Reflections and the Focusing Effect from an Ideal Three-Dimensional Rough Surface*

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Introduction

We begin with an ideal undulating surface, modeled as a collection of planar facets, and a simple definition of a roughness metric for it. Given the reflectance properties of a shallow-water ocean bottom, reflections from this surface are considered with the aim of expressing the resulting bi-directional radiance distribution function analytically. Focusing effects that affect this distribution from first- and second-order reflections are discussed, incorporating the behavior of any effective shadowing and observation. We ignore polarization effects and limit the analyses to geometric optics.

The Egg-Carton Surface

A simple surface model is that of an egg-carton, which is represented by

\[ z = a \sin \left( \frac{2\pi}{l} x + \frac{2\pi}{r} y \right) \]

where \( a \) is the amplitude of the basic sinusoidal function with length, \( l \); the amplitudes of roughness is expressed as the amplitude-length ratio, \( a/l \), of the basic waveform. A single waveform is that area formed by a depression on the surface, bounded by four peaks and the middle ridge that connects them. The light source is assumed to be infinitely distant and the detector area a virtual hemisphere surrounding the surface. The detector field-of-view (FOV) is adjusted so that the same projected surface area is observed either as the depth is varied or as the roughness is increased.

Figure 1. The egg-carton surface along with a single waveform and its projection. The roughness metric is dependent on the length, \( l \), and amplitude, \( a \), of the basic waveform.

Non-Lambertian Behavior

It has been proposed [1] that a rough diffuse surface increases in brightness as the viewing direction approaches the retrorefection direction (compare Figs. 2 and 3), even in the absence of shadowing and/or obscuration [2], [3]. We show that this is due to focusing by the surface facets towards the retroreflection direction.

Figure 2. Peak for normalized radiance near the retroreflection direction (red dot) for a rough completely diffuse surface with roughness \( a = 0.5a \).

Focusing near the Specular Direction

It has been shown [2] that a peak away from the specular direction occurs at large angles of incidence relative to the normal to the surface gets rough. Furthermore, [2] show similar results for all soils on ocean surfaces from Monte Carlo simulations. Both the peaks in the forward and backward directions have been observed in measurements at smaller angles of incidence by [2]. We show that this is caused by shadowing and obscuration (Fig. 4), but also that the peaks are determined by the roughness scale of the surface.

Second-Order Radiance Reflectance

For a periodically rough diffuse surface that does not consist of contiguous, an overall reflectance function dominated by the first-order pattern is expected. Interferences on the surface allow higher order reflections although they are relatively weak. This is another way of thinking of higher-order reflections make a surface more "diffuse".

Figure 3. As in Fig. 2 (normalized to values in Fig. 5), but for surface roughness \( a = 0.5a \). Radiance decreases with roughness in the retroreflection direction. The peak is not dependent only on illumination direction but is also affected by roughness.

Shifting the Retroreflection Peak

Let \( \theta \) and \( \varphi \) describe zenith and azimuthal directions, respectively, \( r \) the magnitude of the vectors of interest, \( s \) a surface scale parameter, and \( \lambda \) the wavelength. For some detector location, \( \textbf{v} = (\hat{r}, \hat{\varphi}, \hat{\theta}) \), \( \{ \hat{r}, \hat{\varphi}, \hat{\theta} \} \) at any point on the surface, \( \rho \) the material reflectance, \( \omega \) the incident radiance, \( A = 550\text{nm} \) the wavelength of incident light, \( \gamma \) the incoming-airwater transmission factor, \( \tau \) the material reflectance, \( \Omega \) the solid angle subtended by the source (area), and \( \Omega_{s} \) the sum of the radiance transfer factor from an infinitesimally small area on the surface. This integral can be expressed as an elliptic integral of the second kind and its maximization for a given \( s \) value will determine the location of the retroreflection peak, and so we have described a radiance distribution that is dependent on the roughness parameter defined.

Figure 4. An off-specular peak for normalized radiance in the forward direction (shown), for a rough completely specular surface. Shadowing and masking play a more significant role in obscuring the return. The distribution becomes more Lambertian-like as the transfections "diffuse out" the return.

Future Endeavors

We have proposed an expression for the peak close to the retroreflection direction for a rough diffuse surface that is dependent on a roughness metric that is clearly defined. An expression for the full bidirectional radiance distribution that includes higher-order reflections would be desired, as well as that for a specular surface in both backward and forward directions. While geometrical effects play a significant role in the radiance distribution, more insight on real-world reflections might be gained by considering polarization and including wavelength-dependent effects.

References


*Please see http://www.people.cornell.edu/pages/wrc22/ for more information.