four-year period between 1980-2003. Throughout the specification, the number of observations is 1652, and the number of country pairs are 276. In other words, there are 7 observations for each country pair. Column (1) contains the pooled OLS estimate of financial integration. The estimate is positive and significant, suggesting that financial integration is associated with higher comovements of stock markets on the next period. Inclusion of period dummies does not change this result as shown in column (2) of Table 4.3. Columns (3) and (4) report the panel 2SLS estimates with and without period dummies. In both regressions, the coefficients on financial integration are positive and significant, which is consistent with the OLS estimates. Finally, columns (5) and (6) contains the panel 2SLS regression results with inclusion of country-pair effects. The results of the Hausman specification tests are in favor of the country-pair random effects over the fixed-effect specifications. The estimates on financial integration are positive and significant with or without the period dummies. This suggests that financial integration driven by reduction of trade costs leads to higher synchronization across financial markets even after controlling for cross-sectional-specific and period-specific effects.

As sensitivity analyses, I run several regressions with alternative measures of integration and synchronization as well as different time horizons. Table 4.4 reports the sensitivity analysis results. Column (1) contains the panel 2SLS regression results with period dummies and country-pair effects. Instead of using the volatility-corrected correlations, the realized correlations with the Fisher-Z transformation are used as the dependent variable. The estimate on financial integration is positive and significant, and this suggests that my earlier finding does not depend on volatility correction of the realized correlations. Column (2) reports the similar regression results to column (1). The only difference is the use of log distance between two country as instrument instead of trade costs of Fitzgerald (2008). The estimate on financial integration is positive and significant, suggesting that my finding does not rely on the choice instruments. In order to verify that the analysis does not depend on the choice of the financial integration measures, the portfolio equity investments as a share of GDP is used as an independent variable instead of the total assets and liabilities as a share of GDP. The positive and significant estimate on this measure of financial integration supports the robustness of the analysis. Columns (4) and (5) report the regression coefficients for subperiods 1980-1991 and 1992-2003, respectively. As the estimates on financial integration are positive and significant for both specifications, it is confirmed that the positive effects of financial integration on comovements of financial market does not rely on the choice of periods. Finally, a different time horizon is examined in column (6). The averages of variables for each eight-year period over 1980-2003 are used in the panel 2SLS regressions. The estimate on financial integration is still positive and significant, suggesting that the result does not depend on the choice of time horizons.

4.5 Conclusions

In this chapter, I investigate whether financial integration increases the degree of stock market synchronization. I provide evidence of strong linkage between de facto measures of financial integration and the correlation of stock returns among 24 countries over the period 1980-2003. A country-pair panel instrumental variables framework is employed to explain time-varying correlations among national stock returns, by utilizing the Fitzgerald (2008) dataset on trade costs. Although financial integration has an ambiguous impact on synchronization at a theoretical level, it is found that financial integration driven by reduction of trade costs leads to higher degree of synchronization across stock markets.

Figures and Tables

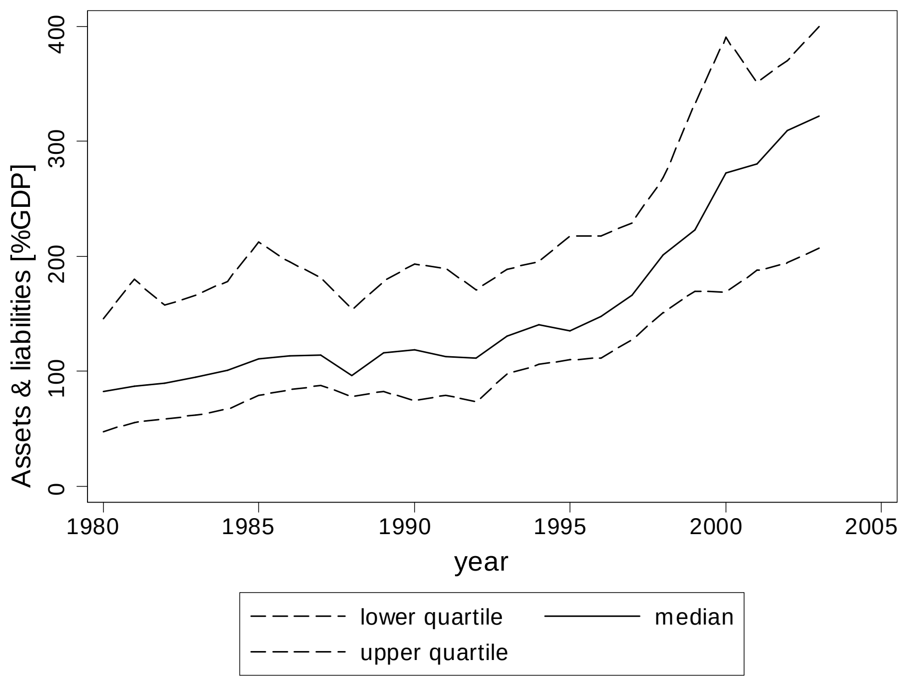

Figure 4.1: Financial Integration over Time

This figure shows cross-country medians and quartiles of the de facto measure of financial integration, which is based on the gross stocks of foreign assets and liabilities as a ratio to GDP. The raw data is taken from Lane and Milesi-Ferretti (2007).

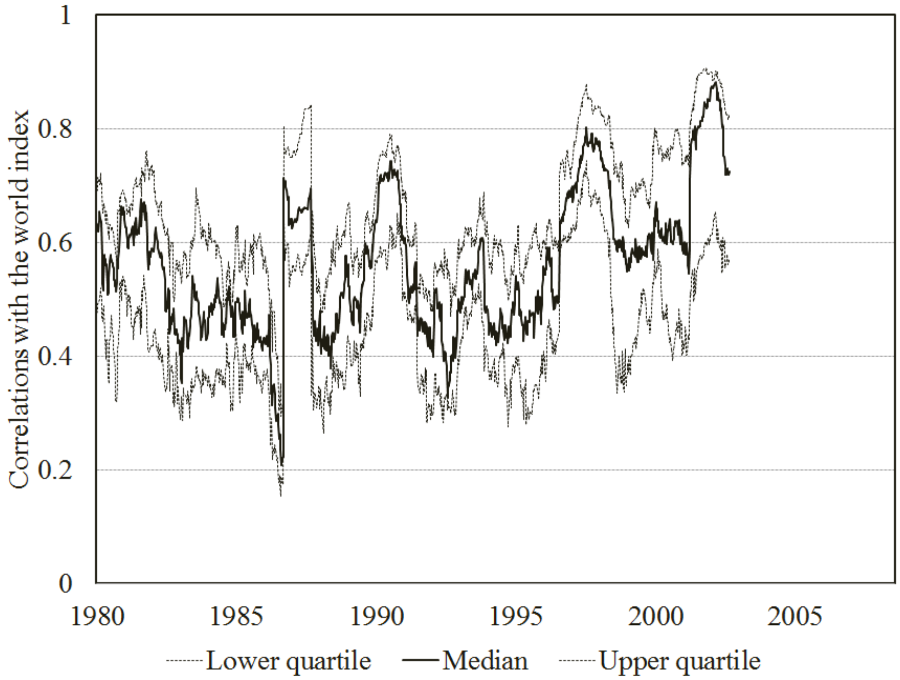

Figure 4.2: Stock Market Synchronization over Time

This figure depicts the rolling correlations between national and world stock returns over time. The lower quartile, median, and upper quartile values of 24 countries are taken for one-year rolling window; the solid line is the median, and the two dotted lines are the lower and upper quartiles. Stock return data is computed using the MSCI World index, and the MSCI country indexes, complied from DataStream. Observations are weekly and converted into US dollars using nominal exchange rates.

	Obs.	Mean	St. Dev.	Min	P25	Median	P75	Max
CORR1	1928	0.35	0.20	-0.47	0.23	0.34	0.48	0.93
CORR2	1928	0.28	0.18	-0.27	0.15	0.26	0.39	0.92
CORR3	1928	0.61	0.45	-0.55	0.31	0.55	0.85	3.27
FININT1	1928	10.04	1.16	7.19	9.20	9.94	10.80	14.21
FININT2	1928	4.77	2.42	-3.17	3.09	4.84	6.61	11.37
TRAINT	1928	0.61	1.28	-4.89	-0.24	0.55	1.48	4.21
TCOST	1928	3.04	0.47	1.61	2.79	3.05	3.42	3.86
DIST	1928	7.66	1.13	4.93	6.74	7.63	8.64	9.42
FXVOL	1928	0.11	0.10	0.00	0.04	0.09	0.14	0.73

Table 4.1: Sample Statistics

The table reports summary statistics of the main variables used in the regression analysis. $CORR1$ is the realized correlations between national stock returns for each four-year period. $CORR2$ is the volatility-corrected correlations between national stock returns (aka Forbes and Rigobon, 2002). $CORR3$ is the volatility-corrected correlations between national stock returns after the Fisher-Z transformation. $FINIINT1$ is calculated as the logarithm of the product of two countries' respective measures of financial integration, that is, the sum of assets and liabilities as a percent of GDP. $FININT2$ is the same as $FININT1$ except that financial integration is measured by the sum of portfolio equity assets and liabilities as a percent of GDP. $TRAINT$ denotes the logarithm of bilateral imports and exports as a percent of the two countries' GDP. $TCOST$ is a bilateral index of trade costs taken from Fitzgerald (2008). $DIST$ is the logarithm of distance between the two countries' capitals. $FXVOL$ is the standard deviations of percent changes in the end-year exchange rates over each four-year period.

Variables	(1) OLS	(2) OLS	(3) OLS	(4) 2SLS Reduced form	1st stage: financial integration	1st stage: trade integration	2nd stage	(5) 2SLS
Financial integration	0.231***	0.070***	0.057***				0.436**	0.310**
(*FININT1*)	(0.017)	(0.021)	(0.020)				(0.171)	(0.140)
Trade integration		0.127***	0.117***				0.046	0.097***
(*TRAINT*)		(0.013)	(0.018)				(0.075)	(0.034)
Exchange rate volatility		-1.531***	-1.616***					-0.023
(*FXVOL*)		(0.255)	(0.264)					(0.887)
Trade costs			-0.040	-0.428***	-0.759***	-2.217***		
(*TCOST*)			(0.055)	(0.052)	(0.155)	(0.129)		
Common Language			0.115**	0.142**	0.361*	-0.249		
(*COMLANG*)			(0.045)	(0.057)	(0.187)	(0.154)		
Joint EU members			-0.101	0.294**	0.568***	0.322		
(*EU*)			(0.111)	(0.132)	(0.261)	(0.285)		
Observations	276	276	276	276	276	276	276	276
R-squared	0.335	0.613	0.626	0.398	0.219	0.607	0.108	0.407
1st stage F-score					25.15	153.09		
p-value					0	0		

Table 4.2: Cross-sectional Estimates, 1992-2003

This table shows the results of cross-sectional regressions. The dependent variable is $CORR3$, the volatility-corrected correlations between national stock returns. All variables except the exchange rate volatility are averages over the period 1992-2003. The exchange rate volatility is the standard deviations of percent changes in the end-year exchange rates for the period 1992-2003. The robust standard errors are reported in brackets. The symbols *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

Variables	(1) OLS	(2) OLS	(3) 2SLS	(4) 2SLS	(5) RE 2SLS	(6) RE 2SLS
Financial integration	0.147***	0.116***	0.311***	0.348***	0.263***	0.343***
(*FININT*)	(0.009)	(0.010)	(0.069)	(0.072)	(0.031)	(0.065)
Trade integration	0.103***	0.109***	0.112***	0.092***	0.136***	0.093***
(*TRAINT*)	(0.009)	(0.009)	(0.029)	(0.026)	(0.027)	(0.026)
Exchange rate volatility	-0.109*	-0.349***	-0.074	-0.194**	-0.004	-0.217**
(*FXVOL*)	(0.063)	(0.067)	(0.082)	(0.088)	(0.092)	(0.095)
Trade costs	-0.135***	-0.108***				
(*TCOST*)	(0.027)	(0.026)				
Common Language	0.063**	0.087***				
(*COMLANG*)	(0.028)	(0.026)				
Joint EU members	0.328**	0.203				
(*EU*)	(0.161)	(0.163)				
Period dummy	No	Yes	No	Yes	Yes	Yes
Country-pair effects	No	No	No	No	No	Yes
Observations	1,652	1,652	1,652	1,652	1,652	1,652
County pairs					276	276
R-squared	0.459	0.529	0.341	0.367		
Hausman test					0.34	3.86
p-value					0.95	0.92

Table 4.3: Panel Estimates, Four-year averages, 1980-2003

This table shows the results of panel regression with four-year data. The dependent variable is $CORR3$, the volatility-corrected correlations between national stock returns. All variables except the exchange rate volatility are averages for each four-year period over 1980-2003. The exchange rate volatility is the standard deviations of percent changes in the end-year exchange rates for each four-year period. The robust standard errors are reported in brackets. The symbols *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

Variables	(1) Dep: *CORR2*	(2) *DIST* instead of *TCOST*	(3) 1980-1991	(4) 1992-2003	(5) 8-year averages
Financial integration	0.088***	0.391***	0.269**	0.391***	0.304**
(*FININT*)	(0.025)	(0.083)	0.109	(0.084)	(0.128)
Trade integration	0.054***	0.083**	0.124**	0.083***	0.094*
(*TRAINT*)	(0.010)	(0.040)	(0.051)	(0.028)	(0.051)
Exchange rate volatility	-0.100***	-0.316***	-0.283	0.073	-0.728**
(*FXVOL*)	(0.032)	(0.092)	(0.184)	(0.224)	(0.315)
Period dummy	Yes	Yes	Yes	Yes	Yes
Country-pair effects	Yes	Yes	Yes	Yes	No
Observations	1,652	1,652	548	1,104	447
County pairs	276	276	276	276	276

Table 4.4: Sensitivity Analysis

This table shows the results of panel regression with four-year data, unless otherwise indicated. The dependent variable is $CORR3$, the volatility-corrected correlations between national stock returns, except for column (1). In Column (1), the dependent variable is CORR2, the correlations between national stock returns without volatility correction aka Forbes and Rigobon (2002). All variables except the exchange rate volatility are averages for each four- or eight-year period over 1980-2003. The exchange rate volatility is the standard deviations of percent changes in the end-year exchange rates for each four-year period. The robust standard errors are reported in brackets. The symbols *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

APPENDIX A

PROOFS AND DERIVATIONS

Section 2.2.5: The market clearing condition for the Home goods implies

$$Y^H(t)\,\boldsymbol{\sigma}^{Y^H} = c_H^H(t)\,\boldsymbol{\sigma}^{c_H^H}(t) + c_F^H(t)\,\boldsymbol{\sigma}^{c_F^H}(t) \tag{A.1}$$

Apply Ito's lemma to the first order conditions:

$$\begin{aligned} \boldsymbol{\sigma}^{c_H^H}(t) &= \frac{1}{\gamma}\left[\boldsymbol{\sigma}^{\delta_H}(t) - \boldsymbol{\theta}_H(t)\right] \\ \boldsymbol{\sigma}^{c_F^H}(t) &= \frac{1}{\gamma}\left[\boldsymbol{\sigma}^{\delta_F}(t) - \boldsymbol{\theta}_F(t)\right] \end{aligned} \tag{A.2}$$

Plug these in the the market clearing condition for the Home goods:

$$Y^H(t)\,\boldsymbol{\sigma}^{Y^H} = c_H^H(t)\,\frac{1}{\gamma}\left[\boldsymbol{\sigma}^{\delta_H}(t) - \boldsymbol{\theta}_H(t)\right] + c_F^H(t)\,\frac{1}{\gamma}\left[\boldsymbol{\sigma}^{\delta_F}(t) - \boldsymbol{\theta}_F(t)\right] \tag{A.3}$$

The market prices of risks are given by solving for $\boldsymbol{\theta}_H(t)$ and $\boldsymbol{\theta}_F(t)$ using $\boldsymbol{\sigma}^{\lambda}(t) = \boldsymbol{\theta}_F(t) - \boldsymbol{\theta}_H(t)$.

Section 2.3.1-a

The variance of terms-of-trade is given by

$$\begin{aligned} VAR\left(\frac{dp(t)}{p(t)}\right) &= VAR\begin{pmatrix} -A(t)\left(\frac{d\delta_H(t)}{\delta_H(t)} - \frac{d\delta_F(t)}{\delta_F(t)} - \frac{d\lambda(t)}{\lambda(t)}\right) \\ +\frac{dY^H(t)}{Y^H(t)} - \frac{dY^F(t)}{Y^F(t)} \end{pmatrix} \\ &= \left\{\begin{array}{c} [A(t)]^2\left\|\boldsymbol{\sigma}_{\delta_H} - \boldsymbol{\sigma}_{\delta_F} - \boldsymbol{\sigma}_{\lambda}\right\|^2 \\ +\left\|A(t)\,\boldsymbol{\sigma}_{\lambda} + \boldsymbol{\sigma}_{Y^H} - \boldsymbol{\sigma}_{Y^F}\right\|^2 - [A(t)]^2\left\|\boldsymbol{\sigma}_{\lambda}\right\|^2 \end{array}\right\} dt \end{aligned} \tag{A.4}$$

The covariance between the two gain processes is

$$\begin{aligned} & COV\left(\frac{dS^H(t) + Y^H(t)\,dt}{S^H(t)}, \frac{dS^F(t) + p(t)\,Y^F(t)\,dt}{S^F(t)}\right) \\ =\ & COV\left(\frac{dY^H(t)}{Y^H(t)}, \frac{dY^F(t)}{Y^F(t)}\right) + COV\left(\frac{dY^H(t)}{Y^H(t)}, \frac{dp(t)}{p(t)}\right) \\ =\ & \left[A(t)\,\boldsymbol{\sigma}_{Y_H}\cdot\boldsymbol{\sigma}_{\lambda} + \left\|\boldsymbol{\sigma}_{Y^H}\right\|^2\right] dt \end{aligned} \tag{A.5}$$

The variance of the Home gain process is

$$VAR\left(\frac{dS^{H}(t)+Y^{H}(t)\,dt}{S^{H}(t)}\right) = VAR\left(\frac{dS^{H}(t)}{S^{H}(t)}\right) = VAR\left(\frac{dY^{H}(t)}{Y^{H}(t)}\right) = \left\|\boldsymbol{\sigma}_{Y^{H}}\right\|^{2} dt \tag{A.6}$$

The variance of the Foreign gain process is

$$\begin{aligned} & VAR\left(\frac{dS^{F}(t)+p(t)\,Y^{F}(t)\,dt}{S^{F}(t)}\right) \\ = \; & VAR\left(\frac{dY^{F}(t)}{Y^{F}(t)}+\frac{dp(t)}{p(t)}\right) \\ = \; & VAR\left(-A(t)\left(\frac{d\delta_{H}(t)}{\delta_{H}(t)}-\frac{d\delta_{F}(t)}{\delta_{F}(t)}-\frac{d\lambda(t)}{\lambda(t)}\right)+\frac{dY^{H}(t)}{Y^{H}(t)}\right) \\ = \; & \left\|-A(t)\left(\boldsymbol{\sigma}^{\delta_{H}}-\boldsymbol{\sigma}^{\delta_{F}}-\boldsymbol{\sigma}^{\lambda}\right)+\boldsymbol{\sigma}^{Y^{H}}\right\|^{2} dt \end{aligned} \tag{A.7}$$

The correlation between two stock returns is given by

$$\rho_{S^{H}S^{F}}(t)=\frac{\boldsymbol{\sigma}_{S^{H}S^{F}}(t)}{\left\|\boldsymbol{\sigma}_{S^{H}}(t)\right\|\left\|\boldsymbol{\sigma}_{S^{F}}(t)\right\|}. \tag{A.8}$$

Section 2.3.1-b: I consider the case with asymmetry related to the center of perceived output growth rates. Define the differences in the initial levels as $\tau \equiv \mu_{H}^{Y^{H}}(0)-\mu_{F}^{Y^{H}}(0)=\mu_{F}^{Y^{F}}(0)-\mu_{H}^{Y^{F}}(0)$. Since this value τ is a difference between perceived output growth across country, it is reasonable to assume that the value is small enough to make all perceived rates to equal to or greater than 0.Define the degree of consumption home bias $a \equiv \frac{a_{H}-a_{F}}{2}>0$. At the symmetric equilibrium, we get by combining equations 2.18. 2.20, and 2.22:

$$\lambda(0)=\frac{1+2a}{1-2a}>0.$$

Equation 2.25 is

$$A(0) = \frac{-2a}{1+4a^2} < 0. \tag{A.9}$$

Thus, the derivatives of λ and A with respect to a are

$$\frac{d\lambda(0)}{da} = \frac{4}{(1-2a)^2} > 0, \text{ and} \tag{A.10}$$

$$\frac{dA(0)}{da} = \frac{-2(1+2a)(1-2a)}{(1+4a^2)^2} < 0. \tag{A.11}$$

From equations 2.9 and 2.21, the vectors of partial derivative of $\boldsymbol{\sigma}^{\lambda}$ with respect to a is given by

$$\frac{d\boldsymbol{\sigma}^{\lambda}(0)}{da} = \mathbf{0}. \tag{A.12}$$

By combining this with equations 2.24, the vectors of partial derivative of $\boldsymbol{\sigma}^{p}(t)$ with respect to a is given by

$$\frac{d\boldsymbol{\sigma}^{p}}{da} = -\frac{dA(0)}{da}\boldsymbol{\sigma}^{\lambda}(0) \tag{A.13}$$

Using equations 2.28, A.10, and A.12, the derivative of $\sigma_{S^H S^F}$ with respect to a is expressed as

$$\frac{d\sigma_{S^H S^F}(0)}{da} = -\frac{dA(0)}{da}\boldsymbol{\sigma}_{Y_H} \cdot \boldsymbol{\sigma}^{\lambda}(0). \tag{A.14}$$

Similarly, from equations 2.27, A.12 and A.13, the derivative of $\|\boldsymbol{\sigma}_{S^F}(t)\|$ with respect to a is

$$\frac{d\left\|\boldsymbol{\sigma}^{S^F}(0)\right\|}{da} = \left\|\boldsymbol{\sigma}^{S^F}(0)\right\|^{-1}\boldsymbol{\sigma}^{S^F}(0) \cdot \frac{d\boldsymbol{\sigma}^{p}(0)}{da} \tag{A.15}$$

Finally, using equations 2.29,A.14, and A.15, we get the derivative of the equilibrium correlations of stock returns $\rho_{S^H S^F}$ with respect to a:

$$\begin{aligned} \frac{d\rho_{S^H S^F}(0)}{da} &= \frac{1}{\|\boldsymbol{\sigma}_{S^H}(0)\| \|\boldsymbol{\sigma}_{S^F}(0)\|}\begin{bmatrix} -\frac{dA(0)}{da}\boldsymbol{\sigma}_{Y_H} \cdot \boldsymbol{\sigma}^{\lambda}(0) \\ -\frac{\sigma_{S^H S^F}(0)}{\|\boldsymbol{\sigma}_{S^F}(0)\|}\frac{d\|\boldsymbol{\sigma}_{S^F}(0)\|}{da} \end{bmatrix} \\ &< 0. \end{aligned}$$

Section 2.3.1-c: From equations 2.9 and 2.21, the vectors of partial derivative of $\boldsymbol{\sigma}^{\lambda}$ with respect to τ is given by

$$\frac{\partial \boldsymbol{\sigma}^{\lambda}(0)}{\partial \tau} = \begin{pmatrix} -\frac{1}{\sigma^{Y^H}} \\ \frac{1}{\sigma^{Y^F}} \\ 0 \end{pmatrix}. \tag{A.16}$$

By combining this with equations 2.24, the vectors of partial derivative of $\boldsymbol{\sigma}^{p}(t)$ with respect to τ is given by

$$\frac{\partial \boldsymbol{\sigma}^{p}}{\partial \tau} = -A(0)\frac{\partial \boldsymbol{\sigma}^{\lambda}(0)}{\partial \tau} \tag{A.17}$$

Using equations 2.28, A.10, and **??**, the derivative of $\sigma_{S^H S^F}$ with respect to τ is expressed as

$$\frac{d\sigma_{S^H S^F}(0)}{d\tau} = -A(0)\boldsymbol{\sigma}_{Y_H}\cdot\frac{\partial \boldsymbol{\sigma}^{\lambda}(0)}{\partial \tau}. \tag{A.18}$$

Similarly, from equations 2.27, **??** and **??**, the derivative of $\|\boldsymbol{\sigma}_{S^F}(t)\|$ with respect to τ is

$$\begin{aligned}\frac{d\left\|\boldsymbol{\sigma}^{S^F}(0)\right\|}{d\tau} &= \frac{d}{d\tau}\left(\boldsymbol{\sigma}^{Y^F}\cdot\boldsymbol{\sigma}^{Y^F}+2\boldsymbol{\sigma}^{Y^F}\cdot\boldsymbol{\sigma}^{p}(0)+\boldsymbol{\sigma}^{p}(0)\cdot\boldsymbol{\sigma}^{p}(0)\right)^{\frac{1}{2}} \\ &= \left\|\boldsymbol{\sigma}^{S^F}(0)\right\|^{-1}\boldsymbol{\sigma}^{S^F}(0)\cdot\frac{d\boldsymbol{\sigma}^{p}(0)}{d\tau}\end{aligned} \tag{A.19}$$

Finally, using equations 2.29,A.18, and **??**, we get the derivative of the equilibrium correlations of stock returns $\rho_{S^H S^F}$ with respect to a:

$$\begin{aligned}\frac{d\rho_{S^H S^F}(0)}{d\tau} &= \frac{1}{\|\boldsymbol{\sigma}_{S^H}(0)\|\,\|\boldsymbol{\sigma}_{S^F}(0)\|}A(0)\left[-\boldsymbol{\sigma}_{Y_H}+\sigma_{S^H S^F}(0)\boldsymbol{\sigma}^{S^F}(0)\right]\cdot\frac{\partial \boldsymbol{\sigma}^{\lambda}(0)}{\partial \tau} \\ &< 0.\end{aligned} \tag{A.20}$$

Section 2.3.2: The equilibrium wealth is

$$\begin{aligned}W_H(t) &= E_{H,t}\left[\int_t^T \frac{\xi_H(s)}{\xi_H(t)}\left(c_H^H(s)+p(s)\,c_H^F(s)\right)dt\right] \\ &= \frac{\delta_H(t)\,Y^H(t)}{\delta_H(t)\,a_H+\lambda(t)\,\delta_F(t)\,a_F}\frac{1-e^{-\rho(T-t)}}{\rho}\end{aligned} \tag{A.21}$$

$$W_F(t) = E_{F,t}\left[\int_t^T \frac{\xi_F(s)}{\xi_F(t)}\left(c_F^H(s) + p(s)\,c_F^F(s)\right)dt\right]$$
$$= \frac{\lambda(t)\,\delta_F(t)\,Y^H(t)}{\delta_H(t)\,a_H + \lambda(t)\,\delta_F(t)\,a_F}\frac{1-e^{-\rho(T-t)}}{\rho}$$

Market clearing condition for wealth implies that

$$S^H(t) + S^F(t) = W_H(t) + W_F(t), \tag{A.22}$$
$$E_{H,t}\left[\int_t^T e^{-\rho(s-t)}\lambda(s)\,ds\right] = \lambda(t)\,\frac{1-e^{-\rho(T-t)}}{\rho}.$$

Substitute it to the equilibrium stock prices gives

$$S^H(t) = Y^H(t)\,\frac{1-e^{-\rho(T-t)}}{\rho} \tag{A.23}$$
$$S^F(t) = p(t)\,Y^F(t)\,\frac{1-e^{-\rho(T-t)}}{\rho}$$

Apply Ito's Lemma to the equilibrium stock prices

$$\frac{dS^H(t)}{S^H(t)} = [\text{Ito terms}]\,dt + \frac{dY^H(t)}{Y^H(t)} \tag{A.24}$$
$$\frac{dS^F(t)}{S^F(t)} = [\text{Ito terms}]\,dt + \frac{dY^F(t)}{Y^F(t)} + \frac{dp(t)}{p(t)}$$

APPENDIX B

APPROXIMATIONS FOR THE TERMS OF TRADE

The relative output ratio b is defined for convenience as:

$$b(t) = \frac{Y^H(t)}{Y^F(t)}. \tag{B.1}$$

The market clearing condition for Home goods (equation 3.8) gives

$$\begin{aligned} & \alpha(\tau, p(t))^{-\frac{1-\gamma}{\gamma}\frac{1}{1-\psi}-1}\left[(1-a)(1+\tau)^{1-\psi} p(t)^{1-\psi} b(t) - ap(t)\right] \\ = \;& \beta(\tau, p(t))^{-\frac{1-\gamma}{\gamma}\frac{1}{1-\psi}-1}\left[a(1+\tau)^{1-\psi} p(t) - b(t)(1-a)p(t)^{1-\psi}\right]\lambda^{\frac{1}{\gamma}} \end{aligned} \tag{B.2}$$

where $\alpha(\tau, p(t)) = a + (1-a)(1+\tau)^{1-\psi} p(t)^{1-\psi}$ and $\beta(\tau, p(t)) = a(1+\tau)^{1-\psi} + (1-a)p(t)^{1-\psi}$.

Define $F(p(t), \tau)$ such that $F(p(t), \tau)$ satisfies

$$\begin{aligned} F(p(t), \tau) = \;& \alpha(\tau, p(t))^{-\frac{1-\gamma}{\gamma}\frac{1}{1-\psi}-1}\left[(1-a)(1+\tau)^{1-\psi} p(t)^{1-\psi} b(t) - ap(t)\right] \\ & -\lambda^{\frac{1}{\gamma}}\beta(\tau, p(t))^{-\frac{1-\gamma}{\gamma}\frac{1}{1-\psi}-1}\left[a(1+\tau)^{1-\psi} p(t) - b(t)(1-a)p(t)^{1-\psi}\right] \end{aligned} \tag{B.3}$$

Using partial derivatives of $F(p(t), \tau)$ with respect to p and τ, the values of p, $\frac{\partial F}{\partial \tau}, \frac{\partial F}{\partial p}$ are evaluated at $\tau = 0$:

$$\begin{aligned} [p(t)]_{\tau=0} &= \left(\frac{1-a}{a}\right)^{\frac{1}{\psi}} b(t)^{\frac{1}{\psi}} \\ \left[\frac{\partial F(p(t), \tau)}{\partial \tau}\right]_{\tau=0} &= (1-\psi)\,\delta(b(t))\left(\frac{1-a}{a}\right)^{\frac{1}{\psi}} b(t)^{\frac{1}{\psi}}\left(1-\lambda^{\frac{1}{\gamma}}\right) \\ \left[\frac{\partial F(p(t), \tau)}{\partial p(t)}\right]_{\tau=0} &= -\delta(b(t))\,\psi\left(1+\lambda^{\frac{1}{\gamma}}\right) \end{aligned} \tag{B.4}$$

where $\delta(b(t)) = a^{-\frac{1-\gamma}{\gamma}\frac{1}{1-\psi}}\left[1+\left(\frac{1-a}{a}\right)^{\frac{1}{\psi}} b(t)^{\frac{1-\psi}{\psi}}\right]^{-\frac{1-\gamma}{\gamma}\frac{1}{1-\psi}-1}$.

Similarly, $\frac{\partial^2 F}{\partial \tau^2}$, and $\frac{\partial^2 F}{\partial p^2}$ are evaluated at $\tau = 0$:

$$\left[\frac{\partial^2 F(p(t),\tau)}{\partial \tau^2}\right]_{\tau=0} = -(1-\psi)\,\delta(b(t))\left(\frac{1-a}{a}\right)^{\frac{1}{\psi}} b(t)^{\frac{1}{\psi}} \left[\begin{array}{c} 2\left(\frac{1}{\gamma}-\psi\right)\frac{\left(\frac{1-a}{a}\right)^{\frac{1}{\psi}} b(t)^{\frac{1-\psi}{\psi}}-\lambda^{\frac{1}{\gamma}}}{1+\left(\frac{1-a}{a}\right)^{\frac{1}{\psi}} b(t)^{\frac{1-\psi}{\psi}}} \\ +\psi\left(1-\lambda^{\frac{1}{\gamma}}\right) \end{array}\right] \quad \text{(B.5)}$$

$$\left[\frac{\partial^2 F(p(t),\tau)}{\partial p(t)^2}\right]_{\tau=0} = \psi\left(1+\lambda^{\frac{1}{\gamma}}\right)\delta(b(t))\left[\begin{array}{c} 2\left(\frac{1}{\gamma}-\psi\right)\frac{b(t)^{-1}}{1+\left(\frac{1-a}{a}\right)^{\frac{1}{\psi}} b(t)^{\frac{1-\psi}{\psi}}} \\ -(1-\psi)\left(\frac{1-a}{a}\right)^{-\frac{1}{\psi}} b(t)^{-\frac{1}{\psi}} \end{array}\right] \quad \text{(B.6)}$$

Given these values, $\frac{dp}{d\tau}$ and $\frac{d^2p}{d\tau^2}$ are evaluated at $\tau=0$ as follows:

$$\begin{aligned} \left[\frac{dp}{d\tau}\right]_{\tau=0} &= \frac{\left[\frac{\partial F}{\partial \tau}\right]_{\tau=0}}{\left[\frac{\partial F}{\partial p}\right]_{\tau=0}} \\ &= \frac{\psi-1}{\psi}\frac{1-\lambda^{\frac{1}{\gamma}}}{1+\lambda^{\frac{1}{\gamma}}}[p(t)]_{\tau=0} \end{aligned} \quad \text{(B.7)}$$

$$\begin{aligned} \left[\frac{d^2p(t)}{d\tau^2}\right]_{\tau=0} &= \frac{\left[\frac{\partial^2 F(p(t),\tau)}{\partial \tau^2}\right]_{\tau=0}}{\left[\frac{\partial F(p(t),\tau)}{\partial p(t)}\right]_{\tau=0}} - \frac{\left[\frac{\partial^2 F(p(t),\tau)}{\partial p(t)^2}\right]_{\tau=0}}{\left[\frac{\partial F(p(t),\tau)}{\partial p(t)}\right]_{\tau=0}}\left(\left[\frac{dp}{d\tau}\right]_{\tau=0}\right)^2 \\ &= \left\{\begin{array}{c} (\psi-1)\frac{1-\lambda^{\frac{1}{\gamma}}}{1+\lambda^{\frac{1}{\gamma}}}\left[\frac{1-\lambda^{\frac{1}{\gamma}}}{1+\lambda^{\frac{1}{\gamma}}}\left(\frac{\psi-1}{\psi}\right)^2-1\right]\left(\frac{1-a}{a}\right)^{\frac{1}{\psi}} b(t)^{\frac{1}{\psi}} \\ +2\left(\frac{1}{\gamma}-\psi\right)\frac{\psi-1}{\psi}\frac{\lambda^{\frac{1}{\gamma}}}{1+\lambda^{\frac{1}{\gamma}}}\frac{\left(\frac{1-a}{a}\right)^{\frac{1}{\psi}} b(t)^{\frac{1}{\psi}}}{1+\left(\frac{1-a}{a}\right)^{\frac{1}{\psi}} b(t)^{\frac{1-\psi}{\psi}}} \\ +2\left(\frac{1}{\gamma}-\psi\right)\frac{\psi-1}{\psi}\left[\frac{\psi-1}{\psi}\left(\frac{1-\lambda^{\frac{1}{\gamma}}}{1+\lambda^{\frac{1}{\gamma}}}\right)^2-\frac{1}{1+\lambda^{\frac{1}{\gamma}}}\right]\frac{\left(\frac{1-a}{a}\right)^{\frac{2}{\psi}} b(t)^{\frac{2-\psi}{\psi}}}{1+\left(\frac{1-a}{a}\right)^{\frac{1}{\psi}} b(t)^{\frac{1-\psi}{\psi}}} \end{array}\right\} \end{aligned} \quad \text{(B.8)}$$

The first-order approximation of terms of trade is given:

$$\begin{aligned} p(t) &= [p(t)]_{\tau=0} + \tau\left[\frac{dp(t)}{d\tau}\right]_{\tau=0} + o(\tau) \\ &= \left(1+\tau\frac{\psi-1}{\psi}\frac{1-\lambda^{\frac{1}{\gamma}}}{1+\lambda^{\frac{1}{\gamma}}}\right)\left(\frac{1-a}{a}\frac{Y^H(t)}{Y^F(t)}\right)^{\frac{1}{\psi}} + o(\tau) \end{aligned} \quad \text{(B.9)}$$

and, finally, the second-order approximation of terms of trade is:

$$p(t) = [p(t)]_{\tau=0} + \tau\left[\frac{dp(t)}{d\tau}\right]_{\tau=0} + \frac{\tau^2}{2}\left[\frac{d^2p(t)}{d\tau^2}\right]_{\tau=0} + o\left(\tau^2\right) \tag{B.10}$$

$$= \left\{\begin{array}{c} \left(\frac{1-a}{a}\right)^{\frac{1}{\psi}} b(t)^{\frac{1}{\psi}} \\ +\left[\tau + \frac{\tau^2}{2}\psi\left(\frac{1-\lambda^{\frac{1}{\gamma}}}{1+\lambda^{\frac{1}{\gamma}}}\left(\frac{\psi-1}{\psi}\right)^2 - 1\right)\right]\frac{\psi-1}{\psi}\frac{1-\lambda^{\frac{1}{\gamma}}}{1+\lambda^{\frac{1}{\gamma}}}\left(\frac{1-a}{a}\right)^{\frac{1}{\psi}} b(t)^{\frac{1}{\psi}} \\ +\tau^2\left(\frac{1}{\gamma}-\psi\right)\frac{\psi-1}{\psi}\frac{\lambda^{\frac{1}{\gamma}}}{1+\lambda^{\frac{1}{\gamma}}}\frac{\left(\frac{1-a}{a}\right)^{\frac{1}{\psi}}b(t)^{\frac{1}{\psi}}}{1+\left(\frac{1-a}{a}\right)^{\frac{1}{\psi}}b(t)^{\frac{1-\psi}{\psi}}} \\ +\tau^2\left(\frac{1}{\gamma}-\psi\right)\frac{\psi-1}{\psi}\left[\frac{\psi-1}{\psi}\left(\frac{1-\lambda^{\frac{1}{\gamma}}}{1+\lambda^{\frac{1}{\gamma}}}\right)^2 - \frac{1}{1+\lambda^{\frac{1}{\gamma}}}\right]\frac{\left(\frac{1-a}{a}\right)^{\frac{2}{\psi}}b(t)^{\frac{2-\psi}{\psi}}}{1+\left(\frac{1-a}{a}\right)^{\frac{1}{\psi}}b(t)^{\frac{1-\psi}{\psi}}} \end{array}\right\}$$

By applying Ito's Lemma to the first- and second approximations of terms of trade, the diffusions for both approximations are given:

$$\frac{D_t p(t)}{p(t)} = \frac{\boldsymbol{\sigma}^{Y^H} - \boldsymbol{\sigma}^{Y^F}}{\psi} + \mathbf{o}(\tau) \tag{B.11}$$

up to the first-order, and

$$\frac{D_t p_2(t)}{p_2(t)} = \left\{\begin{array}{c} 1 + \tau\frac{\psi-1}{\psi}\frac{1-\lambda^{\frac{1}{\gamma}}}{1+\lambda^{\frac{1}{\gamma}}} - \frac{\tau^2}{2}(\psi-1)\frac{1-\lambda^{\frac{1}{\gamma}}}{1+\lambda^{\frac{1}{\gamma}}} + \frac{\tau^2}{2}\left(\frac{\psi-1}{\psi}\frac{1-\lambda^{\frac{1}{\gamma}}}{1+\lambda^{\frac{1}{\gamma}}}\right)^2(\psi-1) \\ +\frac{1+\psi p_0(t)b(t)^{-1}}{\left[1+p_0(t)b(t)^{-1}\right]^2}\tau^2\frac{\psi-1}{\psi}\frac{\lambda^{\frac{1}{\gamma}}}{1+\lambda^{\frac{1}{\gamma}}}\left(\frac{1}{\gamma}-\psi\right) \\ -\frac{2-\psi+p_0(t)b(t)^{-1}}{\left[1+p_0(t)b(t)^{-1}\right]^2}p_0(t)\,b(t)^{-1}\,\tau^2\frac{\psi-1}{\psi}\left(\frac{1}{\gamma}-\psi\right)\left[\frac{1}{1+\lambda^{\frac{1}{\gamma}}} - \frac{\psi-1}{\psi}\left(\frac{1-\lambda^{\frac{1}{\gamma}}}{1+\lambda^{\frac{1}{\gamma}}}\right)^2\right] \end{array}\right\} \tag{B.12}$$

for the second-order.

APPENDIX C

MALLIAVIN DERIVATIVES

In this appendix, I derive explicit expression for the Malliavin derivatives. The Malliavin derivative of the first-order approximation of the terms of trade process is

$$\frac{D_t p(t)}{p(t)} = \frac{1}{\psi}\left[\frac{D_t Y^H(t)}{Y^H(t)} - \frac{D_t Y^F(t)}{Y^F(t)}\right]. \tag{C.1}$$

The Malliavin derivative of the wealth weight for Home agent h is

$$\frac{D_t h(t)}{h(t)} = \frac{1-\gamma}{\gamma}\frac{[1-h(t)]\,a(1-a)\,p(t)^{1-\psi}\left[1-(1+\tau)^{2(1-\psi)}\right]}{\left[a(1+\tau)^{1-\psi}+(1-a)\,p(t)^{1-\psi}\right]\left[a+(1-a)(1+\tau)^{1-\psi}p(t)^{1-\psi}\right]}\frac{D_t p(t)}{p(t)}. \tag{C.2}$$

The Malliavin derivatives of the consumption expenditure κ_H and κ_F are

$$\begin{aligned}\frac{D_t\kappa_H(t)}{\kappa_H(t)} &= \frac{D_t h(t)}{h(t)} + \frac{Y^H(t)}{Y^H(t)+p(t)\,Y^F(t)}\frac{D_t Y^H(t)}{Y^H(t)} \\ &\quad + \frac{p(t)\,Y^F(t)}{Y^H(t)+p(t)\,Y^F(t)}\left(\frac{D_t Y^F(t)}{Y^F(t)} + \frac{D_t p(t)}{p(t)}\right), \\ \frac{D_t\kappa_F(t)}{\kappa_F(t)} &= \frac{D_t\kappa_H(t)}{\kappa_H(t)} - \left(\frac{1}{1-h(t)}\right)\frac{D_t h(t)}{h(t)}.\end{aligned} \tag{C.3}$$

The Malliavin derivative of the state price density process is

$$\frac{D_t\xi(t)}{\xi(t)} = -(1-\gamma)\frac{(1-a)(1+\tau)^{1-\psi}p(t)^{1-\psi}}{a+(1-a)(1+\tau)^{1-\psi}p(t)^{1-\psi}}\frac{D_t p(t)}{p(t)} - \gamma\frac{D_t\kappa_H(t)}{\kappa_H(t)} \tag{C.4}$$

The Malliavin derivatives of equity prices are

$$\frac{D_t S^H(t)}{S^H(t)} = -\frac{D_t \xi(t)}{\xi(t)} + \frac{D_t Y^H(s)}{Y^H(s)} + \frac{E_t\left[\int_t^T \xi(s) Y^H(s) \frac{D_t \xi(s)}{\xi(s)} ds\right]}{E_t\left[\int_t^T \xi(s) Y^H(s) ds\right]} \tag{C.5}$$

$$\frac{D_t S^F(t)}{S^F(t)} = -\frac{D_t \xi(t)}{\xi(t)} + \frac{D_t Y^F(t)}{Y^F(t)} \tag{C.6}$$

$$+\frac{E_t\left[\int_t^T \xi(s) Y^F(s) p(s) \left(\frac{D_t p(s)}{p(s)} + \frac{D_t \xi(s)}{\xi(s)}\right) ds\right]}{E_t\left[\int_t^T \xi(s) p(s) Y^F(s) ds\right]} \tag{C.7}$$

Finally, the Malliavin derivatives of wealth processes are

$$\frac{D_t W_H(t)}{W_H(t)} = -\frac{D_t \xi(t)}{\xi(t)} + \frac{E_t\left[\int_t^T \xi(s) \kappa_H(s) \left(\frac{D_t \kappa_H(s)}{\kappa_H(s)} + \frac{D_t \xi(s)}{\xi(s)}\right) ds\right]}{E_t\left[\int_t^T \xi(s) \kappa_H(s) ds\right]} \tag{C.8}$$

$$\frac{D_t W_F(t)}{W_F(t)} = -\frac{D_t \xi(t)}{\xi(t)} + \frac{E_t\left[\int_t^T \xi(s) \kappa_F(s) \left(\frac{D_t \kappa_F(s)}{\kappa_F(s)} + \frac{D_t \xi(s)}{\xi(s)}\right) ds\right]}{E_t\left[\int_t^T \xi(s) \kappa_F(s) ds\right]} \tag{C.9}$$

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