Hans Bethe and the Theory of Nuclear Matter

Post-war research helped take nuclear physics from the domain of phenomenology and models to that of many-body theory based on the underlying nucleon–nucleon interaction.

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Prior to World War II, nuclear physics was a phenomenological science, and Hans Bethe was unrivaled in his comprehensive mastery of nuclear phenomena, experimental data, and descriptive models. As described in other articles in this special issue of PHYSICS TODAY, by applying the emerging phenomenology Bethe achieved remarkable successes that ranged from understanding the energy production in stars to guiding the harnessing of nuclear fission as part of the Manhattan Project.

The post-war era offered Hans the opportunity to return to nuclear physics and approach the subject from a deeper theoretical perspective: understanding the many-body structure and properties of nuclei directly in terms of the underlying nuclear interaction. He was freed from the applied-physics demands of the war effort and could again pursue theoretical physics for its own sake. His goals were to understand why the shell model worked in the presence of nuclear forces containing strongly repulsive short-range components; to understand nuclear collective motion; and to calculate the binding energies, excitation energies, nuclear charge distributions, and deformations of atomic nuclei. Once terrestrial nuclei were sufficiently understood, he could use that knowledge as a basis for studying the properties of dense matter in supernovae and neutron stars.

Foundations

One of the keys to his success in confronting complex problems was that Hans could complement his extensive knowledge of a field with an acute ability to separate the essentials from the nonessentials. So it is illuminating to see what he regarded as the essential issues in nuclear physics, and how those guided his approach to the nuclear many-body problem.

I had an inside view of that approach when I took his nuclear-physics course in the fall of 1967 as his graduate research student at Cornell University. That same fall, Hans received the Nobel Prize in Physics. The announcement was made on a lecture day, so we students had the pleasure of watching his multitalented secretary, Velma Ray, tuck his tie neatly under his collar for the photographers. Then we got to hear Hans inform them politely but firmly that they needed to finish their task quickly because he had to start his lecture (see figure 1). Another fond memory connected with the prize was Hans's crash program for learning everything that had happened in stellar evolution since his 1939 paper on energy production in stars.[1] My reward for helping prepare the graphs for his Nobel lecture[2] was a detailed explanation of the physics each graph displayed.

As his starting point for nuclear physics, Hans chose the nuclear force, whose long-range behavior was given uniquely by a one-pion exchange potential. Before the war, Hans had worked out, in his own characteristic physical way, the spin-dependent part of that long-range potential, and he used it with an appropriate cutoff to calculate the properties of the deuteron.[3] The short-range potential could not be calculated by any known theory, so Hans thought about the most physical way to deal with the problem. The first step was to understand that very different potentials produce the same low-energy scattering behavior; that insight led to his famous simplified derivation of effective-range theory.[4] He then adopted the pragmatic approach of using physical arguments to parameterize the strong short-range repulsion, determining the parameters from scattering phase shifts, and then studying the many-body physics the complete potential produced.

An essential feature of nuclear physics, in Hans's view, was that nuclei behaved like quantum liquid drops. Moreover, the binding energy per particle and equilibrium density of the quantum liquid could be deduced from electron scattering experiments and the few-parameter semi-empirical mass formula derived from measurements of nuclear masses. Thus, the central task of nuclear many-body theory was to calculate, from the two-body potential, the properties of nuclear matter—an infinite system of neutrons and protons at equal density interacting through nuclear forces but without Coulomb interactions. Once nuclear matter was properly understood, one could apply many-body theory to finite nuclei and to the material of neutron stars.

Hole-line expansion

The starting point of Hans's solution of the nuclear-matter problem was Keith Brueckner's pioneering approach to solving the two-body scattering problem in the nuclear medium. Brueckner rearranged perturbation theory so that the contribution to the total energy at each order was proportional to the number of particles. Thus, the energy per particle was manifestly finite even in the limit of nuclear matter. Hans, in his usual style, embarked on a systematic program to calculate properties of nuclear matter; his approach was based on a diagrammatic expansion of perturbation theory in a series ordered by the number of interacting particles, that is, the number of so-called hole lines in the contributing diagrams.

The first term in the series involved two particles and incorporated a sum of all two-body scattering processes in the background of other particles, as in Brueckner theory. Hans calculated the reaction matrix, which sums all two-body ladder diagrams. These correspond to processes in which two particles in the Fermi sea rescatter any number of times into unoccupied intermediate states and fi-

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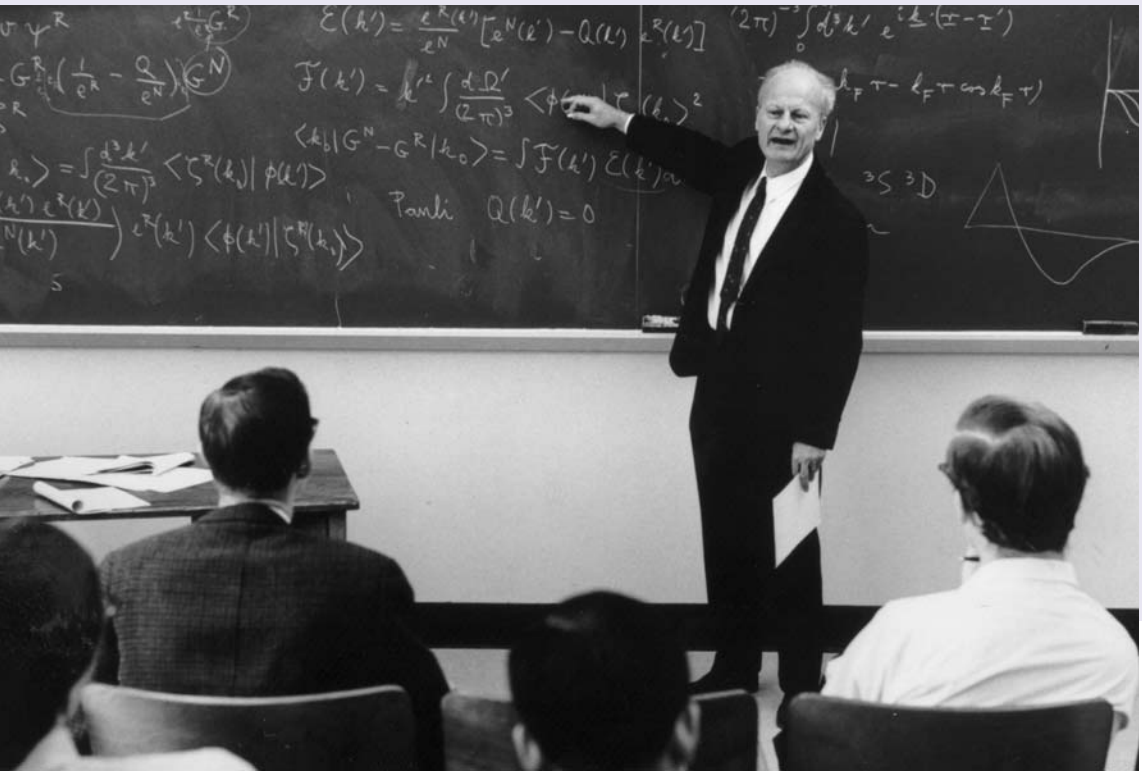

Figure 1. On the day he learned that he would receive the 1967 Nobel Prize in Physics, Hans Bethe insisted on teaching his usual nuclear-physics class. The day's lecture included some topics discussed in this article. Just below Hans's hand is an equation for the reaction matrix, and the graph at the extreme right shows both the probability depletion at small separation, or "wound," in the two-particle wavefunction and the "healing" of that wound at large separation. (Courtesy of Cornell University.)

nally return to the original two states. He found it most physical to formulate his calculation in terms of the two-body wavefunction. As a consequence of the Pauli principle, there is no scattering phase shift. The two-body wavefunction thus has a "wound," where the probability density is depleted due to the short-range repulsive potential, that "heals" to the free, independent-particle wavefunction at large relative separations.[5]

Analytic solutions of special cases were a regular source of insight for Hans, so with Jeffrey Goldstone he solved the case of an infinite hard-core potential. Their solution became a touchstone for calculations with realistic potentials. With Baird Brandow and Albert Petschek, he introduced an approximation in which the particle energy spectrum was treated as quadratic in momentum. That approximation enabled them to convert the integral scattering equation into a differential equation that is easily solved, treat the corrections perturbatively, and include the full complexity of realistic nucleon–nucleon potentials at the two-body level.

Goldstone derived the linked cluster theorem using many-body diagrams,[6] and the resulting Goldstone diagrams became Hans's organizing language for the nuclear many-body problem. In contrast to Feynman diagrams, Goldstone diagrams retain a fundamental distinction between hole lines, which represent propagators for normally occupied single-particle states, and particle lines, which represent propagators for unoccupied states.

Hans showed that the Goldstone-diagram expansion for the binding energy of nuclear matter does not converge in powers of the reaction matrix. Rather, he demonstrated, one must rearrange the expansion in powers of the density or, equivalently, in the number of independent hole lines. Hence, as the next step in the hierarchy, he formulated what is now called the Bethe–Faddeev equation, which sums all three-body ladder diagrams. In this case, three particles in the Fermi sea pairwise scatter to unoccupied states any number of times via the reaction matrix before finally returning to the original three states. Hans's approach generalized to the nuclear medium Ludwig Faddeev's technique for three-body scattering in free space. Hans found it most physical to formulate the problem in terms of the three-body wavefunction, and developed the analytical tools to evaluate the three-body contributions to the binding energy.[7] Terms in the density expansion that correspond to greater numbers of hole lines are treated analogously.

Hans and coworkers fleshed out the basic ideas just sketched. Hans's student Roderick Reid constructed a potential that had a strongly repulsive core and fit experimental phase shifts. Former student Benjamin Day used that potential in an extensive set of nuclear-matter calculations. Overall, the results were quite impressive. Indeed, the hole-line expansion converged as expected—the three-hole-line contributions changed the total potential energy by approximately 13% and the four-hole-line contributions changed it by an estimated 3%. The binding energy per particle of 17 MeV was quite close to the mass-formula value of 16 MeV. The equilibrium density, however, was approximately 30% higher than the value inferred from electron scattering experiments on nuclei. Hans interpreted the convergence as validation of the nuclear-matter theory, and attributed the incorrect equilibrium density to the omission of explicit three-body forces.

Nuclei and neutron stars

Hans's view was that once nuclear matter was under control, finite nuclei and the matter in neutron stars would follow. In keeping with his love of tractable analytical approximations, his starting point for finite nuclei was the Thomas–Fermi approximation. In that scheme, each small region of the nucleus is treated as if it contained nuclear matter with the same density. Hans's theory gave a qualitative description of the surface energy and surface thickness of large nuclei.

After pulsars were observed and subsequently interpreted as neutron stars, Hans applied nuclear-matter ideas to the charge-neutral matter in those stars. With Gordon Baym and Chris Pethick, he considered the range of configurations that result as the density in a star increases from low densities characterized by well-separated nuclei up to densities typically found inside nuclei.[8] The

new feature is that as the density increases, the Fermi energy of the electrons increases, and so it becomes energetically favorable for an electron and proton to combine to make a neutron and also a neutrino that escapes from the star.

Using results for nuclear matter at unequal neutron and proton densities, the researchers were able to calculate the equation of state of matter as neutrons begin to "drip" out of nuclei and form a low-density neutron gas between them. Hans and collaborators could then follow the increase of the neutron density and calculate the eventual merging of individual nuclei into a uniform gas with a high density of neutrons, a low density of protons, and an equally low density of electrons.

With Mikkel Johnson, Hans used nuclear-matter theory to calculate the equation of state of dense matter for several potentials similar to the Reid potential. The two then teamed with Robert Malone to work out the properties of neutron stars based on each equation of state. Their approach, based on phenomenological nuclear potentials fit to phase shifts, remains the most viable method for treating the equation of state for densities up to several times those occurring in nuclei.

Figure 2. Enjoying a moment at the 1967 Nobel Prize ceremonies in Stockholm are Hans Bethe and his wife Rose. (Courtesy of Cornell University.)

Legacy

The body of work Hans contributed to nuclear physics spanned several decades and played an essential role in laying the foundation for an understanding of nuclear structure in terms of the underlying nuclear force. What physicists learned subsequently about strong interactions only reinforces the wisdom of Hans's approach. The nucleon is now understood to be a composite system composed of quarks and gluons, with a spatial size of about one fermi (10^{-15} m). Nucleons, in turn, combine to form a nucleus that typically has a size of several fermis. Because the scales of the nucleon and nucleus are not well separated, one cannot derive an unambiguous nuclear potential at short distance: The only option is to use the pion contribution to describe long-distance behavior and to use an effective theory derived from scattering properties for short-distance behavior.

Furthermore, modern local effective field theory necessarily gives rise to many-body forces whose parameters must be determined from properties of many-body systems. That result is consistent with the idea that a three-body force should be introduced to make the equilibrium density of nuclear matter agree with the value determined from finite nuclei. Indeed, physicists working with mean-field theories containing an effective interaction derived from nuclear matter and a three-body interaction that yields proper equilibrium densities have found them to be extremely successful in determining the binding energies, excitation energies, nuclear charge distributions, and deformations of nuclei throughout the periodic table.

Hans's approach to theoretical physics offers many valuable lessons. He refused to be stymied by a complex problem or incomplete information, and never hesitated to make a physical approximation, if necessary, so that he could proceed toward his objective. He was fearless in introducing a physical cutoff to avoid singularities—whether it be his truncation of the short-range one-pion potential in nuclear physics or his truncation of the high-frequency fluctuations of the electric field as part of his famous estimate of the Lamb shift, which he formulated before renormalization theory was developed for quantum electrodynamics. Analytical calculations, often coupled with simplifying approximations, were among his standard tools for obtaining insight into complicated problems. When, during my student days, I once expressed admiration for the way he reduced a problem to an analytically tractable form, he quipped "Yes, when you get old, you have to reuse the same old tricks."

His methodical approach to complicated problems like the many-body theory of nuclei was truly impressive. Starting from a clear vision of the essential issues and committing to the necessary approximations, he would map out a conceptually clear but calculationally complicated program, and then systematically plow through a huge number of steps.

One can learn much from his expositions. In his publications, he explained every aspect of a problem clearly and completely, without suppressing details. He carefully studied the work of others and cited it meticulously. Once, after he suggested that I include all the relevant details in a paper I was writing, I expressed concern about consequent page charges. He replied that part of the cost of research is publication. He was also admirably straightforward in discussing errors. In a review article about summing three-particle ladder diagrams, he started a paragraph with "Bethe originally proposed . . ." and went on to say "This argument is wrong," followed by a gracious footnote crediting a discussion with colleague Thorolf Dahlblom.

Out of the ordinary

Hans was an extraordinary adviser and mentor. When a student went to see him in his office, he would always interrupt a physics calculation to talk. During my years of graduate study, he traveled frequently to the national labs and the federal government in Washington, DC, to advise them on research, arms control, and the Nuclear Test Ban Treaty. So I was deeply impressed that he would come to Newman Laboratory on campus on Saturdays and make himself available to students. The flip side of that availability was that, as I learned early on, one should be prepared to receive a call from him at 5:00pm on a Saturday afternoon to discuss a research idea.

In this age when it appears obligatory to expend so much professional time on university and national committees, fundraising, writing and reviewing proposals, and when the pendulum has swung so far toward promoting discussions, seminars, and interactions at every available opportunity, it is inspiring to recall Hans's enthusiasm for sitting quietly alone in his office focusing intensely on his work. That modus operandi served him well. Indeed, I vividly remember discussing a nuclear-physics problem with another of his graduate students in the coffee room. Hans overheard our discussion and said: "I don't see why you are talking about this problem when either of you is capable of sitting down and solving it." I also remember the pleasure he took in performing his legendary numerical calculations in his head; Hans could remember logarithms or expand functions as necessary and quote answers to several decimal places.

He and his wife Rose, seen together in figure 2, were always warm and hospitable to students and visitors. When Judit Nemeth arrived from Hungary as a postdoc, Hans personally met her at the airport and helped her get settled. Rose's dinner parties were wonderful affairs where colleagues, friends, students, and visitors were warmly welcomed and treated like family.

I also recall with gratitude Hans's response to a request I made in 1968, during the height of the Vietnam War. I had asked permission to interrupt my graduate studies to go to New Hampshire with some other Cornell graduate physics students and work for an extended period on Eugene McCarthy's antiwar primary campaign. He responded that ordinarily a young scientist like me should devote himself exclusively to his work so as to have maximal influence later. But in this case, the war was so terrible that I should go with his blessing. Looking back, I find it noteworthy that in Concord, Nashua, and other New Hampshire towns where Cornell physicists labored to explain to nonacademic citizens why we believed the war was wrong, people voted solidly for McCarthy and that the New Hampshire primary marked the beginning of the end of the war. Hans's wisdom in balancing profession and patriotism is as relevant today as it was then.

In theoretical physics and in life, Hans continues to be a source of inspiration for all those whose lives he touched.

References

1. H. A. Bethe, *Phys. Rev.* **55**, 434 (1939).
2. H. A. Bethe, http://nobelprize.org/physics/laureates/1967/bethe-lecture.html.
3. H. A. Bethe, *Phys. Rev.* **57**, 260 (1940); **57**, 390 (1940).
4. H. A. Bethe, *Phys. Rev.* **76**, 38 (1949).
5. H. A. Bethe, *Phys. Rev.* **103**, 1353 (1956).
6. J. Goldstone, *Proc. R. Soc. London, Ser. A* **239**, 267 (1957).
7. See R. Rajaraman, H. A. Bethe, *Rev. Mod. Phys.* **39**, 745 (1967).
8. G. Baym, H. A. Bethe, C. J. Pethick, *Nucl. Phys.* **A175**, 225 (1971). ■

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