How it was to study and to teach mathematics in Cornell at the end of 19th century?

Preliminary report partially supported by National Science Foundation's National Science, Technology, Engineering, and Mathematics Education Digital Library (NSDL) Program under grant DUE-0226238.

(Based on a talk given at AMS-MAA Joint Conference Special Session in History of Mathematics, January 18, 2003, Baltimore.)

by Daina Taimina
Senior Research Associate, Cornell University

Abstract:
Cornell University's Kroch Library Rare Book and Manuscript Division has a collection called "Department of Mathematics records 1877-1976". It was used already as case studies of the emergence of mathematical research at Cornell University in several publications; but I will talk about my experience going through these records and trying to imagine what mathematics students had learned before entering Cornell University (looking at entrance exams they were given). The earlier publications reported that mathematics entrance requirements to Cornell "were minimal by today's standards" but I found that this was not the case. Many of the students taking the entrance exams were engineering students. At that time the Reuleaux kinematic models collection was used to bring mathematical ideas into engineering curriculum.

There are several publications about teaching mathematics in Cornell University at its beginnings. I just mention three that are most referenced and I think best known:


After reading all these papers which give a good insight in early history of teaching mathematics in American colleges, why did I still have an interest in looking through old Mathematics Department scrapbooks?

When I am teaching history of mathematics (Math 403) in Cornell, at least one of the classes with my students is at Kroch Library Rare Book and Manuscript Division. Besides all other treasures we can see there, always there is a special interest for mathematics students about old Mathematics Department scrapbooks. Those are recycled Scientific American bindings on which the Mathematics Department secretary accurately pasted internal departmental correspondence and all exams given during the academic year including lots of entrance exams. Looking on this curious way of preserving mathematics department records there are always questions about entrance exams. When my current students look at them, their first reaction is a question: Did they want to scare students away from Cornell?
What do we know about entrance requirements for Cornell University Mathematics Department?

The prerequisites in the first year for entrance into Cornell were minimal by today's standards, but they were consonant with those in place at other American colleges. Only arithmetic and algebra through quadratics were required, and "some students were admitted with only arithmetic". With such minimal prerequisites, what mathematics did these early Cornell students take?

In the freshman year, plane geometry, algebra, and solid geometry were required of all students. During the first part of the sophomore year, trigonometry was required, "including a little of mensuration, surveying, and navigation". Those in engineering or architecture also took one or two terms of analytic geometry, three terms of calculus, and one term of synthetic geometry. Later in the first decade at Cornell, when the entrance requirements were increased to include plane geometry, freshman mathematics changed by dropping plane geometry and adding trigonometry. (Cochell)  {Cochell's quotes are from the above Cajori reference.}

Let us look at those entrance exams -- were they really minimal by today's standards?

Cornell University Entrance Examinations

Arithmetic  9-11 AM Friday, September 23, 1887

1. Define bank discount, compound interest, reduction ascending, a decimal fraction, a circulating decimal.
2. Extract the cube root of 5 to five places of decimals.
3. A common lead pencil is 1 3/4 decimeters in length; 32 186 of them arranged in a line would extend how many miles?
4. Simplify: \[
\begin{array}{cccc}
1.75 & 1.71 & 4285 & \\
.63 & .31 & .75 & \\
\end{array}
\]
   \[
\begin{array}{cccc}
2.6 & 2.0 & 95 & 238 & \\
& & & \\
\end{array}
\]

5. A merchant sold a quantity of lumber, and received a note payable in 6 months; he had his note discounted at a bank at 6%, and received $ 4572.40. What was the amount of his note?
6. A, B, C contract to build a piece of railroad for $ 3500. A employs 30 men 50 days, B employs 50 men 36 days; and C employs 48 men and 10 horses 45 days, (each horse to be reckoned equal to one man), and is also to have $ 115.50 for overseeing the work. How much is each man to receive?
Advanced Algebra. 10:30-1:15 P.M. Monday, September 26, 1887

1. Define: a convergent and a divergent series, a harmonic progression and a geometric progression, a continued fraction, an incomensurable number; undetermined coefficients.

2. Find \( \sqrt{30} \), given \( \log 412 = 2.6149, \log 413 = 2.6160, \log 237 = 2.3747, \log 236 = 2.3729 \).

3. Find the scale and sum of the recurring series

4. The series in question 3 is the development of the fraction \( (3-10x):(1-3x)(1-2x) \); resolve this fraction into partial fractions; develop each partial fraction; then show that the co-efficient of \( x^n \) in the above recurring series is \( \frac{423}{n} \), verify this result by putting successively equal to 0, 1, 2, 3, 4.

5. Write down that term of the expansion of \( \sqrt{\frac{x^4}{1-x^2}} \) which does not contain \( x \) when \( n \) is even.

6. Find the three roots of unity, one real and two imaginary.

The equation \( \sqrt{\frac{x^4}{1-x^2}} \) has a real commensurable root, a real incommensurable root, and two imaginary roots of the form \( \sqrt{\frac{x^4}{1-x^2}} \); find all these roots, getting \( P \) and \( Q \) to three decimal places.
Entrance Examination
Elementary Algebra
3-6 P.M. Saturday, 24 September 1887.

1. Multiply \(1 + \frac{1}{2} + \frac{1}{3} + \cdots \frac{1}{x} \) by itself, and find the value of the result if \(1 + \frac{1}{2} = 3\).
2. (a) Prove that \((\alpha - \beta)^n = \alpha^n - \binom{n}{1} \alpha^{n-1} \beta + \binom{n}{2} \alpha^{n-2} \beta^2 - \cdots + (-1)^n \beta^n\).
(b) Simplify \(\frac{(\alpha - \beta)^n}{\alpha - \beta} \times \frac{(\alpha + \beta)^n}{\alpha + \beta}\).
3. Simplify \(\frac{a^2 + b^2}{a^2 - b^2} \times \frac{a^2 + b^2}{a^2 - b^2}\).
4. Solve the simultaneous equations \(x^2 + 2xy + y^2 = 4\), \(x^2 + y^2 = 4\).
5. Factor the expression \(x^2 + x^2 + x^2 - x^2 - x^2 - x^2\).
6. Any odd number may be represented by \(2n+1\) when \(n\) is odd or even. Prove that the difference of the squares of any two odd numbers is exactly divisible by 8.
7. Solve the quadratic \(ax^2 + bx + c = 0\), and show what relation must hold between the co-efficients in order
(a) that the roots shall be equal;
(b) that the roots both shall be infinite;
(c) that the roots both shall be zero.

Entrance Examination
Solid Geometry & Conic sections. 8-10 1/2 A.M. Wednesday, 20 June 1888.
[Omit one question]

1. Define: parallel planes, a dihedral angle, a truncated prism, a pyramid, a symmetrical figure, a cone of revolution, a spherical polygon, an ellipse, a normal of an ellipse, a subtangent of a parabola.
2. If two straight lines be intersected by three parallel planes, their corresponding segments are proportional.
3. Two triangular prisms having equivalent bases and equal altitudes are equivalent.
4. Through any 4 points not in the same plane, one and but one spherical surface can be made to pass.
5. The volume of a spherical sector is equal to the area of the zone that forms its base, multiplied by one third the radius of the sphere.
6. Find the locus of the points in space, that are at a distance \(a\) from a point A, and at a distance \(b\) from a point B.
7. The subnormal of a parabola is equal to one-half the parameter.
8. The transverse semi-axis of an ellipse is a mean proportional between the distance from the centre to the foot of the tangent, and the distance from the centre to the foot of the ordinate of the point of contact.

Entrance Examinations
Plane Geometry. 9-11 A.M. Tuesday 19 June 1888

1. Define: a plane surface, a lemma, a locus, a pentagon, an isosceles triangle, symmetry with respect to an axis, a chord of a circle, a regular polygon, a parallelogram, a straight line tangent to a circle.
2. (a) The sum of the three angles of a triangle is equal to two right angles. (b) In any triangle ABC if AD is drawn perpendicular to BC and AE bisecting the angle BAC, the angle DAE is equal to one half the difference of the angles B and C.
3. An angle inscribed in a circle is measured by one-half intercepted arcs: three cases.
4. In an obtuse angled triangle, the square of the side opposite to the obtuse angle, is equal to the sum of squares of the other two sides increased by twice the product of one of these sides and the projection of the other upon that side.

5. Construct a square equivalent to a given parallelogram, or to a given triangle; and demonstrate.

6. Prove that the circumferences of two circles are to each other as their radii, and that their areas are to each other as their squares of their radii. If areas of two circles be as 9 to 4, and the diameter of the first circle be 10 inches, what is the diameter of the other.

Entrance examination
Trigonometry 3 - 6 P.M. Monday, 26 September 1887

1. Trace the changes in the magnitude and quality of the sine, cosine and tangent of an angle, as the angle increases from to .

2. In a plane triangle ABC right angled at C, show that

   \[ \sqrt{bc} + \sqrt{bc} - \sqrt{\frac{1}{4} + \frac{1}{4}}. \]

3. Prove the following formula for the area of a plane triangle

   \[ \sqrt{s(s-a)(s-b)(s-c)}. \]

4. Find the angle at which the side of a pyramid is inclined to the base, the sides being equilateral triangles and the base a square; thence find the diedral angle of a regular octahedron.

5. In a right spherical triangle, prove that if the hypothenuse is by than a quadrant, the other two sides on of the same species, if the hypothenuse is greater than a quadrant, they are of different species, if the hypothenuse is equal to the quadrant; another side is equal to a quadrant and the opposite angle is right angle.

6. In any spherical triangle ABC, assuming the formulas:

   \[ \sqrt{s(s-a)(s-b)(s-c)}. \]

   Derive any two of the following:

   \[ \cos(\theta) \sin(\phi), \cos(\theta) \cos(\phi); \cos(\theta) \sin(\phi) \]

7. Find the distance in degrees, minutes and seconds on the arc of a great circle from a point in latitude N, longitude W to a point in latitude S, longitude E.

Before each exam students were given following directions:

As the time for reading each paper is in general necessarily very short, your chances of success in mathematical examinations will be much improved by the observation of the following

Directions
1. Write legibly, without crowding.
2. Arrange your work clearly, and give it all.
Cancel and simplify wherever you can.
Put your results into their simplest forms.
Additional credit is given for neat papers in which no words are misspelled.

2. You need not to copy the questions, but only their numbers. Answer them in their order: if one be temporarily skipped, leave a space. If by accident, anything be misplaced, indicate in its proper place where it may be found. Allow yourself time to review carefully what you have written.
3. Number your pages: and at the top of each page write your name and the number of your seat. Hand your paper, unfolded to the professor in charge of the room.
4. Leave all your books and papers on window-sill, and after the questions are distributed, do not leave your seat without permission nor communicate in any manner with your neighbor. If you want anything, raise your hand.
5. Of course the least attempt at fraud would annul your examination, and might expose you to other disagreeable consequences.

Which entrance exams students had to take to start their studies in Cornell? Well, on one of the pages of scrapbooks we can find following:

Requirements for Admission.
I. - Age and Character -All candidates for admission to the University must be of good mental character, and at least sixteen years of age; if women, then seventeen years of age.

II. General Requirements
(a) Arithmetic, including the Metric System.
(b) Geography, political and physical.
(c) English Grammar, including Orthography and Syntax.
(d) Plane Geometry.
(e) Human Physiology.
(f) Algebra, the Elements, including Radicals and Quadratic Equations.
No other subject is required of students in Agriculture, Architecture, Civil Engineering or Mechanical Arts; nor of Optional Studies.

III. - Special Requirements for the Course in Arts.
(a) Latin: the Grammar, including Prosody; Composition (Arnold's first twelve chapters); four Books of Caesar or Sallust's Catiline; eight Orations of Cicero, or five Orations and the De Senectute; Vergil's Eclogues and six Books of Aeneid.
(b) Greek: the Grammar (Goodwin's); writing Greek with accents; first one hundred and eleven pages of Goodwin's Greek Render (or four Books of Xenophon's Anabasis); the first three Books of the Iliad, omitting the Catalogue of Ships; the History of Greece.

IV. - Special Requirements for the Courses in Literature and Philosophy.
(a) Latin as above, III (a), and either
(b) French: the Grammar; translation of English into French; three books of Voltaire's Charles XII, or its equivalent; or
(c) German: the Grammar; translation of English into German; seventy five pages of Whitney's Reader, or its equivalent; or
(d) Mathematics: Solid Geometry, including Conic Sections; Algebra entire; Plane and Spherical trigonometry.

V. - Special Requirements for the courses in Chemistry and Physics, Mathematics, Science, and Science and Letters.
(a) French as above IV (b); or
(b) German as above IV (c), or
(c) Mathematics as above IV (d).
VI. - Special Requirements for the courses in Natural History.

(a) Plane Trigonometry.
(b) Latin: the Grammar; four books of Caesar or its equivalent.
(c) Greek: the Alphabet, and enough of the language to enable the student to recognize, analyze and form scientific technical terms; and either
(d) French as above IV (b), or
(e) German as above IV (c)

After exams students got a note telling what were their results:

The results of your recent mathematical examinations are as follows:

Arithmetic
Alegebra through quadratics
Advanced algebra
Plane Geometry
Solid and Spherical Geometry
Plane Trigonometry
Spherical Trigonometry
Plane Analytic Geometry
Analytic Geometry of three dimensions
Differential Calculus
Integral Calculus
Astronomy

It is recommended however that you review with care the following topics

If you want to go on with the classes in.......... it will be necessary for you to be re-examined in .......... 

What we can see from those examples of exams and also all other exams available in Mathematics Department Scrapbooks? It follows from Requirements and exams students had to be well prepared to come to Cornell. I would not call those entrance exams elementary comparing with modern day standards. It seems to me that Florian Cajori (1890) when he wrote about entrance requirements looked only at the part II "General Requirements". And so this quote went on for more than 100 years, as in the above Cochell quote (1998). The quote may be accurate for the 1868, but certainly was not true twenty years later.

I have not done very detailed research how many students were actually applying to Cornell and how many of them have been admitted over the years. Just an example from June, 1891, the following numbers took each exam: Solid Geometry, total 53, Elementary Algebra, total 63, Plane Geometry, total 65, Arithmetic, total 65, Plane Trigonometry, total 7, Plane and Spherical Trigonometry 5. I should make a note here that these are results of entrance exams in June in Cornell, but there were also entrance exams in September and students could take entrance exams to enter Cornell also in Boston, Cleveland and Chicago.

Looking at the results of these exams one can see that there were not many excellent students. Average grade on those exams is between 60-65. And as we can see
from other notes in scrapbooks even those students who "were conditioned in entrance exams" could still be admitted to Cornell and start their studies.

Ithaca, NY  
Oct.7, 1879

Arrangements have been made for forming classes in arithmetic, Elementary Algebra and Plane Geometry to be taught by professors of the University for the purpose of helping students who are conditioned in those subjects to prepare for the entrance examinations in January. In either of these classes instruction will be given twice a week, and the tuition is fixed at $5 for the term.

You may find it for your advantage to attend, if so, please come to my room No. 34 North University Building, between 12 and 1 Thursday October 9.

Very respectfully J. E. Oliver.

Ten years later we can see that students no more received a note from a chair but secretary sent it:

All Entrance conditions in mathematics are to be made up at the Entrance Examinations on Tuesday morning January 8, 1878.

1878 January 4 - of about 30 students conditioned in the 1877 Entrance Algebra and who were in University last term, 16 took the term Algebra in class. Of them 2 passed honorably, and 2 creditably, while 5 were conditioned, dropped or excused from Examination:- the term work began with H.C.D, and the examination was partly upon topics required for admission, - we think that hereafter it would be well to regard Entrance conditions in Algebra as made up when students pass "creditably" for the term.

Or the other note from scrapbooks:

M .........

You will please take a notice that you were conditioned in

At your entrance examinations.

Under the rules of the University [Rules for students, § 12] you are required to make up these conditions "at such time before the close of the third term as the professors in charge of the department may direct", and you are hereby notified that conditions in Elementary Algebra must be made up at the entrance examination in January 18, and that conditions in Plane Geometry and Arithmetic must be made up at the entrance examinations either in January or June 188... If however you shall at the end of this term pass creditably your examination in Geometry you will be thereby relieved from your condition in Plane Geometry.

Secretary of Mathematics Department, Cornell University, 188...

More about forming Cornell mathematics department see:  
http://www.math.cornell.edu/General/History/historyP3.html

Professors interacted with students not only in their classes. After spending some time in Europe, Prof. Oliver returned to Ithaca with new ideas how to raise students' interest about mathematics. One of the ideas was to organize a "Mathematical Club" that was more like what we now know as seminars. This club continues to this day and is called the Oliver Club. The Oliver Club is today the main department research. From "Minutes of the Cornell Mathematical Club"
The first regular meeting of the Mathematical Club was called to order by Professor Oliver at eight o'clock Saturday evening, January 24, 1891. On a motion it was decided to proceed with the business of organization before listening to the mathematical reports. After the business having been disposed of, Mr. Towler gave the club an interesting talk on Riemann's plane. In the discussion that followed Professor Oliver mentioned the great beauty and use of this model of representing a complex variable.

Next time Professor Hathaway spoke about imaginary numbers. On the third meeting presenters were two of the best students of that time: R. Shoemaker spoke on "The application of the principles of the methods of Limits, Ratio, and Infinitesimals to the theorems of Plane and Solid Geometry" and V. Snyder discussed the various methods of calculus. Next meeting was devoted to Clifford's representation of an imaginary exponential. It followed by a meeting on which Saurel reported on step analysis. (term "steps" at that time was used for "vectors"). Professor Mahon gave the Club some interesting problems from "The Educational Times". On April 25, 1891 was the first presentation by a woman - Miss Palmie discussed the subject of multiple integrals. The next academic year, 1891/92, it was decided that club will meet in two sections but sometimes both sections were combined.

Recently, I joined the team of NSF funded grant "KMODD-L" for digitizing Franz Reuleaux kinematic model collection. Collections of Reuleaux models were widely used in Europe, especially in Germany, before the Second World War. Most and maybe all have been lost in the destruction of 1941-45. The Cornell Reuleaux Collection seems to be the last remaining large set of 219 models out of the original 800 that Reuleaux had built in his laboratory in Berlin over a century ago. First president of Cornell A. D. White acquired it in 1882.

The Centennial Exposition in 1876 in Philadelphia was responsible for a national quickening in mechanical matters and for a growing sense of latent power. The big central Corliss engine of Machinery Hall was a splendid object lesson and this Exposition was signalized by the single valve automatic engine with flywheel governor designed by John C. Hoadley, by Professor Sweet's design of the Straight-Line engine, and by a series of boiler tests by Charles E. Emery, Charles T. Porter and Joseph Belknap. These all marked epochs in the engineering history of the United States. Moreover, in the fifteen years since the Civil War the enormous increase in size and productivity of industrial plants had just begun. The Land Grant colleges had their graduates of a dozen years practicing their profession and by the natural processes of promotion the products of the older schools of engineering had attained positions of trust and influence.


More on Reuleaux collection see:
http://techreports.library.cornell.edu:8081/Dienst/UI/1.0/Display/cul.htmm/2002-2

Franz Reuleaux incorporated mathematics into design and invention of machines in his work Kinematics of Machinery (1876), Dover, 1964. The best known is "Reuleaux triangle" which is one of the curves with constant width. It is mentioned in literature that first to discuss curves with constant width was Leonard Euler:


Other authors point to

This reference doesn't refer to constant width, but fig. 71 looks very like a Reuleaux triangle. Another early reference to Reuleaux triangle is

M. E. Barbier. Note sur le problème de l'aiguille et jeu du joint couvert. J. Math. pures appl. (2) 5 (1860) 273-286,

which mentions that perimeter of the curve of constant width equals times the width.

Reuleaux in his Kinematics gave the first complete analysis of such triangles and he also noticed that such curves could be generated from any regular polygon with an odd number of sides. Reuleaux applied to such curves Poinsot's theory of rolling (centrodes,
roulette). Many of the models show change in motion: circular to trigonometric (slider crank), circular to elliptical (double slider crank), circular to straight-line motion (straight-line mechanisms) -- analysis of these motions involves calculus and inversive geometry. Gear mechanisms in the collection use epicycloid and hypocycloid.

Several of Reuleaux models were included in Walter Dyck's Katalog mathematischer und mathematisch-physikalischer Modelle, Apparate und Instrumente (1892), Georg Olms Verlag, New York 1994.

Walter von Dyck (1856-1934) was appointed Director of the Munich Polytechnikum in 1900 and under his inspired leadership the institution rose to university status becoming the Technische Hochschule of Munich. He served as rector of the Technische Hochschule for two terms, the first from 1903 to 1906 and the second from 1919 to 1925. There was another important project in which von Dyck played an important role. This was the creation of the Deutsches Museum of Natural Science and Technology. The idea was first suggested by Oskar von Miller who was an electrical engineer who was instrumental in setting up the electric power industry in Germany. In 1903 Miller enlisted the help of von Dyck and of Carl von Linde who had been appointed extraordinary professor of machine design at the Munich Polytechnikum in the same year as von Dyck was appointed. They proposed that a museum be built in Munich which would both preserve technological artifacts and let visitors learn about the scientific principles through interactive displays. The Deutsches Museum was first of its kind and its ideas were soon copied by other science museums around the world. Not only was von Dyck one of the three to establish and develop the museum in its early stages, but he was also appointed as the second Director of the Museum in 1906. For his catalogue Dyck chose from the Reuleaux models three with rolling on a sphere (cycloid on a sphere).

Another mathematical application, which appears in the Reuleaux model collection, is the use of circle involutes in pumps and combining other geometrical figures into pumps.

In the late 19th century there were two contrasting approaches to the education of mechanical engineers in US. As an example of the complexities of the school-versus-shop controversy in mechanical engineering education, Monte A. Calvert in his book "The Mechanical Engineer in America, 1830-1910" (Baltimore, 1967, p.87-105) discussed the case of Sibley College of Engineering at Cornell University, which had in succession, leading proponents of both approaches, John Edson Sweet and Robert Henry Thurston. Sweet started to work in Sibley College in 1873. His idea of education was to have a master craftsman surrounded by humble, eager, admiring apprentices, learning about mechanics, industry, and life. He hated the routine and standardized nature of much of educational practice, and spoke of "examinations and diplomas" as the two great evils of schools and colleges. He said:

It is not always the uneducated, the insane or the stupid who produce failures, nor the best educated, most thoughtful, or most experienced who bring out everything according to the original intention. The unexpected comes to good and bad alike, and so in our teachings to the young and our planning for ourselves, is it not well to have our statements and our speculations pretty well saturated with the elements of uncertainty? (ASME, Trans., VII (1885-86), 156)

It is known that Sweet acquired a small but devoted group of students, who literally worshipped him. But, in general, Cornell's mechanical education was criticized in technical journals. Sweet became disgusted and quit in a huff in 1879. A. D. White in 1882 made an effort to answer charges pointing out that Sibley College had two professors and much new equipment. Well, that is the year when Cornell University
acquired the Reuleaux kinematic model collection, clearly trying to follow European examples. In 1885 Robert H. Thurston came to Sibley College and he went to work at once to raise entrance requirements in order to eliminate much of the elementary work in the first two years of the curriculum. The curriculum was reorganized to include basic science, higher mathematics, and language in the first two years. His aim was to perfect his model of the pure technical school, and, as he wrote to A. D. White in 1885:

the more rapidly the lower portion of the work is forced back into the preparatory and lower schools, and the more colleges reach upwards into the higher fields of investigation, the more they will accomplish for the world.