The Isotopic Age of Runoff in Natural Flow Systems

C. Duffy, Penn State University
Isotopic Age of Watershed Runoff
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This research shows how the theoretical “mean age” of a solute can be predicted.

An experiment conducted at the NSF-funded Shale Hills Critical Zone Observatory is testing the theory and develop practical tools for answering the questions such as: Where is the water? And how long did it take to get there?
"Theories for Age of Waters: A Sampling"

The concept of "age" in mass transport research has a long history:

- Nauman (1965) chemical engineering
- Erikson (1971) compartment models
- Allison and Hughes (1973) resource assessment
- Bolin and Rhode (1973) atmospheric science
- Allison and Hughes (1973) resource assessment
- Goode (1996) groundwater
- Delhez et al. (1999) and Gourgue et al. (2006) in Ocean systems
- IAEA publications (2001) isotope methods
Groundwater **Age** is an extensive property of a dissolved solutes and the flow regime.

The **Age** is defined relative to the time since the solute entered the system.

The **Age** of any solute can be calculated in an Eulerian framework.

**Age** is subject to the usual processes of advection, dispersion, diffusion, reaction.

\[ Q(t), C(t) \]

\[ c(t, \tau, x_o : q(t)) \]
The Age Distribution at a Point $X_0$

$c(t, \tau, x_0)$
A Transport Model for Age Distribution

Rotenberg 1972, J, of Theoretical Biology, 37, 291-305

\[ DM(t, \tau) \frac{1}{V} = \left( \frac{\partial M}{\partial t} + \frac{\partial M}{\partial \tau} \right) \frac{1}{V} \]
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\[
\frac{\partial c}{\partial t} + \frac{\partial c}{\partial \tau} = \Gamma_c - L(c)
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\frac{\partial c}{\partial t} + \frac{\partial c}{\partial \tau} = \Gamma_c \rightarrow L(c)
\]

\[
L(c) \rightarrow D \frac{\partial^2 c}{\partial x^2} - u \frac{\partial c}{\partial x}
\]

or

\[
L(c) \Rightarrow \frac{Q_i}{V}(c_i - c)
\]
Transport Model in Terms of Moments
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\frac{\partial \mu_n}{\partial t} = n\mu_{n-1} + \Gamma_{\mu_n} - L(\mu_n)
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Transport Model in Terms of Moments

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**Coupling Moment**

**Transport Operator**

\[ \frac{\partial \mu_n}{\partial t} = n\mu_{n-1} + \Gamma_{\mu_n} - L(\mu_n) \]
Transport Model in Terms of Moments

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Coupling Moment  \quad \text{Transport operator}

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Source terms
Transport Model in Terms of Moments

\[ \frac{\partial c}{\partial t} + \frac{\partial c}{\partial \tau} = \Gamma_c - L(c) \]

Coupling Moment \hspace{1cm} Transport Operator

\[ \frac{\partial \mu_n}{\partial t} = n\mu_{n-1} + \Gamma_{\mu_n} - L(\mu_n) \]

Age \[= \frac{\mu_1}{\mu_0} \]
A Concentration-Age-Flow Dynamical Model

For a volume-averaged system

Duffy, 2010  Hydrologic Processes
A Concentration-Age-Flow Dynamical Model

For a volume-averaged system

\[
\frac{dV}{dt} = Q_i - Q
\]
\[
\frac{dC}{dt} = \frac{Q_i}{V} (C_i - C) + \Gamma_c
\]
\[
\frac{d\alpha}{dt} = C - \frac{Q_i}{V} \alpha + \Gamma_\alpha
\]
\[
A(t) = \frac{\alpha(t)}{C(t)}
\]

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A Concentration-Age-Flow Dynamical Model

For a volume-averaged system

\[
\begin{align*}
\frac{dV}{dt} &= Q_i - Q \\
\frac{dC}{dt} &= \frac{Q_i}{V}(C_i - C) + \Gamma_c \\
\frac{d\alpha}{dt} &= C - \frac{Q_i}{V} \alpha + \Gamma_\alpha \\
A(t) &= \alpha(t) / C(t)
\end{align*}
\]

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Duffy, 2010  Hydrologic Processes
Unit Step Input $C_i$ and $Q_i$
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For constant inputs age reaches a constant value at large time
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For constant inputs age reaches a constant value at large time

$$A(\infty) = \frac{V(\infty)}{Q_i}$$

$$C(\infty) = Q(\infty) = Q_i = 1$$
Unit Pulse Input: $C_i$ and $Q_i$
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During drought periods the Age of water increases like a clock
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During drought periods the Age of water increases like a clock

$$A(t \to \infty) \sim t$$
Unit Pulse Input: $C_i$ and $Q_i$

During drought periods the Age of water increases like a clock:

$$A(t \to \infty) \sim t$$
Random Watershed Inputs: $C_i$ and $Q_i$
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For random inputs, the age of water depends on the flow and solute inputs.
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For random inputs, the age of water depends on the flow and solute inputs.
Concentration-Age-Flow Distributed Dynamical Model

\[ \theta_s \frac{\partial h}{\partial t} = \frac{\partial}{\partial x} Kh \frac{\partial h}{\partial x} + \varepsilon \]

\[ \frac{\partial C}{\partial t} + \hat{u}(x,t) \frac{\partial c}{\partial x} = D(x,t) \frac{\partial^2 c}{\partial x^2} + k(C_i - C) \]

\[ \frac{\partial \alpha}{\partial t} + \hat{u}(x,t) \frac{\partial \alpha}{\partial x} = D(x,t) \frac{\partial^2 \alpha}{\partial x^2} + C - k\alpha \]

\[ \text{Age}(x,t) = \frac{\alpha(x,t)}{C(x,t)} \]
Concentration-Age-Flow Distributed Dynamical Model

\[ \theta_s \frac{\partial h}{\partial t} = \frac{\partial}{\partial x} Kh \frac{\partial h}{\partial x} + \varepsilon \]

\[ \mu_o \Rightarrow \]

\[ \frac{\partial C}{\partial t} + \hat{u}(x,t) \frac{\partial c}{\partial x} = D(x,t) \frac{\partial^2 c}{\partial x^2} + k(C_i - C) \]

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\[ \mu_o \Rightarrow \quad \frac{\partial C}{\partial t} + \hat{u}(x,t) \frac{\partial c}{\partial x} = D(x,t) \frac{\partial^2 c}{\partial x^2} + k(C_i - C) \]

\[ \mu_1 \Rightarrow \quad \frac{\partial \alpha}{\partial t} + \hat{u}(x,t) \frac{\partial \alpha}{\partial x} = D(x,t) \frac{\partial^2 \alpha}{\partial x^2} + C - k\alpha \]

\[ \text{Age}(x,t) = \frac{\alpha(x,t)}{C(x,t)} \]
Concentration-Age-Flow Distributed Dynamical Model

\[ \theta_s \frac{\partial h}{\partial t} = \frac{\partial}{\partial x} Kh \frac{\partial h}{\partial x} + \varepsilon \]

\[ \mu_o \Rightarrow \frac{\partial C}{\partial t} + \hat{u}(x,t) \frac{\partial c}{\partial x} = D(x,t) \frac{\partial^2 c}{\partial x^2} + k(C_i - C) \]

\[ \mu_1 \Rightarrow \frac{\partial \alpha}{\partial t} + \hat{u}(x,t) \frac{\partial \alpha}{\partial x} = D(x,t) \frac{\partial^2 \alpha}{\partial x^2} + C - k\alpha \]

\[ \mu_1 \Rightarrow \frac{\alpha(x,t)}{C(x,t)} \]

\[ \mu_o \Rightarrow \text{Age}(x,t) = \frac{\alpha(x,t)}{C(x,t)} \]
Age(x,t)

Distance x

Time t

Exponential Model + Boussinesq
Shale Hills Critical Zone Observatory
Isotope Network
Prediction of Pathways and Time Scales at the Watershed Scale
Instrumentation for Iso.Net
Instrumentation for Iso.Net
Instrumentation for Iso.Net
>5000 Stable Isotope Samples 2008-2012

Shale Hills MWL - All Data Points

\[ y = 54x + 10 \]

\[ y = 8.0014x + 8.3843 \]

\[ R^2 = 0.96454 \]
Precipitation 2008-2012
**Integrated Hydrologic Modeling System**

- **Control Volume Kernel:** Semi-Discrete Finite Volume formulation of conservation equations. Finite Volume Method ensures mass balance locally (in each control volume) and globally.

The system of ODEs is solved using state-of-the-art solver with adaptive time steps

Kumar et. al., 2009
CZO Hi-Res Data Products

NCALM Lidar→ model grid
Lin and NRCS→ GPR bedrock
Eissenstat→ tree survey

Shale Hills Critical Zone Observatory
All Trees by Species
Soil Mapping

SSURGO (NRCS)

CZO Hi-Res soil survey (Lin & NRCS)
Discharge

![Graph showing discharge and precipitation for 2009. The x-axis represents the months of 2009, and the y-axis represents discharge (m$^3$ day$^{-1}$) and precipitation (m day$^{-1}$). There are three lines on the graph: one for precipitation, one for RTHnet, and one for Flux-PIHM. The graph highlights the peak discharge occurring in November.]
Water Table Depth

![Graph showing water table depth over time in 2009 with two lines: one labeled RTHnet and the other labeled Flux-PIHM.](image-url)
Soil moisture

![Graph showing soil moisture content over 2009 with different curves representing RTHnet and Flux-PIHM data.]
Evapotranspiration
Observed $\delta$ at Shale Hills (Holmes et. al. 2011)

- $\delta$ representations were converted to standard concentration units [Dewitt et. al. 1980]
Simulated Streamflow D2H.
Spatial Mean Watershed Age = 210.9 days

JAN – MAR
Spatial Mean Watershed Age = 188.7 days
JUL - SEP

Spatial Mean Watershed Age = 161.6 days
Spatial Mean Watershed Age = 180.1 days
Spatial Mean Watershed Age = 210.9 days

JAN – MAR
Next Steps

IsoRSM experiment over Shale Hills CZO Region

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University of Tokyo, Japan
Regional 10 km res.
Shale Hills - CZO Hydrology Team

Hydrology Group:
Chris Duffy - Civil & Environmental
Xuan Yu - PhD, CEE
Gopal Bhatt - PhD, CEE
Lorne Leonard - PhD, CEE
Evan Thomas, MS, CEE
George Holmes, MS, CEE

Boundary Layer Meteorology Group:
Ken Davis - Atmospheric Science
Yuning Shi - PhD, Atmospheric Science