



Fingered Flow Unlocking the conundrum.

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With data and inspiration from

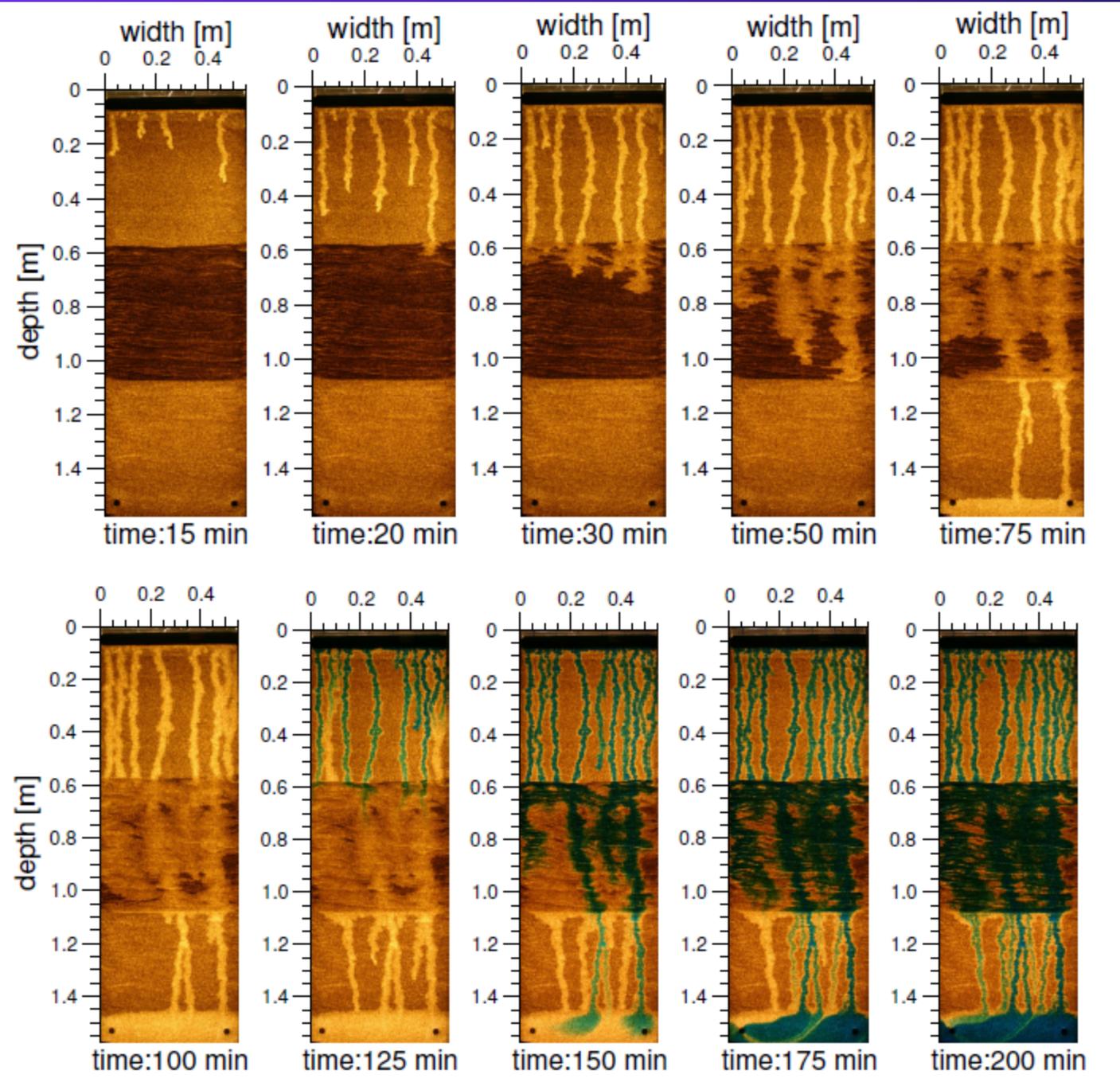
Noam Weisbrod and Martin Schroth

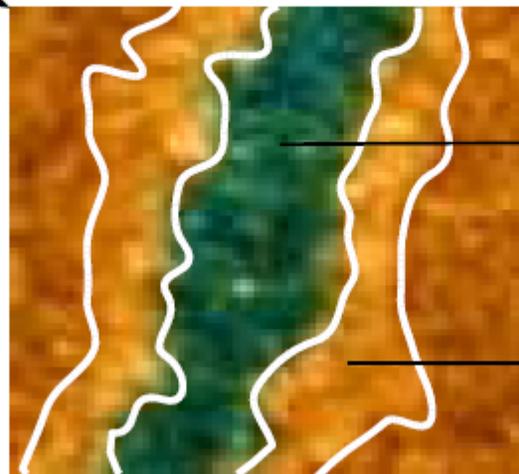
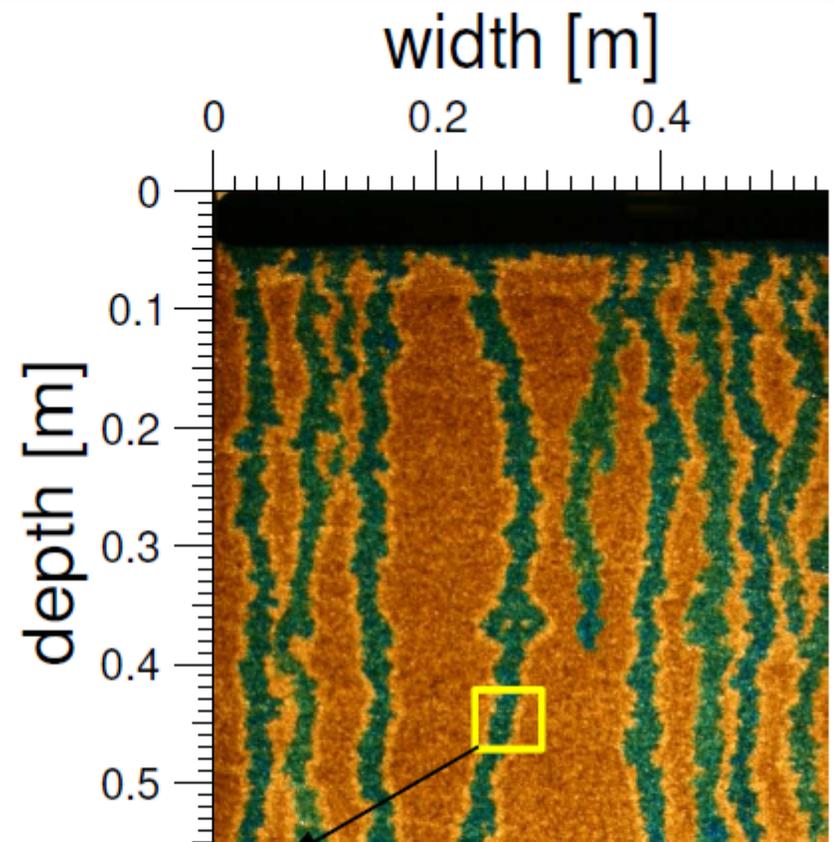
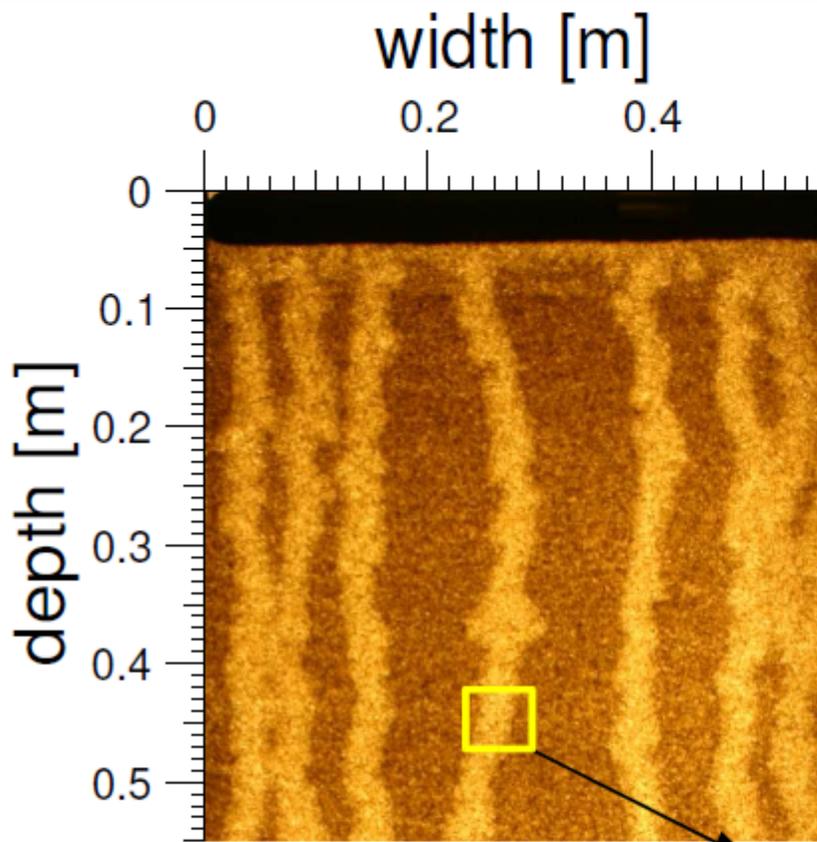
Goals

- ◆ Present the basic conundrum of fingered flow
- ◆ Present a simple explanation of the process
- ◆ Present experimental results for unusual cases
- ◆ Discuss the value of study of basic processes

History

- ◆ 1972 – Hill and Parlange publish the first paper focused on fingered flow
- ◆ 1973 Raats publishes his concept
- ◆ 1975 Philip publishes his concept
- ◆ 1976 Parlange and Hill publish their concept
- ◆ Since then – almost every major soil physicist is required to do so (seriously!)





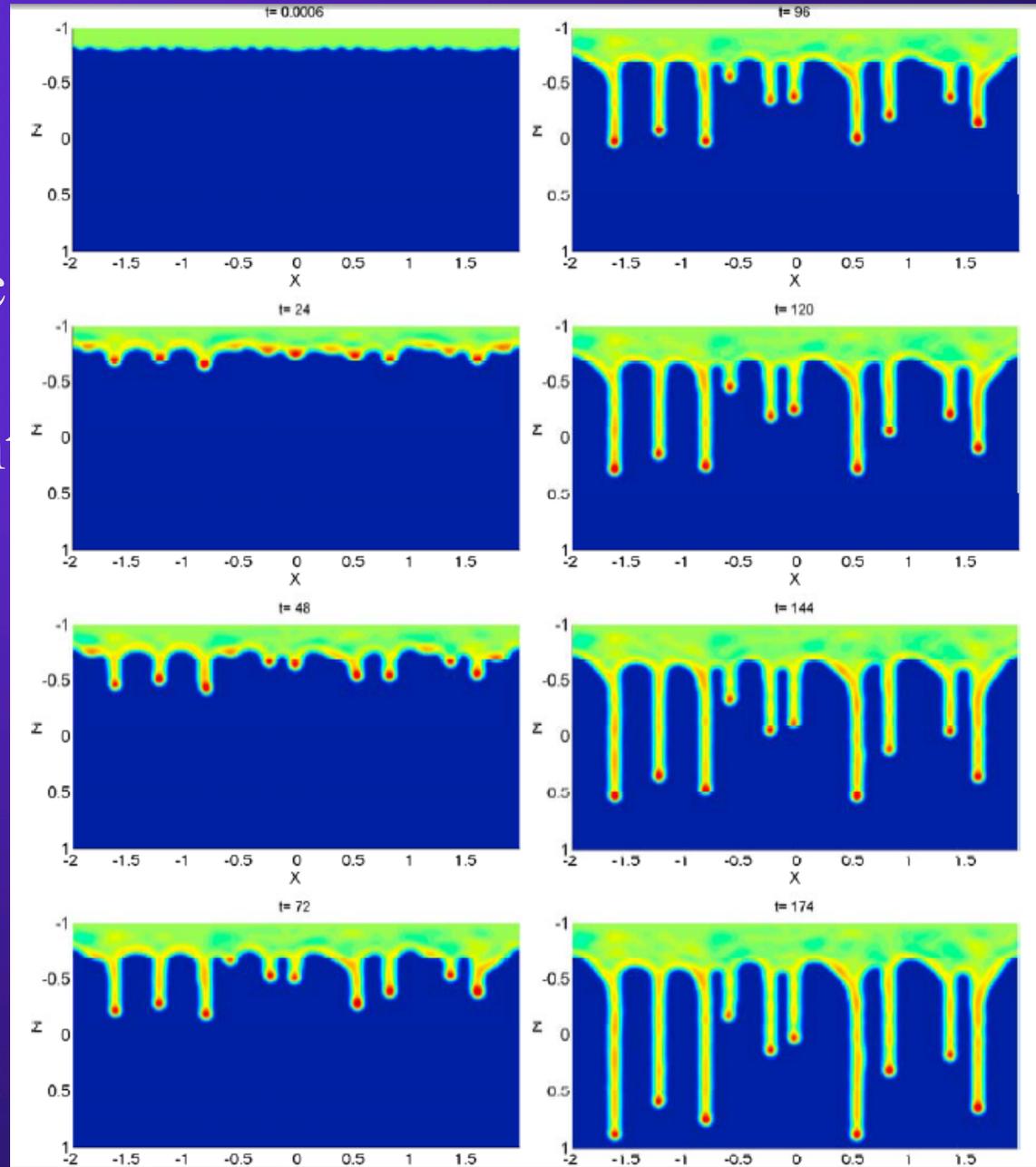
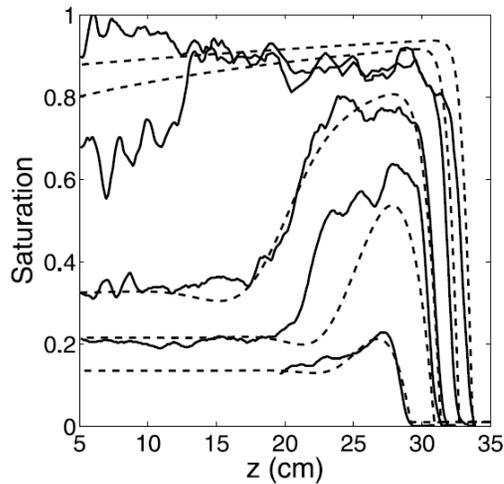
mobile core

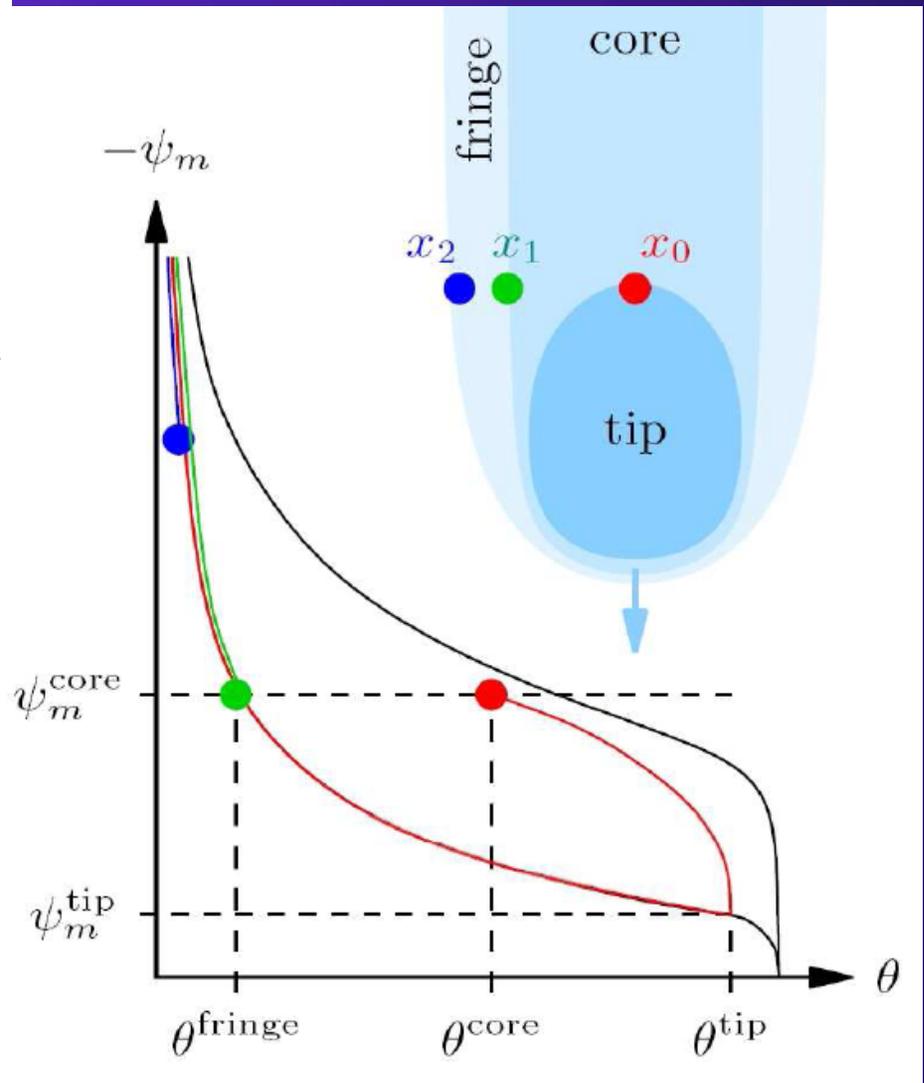
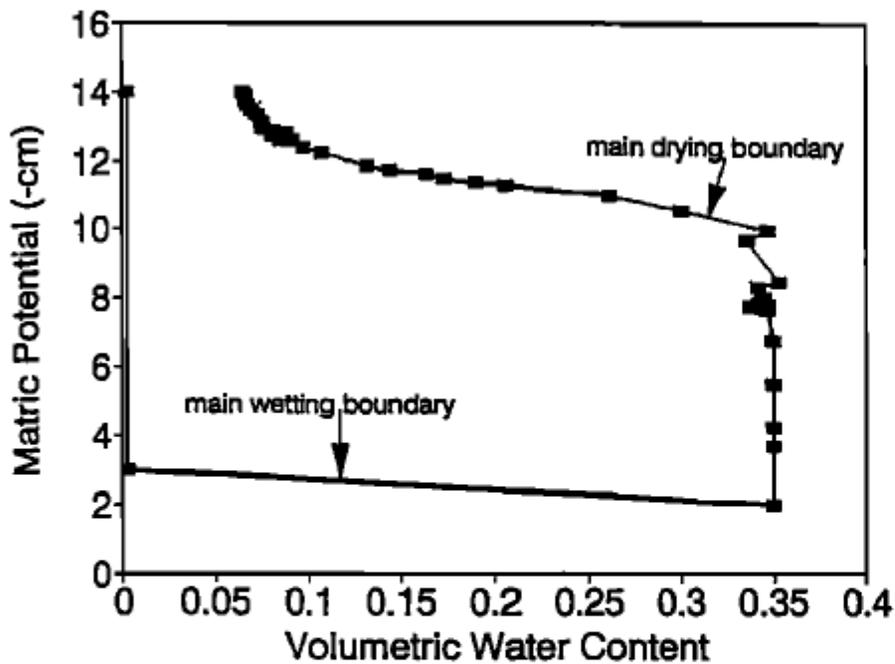
immobile fringe

◆ Cueto-Felgueroso and Juanes, 2009

“Our model predictions, without any hysteretic effects, are in good quantitative agreement with the saturation profiles measured

Experimentally”





- ◆ Liu et al., 1994
- ◆ Rezanezhad et al. 2006 HESS

Eliassi and Glass, 2001

“We define the functional form of the equation of state(i.e., the moisture retention relation) and the constitutive relation (i.e., the hydraulic conductivity function using the standard monotonic models of van Genuchten [1980] and Mualem [1976]...”

The Finger Conundrums

1. Why do fingers form? (instead of horizontal wetting front)
2. What is the relationship between width and soil properties?
3. What is the relationship between width and flux?
4. What is the relationship between width and antecedent water content?
5. Doesn't this violate Richards equation?

The Finger Conundrums

1. Why do fingers form? (instead of horizontal wetting front)

Consider the limiting case – one pore is a whole bucket. Turn it upside down, and the water falls in irregular pattern.

Saffman Taylor gravitational instability:

Gravity destabilized, capillary forces stabilize

The Finger Conundrums

2. What is the relationship between width and soil properties?

Parlange/Hill expression based on most unstable wavelength works (will show data shortly).

Concept: the fastest growing dimension will become dominant.

The Finger Conundrums

3. What is the relationship between width and flux?

Again, Parlange/Hill works quite well.
Fastest growing wavelength is a function of flux (at high flux).

The Finger Conundrums

4. What is the relationship between width and antecedent water content?

HIGHLY controversial, but without a doubt strongly expands finger width. Specifically, if flux goes to zero and films are continuous, finger width goes to infinite.

The Finger Conundrums

5. Doesn't this violate Richards equation?

Not in an essential manner,

but

It make use of smooth (e.g., van Genuchten) water retention functions unacceptable.

A thought Experiment

- ◆ Suppose this is a sand column that was wetted from the top that is now at equilibrium with a step-wise water entry pressure of -4 cm.
- ◆ Is there anything non-physical here? No.
- ◆ Can van Genuchten predict this: no.



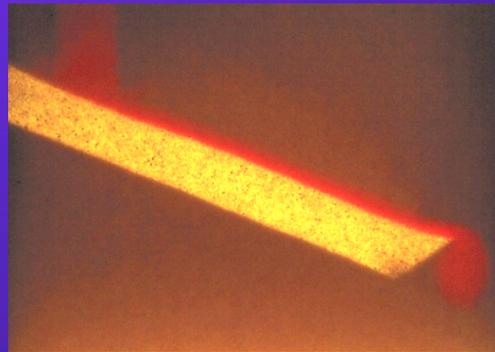
Theta



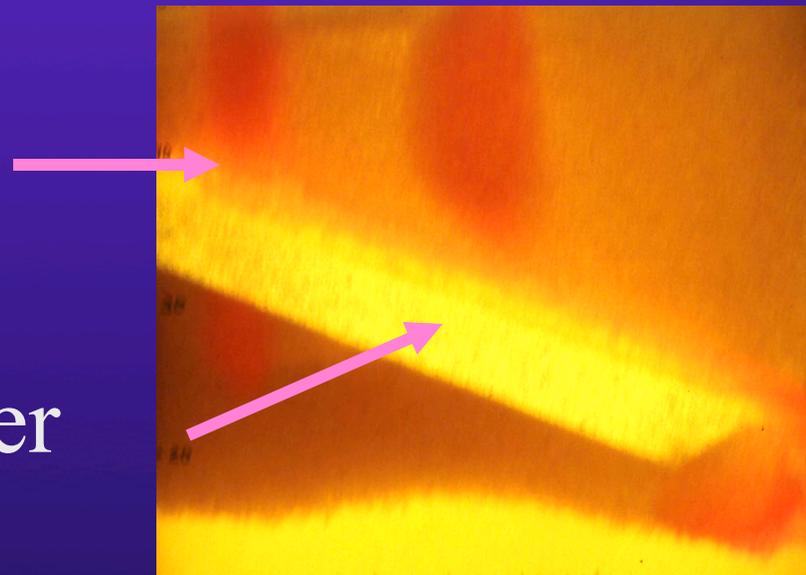
Side bar: NAPL Textural Interfaces

◆ Three oil spill cases

– no water flowing



– little water flowing



– lots of water flowing

Schroth, M. H., J.D. Istok and J.S. Selker. Three-Phase Immiscible Fluid Movement in the Vicinity of Textural Interfaces. *Journal of Contaminant Hydrology*. 32:1-23. 1998.

Schroth, M.H., J.D. Istok, J.S. Selker, M. Oostrom and M.D. White. Multifluid flow in bedded porous media: Laboratory experimental and numerical simulations. *Advances in Water Resources*. 22:169-183. 1998

Thought Experiment (2)

- ◆ So it is hydrostatic with pressure of -4 cm at the bottom and more negative as we go up

Pressure

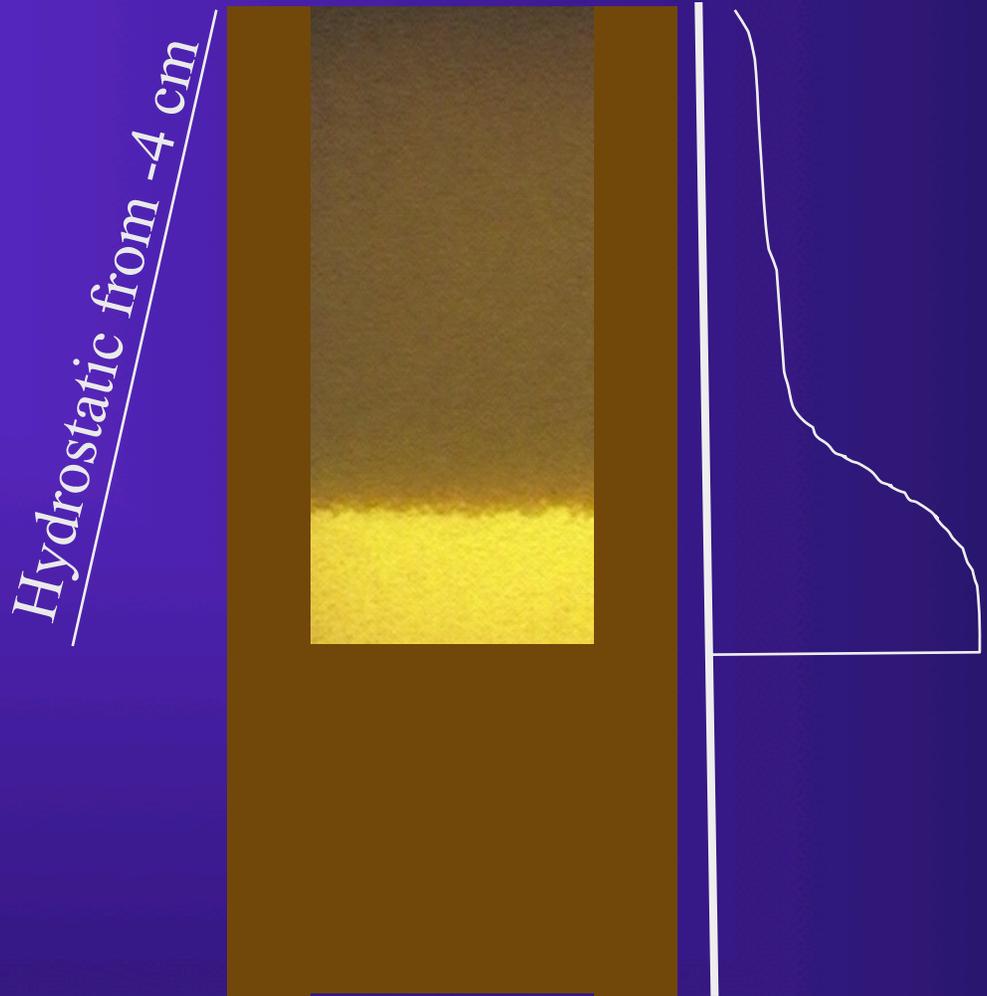


Theta



Thought Experiment (3)

- ◆ Now put dry sand on all sides.
- ◆ The water won't spread, as the pressure is more negative than 4 everywhere in the wet area.



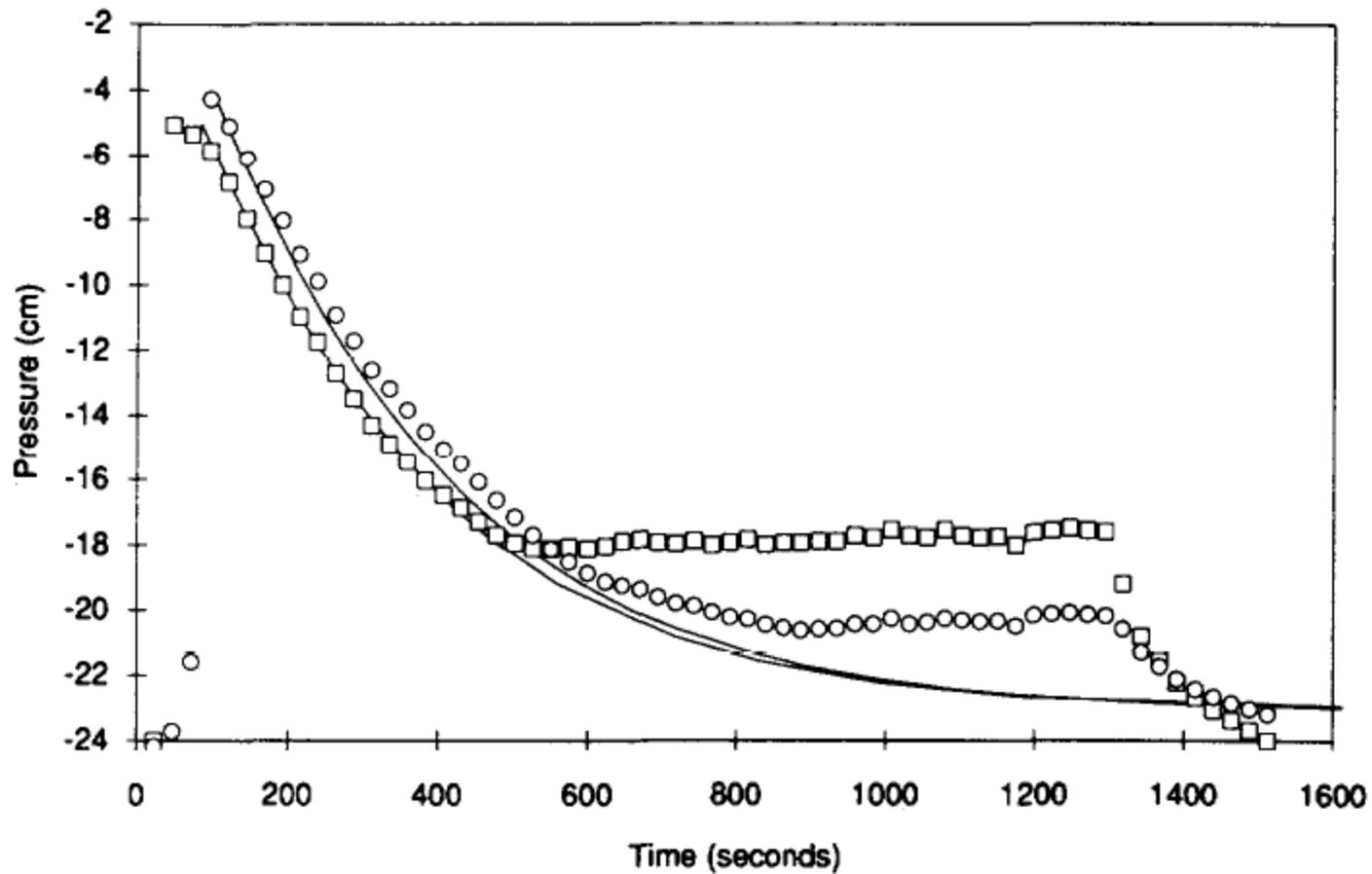
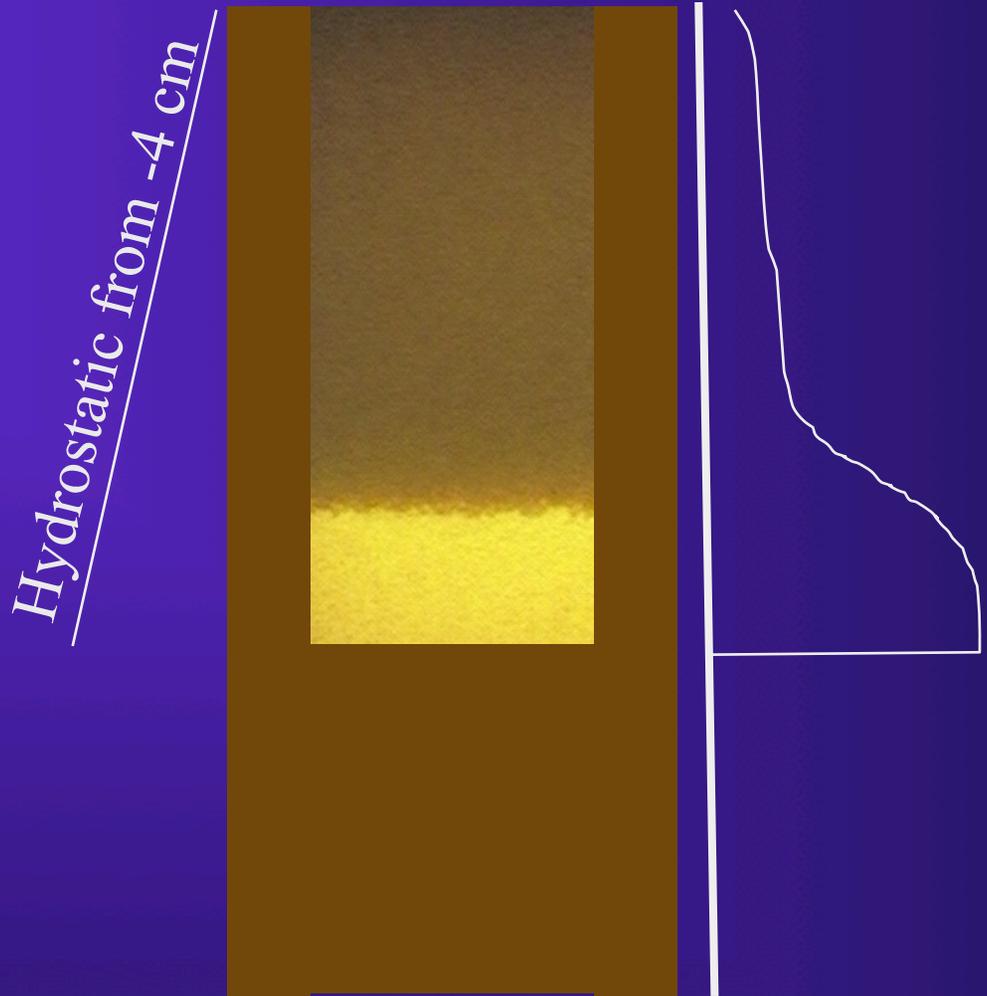


Fig. 4. Plot of matric potential versus time measured through ports 4 (open circles) and 8 (open boxes) in experiment 2 with potential predictions from (12) (solid line). At port 4 the flow becomes one dimensional at about 500 s, with pressure becoming constant. At port 8 the pressure drops more slowly than the prediction for $t > 600$ s, that is, when the finger reached the bottom of the chamber.

Thought Experiment - 4

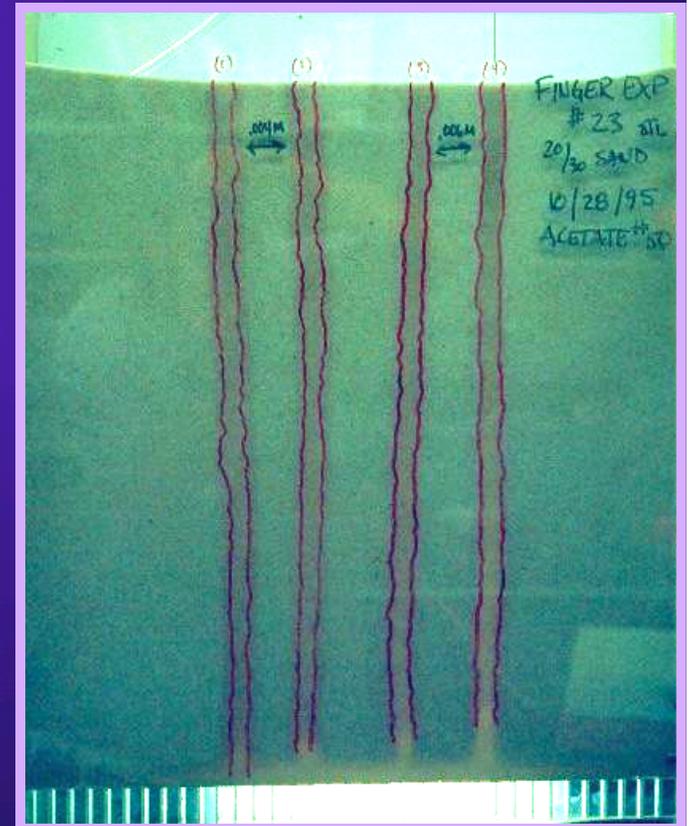
- ◆ Now put one drop of water on top.
- ◆ The water will make its way to the lowest interface, slightly increase the pressure, and drive downward wetting.
- ◆ Wait for some time, and the entire profile will shift downward (maintains overshoot).



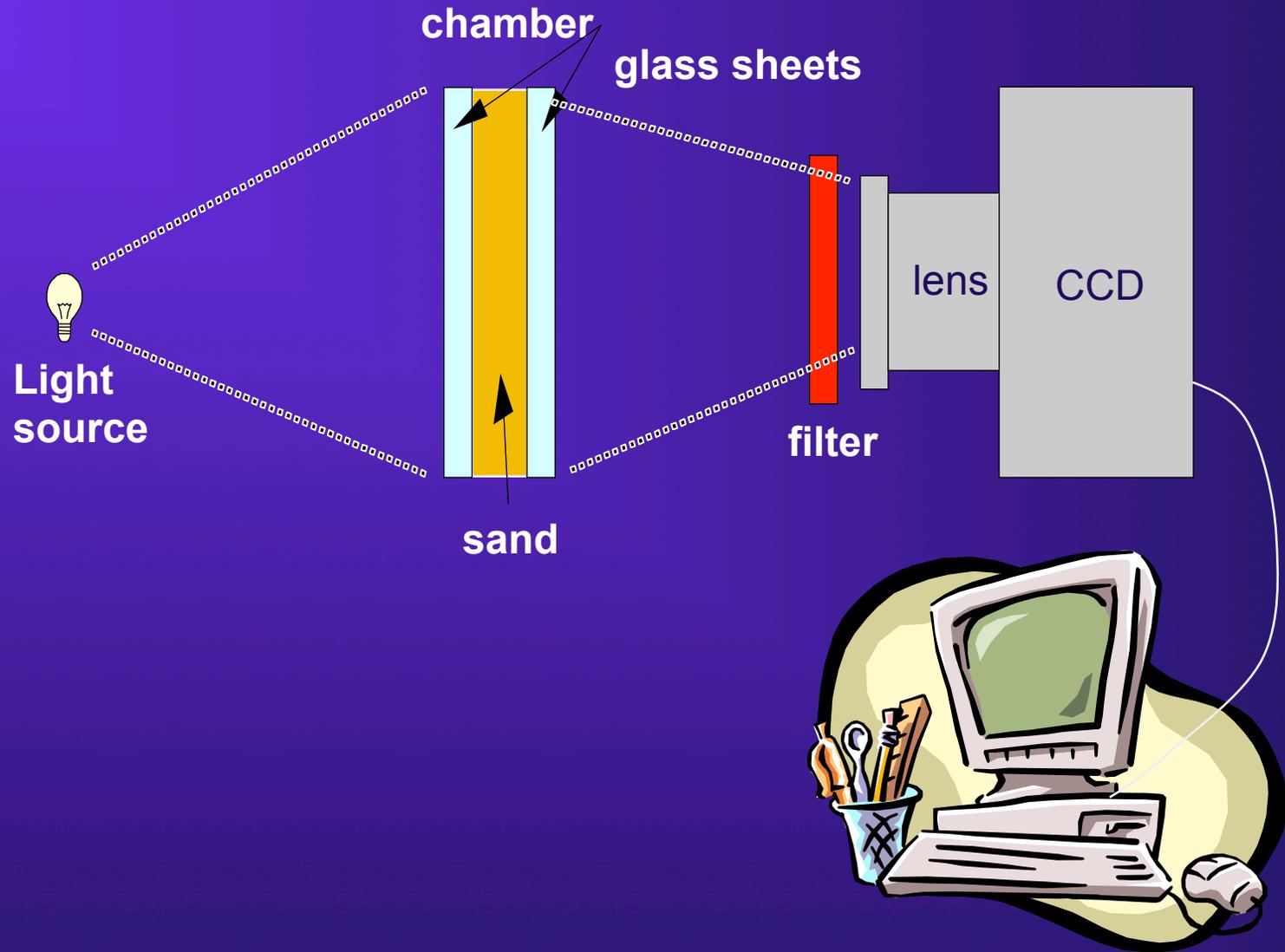
Experimental Observations



Finger width:
viscosity,
surface tension
and texture

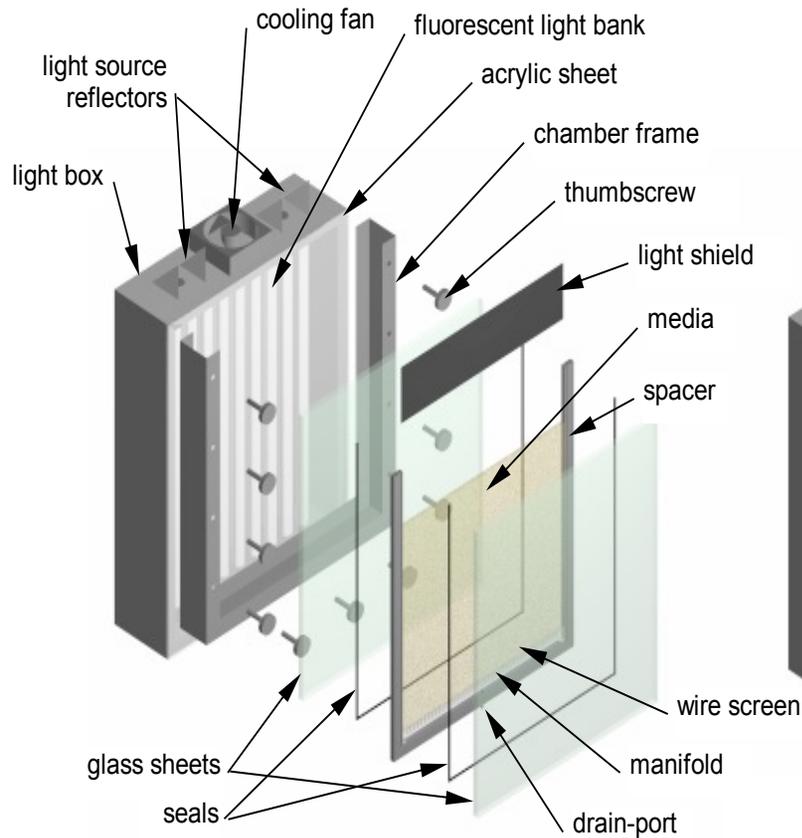


light transmission technique

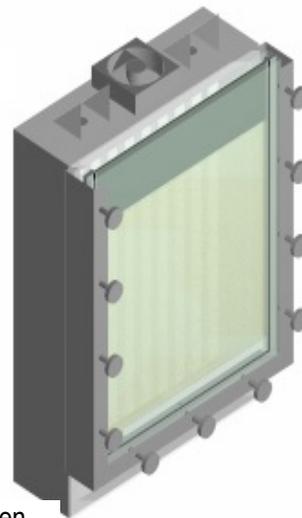


The chamber (1-cm thick pseudo 2D system)

2-D Chamber

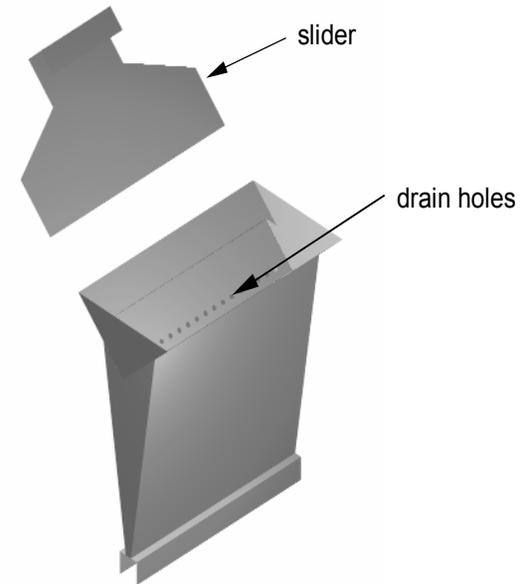


Exploded



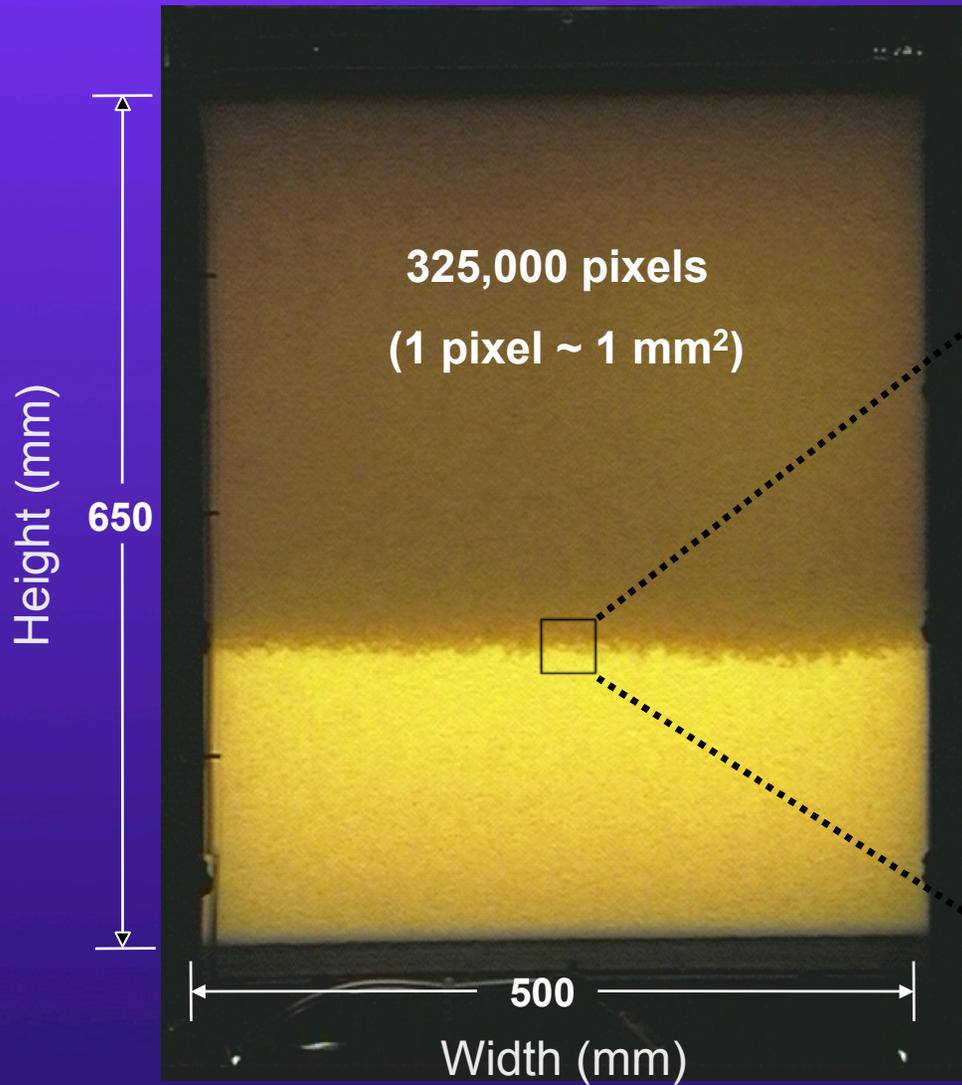
Assembled

Prismatic Funnel



The prismatic funnel, designed to fit the top of the chamber is used to fill the media space with Accusand[®]. Screens inside the funnel randomize the trajectory of the grains as they fall between the glass sheets.

Lots of measurements...



- Fast
- Non-Destructive
- Non-Invasive

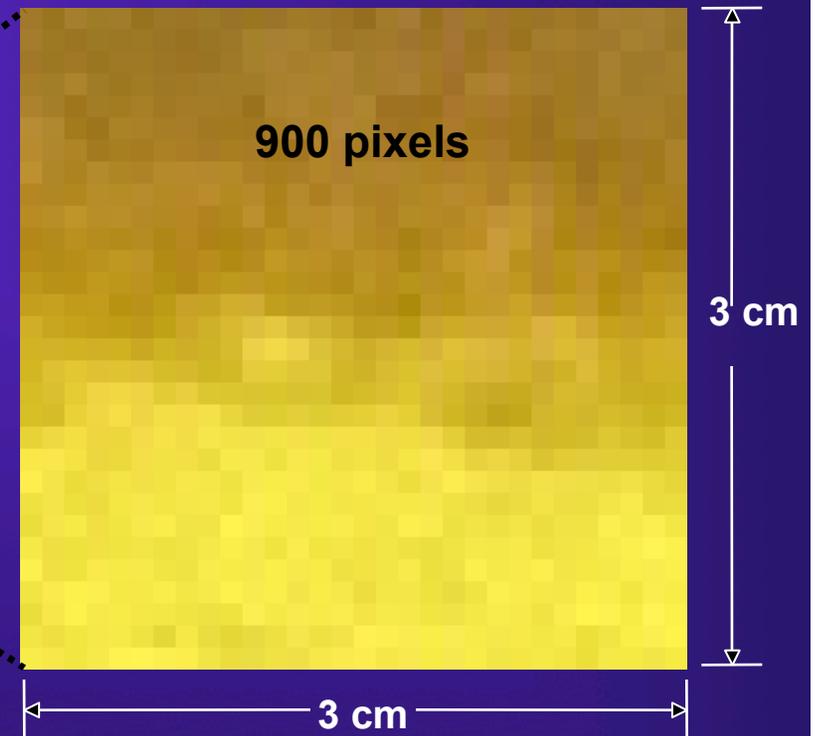


Image processing (**pixel by pixel basis**)

Energy is exponentially attenuated by gas, water, and solid compartments

$$I = I_0 \exp \left[- \left(\sum_{i=1}^n \alpha_s x_s + \sum_{i=1}^n \alpha_a x_a + \sum_{i=1}^n \alpha_w x_w \right) \right] \tau_{sa}^{2(k-p)} \tau_{sw}^{2p}$$

Beer's law component

Fresnel's law component

Where: α is the attenuation coefficient, x is the thickness of each compartment, τ_{sa} is the transmittance of the solid-air interface, τ_{sw} is the transmittance of the solid-water interface, k is the total number of pores, and p is the number of pores filled with water. With light, losses also occur at the interfaces due to the different refractive indices.

$$\Omega = \ln \left(\frac{I}{I_s} \right) / \ln \left(\frac{I_{res}}{I_s} \right)$$

$$S_{eff} = a\Omega^4 - b\Omega^3 + c\Omega^2 - d\Omega + 1$$

$$S = S_{eff} (1 - S_{res}) + S_{res} \quad \Delta S = S - S_0$$

I intensity of transmitted light
 I_0 measured intensity of light source
 Ω log-scaled relative degree of light transmission
 I_s transmitted light from saturated image
 I_{res} transmitted light at residual water content
 S_{eff} effective degree of saturation
 a, b, c, d , polynomial coefficients

Experimental Finger Width

- ◆ Significant variation with media, not much with liquid

Liquid	12/20 (cm)	20/30 (cm)	30/40 (cm)	40/50 (cm)	CV
Water	1.04	1.99	2.78	5.67	0.70
Soltrol	1.36	1.73	2.47	3.99	0.49
Duoprime	0.79	1.64	2.29	3.37	0.54
CV	0.27	0.10	0.10	0.27	

How did d_{50} scaling work out?

- ◆ All that you could hope for -
A factor of three reduction in variability

Liquid	12/20	20/30	30/40	40/50	CV (w/o scaling)
Water	1.04	1.29	1.33	1.81	0.24 (0.70)
Soltr ol	1.36	1.12	1.19	1.28	0.08 (0.49)
Duoprime	0.79	1.07	1.10	1.08	0.15 (0.54)
CV	0.27	0.10	0.10	0.27	

What about surface tension σ ?

◆ Gads! Makes variability >3 times worse!

Liquid	12/20	20/30	30/40	40/50	CV
Water	1.04	1.99	2.78	5.67	0.70
Soltrol	0.49	0.63	0.90	1.45	0.49
Duoprime	0.33	0.69	0.96	1.42	0.54
CV	0.60	0.70	0.69	0.86	
w/o scaling	0.27	0.10	0.10	0.27	

- ◆ Would seem to indicate that surface tension is not involved with finger width.
- ◆ Could this be?

How could this fail??

- ◆ Finger width equation wrong
- ◆ Assumptions for Miller Scaling violated
e.g.- Contact Angle not constant

What if contact angle was non-zero?

- ◆ Not quite what we'd expect for clean sand, but take a look the equations

- ◆ LaPlace

$$P_c = \frac{2\sigma_{gl} \cos \gamma}{r}$$

- ◆ Youngs

$$\cos \gamma = \frac{\sigma_{sg} - \sigma_{sl}}{\sigma_{gl}}$$

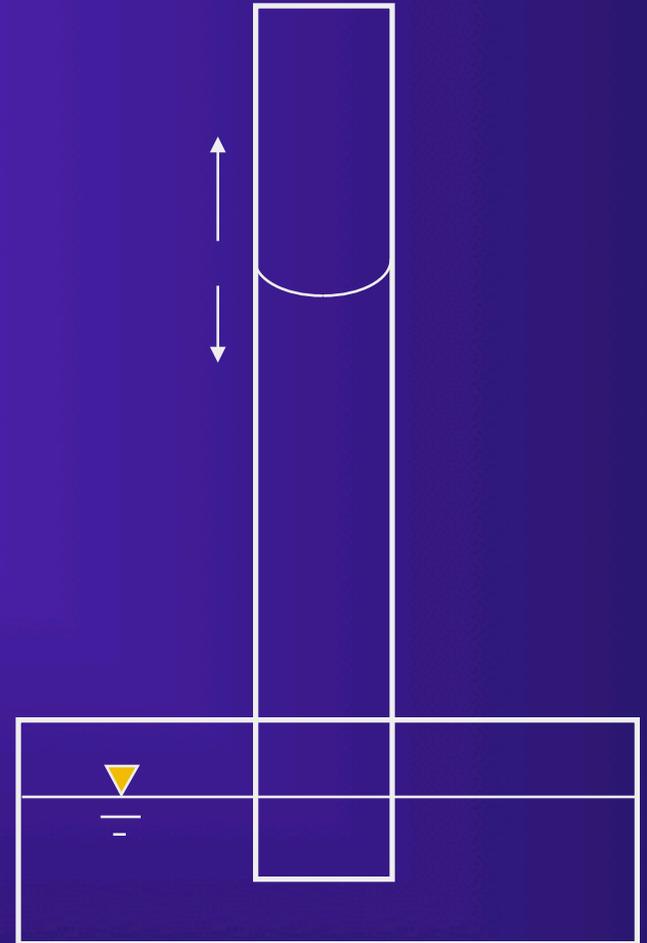
- ◆ Combined

$$P_c = \frac{2(\sigma_{sg} - \sigma_{sl})}{r}$$

no σ_{gl} !

Revisiting the Capillary tube

- ◆ Question:
What drives the meniscus up the Capillary tube?
- ◆ Answer:
the difference between the energy of the solid gas and solid-liquid interfaces



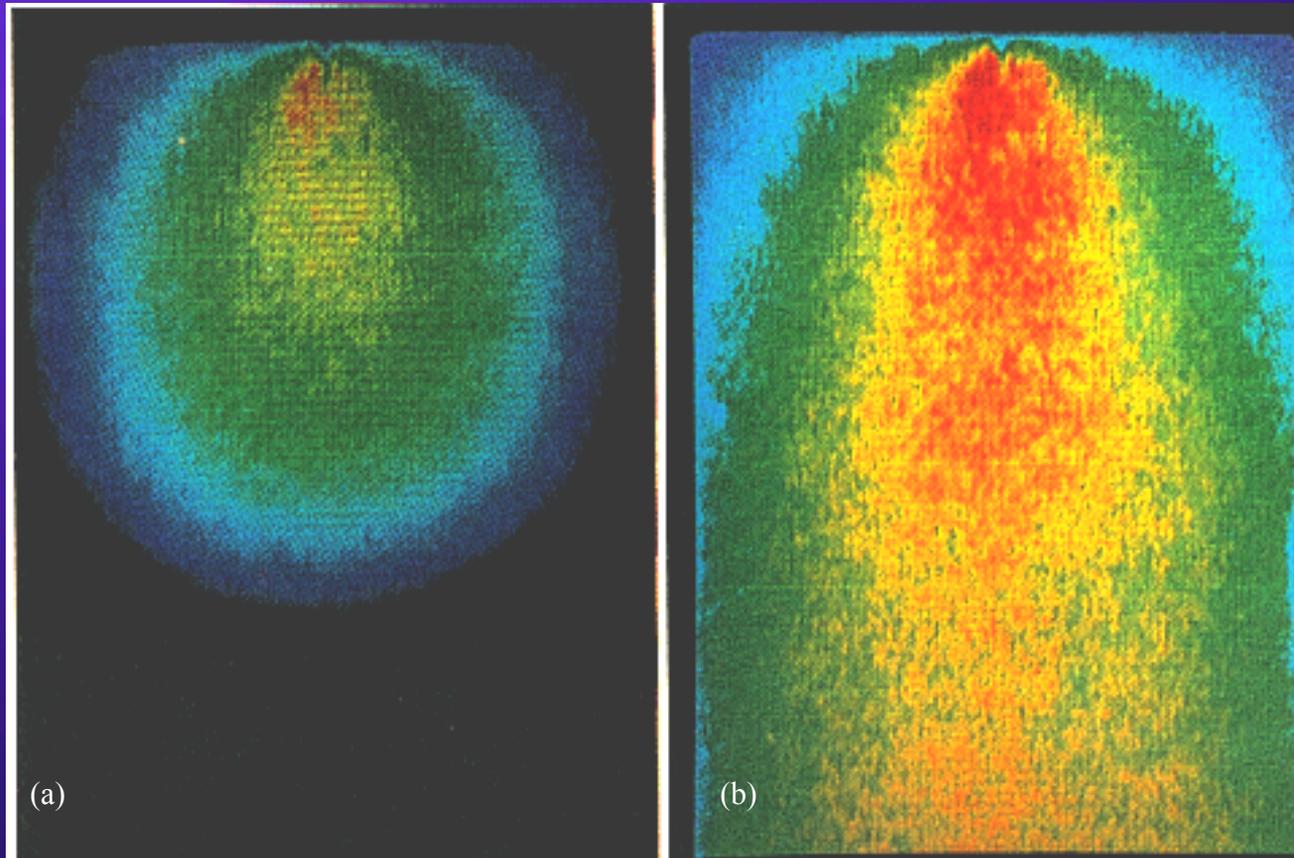
What is going on here?

- ◆ Capillary energy of imbibition is due to liquid/solid - gas/solid interfacial energy.
- ◆ Doesn't scale with gas-liquid interfacial energy! (so long as $\gamma \neq 0$)

Selker, J.S. and M.H. Schroth. Evaluation of Hydrodynamic scaling in porous media using finger dimensions. *Water Resour. Res.* 34:1875-80. 1998.

Fingering in Damp Soils

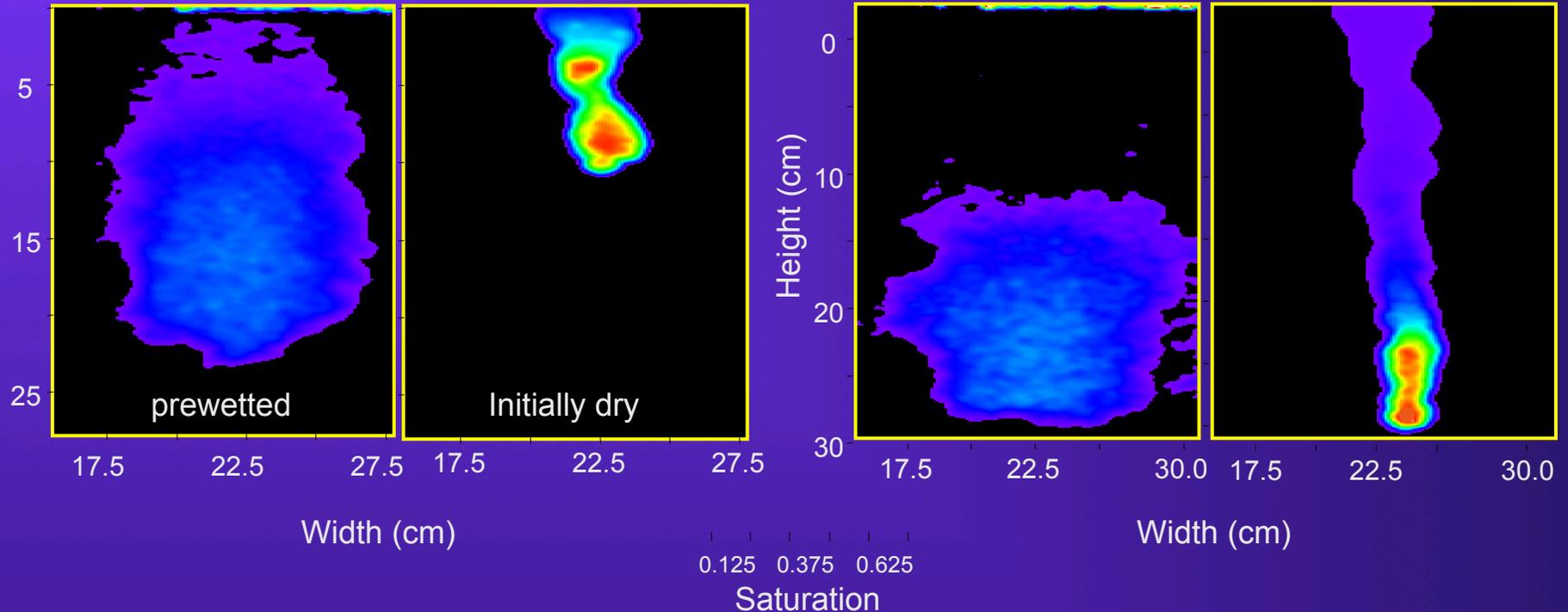
- ◆ Generally observed to be minimal (e.g., Glass and Nichols, 1996)



Infiltration mechanism – dry vs. prewetted (pure water)

20 min from application (20/30)

100 min from application

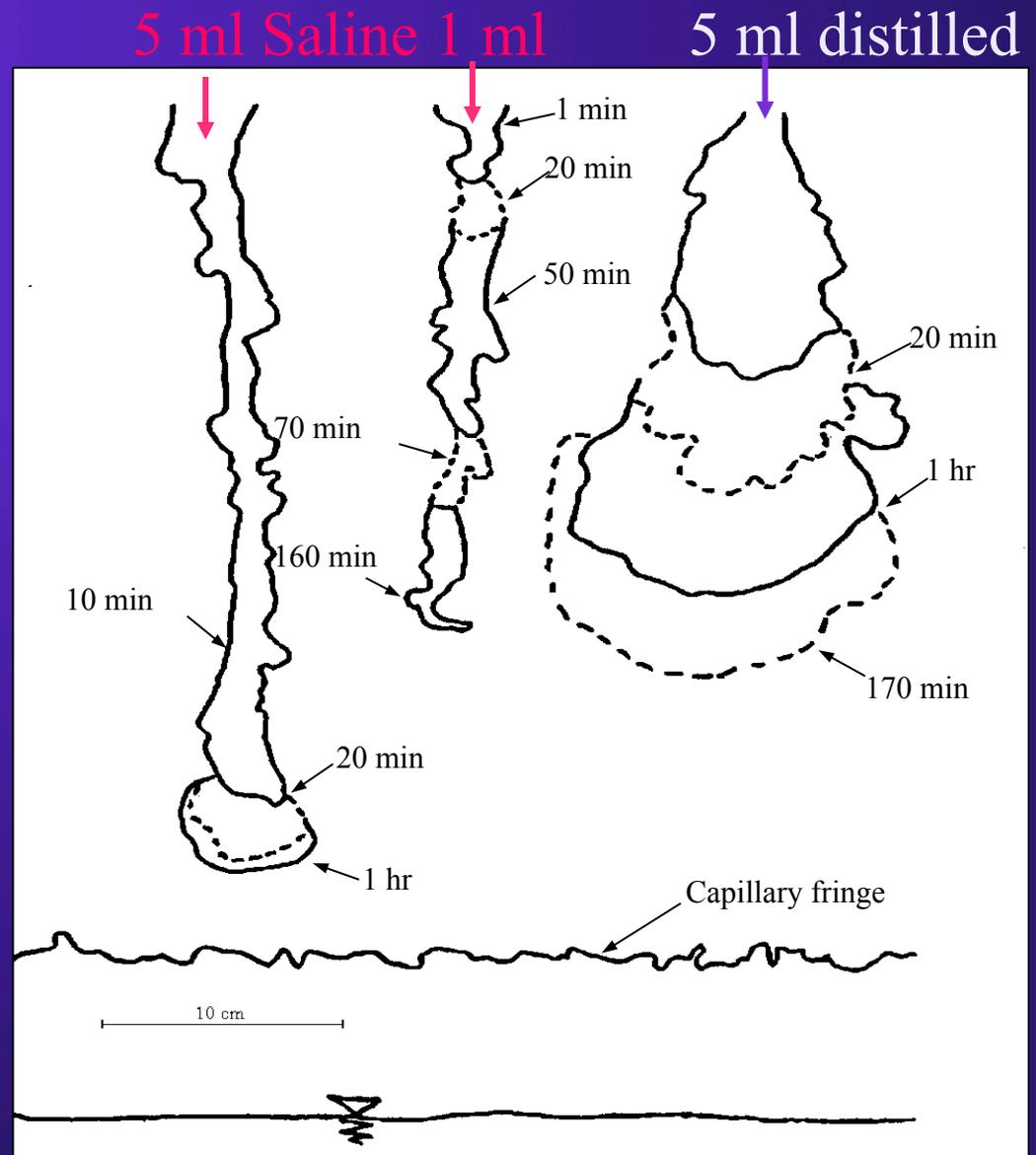


Note that values below $S = 0.02$ (dry) and $S = S_r = 0.065$ (prewetted) set to black. Max $S = 78\%$ in the dry vs. 22 and 18% in the prewetted sand.

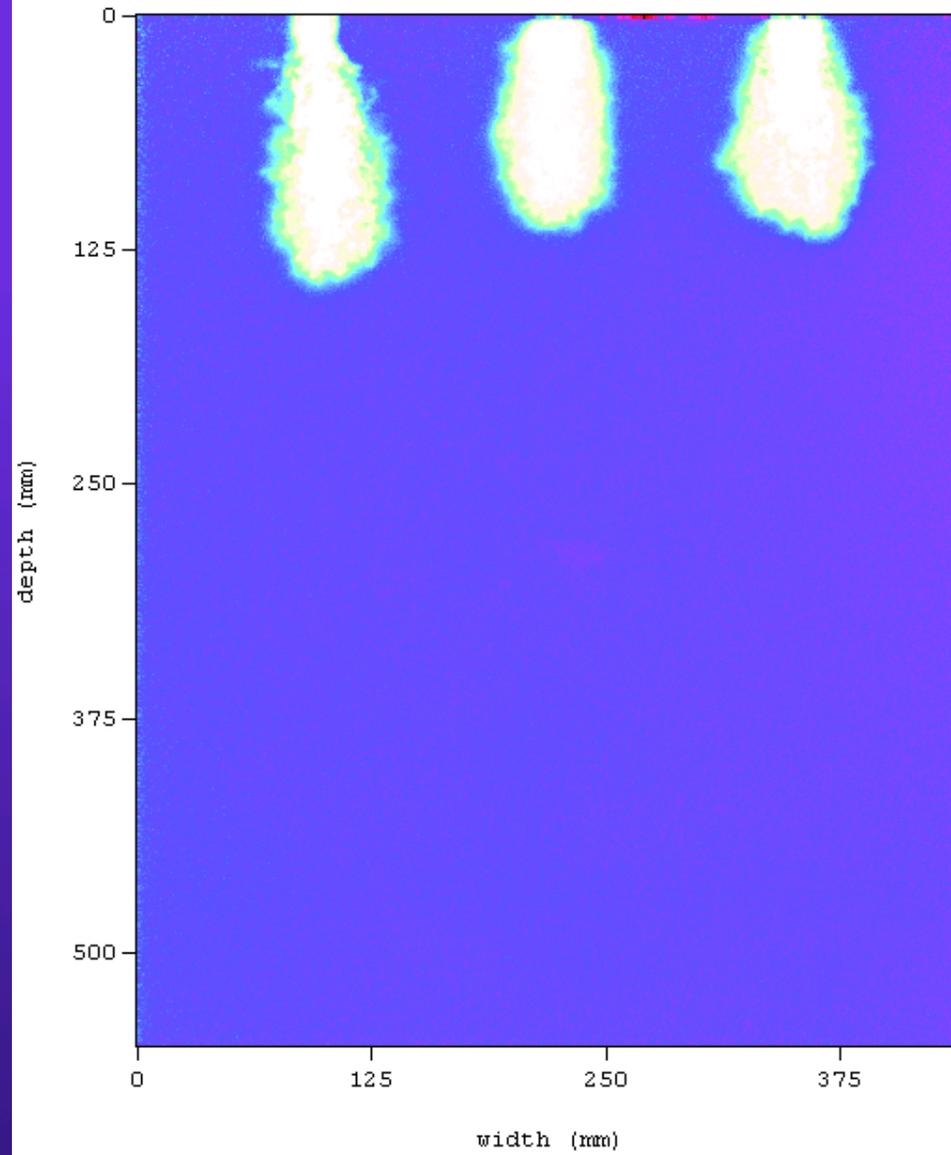
- Contact angle of $\sim 30^\circ$ (dry) vs. 0° (prewetted).
- No classical wetting front in prewetted sand.

Saline Fingers: short time

- ◆ The distilled water plume acted as seen previously
- ◆ The saline plumes formed classic fingers
- ◆ Surface Tension critical



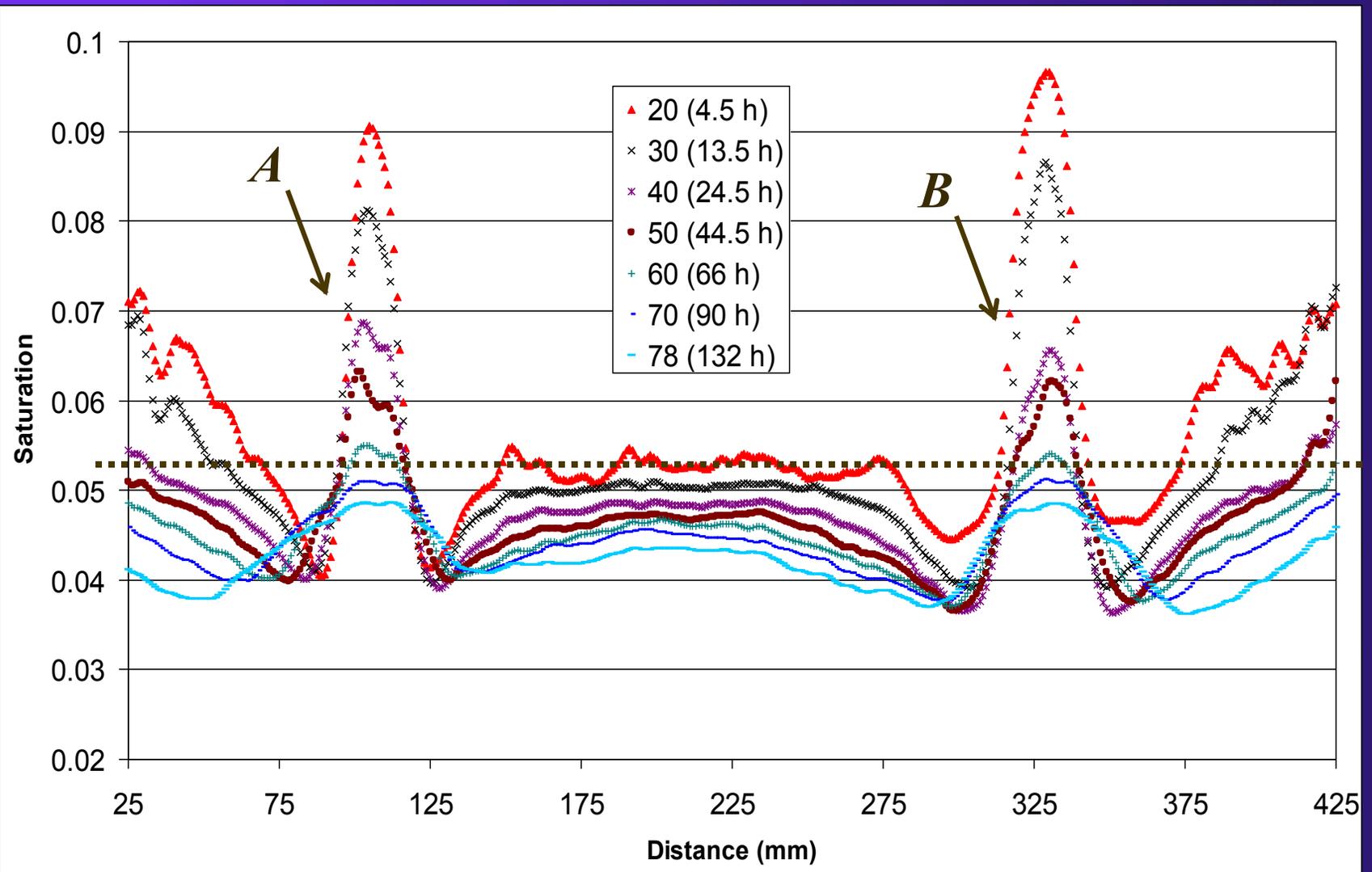
Frame1



-0.125 0.000 0.125 0.250

Predicted change in saturation after addition

A cross section at 20 cm depth. Note the saturation dips (below S_{res}) that develop around the solutions.

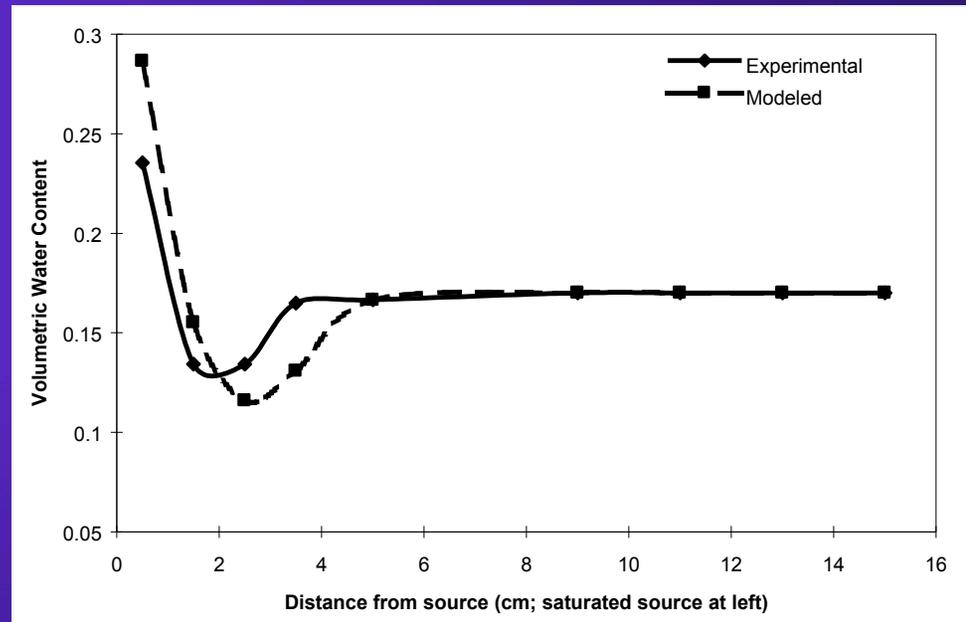


This is not hard to predict

◆ Model and experiment results agree well

◆ Note:

- Diffusive process, so follows $t^{0.5}$
- in 50 years expect about 1.5 m vapor extraction radius



Kelly, S.F, and J.S. Selker. Osmotically driven water vapor transport in unsaturated soil. *Soil Sci. Soc. Am. J.* 65:1634-1641. 2001.

Weisbrod, N, M.R. Niemet, and J.S. Selker. Imbibition of saline solutions into dry and prewetted porous media. *Advances in Water Resources.* 25:841-855. 2002.

Conclusions

- ◆ Subtle issue in choice of retention functions caused massive diversion of attention
- ◆ Contact angle is subtle and important in imbibition processes
- ◆ Saline transport in vadose zone implies vapor transport, and in turn, colloidal transport
- ◆ Simple model systems are exceptionally valuable in process identification