Reflections and the Focusing Effect from an Ideal Three-Dimensional Rough Surface*

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Introduction

We begin with an ideal undulating surface, modelled as a collection of planar facets, and a simple definition of a roughness metric for it. Given the reflectance properties of a shallow-water ocean bottom, reflections from this surface are considered with the aim of expressing the resulting bi-directional radiance distribution function analytically. Focusing effects that affect this distribution from first- and second-order reflections are discussed, incorporating the behavior of any effective shadowing and obscuration. We ignore polarization effects and limit the analyses to geometric optics.

The Egg-Carton Surface

A simple surface model is that of an egg-carton, which is represented by

$$z = a^2 \sin\left(2\pi\frac{x}{l}\right)\cos\left(2\pi\frac{y}{l}\right) + a,$$

where a is the amplitude of the basic sinusoidal function with length, l. The macro-scale roughness is expressed as the amplitude-to-length ratio, $\sigma = \frac{a}{l}$ of the basic waveform (Fig. 1). A single waveform is that area formed by a depression on the surface, bounded by four peaks and the saddle ridges that connect them. The light source is assumed to be infinitely distant and the detector is on a virtual hemisphere surrounding the surface. The detector field-of-view (FOV) is adjusted so that the same projected surface area is observed either as the depth is varied or as the roughness is increased.

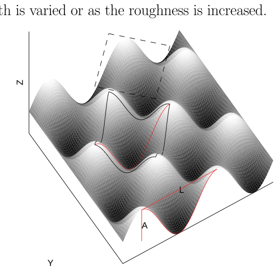

Figure 1. The egg-carton surface along with a single waveform and its projection. The roughness metric is dependent on the length, l, and amplitude, a, of the basic waveform.

Non-Lambertian Behavior

It has been proposed [1] that a rough diffuse surface increases in brightness as the viewing direction approaches the retroreflection direction (compare Figs. 2 and 3), even in the absence of shadowing and/or obscuration [2], [3]. We show that this is due to focusing by the surface facets towards the retroreflection direction.

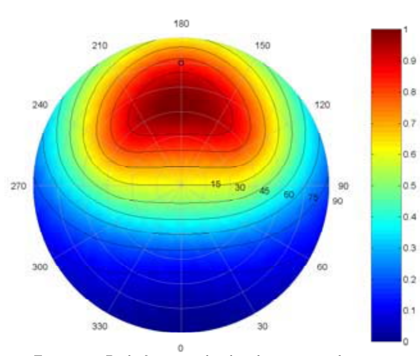

Figure 2. Peak for normalized radiance near the retroreflection direction (red dot) for a rough completely diffuse surface with roughness $\sigma = 0.50$.

The phase angles with respect to the facet normals determine the radiance distribution that results. Shadowing and obscuration are equivalent to the geometrical attenuation factor considered for specular reflectance in [2], similar to an analysis of morphological effects using triangular waves in [4].

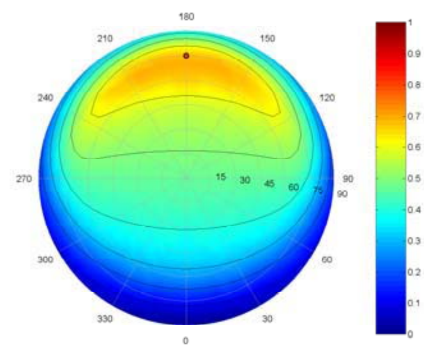

Figure 3. As in Fig. 2 (normalized to values in Fig. 2), but for surface roughness $\sigma = 0.30$; radiance decreases with roughness in the retroreflection direction. The peak is not dependent only on illumination direction but is also affected by roughness.

Shifting the Retroreflection Peak

Let θ and ϕ describe zenith and azimuthal directions, respectively, r the magnitudes of the vectors of interest, s a surface scale parameter, and l the waveform length. For some detector location, $\hat{\xi}_v = \{(\theta_v, \phi_v, r_v)\,|0 \le \theta_v \le \pi, 0 \le \phi_v \le 2\pi, 0 < \frac{s}{l} << r_v\}$, we have the radiance distribution expressed as

$$L(\theta_n, \phi_n) = \iint \frac{1}{\pi}\,\rho(\lambda)\;L_1(\lambda, \tau, \Omega_s)\;\exp\left[-k_w(\lambda)\,D\left(\hat{\xi}_1, \hat{\xi}_v\right)\right] |\cos\hat{\xi}_1|\;G\left(\hat{\xi}_1, \hat{\xi}_v, \hat{\xi}_n\right)\;\;\mathrm{d}A,$$

where A is the surface of integration, for all viewing angles $\Omega_v = \{(\theta_v, \phi_v)\,|0 \le \theta_v \le \pi, 0 \le \phi_v \le 2\pi\}$; $\hat{\xi}_1$ the bottom incidence direction, $\hat{\xi}_v$ the return direction, and $\hat{\xi}_n = (\theta_n, \phi_n, r_n)$ the normal, all at any point on the same surface; ρ the material reflectance, L_1 the incident radiance, $\lambda = 550$nm the wavelength of incident light, τ the incoming air-water transmission effect, Ω_s the solid angle subtended by the source (sun), and k_w the water attenuation factor; D the total path length travelled and G the geometrical radiance transfer factor from an infinitesimally small area on the surface. This integral can be expressed as an elliptic integral of the second kind and its maximization for a given $\frac{a}{l}$ ratio will determine the location of the retroreflection peak, and so we have described a radiance distribution that is dependent on the roughness parameter defined.

Focusing near the Specular Direction

It has been shown [2] that a peak away from the specular direction occurs at large angles of incidence (relative to the normal) as the surface gets rougher. Furthermore, [5] show similar results for oil films on ocean surfaces from Monte Carlo simulations. Both the peaks in the forward and backward directions have been observed in measurements at smaller angles of incidence by in [6]. We show that this is caused by shadowing and obscuration (Fig. 4), but also that the peaks are determined by the roughness scale of the surface.

Second-Order Radiance Reflectance

For a periodically rough diffuse surface that does not consist of cavities, an overall reflectance function dominated by the first-order pattern is expected. Interreflections on the surface allow for higher order reflections although they are relatively weak. This is another way of thinking how higher-order reflections make a surface more "diffuse". Second-order contributions are strongest closest to nadir viewing as the return angles are smallest overall.

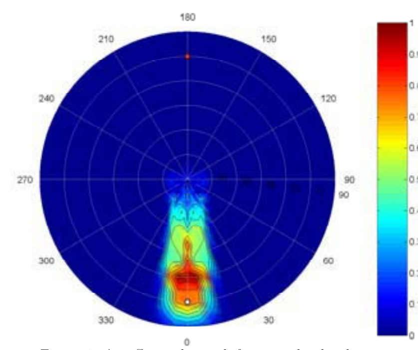

Figure 4. An off-specular peak for normalized radiance in the forward direction (white dot) for a rough completely specular surface. Shadowing and masking play a more significant role in the resulting pattern as the roughness increases, and as the roughness scale decreases.

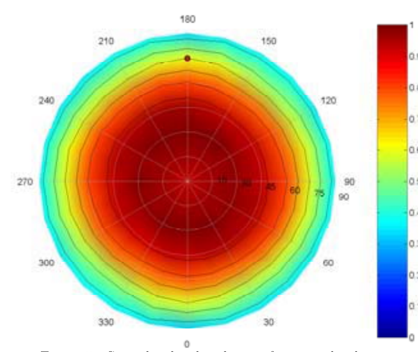

Figure 5. Second-order distribution for normalized radiance of a rough completely diffuse surface. The distribution becomes more Lambertian-like as the interreflections "diffuse out" the return.

Future Endeavors

We have proposed an expression for the peak close to the retroreflection direction for a rough diffuse surface that is dependent on a roughness metric that is clearly defined. An expression for the full bi-directional radiance distribution that includes higher-order reflections would be desired, as well as that for a specular surface in both backward and forward directions. While geometrical effects play a significant role in the radiance distribution, more insight on real-world reflectances might be gained by considering polarization and including wavelength-dependent effects.

References

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*Please see http://www.people.cornell.edu/pages/wrc22/ for more information.