Selected Aspects of Hydraulic and Hydrologic Soil Erosion Processes

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OVERVIEW

1. Introduction – Erosion Research History.


INTRODUCTION

4. Established: Natural Runoff Plots; Sediment Gaging Watersheds; Flood Control Reservoirs, Sedimentation Basins 1930s – 1970s by the SCS.
5. Erosion Research Program yielded first topographic relationships in 1940s.
6. Agricultural HDB 282, 1965 (USLE)
8. Agricultural HDB 703, 1997 (RUSLE)
9. RUSLE2 (2002). Website of NRCS and ARS-NSL.
11. Other Process Equation came into being in the 1970s
Universal Soil Loss Equation (USLE)

\[ A = RKSLCP \]

- \( K \) = Soil Erodibility Factor (Soil Scientist)
- \( S \) = Slope Steepness Factor (Agricultural Engineer)
- \( L \) = Slope Length Factor (Agricultural Engineer)
- \( C \) = Cropping Management Factor (Agronomist)
- \( P \) = Erosion Control Practicing Factor (Agricultural Engineer)
- \( R \) = Rainfall/Runoff Factor (Hydrologist)
K-Factor

A parameter that involves a host of processes ranging from detachment of soil, sediment transport, overland flow, infiltration, seepage, etc.
Sediment Transport

Hydro-mechanics vs. Sedimentary Fluid Mechanics
Motivation Of Research

Fig : The development of an organized sediment transport pattern with increasing concentration.
Granular Gravity Flow

Fig: Schematic of experimental facility.
**Granular Gravity Flow**

![Diagram showing granular gravity flow](image)

*Fig. Shallow granular flow in an inclined channel (a) Uniform flow regime (b,c) Mid-inertial flow regime (d,e) Fully inertial flow regime. Difference in particle velocities within one wavelength, $\theta = 7.5^\circ$, $d_{av} = 800 \mu m$ (f).*

Experimental

Fig: Schematic diagram of the experimental facility
Experimental

Fig: Arrangement of twin Photonic probes with module. Upstream and downstream probes are labeled with numbers 1,2 respectively.
Experimental

Fig: Computer controlled rotating sample collector.

Samples, collected continuously by fine meshed sieves, are transferred to aluminum trays, oven dried overnight, and dry weights are determined.
Definition Of Important Parameters:

\[
\langle \alpha \rangle(z) = \frac{1}{T} \int_{0}^{T} \alpha(z, t) \, dt \quad (1)
\]

\[
C(\Delta T) = \frac{1}{T} \int_{0}^{T} \left( \alpha(z_1, t) - \langle \alpha \rangle(z_1) \right) \left( \alpha(z_2, t - \Delta T) - \langle \alpha \rangle(z_2) \right) \, dt \quad (2)
\]

Fig: Photonic probes are area meters (volumetric information may need correction). Hence the measurements represent the light exposed surface area in the plane normal to ‘z’ axis.
# Physical Properties Of Materials

<table>
<thead>
<tr>
<th>Material</th>
<th>Diameter, μm</th>
<th>Density, ρ_s, kg/m³</th>
<th>F_p</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>d_s, min</td>
<td>d_s, max</td>
<td>d_s, mean (d_sm)</td>
</tr>
<tr>
<td>Coarse sand</td>
<td>1000</td>
<td>1400</td>
<td>1200</td>
</tr>
<tr>
<td>Medium sand</td>
<td>600</td>
<td>850</td>
<td>725</td>
</tr>
<tr>
<td>Glass beads</td>
<td>600</td>
<td>1000</td>
<td>800</td>
</tr>
</tbody>
</table>

F_p — Packing factor = (Bulk density/True density)
Saltating Coarse Sand

Fig: Saltating flow of coarse sand particles ($d_{\text{mean}} = 1200$ µm) with increasing sediment transport rates and a water flow rate of 21.6 l/min ($Fr_i = 1.92$). Typical photographs [figs (a)-(d)] of saltating grains for four transport rates $m_t = 6.0$ g/min, 11.8 g/min, 38.1 g/min & 55.3 g/min respectively. Figs (e) & (f) represent changes in the planar concentration with time for figs. (a) & (d), respectively.
Cross-correlation Of Saltating Coarse Sand

Fig: Cross-correlation of saltating coarse sand with transport rates (a) $m_t = 5.95$ g/min (b) $m_t = 11.8$ g/min (c) $m_t = 24.0$ g/min & (d) $m_t = 38.1$ g/min. Water flow rate, $q_l = 21.6$ l/min ($Fr_l = 1.92$). The estimated grain velocity, $V_s$ values for (a),(b),(c) & (d) are 0.31 m/s, 0.35 m/s, 0.31 m/s and 0.22 m/s respectively.
Velocity vs. Concentration Measurements

Fig: Measured velocity vs. concentration relationships for glass beads, coarse and medium sized sand for two hydraulic regimes ($Fr_i = 1.92$ and $Fr_i = 1.45$)
Meandering development: Medium sand

Figs. (a) – (g): Meander formation and its development (all the figures correspond to the same scale (camera placed at a fixed location). Particle size, $d_s = 600 – 850 \, \mu m$. Water flow rate, $q_l = 15.7 \, l/min$ (Fr$_l = 1.45$); solids feed rate is constant, $m_s = 168.9 \, g/min$. Fig. (h) – Fully developed meander structure ($m_t = 10.2 \, g/min$).
Measured Transport Rates

Figure: Sediment transport rates in relation to sediment addition rates. Coarse sand (ds = 1000-1400 µm) water flow rate q_l = 15.7 l/min (Fr_l = 1.45)
Grain Mechanics

1. Particle – Fluid Interactions
2. Particle – Particle Interactions
3. Particle – Boundary Interactions.
Sediment Transport Model

Fig: Sediment transport in water over an inclined channel

\[ \tau_s \text{ - unit width dispersive stress in the sediment} \]
\[ \tau \text{ - unit width tractive hydro-dynamic stress on the sediment} \]
\[ h \text{ - saltation height} \]
\[ H \text{ - water depth (flow)} \]
\[ \theta \text{ - bed slope and } x, y, z \text{ are coordinates} \]
Sediment Transport Relations

**Sediment zone:**

**Continuity:**

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} (\rho u) = 0 \tag{3}$$

**Momentum:**

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = - \frac{h}{\rho_s \rho d_s} \frac{\partial p}{\partial x} + g \sin \theta - \frac{h}{\rho_s \rho d_s} (\tau_s - \tau) \tag{4}$$

- pressure
- gravitational resistance
- effective resistance

\(\alpha_v\) – volumetric solid concentration: \(u\) – velocity of grains: \(p\) – total pressure on the solid-water system.

$$p = (p_h + p_d); \quad p_h = \rho_w g (H - y); \quad p_d = \frac{10}{3} \mu_\gamma \alpha^2 \tag{5}$$

[Bagnold (1954)]
Sediment Transport Relations

**Sediment free zone** - St. Venant equations

\[
\frac{\partial \eta}{\partial t} + U \frac{\partial \eta}{\partial x} + \eta \frac{\partial U}{\partial x} = 0 \quad (1)
\]

\[
\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} + g \frac{\partial \eta}{\partial x} - g \sin \theta = -T \quad (2)
\]

\[\eta = (H-h)\]; \(T\) - the effective flow resistance and \(U\) - the depth average flow velocity.
Sediment Transport Relations

From the available information and through the solution of eq. (4) the concentration profile $\alpha(X)$ can be expressed as,

$$3 \rho_0 \frac{(u-c)^2}{\alpha^2} \left(1 + \frac{1}{\alpha}\right)^{-4} \frac{d\alpha}{dX} - \frac{5h}{\rho_s d_s} \mu \gamma \alpha^2 \frac{1}{dX} =$$

$$\frac{\rho_{w} \rho_{g} \frac{d\eta}{dX}}{\rho_s d_s} - \rho_0 g \left(1 + \frac{1}{\alpha}\right)^{-3} \sin \theta + \frac{h}{\rho_s d_s} \left(2.25 \mu \gamma \alpha^2 - \tau\right)$$

Sediment Transport Relations

Critical solid concentration

A simple criteria for the transition of grain process from saltation to strip mode is obtained from eq. (6) by setting the coefficient of $\frac{d\alpha}{dX}$ to zero,

The expression for critical linear solid fraction $\alpha$ may be given as

\[
\frac{\alpha^6}{(1 + \alpha)^2} = \frac{3 q_m^2}{48 \left( \frac{h}{d_s} \right) C_0^2 \rho_s \rho_w \nu U}
\]

$q_m$ – Sediment mass flow rate per unit width of the channel.
Particle Velocity vs Concentration

\[
u = 10.7 C_m C_0 U \alpha^2 \left( \frac{h}{d_s} \right)^{\frac{3}{2}}
\]

- \( u \) – particle velocity
- \( C_m \) – Coefficient for the particle matrix (Eames et al., 2004)
- \( C_0 \) – maximum possible volumetric concentration
- \( U \) – free stream velocity of water
- \( \alpha \) - linear particle concentration
- \( h \) – saltation height
- \( d_s \) – particle diameter
Particle Velocity vs Concentration

Fig: Particle velocity predictions are plotted against planar solid concentration for three different $h/d_s$ ratios ($h/d_s = 5, 4 & 3$) and for water velocity, $U$ values 0.32 and 0.35 m/s.
Boundary Effect – Conceptual Model of Particle Impact Mechanics

Impact pulse: $\int_{t_1}^{t_2} Fdt = m(uv_1 - uv_2)$ with

Impulse components: $I_n = 2m\sqrt{2gh}$; $I_t = 0.25I_n$

Relationship: $\frac{I_t}{I_n} = \frac{m(u_1 - u_2)}{m(v_1 - v_2)} = \frac{u_1 - u_2}{2v_1} = 0.25$ Large oblique impact.

$t = t_1$ $u_1 = u_0$ $t = t_2$ $u_2 = u_t = e_t u_0$ $e_t$ = coefficient of restitution

Result: $h = 206.6(1 - e_t)^2 u_0^2$
Boundary Effect

Assume \( u = \frac{u_0 + u_t}{2} \)

then \( h = 8.26 \times 10^2 \left( \frac{1 - e_t}{1 + e_t} \right)^2 u^2 \)

which upon substitution in the particle velocity - linear concentration relationship yields

\[
 u = \frac{1.13 \times 10^{-4} d_s}{C_m C_0 U \alpha^{1.5}} \left( \frac{1 + e_t}{1 - e_t} \right)^2
\]
Results: Coefficient of Restitution

Fig: Estimated tangential restitution coefficients ($e_t$) plotted against measured solids concentration. The data from saltating experiments using two sizes of sand grains and spherical glass beads. Two hydraulic conditions with Froude numbers $F_r = 1.45$ and 1.92.

Hydrology

Infiltration and Soil Water Movement

Method of Weighted Residuals in conjunction with a Spectral Series Solution

\[
\theta = a_0 \cdot (\delta - z)^{\alpha} + a_1 \cdot (\delta - z)^{\alpha+1} + a_2 \cdot (\delta - z)^{\alpha+2} + \ldots
\]

\[
= \eta^{\alpha} \cdot \sum_{i=0}^{N} a_i \cdot \eta^i = \eta^{\alpha} \cdot f(t, \eta)
\]

where

\[
\eta = (\delta - z)
\]
Moments of Residual Terms:

\[
\int_{0}^{\delta} z^m \cdot \text{Re} \, dz = 0 \quad m = 1, 2, \ldots N - 2
\]

where

\[
\text{Re} = \frac{\partial \theta_n}{\partial t} - \frac{\partial}{\partial z} \left[ D(\theta) \cdot \frac{\partial \theta_n}{\partial z} \right]
\]

and

\[
D(\theta) = \theta^n \frac{a}{(\theta_s - \theta)^{n/s}} = \theta^n \cdot F(\theta)
\]

Ahuja-Swartzendruber Relationships
Ahuja-Swartzendruber Relationships

\[ Function = \theta^n \]

\[ F(\theta) \]

\[ D(\theta) = \frac{a\theta^n}{(\theta_s - \theta)^{n/s}} \]

VOL. SOIL WATER CONTENT, \( \theta \)
Substitution of these relationships in the Richards Equation yields:

\[(i)\]
\[
\alpha = \frac{1}{n}
\]

This relationship shows the interdependence of the shape of the water content profile and the diffusivity function through the parameter \(a\).

\[(ii)\]
\[
\frac{d\delta}{dt} = \alpha \cdot f^n(t,0) \cdot F(0)
\]

This relationship refers to the dynamic equation governing the growth of the wetting front.
Applications

1. Infiltration in crusted soils
2. Infiltration in swelling, shrinking, and cracking soils
Horizontal Infiltration

![Graph showing soil water content vs. reduced depth for Columbia Silt Loam.](image)
Vertical Infiltration
Application 3: Infiltration into swelling-shrinking-cracking soils

Physical Image

Cumulative Infiltration
Summary

This presentation shows how erosion research, in a time span of about 80 years, has changed from a purely regressive relationship approach to a highly complex physical process based emphasis.