GRATING-PAIR CHIRPED PULSE AMPLIFICATION SYSTEM FOR A LOW POWER FEMTO-SECOND PULSE SOURCE

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Master of Science

by
Ishan Sharma
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ABSTRACT

A grating-pair Chirped Pulse Amplification (CPA) system is simulated and built with the goal to amplify a 535 fs sech\(^2\) intensity pulse with a pulse energy of 3.36 pJ to a final output pulse of 400 fs width and 30 nJ of energy. The MATLAB simulation code, schematic design of the setup, experimental measurements, and analyses are presented. It is the hope that the reader will be able to easily simulate and build a CPA system by using the code and information presented in this thesis.
BIOGRAPHICAL SKETCH

Ishan Sharma, a resident of Edmonton, Alberta, Canada, joined Cornell University, Ithaca, NY in August of 2006 as an undergraduate in the College of Engineering. He affiliated with the School of Applied and Engineering Physics, while taking courses in both the School of Applied and Engineering Physics and the Department of Electrical and Computer Engineering. On May 30, 2010, Ishan graduated Class of 2010 from Cornell University with a Bachelor of Science degree in Applied and Engineering Physics with a minor in Electrical and Computer Engineering. Upon graduation, Ishan chose to further his understanding of the physical world and hence enrolled at Cornell University again in August of 2010. While pursuing a Master of Science degree in Applied Physics, he took courses in various subjects including nanofabrication, quantum optics, nonlinear optics, and solid state physics. In addition to taking courses, he has been working in the research laboratory of Prof. Chris Xu in the School of Applied and Engineering Physics. Ishan likes to balance his academic life with playing the piano, learning to play the guitar, exercising at the gym, cooking and taking easy strolls on the beautiful Cornell campus and the surrounding areas.
This thesis is dedicated to my parents, Rashmi and Pankaj Sharma, who supported me very generously in my academic and extracurricular pursuits. I would not have made it here without your unconditional support. I hope you forgive me for my mistakes and I hope I have been able to make you proud.
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1.1 The Technique - Chirped Pulse Amplification

Fiber-based ultrafast technology is more robust and compact than its solid-state counterpart. Ultrashort-pulse solid state systems are quite complex with a large number of components inside a long free-space optical cavity. The fiber-based systems, on the other hand, use fiber as a gain medium thus allowing a compact integrated cavity design without the need for constant re-alignment. Despite their advantages, fiber based amplification systems are more prone to peak-power induced nonlinear effects than solid state systems. An order-of-magnitude comparison of typical mode sizes of $\sim 10 \, \mu m$ versus $1 - 3 \, mm$ and signal propagation lengths of $\sim 1 - 10 \, m$ versus $1 - 10 \, cm$ in fiber-based and bulk solid-state medium, respectively, clearly shows that a fiber amplifier is deemed to be $10^6 - 10^7$ times more sensitive to nonlinear effects. Therefore, the key to avoiding these nonlinearities is to scale down the peak intensity inside the fiber core, so that the nonlinear effects can be reduced. This can be accomplished by a process called Chirped-Pulse Amplification (CPA) [1].

To outline the process of CPA (see Figure 1.1), an initial transform-limited pulse\(^1\), given in Figure 1.1(a), is passed through a pulse stretcher, which causes the temporal pulse-width to increase as shown in Figure 1.1(b). Due to energy conservation, the total pulse energy should remain the same; however, the increased

---

\(^1\)A transform-limited pulse is the shortest pulse permitted by its bandwidth as dictated by the Fourier Transform. In other words, the time-bandwidth product is a minimum for a transform-limited pulse. The Full-Width-at-Half-Maximum (FWHM) time-bandwidth product is $\sim 0.315$ for a sech\(^2\) pulse and is $\sim 0.44$ for a Gaussian pulse
temporal pulse-width would require the peak-power to decrease (the energy would be spread over a longer time period). This pulse can then be passed through a fiber-based amplifier, such as an Erbium-Doped Fiber Amplifier (EDFA) or an Erbium-Doped Waveguide Amplifier (EDWA), to increase the pulse energy to the desired value as shown in Figure 1.1(c). Although the pulse energy is increased, the peak-power remains low enough such that the nonlinearities are kept at a minimum. Finally, the pulse is compressed back to its transform-limited width as shown in Figure 1.1(d). Thus the result is an amplified transform limited pulse with minimal nonlinearities.

1.2 The Specifications

The input pulse is obtained from the seed monitoring output of the Calmar Laser Model FLCPA-01C with a measured spectral width of $\Delta \lambda = 4.713$ nm and a repetition rate of 20 MHz. Assuming a transform-limited pulse, a sech$^2$ pulse-shape has a pulse-width $T_{fwhm} = 535$ fs and a Gaussian pulse-shape has a pulse-width $T_{fwhm} = 747$ fs. The average power (including connector losses) is 67.10 $\mu$W or -11.73 dBm. This means that the pulse energy is $E_p = 3.36$ pJ.

The desired output is an unchirped pulse with FWHM pulse width of 400 fs, a pulse energy of 30-50 nJ, and a repetition rate of 20 MHz.
Figure 1.1: The evolution of a pulse in 1.1(a) as it passes through a stretcher, 1.1(b), a fiber based amplifier, 1.1(c), and a compressor to give an amplified pulse of the initial width, 1.1(d), as an output. Note: The nonlinearities are ignored for simplicity.
CHAPTER 2
THEORY AND SIMULATION

2.1 The Pulse-Propagation Equation

An electromagnetic pulse can be written mathematically as:

\[
E(\mathbf{r}, t) = \frac{1}{2} \hat{x}[E(\mathbf{r}, t) \exp(-i\omega_0 t) + \text{c.c.}]
\] (2.1)

where \(\hat{x}\) is the polarization unit vector, \(E(\mathbf{r}, t)\) is a function of space and a slowly varying function of time, and \(\omega_0\) is the center frequency. The slowly varying envelope approximation is used here since the pulse width used (~600 fs) is two orders of magnitude larger than the optical period (~5 fs for 1.55 \(\mu\)m wavelength light).

To study the pulse propagation, it is insightful to take the Fourier transform of the slowly varying amplitude, \(E(\mathbf{r}, t)\), of the electric field:

\[
\tilde{E}(\mathbf{r}, \omega - \omega_0) = \int_{-\infty}^{\infty} E(\mathbf{r}, t) \exp(i(\omega - \omega_0)t) \, dt
\] (2.2)

The frequency domain amplitude of the electric field given in Equation 2.2 can be written in a form that separates it into an \(x\)- and \(y\)- dependant part, and a \(z\)- dependent part as shown below:

\[
\tilde{E}(\mathbf{r}, \omega - \omega_0) = F(x, y)\tilde{A}(z, \omega - \omega_0) \exp(i\beta_0 z)
\] (2.3)
which solves the Helmholtz equation,

\[ \nabla^2 \tilde{E} + \varepsilon(\omega)k_0^2 \tilde{E} = 0 \]  

(2.4)

where \( k_0 = \omega/c \) and \( \varepsilon(\omega) \) is the dielectric constant. In Equation 2.3, \( F(x, y) \) is the transverse modal distribution\(^1\), \( \tilde{A}(z, \omega - \omega_0) \) is a slowly varying function of \( z \) (commonly referred to as the pulse-shape, i.e. gaussian, hyperbolic secant, etc.), and \( \beta_0 \) is the wave number.

Combining Equations 2.1, 2.2 and 2.3 gives:

\[ \mathbf{E}(r, t) = \frac{1}{2} \hat{x}[F(x, y)A(z, t) \exp(i(\beta_0 z - \omega_0 t)) + c.c.] \]  

(2.5)

where \( A(z, t) \) is the inverse Fourier transform of \( \tilde{A}(z, \omega - \omega_0) \) as given by:

\[ A(z, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{A}(z, \omega - \omega_0) \exp(-i(\omega - \omega_0)t) \ d\omega \]  

(2.6)

With the electric field expressed in the form of Equation 2.5, the analysis given in Section 2.3 of Agrawal can followed to derive the generalized pulse-progation equation (also referred to as the generalized Nonlinear Schrödinger Equation (NLSE): \([2]\)

\[ \frac{\partial A}{\partial z} + \frac{\alpha}{2} A + \frac{i \beta_2}{2} \frac{\partial^2 A}{\partial T^2} - \frac{\beta_3}{6} \frac{\partial^3 A}{\partial T^3} = i \gamma \left( |A|^2 A + \frac{i}{\omega_0} \frac{\partial}{\partial T} (|A|^2 A) - T_R A \frac{\partial |A|^2}{\partial T} \right) \]  

(2.7)

where \( \alpha \) is the absorption coefficient of the medium, \( \gamma \) is the nonlinear parameter, \( 1 \)

\( For a single-mode fiber, \( F(x, y) \) corresponds to the modal distribution of the fundamental fiber mode \( HE_{11} \), often approximated by a Gaussian distribution in \( x- \) and \( y- \) for simplicity. \)
\(\beta_2\) and \(\beta_3\) are the second- and third-order dispersion terms, respectively, \(T\) is the retarded time, \(T \equiv t - z/v_g\), in the frame of reference moving with the pulse at group velocity \(v_g\), and \(T_R\) is the first moment of the nonlinear repose function, \(R(t)\) defined by:

\[
T_R \equiv \int_0^\infty tR(t)\,dt
\]

(2.8)

where \(R(t)\) is the nonlinear response function dependent on the electronic and nuclear responses due to Raman scattering most dominant in ultrashort pulses (<1ps). For more information on Raman scattering, refer to Chapter 8 and 12 of Agrawal [2].

To obtain the second- and third-order dispersion terms, the wave number \(\beta(\omega)\) is Taylor expanded around the carrier frequency \(\omega_0\) as given by:

\[
\beta(\omega) = \beta_0 + (\omega - \omega_0)\beta_1 + \frac{1}{2}(\omega - \omega_0)^2\beta_2 + \frac{1}{6}(\omega - \omega_0)^3\beta_3 + \ldots
\]

(2.9)

where the various \(\beta_m\) parameters are given by:

\[
\beta_m = \left(\frac{d^m \beta}{d\omega^m}\right)_{\omega=\omega_0} \quad (m = 1, 2, \ldots)
\]

(2.10)

### 2.2 The Nonlinear Schrödinger Equation Solver

Although a simplified version of the NLSE can be solved analytically for certain pulse shapes, for example the Gaussian pulse shape, the solution to the generalized NLSE involving other pulse shapes, such as sech^2, has to be obtained numerically. More details on this can be found in Section 2.4 of Agrawal [2]. The numerical
NLSE solvers have been coded in various programming languages. The solver that has been used in this project is from Appendix 1 of Travers et al [3]. This particular solver is written in the MATLAB scripting language. A slightly modified version of the source code is given as Listing A.1 in Appendix A.

2.3 The Dispersive and Nonlinear Effects

A closer look at the NLSE given as Equation 2.7 reveals that there are three main effects that govern the evolution of a pulse through a medium. On the left hand side of Equation 2.7, the second term depicts the loss in the medium, while the third and fourth terms depict the effects of the second- and third-order dispersion. All the terms on the right hand side of Equation 2.7 summarize the nonlinear effects on the pulse propagation. Depending on the initial pulse width $T_0^2$ and the pulse peak power $P_0$, dispersive or nonlinear effects may dominate. Hence, it is insightful to express the length scales over which each phenomenon is dominant.

The dispersive effects are dominant at the dispersion length, $L_D$ given by,

$$L_D = \frac{T_0^2}{|\beta_2|} \tag{2.11}$$

while the nonlinear effects are dominant at the nonlinear length, $L_{NL}$, given by,

$$L_{NL} = \frac{1}{\gamma P_0} \tag{2.12}$$

$^2T_0$ is the half-width at the 1/e intensity point and is related to the FWHM pulse width by the following factors:

<table>
<thead>
<tr>
<th>Function</th>
<th>$T_{fwhm}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>sech$^2$</td>
<td>$T_{fwhm} \approx 1.763T_0$</td>
</tr>
<tr>
<td>Gaussian</td>
<td>$T_{fwhm} \approx 1.665T_0$</td>
</tr>
</tbody>
</table>
In the given system, with the specifications that were outlined in Section 1.2, the dispersion length can be calculated to be,

\[ L_D \approx \frac{(0.535 \text{ ps}/1.763)^2}{|20 \text{ ps}^2/\text{km}|} = 4.6 \text{ m} \tag{2.13} \]

where a sech$^2$ pulse is assumed, and the $\beta_2$ of single mode fiber at $\lambda = 1.55 \mu m$ is taken to be \( \approx 20 \text{ ps}^2/\text{km} \) [2]. Similarly, the nonlinear length can also be calculated to be,

\[ L_{NL} \approx \frac{1}{(2 \text{ W}^{-1} \text{km}^{-1})(9.88 \times 10^4 \text{ W})} = 5.1 \text{ mm} \tag{2.14} \]

where a 30 nJ sech$^2$ pulse is assumed to calculated the peak power, and the $\gamma$ of single mode fiber at $\lambda = 1.55 \mu m$ is taken to be \( \approx 2 \text{ W}^{-1} \text{km}^{-1} \) [2]. From the calculated values for $L_D$ and $L_{NL}$, it is evident that if the pulse were to be passed directly (i.e., without stretching) through a couple of EDFAs, which can contain upto 30 m of fiber each, the pulses would accumulate both dispersion and nonlinearity. Therefore, it is essential to stretch the pulse before amplification.

### 2.4 The Simulation

The entire setup can be simulated given the input pulse specifications, the fiber parameters and the dispersion of the stretcher-compressor system. The simulation has been coded in MATLAB so that the NLSE solver function given in Listing A.1 can be easily called. The entire simulation can be split into 6 stages as follows:

I Pre-Amplification

II Propagation to Stretcher
III Pulse Stretcher

IV Amplification

V Propagation to Compressor

VI Pulse Compressor

The simulation code for each of these stages is discussed below along with the results of the simulations. This code, written by Ishan Sharma, is presented in its entirety as Listing B.1 Appendix B.

2.4.1 Initial Pulse Parameters

Since the core of this program is a numerical NLSE solver, the initial pulse is defined in terms of discrete points in the time domain. First, a suitable time window is chosen to maintain a good balance between the steadfastness of the numerical methods and the use of the computer’s memory or computation time. After several tries, a time window of 1400 ps was found to be acceptable. Similarly, a suitable number of grid points is chosen to maintain a good balance between resolution and the use of the computer’s memory or computation time - $2^{19}$ was found to be a good number. The time grid, in the units of picoseconds, is then created of the defined time length with the specified resolution. Since the NLSE assumes a retarded time, i.e., a time frame travelling with the pulse, the time grid is centered about 0 with an equal number of negative and positive time grid values. The frequency grid, in the units of radians, is created by using the relation $\omega = 2\pi/T$. The MATLAB code for the time and frequency grids is given in Listing 2.1 below.
Listing 2.1: MATLAB code for creating the Time and Frequency Grids

1 % Definitions of the time and frequency grids
2 %
3
4 cpt = cputime; % time since MATLAB started (initial reference time)
5 n = 2^19; % number of grid points
6 twidth = 1400; % width of time window [ps]
7 c = 299792458*1e9/1e12; % speed of light [nm/ps]
8 wavelength = 1550; % reference wavelength [nm]
9 w0 = (2.0*pi*c)/wavelength; % reference frequency [rads/ps]
10 dT = twidth/n; % time interval
11 T = (-n/2:n/2-1)*dT; % time grid (NB: now a point at T=0)
12 V = 2*pi*(-n/2:n/2-1)/(n*dT); % frequency grid, ' implies transpose
13 Vabs = V + w0; % absolute frequency grid
14 WL = 2*pi*c./Vabs; % wavelength grid

The initial pulse in the simulation is based on the spectrum trace of the Calmar Laser Seed Monitoring Output measured by an Optical Spectrum Analyzer (OSA). The specifications are coded as given in Listing 2.2. The user has the option of choosing a sech\(^2\) or a Gaussian shaped intensity pulse. Note that only the slowly-varying envelope of the electric field is defined here, since that forms the input for the NLSE solver. The \texttt{fftshift} and \texttt{fft} functions of MATLAB are used to calculate the Fourier Transform of \(E(t)\); and the \texttt{ifftshift} and \texttt{ifft} functions of MATLAB are used to calculate the Inverse Fourier Transform of \(\tilde{E}(\omega)\).

Listing 2.2: MATLAB code for the Initial Pulse

1 %
2 % Definition of the Input Pulse – Stage 0
3 %
4
5 reprate = 20*10^6; % repetition rate [Hz]
6 FWHM = 0.3127; % FWHM [ps]
After each stage, a few useful figures of merit are calculated. These include the pulse energy (to check energy conservation), the pulse intensity profile, and the pulse spectral profile. The Full-Width-at-Half-Maximum temporal and spectral pulse widths are calculated using the \textit{fwhm} function obtained from MATLAB Central File Exchange [4] and presented in its entirety in Appendix C. The intensity pulse and the autocorrelation trace for this stage are plotted and presented in Figure 2.1. The function used to generate the autocorrelation trace is given in its entirety in Appendix D.
2.4.2 Stage I: Pre-Amplification

The pulse pre-amplification is represented by a gain factor that is multiplied to the electric field. The pulse is amplified to an average power of $\sim 1\text{mW}$. As expected, the pulse peak power increases without any change in the pulse width (Figure 2.2(a)), while the autocorrelation trace remains the same (Figure 2.2(b)).

Listing 2.3: MATLAB code for the Stage I

```matlab
1 % = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = =
2 % Pre-Amplifier (EDFA/EDWA) - Stage 1
3 % = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = =
4
5 avgpower_preamp = 1.06*10^-3; % Average power after EDFA [W]
6 E_pulse_preamp = 10^12*avgpower_preamp/reprate; % Calculate the pulse energy after EDFA [pJ]
7 P_peak_preamp = 0.88*E_pulse_preamp/t_FWHM0; % Calculate peak power [W]
8 gain_preamp = sqrt(P_peak_preamp)/sqrt(power); % gain for preamp
9 E1 = E0.*gain_preamp;
10 cx1 = fftshift(fft(E1));
```
Figure 2.2: The simulated intensity profile and autocorrelation trace of the pulse after pre-amplification (Stage I)

2.4.3 Stage II: Propagation through Fiber between the Preamp and the Stretcher

Since fiber amplifiers usually have 10-30 m of fiber through which the pulse propagates after amplification, the NLSE solver is used to simulate the effect of this propagation on the pulse. The output pulse from Stage I is used as input for this stage. A fiber length of 30 m is used as shown in the code presented as Listing 2.4. The output intensity curve and autocorrelation trace is presented in Figure 2.3.

Listing 2.4: MATLAB code for the Stage II

```matlab
1 % = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = =
2 % Propagation through fiber between preamp and stretcher - Stage 2
3 % = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = =
4 5 flength1 = 30; % fibre length [m]
6 7 % Simulation Parameters %
8 nsaves = 20; % number of length ← steps to save field at
9 10 % Propagate Field %
```

13
\[ Z, AT2, AW2, W \] = gnlse(T, E1, w0, gamma, betas, loss, fr, RT, \leftarrow \operatorname{flength1}, n\text{saves});

\[ E2 = AT2(n\text{saves,:}); \]

\[ cx2 = AW2(n\text{saves,:}); \]

Figure 2.3: The simulated intensity profile and autocorrelation trace of the pulse after propagation through fiber between the preamp and the stretcher (Stage II)

As can be seen from Figure 2.3, the pulse is stretched; however, experimentally, it was found that the pulse width is smaller that the input pulse as shown in Figure 4.3. A more accurate simulation can be obtained by incorporating the effects of the EDFA more carefully, perhaps based on Section 4.2.6 of Agrawal [2]. To ensure that the simulation matches the experiment, a \( \text{sech}^2 \) intensity pulse with the pulse width obtained from the experiment (Figure 4.3) is used for the subsequent stages instead of using the output of Stage II (Figure 2.3).

### 2.4.4 Stage III: The Pulse Stretcher

For the pulse stretcher, the function \textit{gratingpair} given as Listing 3.1 is called with the desired grating separation and angle of incidence as the input parameters.
The group-velocity dispersion and the third-order dispersion are the outputs. As mentioned above, a sech\(^2\) intensity pulse with a pulse width of 0.313 ps is used instead of the output pulse from Stage II. The stretcher parameters are as follows: 1000 lines/mm gratings, 30 cm focal length lenses, 16 cm of separation between the gratings and the lenses, and an incidence angle that is 9° away from the Littrow angle. These values were chosen to stretch the pulse to ∼136 ps, which is the same pulse width obtained by Howe et al [5] in a similar experimental setup. The output intensity curve and autocorrelation trace for this stage is presented in Figure 2.4.

Listing 2.5: MATLAB code for the Stage III

```matlab
1 % = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = =
2 % Grating−pair stretcher − Stage 3
3 % = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = =
4
5 [GVD1, TOD1] = gratingpair(1, wavelength, 1.5*lambda_FWHM1, 1000, 0,←
6            30, 16, 16, 16, 50.8−9)
7 cx3=exp((((1i*(1/2)*GVD1).*V.^2)+((1i*(1/6)*TOD1).*V.^3)).*cx1;
8 E3 = ifft(ifftshift(cx3));
```

2.4.5 Stage IV: The Pulse Amplifier

To get the desired pulse energy of 30 nJ, a gain of ∼24.1 is required from this stage. The code is similar to that of the preamp given in Listing 2.3. The output intensity curve and autocorrelation trace for this stage is presented in Figure 2.5.

\[^3\]Upon talking to the author of [5], it was realized that the experimental stretch factor is usually less than the simulated stretch factor. Therefore, the pulse is over-stretched to ensure that the desired experimental pulse width is obtained.
Figure 2.4: The simulated intensity profile and autocorrelation trace of the pulse after the pulse stretcher (Stage III)

Figure 2.5: The simulated intensity profile and autocorrelation trace of the pulse after the amplifier (Stage IV)

2.4.6 Stage V: Propagation through Fiber between the Amplifier and the Compressor

The pulse is propagated through 30 m of fiber (typical fiber length in an EDFA) to get a sense of how the pulse is affected. The code is similar to that of the
pulse propagation between the preamp and the stretcher given in Listing 2.4. The output intensity curve and autocorrelation trace for the pulse propagation between the amplifier and the compressor (Stage V) is presented in Figure 2.6, which shows that the FWHM pulse width after propagation does not change by a significant amount, as desired.

![Figure 2.6: The simulated intensity profile and autocorrelation trace of the pulse after propagation between amplifier and compressor (Stage V)](image)

**2.4.7 Stage VI: The Pulse Compressor**

In this stage, the pulse is compressed back to femtosecond pulse-width. The parameters for the grating-pair were calculated to ensure that the Group Velocity Dispersion of the compressor canceled that of the stretcher. Since the propagation through fiber in Stage V does not affect the pulse significantly, the output from Stage IV is used as input to the compressor. This also ensures that the stretcher and the compressor are matched (small adjustments can be made to compensate
for miscellaneous GVD contributions). As can be seen from the output intensity
curve and autocorrelation trace presented in Figure 2.7, the original pulse width
is recovered with a 30 nJ pulse energy (Note: The distance between the gratings
can be adjusted to achieve the desired 400 fs pulse width).

Listing 2.6: MATLAB code for the Stage VI

```matlab
1 % = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = =
2 % Grating−pair compressor − Stage 6
3 % = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = =
4
5 [GVD2, TOD2] = gratingpair(0, wavelength, lambda_FWHM4, 1200, ←
    1.63687881749541e+001, 0, 0, 0, 74.4)
6 cx6 = exp(((1i*(1/2)*GVD2).*V.ˆ2) +((1i*(1/6)*TOD2).*V.ˆ3)).*cx4;
7 E6 = ifft(ifftshift(cx6));
```

![Figure 2.7: The simulated intensity profile and autocorrelation trace of the pulse after the compressor (Stage VI)](image)

(a)  
(b)
3.1 The Grating-Pair System

There are many ways of stretching a pulse by introducing dispersion, such as propagation through material (fiber), a prism pair or a grating pair; however, the grating-pair is the cleanest method that also allows the design of a matched compressor to get back the initial pulse width. It was realized by Martinez that by placing a telescope between a grating pair, the dispersion is controlled by the effective distance between the second grating and the image of the first grating. When this distance is optically made to be negative, the arrangement has exactly the opposite dispersion of a grating compressor. This forms the basis of a perfectly matched stretcher-compressor pair [6] [7].

3.1.1 The Equations for the Grating-Pair System

When light at a particular wavelength, $\lambda$, is incident on a grating at an angle of incidence, $\gamma$, from the normal to the surface of the grating, the different wavelengths in the beam get diffracted by the diffracted angle, $\theta$, as governed by Equation 3.1. Therefore, the pulse will become negatively chirped (short wavelengths precede longer wavelengths).

$$\sin \gamma + \sin \theta = \frac{\lambda}{d}$$

As calculated by Backus et al, the phase contribution due to a double pass through the grating-pair system, where a pulse travels through the grating-pair once, gets
reflected by a retro-reflector, and passes again through the grating-pair system, is given by Equation 3.2 [8]:

\[
\phi(\omega) = \frac{4\omega L_{\text{eff}}}{c} \left[ 1 - \left( \frac{2\pi c}{\omega d} - \sin \gamma \right)^2 \right]^{1/2}
\] (3.2)

where \( L_{\text{eff}} \) is the effective grating separation, and \( d \) is the groove spacing of the grating in the unit of length. To find the group velocity dispersion (GVD) and the third order dispersion (TOD), Equation 3.2 can be differentiated w.r.t. \( \omega \) twice and thrice, respectively. This gives the expression for GVD in Equation 3.3 and the expression for TOD in Equation 3.4:

\[
\frac{\partial^2 \phi(\omega)}{\partial \omega^2} = \frac{-2\lambda^3 L_{\text{eff}}}{2\pi c^2 d^2} \left[ 1 - \left( \frac{\lambda}{d} - \sin \gamma \right)^2 \right]^{-3/2}
\] (3.3)

\[
\frac{\partial^3 \phi(\omega)}{\partial \omega^3} = \frac{-3 \lambda}{2\pi c} \frac{\partial^2 \phi(\omega)}{\partial \omega^2} \left( \frac{1 + \frac{\lambda}{d} \sin \gamma - \sin^2 \gamma}{1 - \left( \frac{\lambda}{d} - \sin \gamma \right)^2} \right)
\] (3.4)

Equations 3.2, 3.3 and 3.4 are valid for both the stretcher and the compressor with one major difference: while the \( L_{\text{eff}} \) is the distance between the grating surfaces in the case of the compressor, the \( L_{\text{eff}} \) for the stretcher is given by \(-2(f - s)\), where \( f \) is the focal length of the lenses and \( s \) is the distance between the lens and the grating.

### 3.1.2 The MATLAB Function \texttt{gratingpair}

As part of this project, the function \texttt{gratingpair} was coded in MATLAB by Ishan Sharma to take the grating-pair setup parameters as inputs and output the GVD
and TOD. This function also calculates the spatial chirp on the pulse after a single pass through the grating-pair system based on the input bandwidth; the calculated spatial chirp can help in the system design to ensure that the beam is not clipped. The code for this function is included below as Listing 3.1. To calculate the grating-pair parameters required to achieve a desired GVD, a program is given as Listing E.1 in Appendix E.

Listing 3.1: MATLAB code for the function gratingpair

```matlab
function [GVD_ps, TOD_ps] = gratingpair(stretcher_bool, lambda_nm, delta_lambda_nm, sgroove, Lg_cm, f_cm, s1_cm, s2_cm, thetai_deg)

% GRATINGPAIR Calculation of the GVD, TOD and FOD for a grating−pair
coded by Ishan Sharma, 2011.
%
% Calculation of the GVD, TOD and FOD for a grating−pair−telescope
%
% Implementation is as follows:
% [GVD_ps, TOD_ps] = gratingpair(stretcher_bool, lambda_nm, sgroove, Lg_cm, f_cm, s_cm)

% User input variables
stretcher = stretcher_bool; % enter 1 if stretcher (with telescope), else 0
lambda = lambda_nm*10^-9; % center wavelength [m]
delta_lambda = delta_lambda_nm*10^-9; % wavelength spread [m]
f = f_cm*10^-2; % focal length of lenses [m]
s1 = s1_cm*10^-2; % distance between lens and grating 1 [m]
s2 = s2_cm*10^-2; % distance between lens and grating 2 [m]
Lg = Lg_cm*10^-2; % distance between gratings (grating separation) [m]
sgroove; % groove spacing [lines/mm]

% speed of light [m/s]
c = 2.99792458*10^8;
if stretcher == 1
```

21
\[
\begin{align*}
\text{Leff} &= -((f-s1)+(f-s2)) \quad \% \text{effective grating separation} \\
\text{Lg} &= f+f+s1+s2 \quad \% \text{Actual grating separation} \\
\text{if} \quad \text{thetai} \deg \quad \text{==} \quad 0 \quad \% \text{line–width} \\
\text{thetai} &= \text{thetai} \deg \times (\pi/180) \quad \% \text{working at the Littrow angle} \\
\text{thetalittrow} &= \text{asin}(\lambda/(2*d)) \quad \% \text{calculation of the littrow} \\
\text{thetad} &= \text{asin}((\lambda/d)-\sin(\text{thetai})) \quad \% \text{incidence angle} \\
\text{thetad} &= \text{thetad} / (\pi/180) \quad \% \text{working at user–input} \\
\text{GVD} &= -(2*\lambda^3*\text{Leff}/(2*\pi*c^2*d^2))*(1-((\lambda/d)-\sin(\text{thetai}))^{-2})^{-2}/(3/2) \quad \% \text{calculation of GVD, TOD and FOD} \\
\text{TOD} &= -(3/(2*\pi)*\lambda/c)*GVD*(((80*\lambda^2/d^2) + 20 - (48*\lambda^2*\cos(\text{thetai})/d^2) + (16*\cos(2*\text{thetai}) - 4*\cos(4*\text{thetai})) + (32*\lambda*\sin(\text{thetai})/d) + (32*\lambda*\sin(3*\text{thetai})/d))/((8*\lambda/d) + (4*d/(\pi/180) + (4*d*\cos(2*\text{thetai})/\lambda) + (32*\sin(\text{thetai})))^2 - (\text{TOD}*2)) + (32*\pi*\lambda/c)*((1+((\lambda*\sin(\text{thetai}))/d) - (\sin(\text{thetai}))^2) - (1/(\lambda/d - \sin(\text{thetai})))^2))) \quad \% \text{GVD, TOD and FOD} \\
\text{GVD}_p &= \text{GVD}*(10^{12})^2 \quad \% \text{Calculation of spatial chirp} \\
\text{TOD}_p &= \text{TOD}*(10^{12})^3 \quad \% \text{the same for blue and red.} \\
\text{FOD}_p &= \text{FOD}*(10^{12})^4 \quad \% \text{the same for blue and red.} \\
\text{thetai} &= \text{asin}((\lambda_{\min}/d)-\sin(\text{thetai})) \quad \% \text{Assume that the initial beam spot is small (which it is). So} \\
\text{thetai} &= \text{asin}((\lambda_{\max}/d)-\sin(\text{thetai})) \quad \% \text{he is} \\
\text{spatialspread} &= (\tan(\text{thetad} - \text{thetad})*\text{Lg}) + (\tan(\text{thetad} - \text{thetad})*\text{Lg}) \quad \% \text{in} \\
\end{align*}
\]
3.2 CPA Design Schematics

The schematic design for the entire CPA system is given below as Figure 3.1.

![Design Schematics of the CPA system](image)

Figure 3.1: Design Schematics of the CPA system. PC, polarization controller; Coll., fiber collimator; G1, 2-inch$^2$ grating (1000 lines/mm); L, 2-inch double convex lens (30 cm focal length); M, mirror that offsets beam vertically; PUM, pick-up mirror vertically offset; FC, fiber collimator; G2, 2-inch$^2$ grating (1200 lines/mm)

**LASER:** Calmar Laser Model: FLCPA-01C

**PRE-AMP:** Amonics Model: AEDFA-C-23I-B-FA with Diode 1 at 25 mA and
Diode 2 at 25 mA
AMP1: AFC Model: RS-232
AMP2: IPG Photonics Model: EAR-1K-C-W

Since the grating efficiency is polarization sensitive, the fiber-based paddle polarization controllers are used to maximize the power of the diffracted beam. The entire stretcher is first aligned using irises and continuous wave (CW) light. Pulsed light is then used to ensure that the spatially chirped beam is not being clipped. The return beam in the stretcher is displaced slightly above the incident beam. The upward displacement switches to downward displacement through the telescope and the output is collected by the pickup-mirror that is placed below the incident beam. For the fiber collimator, a 0.18 N.A. aspheric lens is used in conjunction with a fiber tip on a Nanomax translation stage. The second grating might need to be tilted to reduce spatial chirp in the output beam. Spatial chirp in the output beam causes decreased bandwidth in the light coupled into fiber. A similar procedure is followed in aligning the compressor.
4.1 Spectrum Measurements and Autocorrelation Traces or Pulse-width Measurements

The spectrum measurements obtained from the Optical Spectrum Analyser (ANDO Model: AQ6317) for each stage are given below in linear and logarithmic scales. To gain information about the pulse width, autocorrelation traces and calculated intensity pulse plots are given. In the cases where the pulse width was too big for the scanning range of the motor available or the pulse peak-power was too low to trigger a two-photon response from the detector, the pulse width traces from the Sampling Oscilloscope (AGILENT Model: 86100A) are given.

4.1.1 Initial Pulse

Since the power from the laser is quite low, an autocorrelation trace could not be done; therefore, only the spectrum measurements are given in Figure 4.1. The average power is measured to be 67.10 µW.

4.1.2 Pre-amplifier

Both the spectrum measurements and the autocorrelation measurements are given for this stage in Figures 4.2 and 4.3. The average power is measured to be 1.06 mW.
4.1.3 Pulse Stretcher

The spectrum measurements are presented in Figure 4.4. It was difficult to obtain an autocorrelation trace, so the Sampling Oscilloscope is used to measure the pulse width, given in Figure 4.5.
Figure 4.3: The measured autocorrelation trace of the pulse after the pre-amplifier

Figure 4.4: The measured spectrum of the pulse after the pulse stretcher in 4.4(a) linear scale and 4.4(b) logarithmic scale

4.1.4 Pulse Compressor

In order to ensure that the pulse compressor works, it is connected directly to the pre-amplifier (thus by-passing the stretcher altogether). If the compressor is designed correctly, then it should have the same stretcher factor as the stretcher,
thereby giving a pulse width close to that of the stretcher. Once again the spectrum was measured (Figure 4.6) along with the pulse width (Figure 4.7) using the Sampling Oscilloscope.

Figure 4.5: The measured pulse width after the pulse stretcher

Figure 4.6: The measured spectrum of the pulse after the pulse compressor (bypassing the stretcher) in 4.6(a) linear scale and 4.6(b) logarithmic scale
4.1.5 Output of the Entire Setup

Upon testing the stretcher and the compressor individually, the entire setup is tested. The optical circuit shown in Figure 3.1 is built; but instead of the IPG Photonics Model: EAR-1K-C-W for AMP2, the MPB Communications Model: EFA-P22F fiber amplifier is used as AMP2. This amplifier can easily be switched out for the higher power amplifier initially suggested. The rest of the setup is kept the same.

The average power out of the pre-amp is 1.06 mW. After coupling the stretched pulse into fiber, the average power is measured to be 0.06 mW. After AMP1, the average power is 8.2 dBm or 6.6 mW and after AMP2, the average power is 21 dBm or 126 mW.

Perhaps due to the traffic near the optical table, the stretcher was found to be out of alignment when performing this measurement. It was difficult to reach the
same optical bandwidth (thus the same stretch factor) as presented previously in Figure 4.4. The spectrum for the output pulse from the stretcher is presented in Figure 4.8. The FWHM pulse width of the output from the stretcher is measured to be 79.3 ps as shown in Figure 4.9.

![Figure 4.8](image)

Figure 4.8: The measured spectrum of the pulse after the pulse stretcher for the final trial in 4.8(a) linear scale and 4.8(b) logarithmic scale

![Figure 4.9](image)

Figure 4.9: The measured pulse width after the pulse stretcher for the final trial

The pulse from AMP2 is passed through the compressor. The distance between
the gratings in the compressor (and hence the GVD) is adjusted until the desired pulse width is obtained\textsuperscript{1}. Upon passing the pulse through the compressor, the autocorrelation trace presented in Figure 4.10 is obtained. The intensity pulse-width, after taking the de-convolution factor for a sech\textsuperscript{2} pulse (1.543) into account, is: \( T_{fwhm} = 3.65/1.543 = 2.37 \text{ ps} \)

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure4.10a}
\caption{(a) Normalized Signal vs. Delay for CPA output pulse with \( T = 2.54 \text{ ps} \).}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure4.10b}
\caption{(b) Normalized Intensity vs. Delay for CPA output pulse with \( T = 3.65 \text{ ps} \).}
\end{figure}

Figure 4.10: The measured autocorrelation trace of the output pulse from the CPA

\textsuperscript{1}Due to time constraints, this adjustment was stopped before the final pulse-width objective was achieved.
The goal of this project was to amplify an ultra-short 20 MHz pulse with a pulse-width of $T_{fwhm} = 535$ ps (assuming a transform-limited sech$^2$-intensity pulse-shape) and a pulse-energy of $E_p = 3.36$ pJ to an output pulse of width 400 fs and energy $30 - 50$ nJ. In order to achieve a high pulse quality, it was important to reduce nonlinearities accumulated by the pulse while propagating through fiber. To achieve this objective, a technique called Chirped Pulse Amplification, where the pulse is stretched, amplified and compressed, is used to ensure that the pulse peak-power is kept low. It was decided that a grating-pair would be used to stretch and compress the pulse. A program was written in MATLAB to simulate both the propagation of the pulse through the stretcher-compressor system and the fiber. The optical circuit for the CPA was then designed as given in Figure 3.1. The spectrum measurements and autocorrelation or pulse-width traces were obtained for each stage in the optical circuit. Finally, the output pulse autocorrelation, showing an intensity FWHM pulse width of 2.37 ps, was presented.

The output pulse still has some chirp, which can easily be removed by adjusting the distance between the gratings in the compressor. The two-photon current produced by the output beam on a detector can then be observed to see the relative change in the final pulse width (Two-photon current $\propto 1/\tau$), i.e. as the pulse width gets shorter, the two-photon current will increase. Moreover, time can also be spent on removing the spatial chirp on the output of the stretcher and increasing the efficiency of coupling into fiber.

In conclusion, the reader is presented with a well-documented and thorough simulation, detailed design schematics, experimental results and observations of the
grating-pair Chirped Pulse Amplification system. It is the hope that upon reading this thesis, one will be able to replicate this setup or use the simulation code to design a new CPA system to suit his/her input and output pulse requirements.
The code for the Nonlinear Schrödinger Equation Solver as presented by Travers et al [3] is given below.

Listing A.1: MATLAB code for the NLSE Solver [3]

1. function \[ \text{gnlse}(T, \text{A}, \text{w0}, \text{gamma}, \text{betas}, \ldots \) \]
2. \% Propagate an optical field using the generalised NLSE
3. \% This code integrates Eqs. A.1, A.4 and A.5.
4. \% For usage see the example of test Dudley.m (below)
6. \% Please cite this chapter in any publication using this code.
7. \% Updates to this code are available at www.scgbook.info
8. \n9. \text{n} = \text{length}(T); \text{dT} = T(2) - T(1); \% grid parameters
10. \text{V} = 2 \times \pi \times (-n/2:n/2-1)'/(n*dT); \% frequency grid
11. \text{B} = \text{0};
12. \text{for i = 1:length(betas)} \% Taylor expansion of betas
13. \text{B} = \text{B} + \text{betas(i)}/\text{factorial(i+1)} \times \text{V} .^\text{(i+1)};
14. \text{end}
15. \n16. \text{if (loss==0)}
17. \text{alpha} = \text{loss}/10/\text{log10}(\exp(1));
18. \text{else}
19. \text{alphadb} = \text{polyval(loss,V)};
20. \text{alpha} = \text{alphadb}/10/\text{log10}(\exp(1)); \% attenuation coefficient
21. \text{end}
22. \n23. \text{L} = \text{i*}B - \text{alpha}/2; \% linear operator (one*i)
24. \text{if abs(w0) > eps} \% if w0>0 then include shock
25. \text{gamma} = \text{gamma}/w0;
26. \text{else}
27. \text{W} = \text{V} + \text{w0}; \% for shock W is true freq
28. \text{end}
29. \text{if abs(w0) > eps} \% if w0>0 then include shock
30. \text{W} = \text{1}; \% set W to 1 when no shock
31. \text{end}
32. \n33. \text{RW} = \text{n*ifft(fftshift(RT.');)}; \% frequency domain Raman
34. \text{L} = \text{fftshift(L)}; \text{W} = \text{fftshift(W)}; \% shift to fft space
35. \% define function to return the RHS of Eq. A.1
36. \text{function R = rhs(z, AW)} \% \text{AW=F(AT).*exp(-Lz)}, or \text{A'}(z,w) ←
37. \% in Eq.1.8
38. \text{AT} = \text{fft(AW.*exp(L+Z))}; \% time domain field, see comments ←
39. \text{above}
40. \text{IT} = \text{abs(AT).^2}; \% time domain intensity
41. \text{if (length(RT)==1)&&(abs(fr)<eps)}
M = iFFT(AT.*IT);
else
    RS = dT*fr*FFT(iFFT(IT).*RW); % Raman convolution
    M = iFFT(AT.*((1-fr).*IT + RS));% response function
end
R = li*gamma*W.*M.*exp(-L*z); % full RHS of Eq. A.1
end

% === define function to print ODE integrator status
function status = report(z, y, flag) %
    status = 0;
    if isempty(flag)
        fprintf('%%05.1f%% complete\n', z/flength*100);
    end
end

% === setup and run the ODE integrator
Z = linspace(0, flength, nsaves); % select output z points
options = odeset('RelTol', 1e-5, 'AbsTol', 1e-12, ...
    'NormControl', 'on', ...
    'OutputFcn', @report);
[Z, AW] = ode45(@rhs, Z, iFFT(A), options); % run integrator, ← integrate region is Z, initial value of AW=IFFT(A) at Z=0

% === process output of integrator
AT = zeros(size(AW(:,1)));
for i = 1:length(AW(:,1))
    AW(i,:) = AW(i,:)*exp(L.*Z(i)); % change variables
    AT(i,:) = FFT(AW(i,:)); % time domain output
    AW(i,:) = fftshift(AW(i,:))./dT; % scale
end
W = V + w0; % the absolute frequency grid
APPENDIX B
MATLAB CODE FOR THE CHIRPED PULSE AMPLIFICATION SYSTEM

The MATLAB code discussed in the previous chapters is given in its entirety below. The code was written for this Master’s Thesis by Ishan Sharma, Cornell University.

Listing B.1: MATLAB code for the Chirped Pulse Amplification System

```matlab
1 % Simulation for the CPA system using grating−pair stretcher and grating−pair compressor. Coded by Ishan Sharma, 2011.
2
3 clear all
4
5 % Definitions of the time and frequency grids
6
7 cpt = cputime; % time since MATLAB ←
8
9 n = 2^19; % number of grid ←
10 twidth = 1400; % width of time ←
11 c = 299792458*1e9/1e12; % speed of light [nm/ps] ←
12 wavelength = 1550; % reference wavelength [nm] ←
13 w0 = (2.0*pi*c)/wavelength; % reference frequency [rads/ps] ←
14 dT = twidth/n; % time interval ←
15 T = (−n/2:n/2−1)*dT; % better than T = linspace(−twidth/2, twidth/2, n), since there is now a point at T=0)
16 V = 2*pi*(-n/2:n/2−1)/(n*dT); % frequency grid, ' ← means transpose
17 Vabs = V + w0; % absolute frequency ←
18 WL = 2*pi*c./Vabs; % wavelength grid ←
20
21 % Definition of the Input Pulse − Stage 0
22
23 repreate = 20*10^6; % repetition rate [Hz] ←
24 FWHM = 0.3127; % FWHM [ps] ←
25 power = 0.88*3.36/FWHM; % peak power of input [W] = 0.88*E_pulse/T_fwhm (NB: E_pulse is in pJ)
```

36
t0 = FWHM/1.763; % sech duration of input [ps]
E = sqrt(power)*sech(T/t0); % sech input field [W(1/2)]

t0 = FWHM/(2*log(2))^-0.5; % guassian duration of input [ps]
E = sqrt(power)*exp(-T.*2/(2*t0.^2)); % guassian input field [W(1/2)]

cx0 = fftshift(fft(E)); % FT of input field
E0 = ifft(ifftshift(cx0)); % inverse_FT of FT of input field (sanity check)

% should match pulse entered above (sanity check) [pJ]
E0_energy = sum(abs(E0).^2.*dT); % should match pulse intensity [W]

I0 = abs(E0).^2; % pulse intensity [W]

t_FWHM0 = fwhm(T,I0); % fwhm pulse width [ps]
spec0 = abs(cx0).^2; % fwhm pulse width [ps]
w_FWHM0 = fwhm(V,spec0); % pulse spectrum
lambda_FWHM0 = (w_FWHM0/(2*pi))*wavelength^2/c; % should match the delta_lambda from OSA [nm]

% Definitions of fiber parameters for use with GNLSE code

radius_mode = 0.5*10.5*10^-6; % Mode Field Radius @ 1550nm for SMF-28 (Ref: Thorlabs.com) [m]
Aeff = pi*radius_mode^2; % Effective mode area (make this more accurate (read 2.3.1 of Agrawal)) [m^2]
n2 = 2.6*10^-20; % nonlinear index

Gamma = 2*pi*n2/(Aeff*wavelength*10^-9); % nonlinear coefficient [1/W/m]

loss = 0; % loss [dB/m]
[betas] = dispersion_smf(c,w0); % call the function to extract betas for SMF

% Raman Response for Silica
fr = 0.18; % fractional Raman

tau1 = 0.0122; tau2 = 0.032;
RT = (tau1^2+tau2^2)/tau1/tau2^2*exp(-T/tau2).*sin(T/tau1); % heaviside step

RT = RT/trapz(T,RT); % normalise RT to unit integral
66 % Pre-Amplifier (EDFA/EDWA) - Stage 1
67 %
68 avgpower_preamp = 1.06*10^-3; % Average power after EDFA [W]
69 E_pulse_preamp = 10^12*avgpower_preamp/reprate; % Calculate the pulse energy after EDFA [pJ]
70 P_peak_preamp = 0.88*E_pulse_preamp/t_FWHM0; % Calculate peak power [W]
71 gain_preamp = sqrt(P_peak_preamp)/sqrt(power); % gain for preamp
72 E1 = E0.*gain_preamp;
73 cx1 = fftshift(fft(E1));
74 % Figures of Merit %
75 E1_energy = sum(abs(E1).^2.*dT); % pulse energy after preamp [pJ]
76 I1 = abs(E1).^2; % pulse intensity [W]
77 t_FWHM1 = fwhm(T,I1); % fwhm pulse width [ps]
78 spec1 = abs(cx1).^2; % pulse spectrum
79 w_FWHM1 = fwhm(V,spec1); % fwhm spectral width
80 lambda_FWHM1 = (w_FWHM1/(2*pi))*wavelength^2/c; % delta_lambda [nm]
81 %
82 % Propagation through fiber between preamp and stretcher - Stage 2
83 %
84 flength1 = 30; % fibre length [m]
85 nsaves = 20; % number of length steps to save field at
86 % Simulation Parameters %
87 %
88 % Propagate Field %
89 [Z, AT2, AW2, W] = gnlse(T, E1, w0, gamma, betas, loss, fr, RT, flength1, nsaves);
90 E2 = AT2(nsaves,:);
91 cx2 = AW2(nsaves,:);
92 % Figures of Merit %
93 E2_energy = sum(abs(E2).^2.*dT); % pulse energy after propagation through Fiber_1 [pJ]
94 I2 = abs(E2).^2; % pulse intensity [W]
95 t_FWHM2 = fwhm(T,I2); % fwhm pulse width [ps]
96 spec2 = abs(cx2).^2; % pulse spectrum
97 w_FWHM2 = fwhm(V,spec2); % fwhm spectral width
98 lambda_FWHM2 = (w_FWHM2/(2*pi))*wavelength^2/c; % delta_lambda [nm]
99 %
100 % Grating-pair stretcher - Stage 3
101 %
102 %
103 %
\[
\begin{align*}
GVD1, \text{TOD1} &= \text{gratingpair}(1, \text{wavelength}, 1.5\lambda_{\text{FWHM1}}, 1000, 0, \rightarrow \ 30, 16, 16, 50.8-9) \\
\text{cx3} &= \exp(((1i\times(1/2)GVD1)\times\text{V}^2) + ((1i\times(1/6)\text{TOD1})\times\text{V}^3)) \times \text{cx1} \\
\text{E3} &= \text{ifft (ifftshift(cx3))} \\
\text{E3\_energy} &= \text{sum}(|\text{E3}|^2 \times \text{dT}) ; \quad \% \text{pulse energy after \rightarrow stretcher} \ [\text{pJ}] \\
\text{I3} &= |\text{E3}|^2 ; \quad \% \text{pulse intensity} \ [\text{W} \rightarrow] \\
\text{t\_FWHM3} &= \text{fwhm(T,I3)} ; \quad \% \text{fwhm pulse width} \ [\rightarrow \ \text{ps}] \\
\text{spec3} &= |\text{cx3}|^2 ; \quad \% \text{pulse spectrum} \\
\text{w\_FWHM3} &= \text{fwhm(V,spec3)} ; \quad \% \text{fwhm spectral \rightarrow width} \\
\lambda_{\text{FWHM3}} &= (w_{\text{FWHM3}}/(2\times\pi))\times\text{wavelength}^2/c ; \quad \% \delta\lambda \ [\text{nm}] \\
\end{align*}
\]

\[
\begin{align*}
\text{E\_pulse\_amp} &= 30\times10^3 ; \quad \% \text{Desired pulse \rightarrow} \\
\text{P\_peak\_amp} &= 0.88\times\text{E\_pulse\_amp}/\text{t\_FWHM3} ; \quad \% \text{Calculate peak \rightarrow} \\
\text{gain\_amp} &= 24.1 ; \quad \% \text{gain for amp} \\
\text{E4} &= \text{E3}\times\text{gain\_amp} ; \\
\text{cx4} &= \text{ffts}h\text{ift (fft(E4))} ; \\
\text{E4\_energy} &= \text{sum}(|\text{E4}|^2 \times \text{dT}) ; \quad \% \text{pulse energy after \rightarrow amplifier} \ [\text{pJ}] \\
\text{I4} &= |\text{E4}|^2 ; \quad \% \text{pulse intensity} \ [\text{W} \rightarrow] \\
\text{t\_FWHM4} &= \text{fwhm(T,I4)} ; \quad \% \text{fwhm pulse width} \ [\rightarrow \ \text{ps}] \\
\text{spec4} &= |\text{cx4}|^2 ; \quad \% \text{pulse spectrum} \\
\text{w\_FWHM4} &= \text{fwhm(V,spec4)} ; \quad \% \text{fwhm spectral \rightarrow width} \\
\lambda_{\text{FWHM4}} &= (w_{\text{FWHM4}}/(2\times\pi))\times\text{wavelength}^2/c ; \quad \% \delta\lambda \ [\text{nm}] \\
\text{flength5} &= 30 ; \quad \% \text{fibre length} \ [\text{m}] \\
\text{nsaves} &= 20 ; \quad \% \text{number of length \rightarrow} \\
\text{steps to save field at} \\
\text{Propagate Field} &= \% \\
\end{align*}
\]
\[ Z, AT5, AW5, W \] = gnlse(T, E4, w0, gamma, betas, loss, fr, RT, \( \rightarrow \) flength5, nsaves);
E5 = AT5(nsaves,:);
cx5 = AW5(nsaves,:);

158 \% Figures of Merit \%
159 E5_energy = sum(abs(E5).^2*dT); % pulse energy after propagation through Fiber_2 [pJ]
I5 = abs(E5).^2;
t_FWHM5 = fwhm(T,I5);
spec5 = abs(cx5).^2;
w_FWHM5 = fwhm(V,spec5);
lambda_FWHM5 = (w_FWHM5/(2*\pi))*wavelength^2/c;

167 \% Grating-pair compressor - Stage 6
168 \% Figures of Merit \%
169
170 [GVD2, TOD2] = gratingpair(0, wavelength, lambda_FWHM4, 1200, 1.63687781749541e+001, 0, 0, 0, 0, 74.4)
cx6\( = \)exp(((1i*(1/2)*GVD2).*V.^2)+((1i*(1/6)*TOD2).*V.^3)).*cx4;
E6 = ifft(ifftshift(cx6));

175 \% Figures of Merit \%
176 E6_energy = sum(abs(E6).^2*dT); % pulse energy after compressor [pJ]
I6 = abs(E6).^2;
t_FWHM6 = fwhm(T,I6);
spec6 = abs(cx6).^2;
w_FWHM6 = fwhm(V,spec6);
lambda_FWHM6 = (w_FWHM6/(2*\pi))*wavelength^2/c;
APPENDIX C
MATLAB CODE FOR THE FWHM FUNCTION

The MATLAB code for the function $fwhm$, called several times by the CPA program given in Appendix B, is given below. The source code for this function, authored by Patrick Egan [4], was obtained from the MATLAB Central File Exchange server and is presented below as per the instructions in the BSD License.

Listing C.1: MATLAB code for the $fwhm$ Function

```
function width = fwhm(x,y)

% function width = fwhm(x,y)
% Full-Width at Half-Maximum (FWHM) of the waveform y(x)
% and its polarity.
% The FWHM result in 'width' will be in units of 'x'
% Rev 1.2, April 2006 (Patrick Egan)

y = y / max(y);
N = length(y);
lev50 = 0.5;

if y(1) < lev50 % find index of center (max or min)← of pulse
    [garbage,centerindex]=max(y);
    Pol = +1;
    disp('Pulse Polarity = Positive')
else
    [garbage,centerindex]=min(y);
    Pol = -1;
    disp('Pulse Polarity = Negative')
end

i = 2;
while sign(y(i)-lev50) == sign(y(i-1)-lev50)
    i = i+1;
end %first crossing is between v(i←
-1) & v(i)
interp = (lev50-y(i-1)) / (y(i)-y(i-1));
tlead = x(i-1) + interp*(x(i)-x(i-1));
i = centerindex+1; %start search for next ←
crossing at center
while ((sign(y(i)-lev50) == sign(y(i-1)-lev50)) & (i <= N-1))
    i = i+1;
end
if i ~ N
    Ptype = 1;
    disp('Pulse is Impulse or Rectangular with 2 edges')
```

41
interp = (lev50−y(i−1)) / (y(i)−y(i−1));
ttrail = x(i−1) + interp*(x(i)−x(i−1));
width = ttrail − tlead;
else
    Ptype = 2;
    %disp('Step−Like Pulse, no second edge')
ttrail = NaN;
width = NaN;
end

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APPENDIX D

MATLAB CODE FOR AN AUTOCORRELATION FUNCTION

The MATLAB code for a function to obtain the autocorrelation trace with the time-domain electric field as the input is given below. This code was authored by Ji Cheng, Cornell University.

Listing D.1: MATLAB code for an Autocorrelation Function

```matlab
function [output]=StrictAutocorrelation(Eft,Ntime,time)
clight=3e5; %unit nm/ps
t=1:Ntime;
dt=time/Ntime;
T=(t−Ntime/2−1)*dt;
w=1:Ntime;
W=2*pi*(w−Ntime/2−1)/time;
input_l=1550; % unit nm
input_w=2*pi*clight/input_l; % unit ps−1
lambdas=2*pi*clight./(W+input_w);
Eft=Eft.*exp(-i*input_w*T);
%figure; plot(T,Eft.*conj(Eft));
Iat=Eft.*conj(Eft);
%figure; plot(T,Iat);
c201=xcorr(Eft.^2);
c20=c201+conj(c201);
c21=xcorr(Iat.*Eft, Eft);
c211=c21+conj(c211);
c221=xcorr(conj(Eft), Iat.*conj(Eft));
c22=c221+conj(c221);
c23=xcorr(Iat);
c2=2*sum(Iat.*Iat)+2*c21+2*c22+4*c23+c20;
C1=xcorr(Iat);
C11=sum(Iat.^2);
Cint=2*C11+4*C1;
output=c2;
Start=Ntime*1/4;
End=Ntime*7/4−1;
indext=1:End−Start+1;
%figure; plot(indext*dt,3*Cint(Start:End)/max(Cint(Start:End)),'k');
% figure; plot(indext*dt,8*c2(Start:End)/max(c2(Start:End)),'k');
```

43
The MATLAB code for a program to calculate the grating-pair parameters including the grating separation and angle of incidence is given below. This code was authored by Ishan Sharma, Cornell University.

Listing E.1: MATLAB code to Calculate Grating-pair Parameters given GVD

1 % Coded by Ishan Sharma, 2011
2
3 lambda = 1550*10^-9; % center wavelength [m]
4 sgroove = 1200; % groove spacing [lines/mm]
5 c = 2.99792458*10^8; % speed of light [m/s]
6 d = 1/(sgroove*10^3); % line-width [m]
7 thetalittrow = asin(lambda/(2*d)); % calculation of the littrow angle [rads]
8 thetalittrow_deg = thetalittrow*180/pi
9 % thetai = thetalittrow; % working at the Littrow angle [rads]
10 % thetai_deg = 50:0.001:90;
11 thetai_deg = 60.4:0.6
12 thetai = thetai_deg.*(pi/180); % working at user-input incidence angle [rads]
13 thetad = asin((lambda/d)-sin(thetai));
14 thetad_deg = thetad/(pi/180)
15 GVD = -3.591320891731296e+001/(10^12)^2;
16
17 L_eff_cm = 100./((-1/GVD)*(2*lambda^3/(2*pi*c^2*d^2))*(1-((lambda/d)-sin(thetai)).^2).^-(3/2))
18 length_cm = L_eff_cm/cos(thetad)
19 collimator_cm = 5/tan(thetai-thetad)
20 % plot(thetai_deg,L_eff)
21 % hold on
22 % plot(thetad_deg,L_eff)
23 % hold off


