5. Fuzzy Optimization

5 Fuzzy Optimization

The precise quantification of many system performance criteria and parameter and decision variables is not always possible, nor is it always necessary. When the values of variables cannot be precisely specified, they are said to be uncertain or fuzzy. If the values are uncertain, probability distributions may be used to quantify them. Alternatively, if they are best described by qualitative adjectives, such as dry or wet, hot or cold, clean or dirty, and high or low, fuzzy membership functions can be used to quantify them. Both probability distributions and fuzzy membership functions of these uncertain or qualitative variables can be included in quantitative optimization models. This chapter introduces fuzzy optimization modelling, again for the preliminary screening of alternative water resources plans and management policies.

1. Fuzziness: An Introduction

Large, small, pure, polluted, satisfactory, unsatisfactory, sufficient, insufficient, excellent, good, fair, poor and so on are words often used to describe various attributes or performance measures of water resources systems. These descriptors do not have 'crisp', well-defined boundaries that separate them from others. A particular mix of economic and environmental impacts may be *more acceptable* to some and *less acceptable* to others. Plan A is *better* than Plan B. The water quality and temperature is *good* for swimming. These qualitative, or 'fuzzy', statements convey information despite the imprecision of the italicized adjectives.

This chapter illustrates how fuzzy descriptors can be incorporated into optimization models of water resources systems. Before this can be done some definitions are needed.

1.1. Fuzzy Membership Functions

Consider a set A of real or integer numbers ranging from say 18 to 25. Thus $A = [18, 25]$. In classical (crisp) set theory, any number x is either in or not in the set A. The statement 'x belongs to A' is either true or false depending on the value of x. The set A is referred to as a crisp set.If one is not able to say for certain whether or not any number x is in the set, then the set A could be referred to as *fuzzy*. The degree of truth attached to that statement is defined by a membership function. This function ranges from 0 (completely false) to 1 (completely true).

Consider the statement, 'The water temperature should be suitable for swimming'. Just what temperatures are suitable will depend on the person asked. It would be difficult for anyone to define precisely those temperatures that are suitable if it is understood that temperatures outside that range are absolutely not suitable.

A membership function defining the interval or range of water temperatures suitable for swimming is shown in Figure 5.1. Such functions may be defined on the basis of the responses of many potential swimmers. There is a zone of imprecision or disagreement at both ends of the range.

The form or shape of a membership function depends on the individual subjective feelings of the 'members' or individuals who are asked their opinions. To define this particular membership function, each individual i could be asked to define his or her comfortable water temperature interval (T_{1i}, T_{2i}). The membership value associated with any temperature value T equals the number of

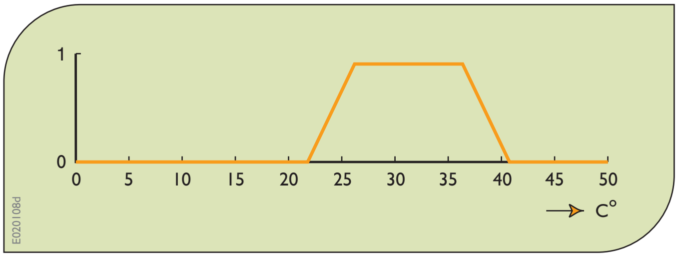

Figure 5.1. A fuzzy membership function for suitability of water temperature for swimming.

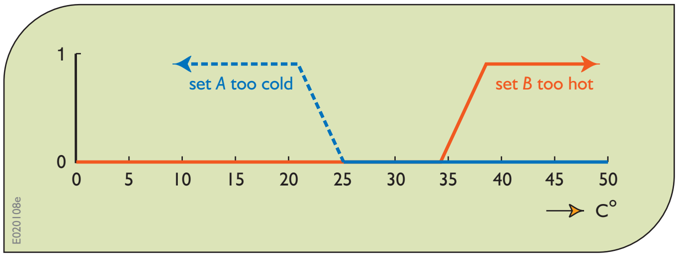

Figure 5.2. Two membership functions relating to swimming water temperature. Set *A* is the set defining the fraction of all individuals who think the water temperature is too cold, and Set *B* defines the fraction of all individuals who think the water temperature is too hot.

individuals who place that T within their range (T_{1i}, T_{2i}), divided by the number of individual opinions obtained. The assignment of membership values is based on subjective judgements, but such judgements seem to be sufficient for much of human communication.

1.2. Membership Function Operations

Denote the membership function associated with a fuzzy set A as $m_A(x)$. It defines the degree or extent to which any value of x belongs to the set A. Now consider two fuzzy sets, A and B. Set A could be the range of temperatures that are considered too cold, and set B could be the range of temperatures that are considered too hot. Assume these two sets are as shown in Figure 5.2.

The degree or extent that a value of x belongs to either of two sets A or B is the maximum of the two individual membership function values. This union membership function is defined as:

$$m_{A\cup B}(x) = \text{maximum}(m_A(x), m_B(x)) \tag{5.1}$$

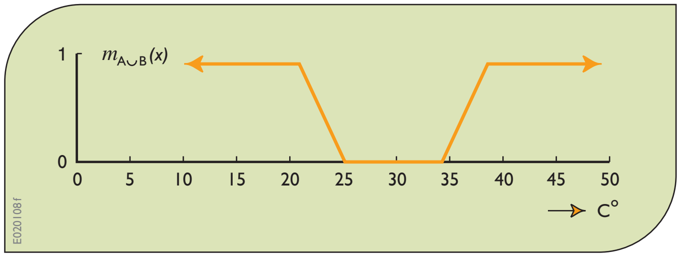

Figure 5.3. Membership function for water temperatures that are considered too cold or too hot.

This union set would represent the ranges of temperatures that are either too cold or too hot, as illustrated in Figure 5.3.

The degree or extent that a value of a variable x is simultaneously in both sets A and B is the minimum of the two individual membership function values. This intersection membership function is defined as:

$$m_{A\cap B}(x) = \text{minimum}\ (m_A(x), m_B(x)) \tag{5.2}$$

This intersection set would define the range of temperatures that are considered both too cold and too hot. Of course it could be an empty set, as indeed it is in this case, based on the two membership functions shown in Figure 5.2. The minimum of either function for any value of x is 0.

The complement of the membership function for fuzzy set A is the membership function, $m^c_A(x)$, of A^c.

$$m^c_A(x) = 1 - m_A(x) \tag{5.3}$$

The complement of set A (defined in Figure 5.2) would represent the range of temperatures considered not too cold for swimming. The complement of set B (also defined in Figure 5.2) would represent the range of temperatures considered not too hot for swimming. The complement of the union set as shown in Figure 5.3 would be the range of temperatures considered just right. This complement set is the same as shown in Figure 5.1.

2. Optimization in Fuzzy Environments

Consider the problem of finding the maximum value of x given that x cannot exceed 11. This is written as:

$$\text{Maximize } U = x \tag{5.4}$$

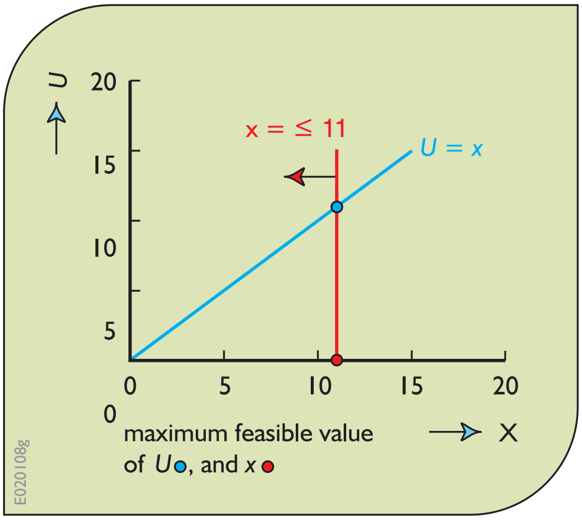

Figure 5.4. A plot of the crisp optimization problem defined by Equations 5.4 and 5.5.

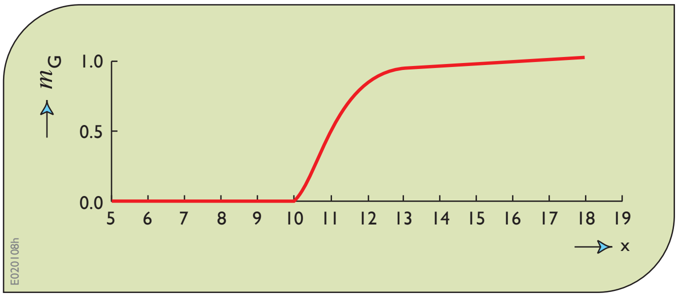

Figure 5.5. Membership function defining the fraction of individuals who think a particular value of *x* is 'substantially' greater than 10.

subject to:

$$x \leq 11 \tag{5.5}$$

The obvious optimal solution, $x = 11$, is shown in Figure 5.4.

Now suppose the objective is to obtain a value of x substantially larger than 10 while making sure that the maximum value of x should be in the vicinity of 11. This is no longer a crisp optimization problem; rather, it is a fuzzy one.

What is perceived to be substantially larger than 10 could be defined by a membership function, again representing the results of an opinion poll of what individuals think is substantially larger than 10. Suppose the membership function for this goal, $m_G(x)$, reflecting the results of such a poll, can be defined as:

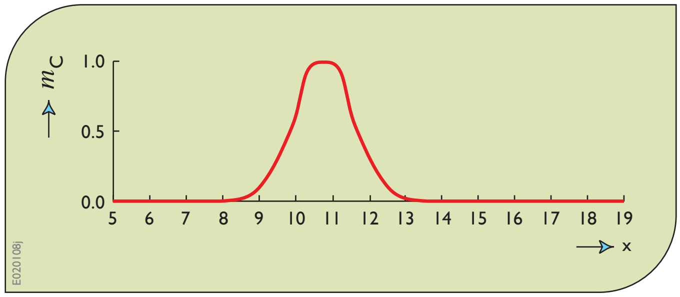

Figure 5.6. Membership function representing the vicinity of 11.

$$m_G(x) = 1/\{1 + [1/(x - 10)^2]\} \quad \text{if } x > 10$$
$$m_G(x) = 0 \text{ otherwise} \tag{5.6}$$

This function is shown in Figure 5.5.

The constraint on x is that it 'should be in the vicinity of 11'. Suppose the results of a poll asking individuals to state what they consider to be in the vicinity of 11 results in the following constraint membership function, $m_C(x)$:

$$m_C(x) = 1/[1 + (x - 11)^4] \tag{5.7}$$

This membership function is shown in Figure 5.6.

Recall the objective is to obtain a value of x substantially larger than 10 while making sure that the maximum value of x should be in the vicinity of 11. In this fuzzy environment the objective is to maximize the extent to which x exceeds 10 while keeping x in the vicinity of 11. The solution can be viewed as finding the value of x that maximizes the minimum values of both membership functions. Thus, we can define the intersection of both membership functions and find the value of x that maximizes that intersection membership function.

The intersection membership function is:

$$\begin{aligned} m_D(x) &= \text{minimum}\{m_G(x), m_C(x)\} \\ &= \text{minimum}\{1/(1 + [1/(x - 10)^2]), \\ &\quad 1/(1 + (x - 11)^4)\} \quad \text{if } x > 10 \\ &= 0 \text{ otherwise} \end{aligned} \tag{5.8}$$

This intersection set, and the value of x that maximizes its value, is shown in Figure 5.7.

This fuzzy decision is the value of x that maximizes the intersection membership function $m_D(x)$, or equivalently:

$$\text{Maximize } m_D(x) = \max \min\{m_G(x), m_C(x)\} \tag{5.9}$$

Using LINGO®, the optimal solution is $x = 11.75$ and $m_D(x) = 0.755$.

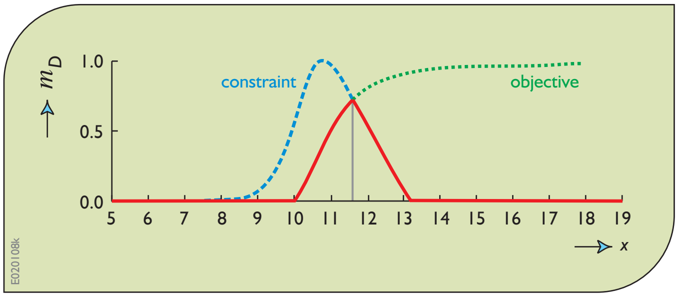

Figure 5.7. The intersection membership function and the value of *x* that represents a fuzzy optimal decision.

3. Fuzzy Sets for Water Allocation

Next consider the application of fuzzy modelling to the water allocation problem illustrated in Figure 5.8.

Assume, as in the previous uses of this example problem, the problem is to find the allocations of water to each firm that maximize the total benefits $TB(\mathbf{X})$:

$$\text{Maximize } TB(\mathbf{X}) = (6x_1 - x_1^2) + (7x_2 - 1.5x_2^2) + (8x_3 - 0.5x_3^2) \quad (5.10)$$

These allocations cannot exceed the amount of water available, Q, less any that must remain in the river, R. Assuming the available flow for allocations, $Q - R$, is 6, the crisp optimization problem is to maximize Equation 5.10 subject to the resource constraint:

$$x_1 + x_2 + x_3 \leq 6 \quad (5.11)$$

The optimal solution is $x_1 = 1$, $x_2 = 1$, and $x_3 = 4$ as previously obtained in Chapter 4 using several different optimization methods. The maximum total benefits, $TB(\mathbf{X})$, from Equation 5.10, equal 34.5.

To create a fuzzy equivalent of this crisp model, the objective can be expressed as a membership function of the set of all possible objective values. The higher the objective value the greater the membership function value. Since membership functions range from 0 to 1, the objective needs to be scaled so that it also ranges from 0 to 1.

The highest value of the objective occurs when there is sufficient water to maximize each firm's benefits. This unconstrained solution would result in a total benefit of 49.17 and this happens when $x_1 = 3$, $x_2 = 2.33$, and $x_3 = 8$. Thus, the objective membership function can be expressed by:

$$m(\mathbf{X}) = [(6x_1 - x_1^2) + (7x_2 - 1.5x_2^2) + (8x_3 - 0.5x_3^2)]/49.17 \quad (5.12)$$

It is obvious that the two functions (Equations 5.10 and 5.12) are equivalent. However, the goal of maximizing objective function 5.10 is changed to that of maximizing the degree of reaching the objective target. The optimization problem becomes:

$$\text{maximize } m(\mathbf{X}) = [(6x_1 - x_1^2) + (7x_2 - 1.5x_2^2) + (8x_3 - 0.5x_3^2)]/49.17$$

subject to:

$$x_1 + x_2 + x_3 \leq 6 \quad (5.13)$$

The optimal solution of (5.13) is the same as (5.10 and 5.11). The optimal degree of satisfaction is $m(\mathbf{X}) = 0.70$.

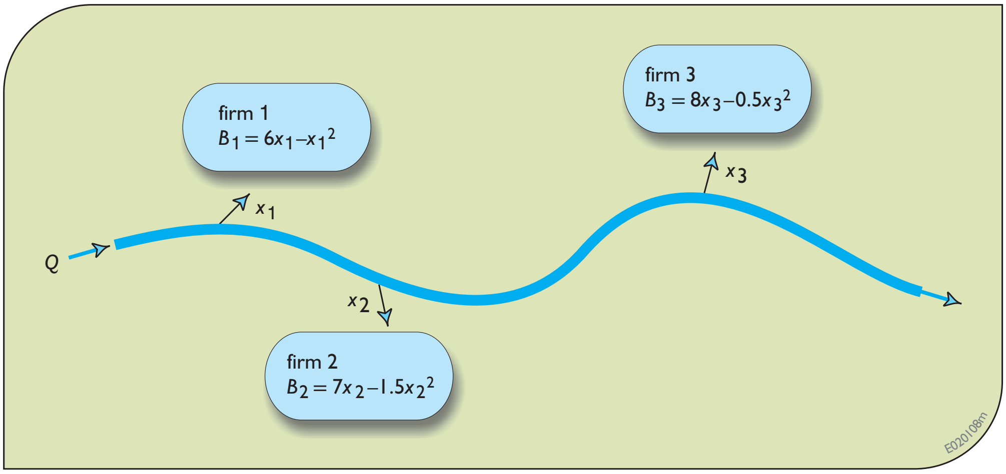

Figure 5.8. Three water-consuming firms *i* obtain benefits B_i from their allocations x_i of water from a river whose flow is *Q*.

Figure 5.9. Membership function for 'about 6 units more or less'.

Next, assume the amount of resources available to be allocated is limited to 'about 6 units more or less', which is a fuzzy constraint. Assume the membership function describing this constraint is defined by Equation 5.14 and is shown in Figure 5.9.

$$m_c(\boldsymbol{X}) = 1 \qquad \text{if } x_1 + x_2 + x_3 \leq 5$$
$$m_c(\boldsymbol{X}) = [7 - (x_1 + x_2 + x_3)]/2 \qquad \text{if } 5 \leq x_1 + x_2 + x_3 \leq 7$$
$$m_c(\boldsymbol{X}) = 0 \qquad \text{if } x_1 + x_2 + x_3 \geq 7 \qquad (5.14)$$

The fuzzy optimization problem becomes:

Maximize minimum ($m_G(\boldsymbol{X})$, $m_C(\boldsymbol{X})$)

subject to:

$$m_G(\boldsymbol{X}) = \left[\left(6x_1 - x_1^2\right) + \left(7x_2 - 1.5x_2^2\right) + \left(8x_3 - 0.5x_3^2\right)\right]/49.17$$
$$m_C(\boldsymbol{X}) = [7 - (x_1 + x_2 + x_3)]/2 \qquad (5.15)$$

Solving (5.15) using LINGO® to find the maximum of a lower bound on each of the two objectives, the optimal fuzzy decisions are $x_1 = 0.91$, $x_2 = 0.94$, $x_3 = 3.81$, $m(\boldsymbol{X}) = 0.67$, and the total net benefit, Equation 5.10, is $TB(\boldsymbol{X}) = 33.1$. Compare this with the crisp solution of $x_1 = 1$, $x_2 = 1$, $x_3 = 4$, and the total net benefit of 34.5.

4. Fuzzy Sets for Reservoir Storage and Release Targets

Consider the problem of trying to identify a reservoir storage volume target, T^S, for the planning of recreation facilities given a known minimum release target, T^R, and reservoir capacity K. Assume, in this simple example, these known release and unknown storage targets must apply in each of the three seasons in a year. The objective will be to find the highest value of the storage target, T^S, that minimizes the sum of squared deviations from actual storage volumes and releases less than the minimum release target.

variable	value	remarks
T^S	15.6	target storage for each period
S_1	19.4	reservoir storage volume at beginning of period 1
S_2	7.5	reservoir storage volume at beginning of period 2
S_3	20.0	reservoir storage volume at beginning of period 3
R_1	14.4	reservoir release during period 1
R_2	27.5	reservoir release during period 2
R_3	18.1	reservoir release during period 3

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Table 5.1. The LINGO® solution to the reservoir optimization problem.

Given a sequence of inflows, Q_t, the optimization model is:

$$\text{Minimize } D = \sum_t^3 \left[(T^S - S_t)^2 + DR_t^2\right] - 0.001T^S \qquad (5.16)$$

subject to:

$$S_t + Q_t - R_t = S_{t+1} \quad t = 1, 2, 3; \quad \text{if } t = 3, t+1 = 1 \qquad (5.17)$$
$$S_t \leq K \qquad t = 1, 2, 3 \qquad (5.18)$$
$$R_t \geq T^R - DR_t \quad t = 1, 2, 3 \qquad (5.19)$$

Assume $K = 20$, $T^R = 25$ and the inflows Q_t are 5, 50 and 20 for periods $t = 1$, 2 and 3. The optimal solution, yielding an objective value of 184.4, obtained by LINGO® is listed in Table 5.1.

Now consider changing the objective function into maximizing the weighted degrees of 'satisfying' the reservoir storage volume and release targets.

$$\text{Maximize } \sum_t (w_S m_{St} + w_R m_{Rt}) \qquad (5.20)$$

where w_S and w_R are weights indicating the relative importance of storage volume targets and release targets respectively. The variables m_{St} are the degrees of satisfying storage volume target in the three periods t, expressed by Equation 5.21. The variables m_{Rt} are the degrees of satisfying release target in periods t, expressed by Equation 5.22.

$$m_S = \begin{cases} S_t/T^S & \text{for } S_t \leq T^S \\ (K - S_t)/(K - T^S) & \text{for } T^S \leq S_t \end{cases} \qquad (5.21)$$

$$m_R = \begin{cases} R_t/T^R & \text{for } R_t \leq T^R \\ 1 & \text{for } R_t > T^R \end{cases} \qquad (5.22)$$

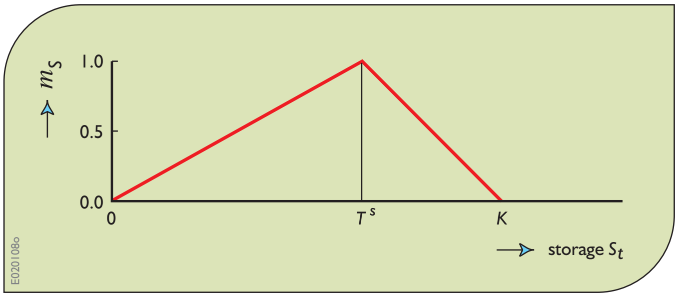

Figure 5.10. Membership function for storage volumes.

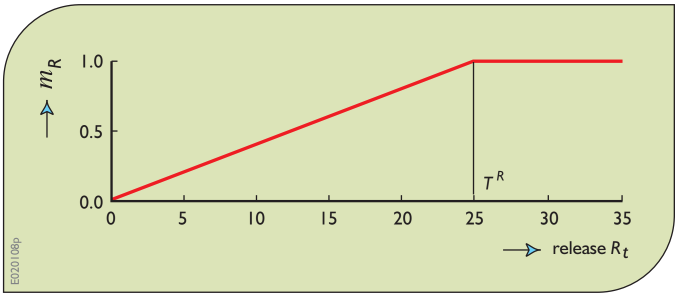

Figure 5.11. Membership function for releases.

Box 5.1. Reservoir model written for solution by LINGO®

```
SETS:
PERIODS /1..3/: I, R, m, ms, mr , s1, s2, ms1, ms2;
NUMBERS /1..4/: S;
ENDSETS
!*** OBJECTIVE ***; max = degree + 0.001*TS;
!Initial conditions;  s(1) = s(TN + 1);
!Total degree of satisfaction; degree = @SUM(PERIODS(t): m(t));
!Weighted degree in period t; @FOR (PERIODS(t):
m(t) = ws*ms(t) + wr*mr(t);
S(t) = s1(t) + s2(t);
s1(t) < TS ;  s2(t) < K – TS  ;
!ms(t) = (s1(t)/TS) – (s2(t)/(K –TS)) =  rewritten in case dividing by 0;
ms1(t)*TS = s1(t);  ms2(t)*(K –TS) = s2(t);  ms(t) = ms1(t) – ms2(t);
 mr(t) < R(t)/TR ;   mr(t) < 1;  S(t+1) = S(t) + I(t) – R(t););
DATA:
TN = 3; K = 20; ws = ?; wr = ?;    I = 5, 50, 20;  TR = 25;
ENDDATA
```

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Equations 5.21 and 5.22 are shown in Figures 5.10 and 5.11, respectively.

This optimization problem written for solution using LINGO® is as shown in Box 5.1.

Given weights $w_S = 0.4$ and $w_R = 0.6$, the optimal solution obtained from solving the model shown in Box 5.1 using LINGO® is listed in Table 5.2.

variable	value	remarks
degree	2.48	total weighted sum membership function values
T^S	20.00	target storage volume
S_1	20.00	storage volume at beginning of period 1
S_2	0.00	storage volume at beginning of period 2
S_3	20.00	storage volume at beginning of period 3
R_1	25.00	reservoir release in period 1
R_2	30.00	reservoir release in period 2
R_3	20.00	reservoir release in period 3
M_1	1.00	sum weighted membership values period 1
M_2	0.60	sum weighted membership values period 2
M_3	0.88	sum weighted membership values period 3
M_1^S	1.00	storage volume membership value period 1
M_2^S	0.00	storage volume membership value period 2
M_3^S	1.00	storage volume membership value period 3
M_1^R	1.00	reservoir release membership value period 1
M_2^R	1.00	reservoir release membership value period 2
M_3^R	0.80	reservoir release membership value period 3

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Table 5.2. Solution of fuzzy model for reservoir storage volumes and releases based on objective 5.20.

If the objective Equation 5.20 is changed to one of maximizing the minimum membership function value, the objective becomes:

$$\text{Maximize } m_{\min} = \text{maximize minimum } \{m_{St}, m_{Rt}\} \quad (5.23)$$

A common lower bound is set on each membership function, m_{St} and m_{Rt}, and this variable is maximized. The optimal solution changes somewhat and is as shown in Table 5.3.

This solution differs from that shown in Table 5.2 primarily in the values of the membership functions. The target storage volume operating variable value, T^S, stays the same in this example.

5. Fuzzy Sets for Water Quality Management

Consider the stream pollution problem illustrated in Figure 5.12. The stream receives waste from sources located at Sites 1 and 2. Without some waste treatment at these sites, the pollutant concentrations at Sites 2 and 3 will exceed the maximum desired concentration. The problem is to find the level, x_i, of wastewater treatment (fraction of waste removed) at Sites $i = 1$ and 2 required to meet the quality standards at Sites 2 and 3 at a minimum total cost. The data used for the problem shown in Figure 5.12 are listed in Table 5.4.

variable	value	remarks
MMF	0.556	minimum membership function value
T^s	20.00	target storage volume
S_1	20.00	storage volume at beginning of period 1
S_2	11.11	storage volume at beginning of period 2
S_3	20.00	storage volume at beginning of period 3
R_1	13.88	reservoir release in period 1
R_2	41.11	reservoir release in period 2
R_3	20.00	reservoir release in period 3
M_1^s	0.556	storage volume membership value period 1
M_2^s	0.556	storage volume membership value period 2
M_3^s	0.800	storage volume membership value period 3
M_1^R	0.556	reservoir release membership value period 1
M_2^R	1.000	reservoir release membership value period 2
M_3^R	0.556	reservoir release membership value period 3

Table 5.3. Optimal solution of reservoir operation model based on objective 5.23.

The crisp model for this problem, as discussed in the previous chapter, is:

$$\text{Minimize } C_1(x_1) + C_2(x_2) \tag{5.24}$$

subject to:

Water quality constraint at site 2:

$$[P_1Q_1 + W_1(1-x_1)]a_{12}/Q_2 \leq P_2^{\max} \tag{5.25}$$

$$[(32)(10) + 250000(1-x_1)/86.4]\ 0.25/12 \leq 20$$

which, when simplified, is: $x_1 \geq 0.78$

Water quality constraint at site 3:

$$\{[P_1Q_1 + W_1(1-x_1)]a_{13} + [W_2(1-x_2)]a_{23}\}/Q_3 \leq P_3^{\max} \tag{5.26}$$

$$\{[(32)(10) + 250000(1-x_1)/86.4]\ 0.15 + [80000(1-x_2)/86.4]\ 0.60\}/13 \leq 20$$

which, when simplified, is: $x_1 + 1.28x_2 \geq 1.79$

Restrictions on fractions of waste removal:

$$0 \leq x_i \leq 1.0 \quad \text{for sites } i = 1 \text{ and } 2 \tag{5.27}$$

For a wide range of reasonable costs, the optimal solution found using linear programming was 0.78 and 0.79, or essentially 80% removal efficiencies at Sites 1 and 2. Compare this solution with that of the following fuzzy model.

To develop a fuzzy version of this problem, suppose the maximum allowable pollutant concentrations in the stream at Sites 2 and 3 were expressed as 'about 20 mg/l or less'. Obtaining opinions of individuals of what

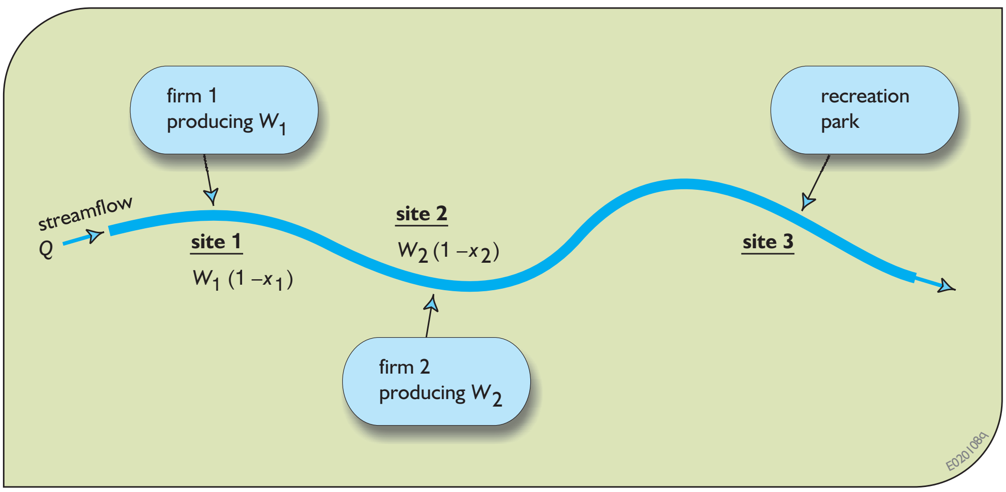

Figure 5.12. A stream pollution problem of finding the waste removal efficiencies (x_1, x_2) that meet the stream quality standards at least cost.

Table 5.4. Parameter values selected for the water quality management problem illustrated in Figure 5.12.

	parameter	unit	value	remark
flow	Q_1	m^3/s	10	flow just upstream of site 1
	Q_2	m^3/s	12	flow just upstream of site 2
	Q_3	m^3/s	13	flow at park
waste	W_1	kg/day	250,000	pollutant mass produced at site 1
	W_2	kg/day	80,000	pollutant mass produced at site 2
pollutant conc.	P_1	mg/l	32	concentration just upstream of site 1
	P_2	mg/l	20	maximum allowable concentration upstream of 2
	P_3	mg/l	20	maximum allowable concentration at site 3
decay fraction	a_{12}	--	0.25	fraction of site 1 pollutant mass at site 2
	a_{13}	--	0.15	fraction of site 1 pollutant mass at site 3
	a_{23}	--	0.60	fraction of site 2 pollutant mass at site 2

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Figure 5.13. Membership function for 'about 20 mg/l or less'.

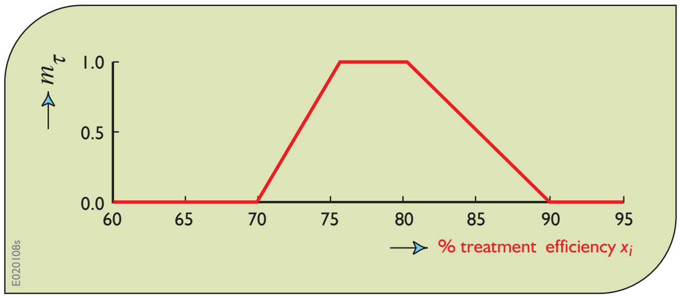

Figure 5.14. Membership function for best available treatment technology.

they consider to be '20 mg/l or less', a membership function can be defined. Assume it is as shown in Figure 5.13.

Next, assume that the government environmental agency expects each polluter to install best available technology (BAT) or to carry out best management practices (BMP) regardless of whether or not this is required to meet stream-quality standards. Asking experts just what BAT or BMP means with respect to treatment efficiencies could result in a variety of answers. These responses can be used to define membership functions for each of the two firms in this example. Assume these membership functions for both firms are as shown in Figure 5.14.

Finally, assume there is a third concern that has to do with equity. It is expected that no polluter should be required to treat at a much higher efficiency than any other polluter. A membership function defining just what differences are acceptable or equitable could quantify this concern. Assume such a membership function is as shown in Figure 5.15.

Considering each of these membership functions as objectives, a number of fuzzy optimization models can be defined. One is to find the treatment efficiencies that maximize the minimum value of each of these membership functions.

$$\text{Maximize } m = \max \min\{m_P, m_T, m_E\} \qquad (5.28)$$

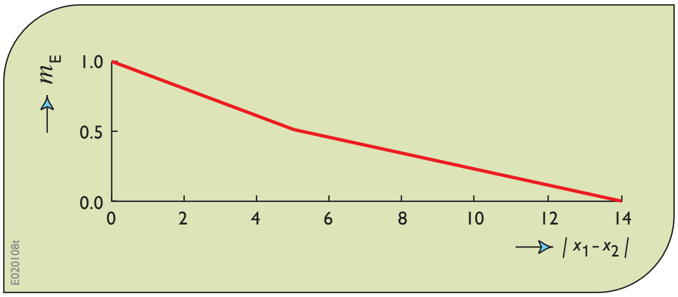

Figure 5.15. Equity membership function in terms of the absolute difference between the two treatment efficiencies.

If we assume that the pollutant concentrations at sites $j = 2$ and 3 will not exceed 23 mg/l, the pollutant concentration membership functions m_{Pj} are:

$$m_{Pj} = 1 - p_{2j}/5 \tag{5.29}$$

The pollutant concentration at each site j is the sum of two components:

$$p_j = p_{1j} + p_{2j} \tag{5.30}$$

where

$$p_{1j} \leq 18 \tag{5.31}$$

$$p_{2j} \leq (23-18) \tag{5.32}$$

If we assume the treatment plant efficiencies will be between 70 and 90% at both Sites $i = 1$ and 2, the treatment technology membership functions m_{Ti} are:

$$m_{Ti} = (x_{2i}/0.05) - (x_{4i}/0.10) \tag{5.33}$$

and the treatment efficiencies are:

$$x_i = 0.70 + x_{2i} + x_{3i} + x_{4i} \tag{5.34}$$

where

$$x_{2i} \leq 0.05 \tag{5.35}$$

$$x_{3i} \leq 0.05 \tag{5.36}$$

$$x_{4i} \leq 0.10 \tag{5.37}$$

Finally, assuming the difference between treatment efficiencies will be no greater than 14, the equity membership function, m_E, is:

$$m_E = Z - (0.5/0.05)\, D_1 + 0.5(1 - Z) + (0.5/(0.14 - 0.05))\, D_2 \tag{5.38}$$

where

$$D_1 \leq 0.05Z \tag{5.39}$$

variable	value	remarks
M	0.93	minimum membership value
X_1	0.81	treatment efficiency at site 1
X_2	0.81	treatment efficiency at site 2
P_2	18.28	pollutant concentration just upstream of site 2
P_3	18.36	pollutant concentration just upstream of site 3
$M^P{}_2$	0.94	membership value for pollutant concentration site 2
$M^P{}_3$	0.93	membership value for pollutant concentration site 3
$M^T{}_1$	0.93	membership value for treatment level site 1
$M^T{}_2$	0.93	membership value for treatment level site 2
M^E	1.00	membership value for difference in treatment

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Table 5.5. Solution to fuzzy water quality management model Equations 5.28 to 5.46.

$$D_2 \leq (0.14 - 0.05)\,(1-Z) \tag{5.40}$$

$$x_1 - x_2 = DP - DM \tag{5.41}$$

$$DP + DM = D_1 + 0.05(1-Z) + D_2 \tag{5.42}$$

Z is a binary 0, 1 variable. (5.43)

The remainder of the water quality model remains the same: Water quality constraint at site 2:

$$[P_1Q_1 + W_1(1-x_1)]\, a_{12}/Q_2 = P_2 \tag{5.44}$$

$$[(32)(10) + 250000(1-x_1)/86.4]\; 0.25/12 = P_2$$

Water quality constraint at site 3:

$$\{[P_1Q_1 + W_1(1-x_1)]\, a_{13} + [W_2(1-x_2)]\, a_{23}\}/Q_3 = P_3 \tag{5.45}$$

$$\{[(32)(10) + 250000(1-x_1)/86.4]\; 0.15 + [80000(1-x_2)/86.4]\; 0.60\}/13 = P_3$$

Restrictions on fractions of waste removal:

$0 \leq x_i \leq 1.0$ for sites $i = 1$ and 2. (5.46)

Solving this fuzzy model using LINGO® yields the results shown in Table 5.5.

This solution confirms the assumptions made when constructing the representations of the membership functions in the model. It is also very similar to the least-cost solution found from solving the crisp LP model.

6. Summary

Optimization models incorporating fuzzy membership functions are sometimes appropriate when only qualitative statements are made when stating objectives and/or constraints of a particular water management problem or issue. This chapter has shown how fuzzy optimization can be applied to some simple example problems associated with water allocations, reservoir operation, and pollution control. This has been only an introduction. Those interested in more detailed explanations and applications may refer to any of the additional references listed in the next section.

7. Additional References (Further Reading)

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