Chapter 3

Attitude Sensing Using a GPS Antenna on a Turntable, Experimental Tests*

3.1 Introduction

The standard GPS attitude sensing method uses measurements of the carrier phase difference between multiple antennas [1]. Various GPS-based attitude determination methods have been developed that try to reduce the necessary number of antennas [2-6]. The goal of such approaches is to simplify the system and make it more robust.

The idea of using a single GPS antenna mounted on a rotating turntable for attitude sensing was suggested in [6], which analyzes the performance of this system through simulations. This sensor uses a single patch antenna that is mounted off-axis on a rotating turntable with its field of view aligned with the rotation axis. The circular motion of the antenna due to the turntable rotation causes a sinusoidal phase modulation of the GPS carrier signal. The amplitude and phase of the modulation can be used to determine the unit direction vector to the tracked GPS satellite as measured in turntable coordinates. A modified phase-locked loop (PLL) is required to extract the amplitude and phase of the carrier-phase modulation. In theory, the three-axis

The attitude of any vehicle can be derived from this system's measured unit direction vectors to two or more GPS satellites. The simulation results for this system showed a $1\sigma$ attitude accuracy on the order of 1.4 deg at an antenna mounting radius of 0.1 m, with the dominant cause of error being multipath.

According to [6], this system has three advantages over standard GPS-based attitude sensing. First, it requires fewer receiver channels – only one per tracked satellite. Second, there is no need to resolve integer phase ambiguities between different antennas. Third, the vector nature of the attitude measurements allows one to use a Wahba-type solution [7] to derive 3-axis attitude given two or more vector measurements. A single turntable/antenna sensor can measure the unit direction vectors to any number of GPS satellites that lie in the field of view of the antenna. Thus, a single-antenna system can be used to determine full three-axis attitude.

This attitude sensing approach is particularly attractive for use on vehicles that already include rotating or otherwise moving parts. The idea is to mount the antenna on the rotating or moving subsystem. This sensor was first conceived for use on a spacecraft that includes a pitch momentum wheel. Typical momentum wheels have a radius of about 10cm and rotate at speeds between 1000 and 4000 rpm with very little vibration. The antenna can be mounted on the momentum wheel, and the momentum wheel can be mounted on the outside of the spacecraft. Other moving subsystems to which this concept can be applied include scanning radar systems and helicopter blades. In the latter example, the antenna has to be mounted near enough to the rotor hub to bound the effects of structural vibrations because once-per-revolution vibrations add an unknown tilt bias to the system.

The main contribution of the present work is to experimentally verify this new attitude sensor concept. In addition, this paper discusses three attitude estimation algorithms that make use of this new sensor’s data. These algorithms are compared in
order to determine how to obtain the best accuracy from the system and to investigate

certain algorithm-dependent constraints on the system’s hardware design.

The remaining sections of this chapter present the hardware configuration of a

prototype system (Section 3.2), a review of the basic attitude sensing model (Section

3.3), three attitude determination algorithms (Section 3.4), experimental results

(Section 3.5), and conclusions.

3.2 Hardware Configuration

This section describes the prototype attitude sensor. It is composed of a
turntable, a GPS antenna mounted off center on the turntable, an axially mounted re-
radiating/re-receiving antenna pair, a motor to rotate the turntable, an angle encoder to
measure the rotation angle of the turntable, a GPS bit-grabber card, a data acquisition
system to record the bit-grabber and angle-encoder data, and software receiver code
that runs on a personal computer, but not in real time. Figure 3.1 shows a photograph
of part of this prototype. A power supply is at the far right of the picture. The
turntable system is at the center. Note the small white patch antenna at the top of the
turntable and the antenna lead coming out from under the turntable. The re-
radiating/re-receiving antenna pair cannot be seen because it is located below the
turntable. It transmits the radio frequency (RF) signal from the rotor to the statically
mounted bit-grabber card. At the far left of the picture is the bit-grabber card. It
contains the receiver's RF front-end and outputs a digitized signal that is processed by
the software receiver. The data from the bit-grabber and the angle encoder pass to the
data acquisition (DAQ) system and then to the software receiver, which post-
processes the collected data.
3.2.1 Turntable with Antenna

The rotating part of the turntable is a 40×15 cm rectangular plate. The turntable base is an aluminum plate with three legs of adjustable height that are useful for leveling the apparatus. A motor and an angle encoder are mounted on the underside of the base plate. The turntable is driven by the motor through a belt gear. The primary antenna is a patch-type antenna, chosen for its small size (about 5 cm diameter) in order to allow placement of its phase center far from the turntable’s spin axis. It is mounted at a radial distance of 15 cm from the center of the turntable. This antenna includes a pre-amp which is powered by a rotor-mounted battery. Its output passes to the re-radiating antenna through a small hole in the middle of the turntable.
The re-radiating antenna is mounted to the bottom of the turntable facing downwards and passively retransmits the signal from the primary antenna. The re-receiving antenna also includes a pre-amp and is located opposite the re-radiating antenna on the static base. It receives the signal from the re-radiating antenna and passes it to the bit-grabber card. The re-radiating/re-receiving antennas are shielded to avoid interference.

It should be noted that the re-radiating/re-receiving antennas are included only to pass the signal from the primary antenna to the bit-grabber across the table’s rotary joint. Therefore, the concept of using a single antenna in attitude sensing remains intact. Figure 3.2 illustrates the function and connections of the three antennas in a signal-flow block diagram of the system.

![Signal-flow block diagram for the prototype system.](image.png)
3.2.2 Bit-grabber/Software Receiver

A bit-grabber/software receiver combination is used in place of a real-time receiver to perform the signal tracking functions that are necessary to deduce attitude data. The bit-grabber corresponds to the RF front end of a real-time receiver. It down-converts and digitizes the L1 signal from the antenna and produces a digitized intermediate frequency signal. The acquisition, pseudorandom number (PRN) code correlation, and tracking of each GPS signal are all implemented by the software receiver in MATLAB, which executes after the fact.

In principle, all of the receiver functions could be implemented in real time in a typical hardware receiver, but it was deemed more efficient to do prototype development using an off-line software receiver. The two reasons for doing this are as follows. First, the receiver needs a special PLL to estimate the sinusoidal phase modulation caused by the turntable rotation. This modulation constitutes the attitude signal. The bit-grabber/software-receiver makes it easy to implement this special PLL. Second, this system requires the acquisition of the angle encoder data and GPS data with known relative time tags. A typical real-time receiver implements the signal tracking in specialized hardware. With such systems, it is a challenge to recognize the exact acquisition time of GPS data and to synchronize angle encoder data with GPS data. The GPS signal from the bit-grabber, however, can be connected to the DAQ card along with the angle encoder data to enable simultaneous data collection.

3.2.3 Data Acquisition System

The bit-grabber card provides a 2-bit digitized signal at 5.714 MHz. A shift register deserializer card is connected to the bit-grabber card. It packs four 2-bit bit-grabber samples into 1 byte of parallel data at the lower frequency of 5.714/4=1.4285 MHz. This 1-byte GPS output and a byte that contains raw angle encoder data are passed to the computer through the DAQ card. The data acquisition is controlled
using a LabVIEW program. This data acquisition system demands a great deal of computer random access memory (RAM) and can save a maximum of 4s worth of data because of memory limitations.

### 3.3 Review of the Attitude Sensing Model

The attitude determination algorithms used in this study for the analysis of experimental results are based on the ideas presented in [6]. This section reviews how the full three-axis attitude can be sensed using the outputs of a special PLL that operates with this system. The details can be found in [6].

Figure 3.3 shows the measurement geometry of this new GPS attitude sensor. The x-y-z axes define a turntable coordinate system which does not rotate with the turntable. The radial offset $r_a$ and the table rotation angle $\psi_a$ define the location of the patch antenna. The rotation angle $\psi_a$ is measured by an angle encoder. $\hat{r}_{table}$ is the unit direction vector to the GPS satellite and is given in table coordinates.

The rotation of the turntable causes an oscillation of the received carrier phase. This oscillation is quantified by $x_c$ and $x_s$, which are, respectively, the in-phase and quadrature components of the received carrier phase oscillation as measured at the table rotation frequency, $\omega_a = d\psi_a / dt$. The vector $\hat{r}_{table}$ to a GPS satellite is related to $x_c$ and $x_s$ by

$$
\hat{r}_{table} = \begin{bmatrix}
\frac{c}{(\omega \tau_a)}x_c \\
\frac{c}{(\omega \tau_a)}x_s \\
\sqrt{1 - \frac{c}{\omega \tau_a}^2} (x_c^2 + x_s^2)
\end{bmatrix}
$$

where $c$ is the speed of light and $\omega_a$ is the nominal transmission frequency of the GPS signal.
Figure 3.3. The measurement geometry for attitude sensing using a GPS antenna mounted on a turntable [6].

The full three-axis attitude can be estimated if two or more non-collinear direction vectors are measured. The vector $\hat{r}_{table}$ to each satellite can be calculated from $x_c$ and $x_s$ using equation (3.1), and the same unit vector in reference coordinates can be derived from ephemeris data and the user position as determined from the navigation solution. Knowledge of two or more such unit vectors in table and reference coordinates gives enough information to enable full three-axis attitude estimation.

The GPS receiver must be able to estimate $x_c$ and $x_s$ for each tracked satellite in order to provide input to the attitude determination system. This can be done using the PLL of [6]. It applies a Kalman filter (KF) to estimate the carrier-phase states,
including \(x_c\) and \(x_s\), along with other typical PLL states, such as the Doppler shift and carrier phase, that would occur if the table were not rotating.

### 3.4 Algorithms for Estimating Attitude

Three different attitude estimation algorithms have been tried in this study. One is the q-method, also commonly known as the quaternion (QUEST) solution [8,9], and the other two are batch filters. One of the batch filters also estimates common-mode errors, but the other does not estimate such errors. Common-mode errors are perturbations to the sinusoidal carrier-phase modulations that are the same for all tracked satellites. Such errors might arise as a result of receiver clock drift.

#### 3.4.1 QUEST Algorithm

The attitude determination problem for this system’s vector-type measurements can be posed as Wahba’s problem [7]

\[
\text{find: } A(q) \quad (3.2a)
\]

\[
\text{to minimize: } J_{\text{QUEST}}\{A(q)\} = \frac{1}{2} \sum_{j=1}^{m} \frac{1}{\sigma_j^2} \{\hat{r}_{\text{table}}^j - A(q)\hat{r}_{\text{ref}}^j\}^T \{\hat{r}_{\text{table}}^j - A(q)\hat{r}_{\text{ref}}^j\} \quad (3.2b)
\]

subject to: \(q^T q = 1\) \quad (3.2c)

where \(q\) is the attitude quaternion representing the transformation from reference coordinates to table coordinates, \(A(q)\) is the direction cosine matrix for that transformation, and \(\hat{r}_{\text{ref}}^j\) is the known unit direction vector to the \(j\)-th GPS satellite in reference coordinates. The QUEST algorithm solves Wahba’s problem expressed in quadratic form, i.e.

\[
\text{find: } q \quad (3.3a)
\]

\[
\text{to minimize: } J_{\text{QUEST}}\{q\} = \frac{1}{2} q^T H_{\text{meas}} q + \sum_{j=1}^{m} \frac{1}{\sigma_j^2} \quad (3.3b)
\]
subject to: \[ q^T q = 1 \] 

where \[ H_{\text{meas}} = \sum_{j=1}^{m} \frac{1}{\sigma_j^2} \left[ \{ I \left[ (\hat{r}_{\text{table}})^T \hat{r}_j \right] - \hat{r}_j^T \hat{r}_{\text{ref}} \} - (\hat{r}_{\text{table}} \times \hat{r}_j)^T \right] - (\hat{r}_{\text{table}} \times \hat{r}_{\text{ref}})^T \]

\[ (3.3d) \]

The optimal solution to Wahba’s problem comes from the necessary condition to minimize the sum of the cost function and the constraint multiplied by the Lagrange multiplier \( \lambda /2 \). The optimality necessary condition is,

\[ (H_{\text{meas}} + \lambda I)q = 0 \quad \text{or} \quad H_{\text{meas}}q = -\lambda q \]

This is an eigenvalue problem. The optimal quaternion is the eigenvector of \( H_{\text{meas}} \) corresponding to its most negative eigenvalue. The QUEST algorithm does not require an initial guess, which is one of its principal strengths.

### 3.4.2 Batch Filter with Common-Mode Error Estimation

Reference [6] suggests that receiver clock phase drift error can significantly worsen the estimated attitude accuracy when the turntable rotation speed is slow. The effects of receiver clock drift on \( x_c \) and \( x_s \) are the same for all channels and thus are referred to as common-mode errors. To obtain the best possible accuracy from this attitude determination sensor, a new algorithm has been developed to estimate and remove the effects of common-mode errors at the same time that it estimates three-axis attitude. This new algorithm uses a batch filter to find the best-fit attitude from the measured \( x \) and \( y \) components of the unit direction vectors to the GPS satellites after compensating for the effects of common-mode errors. This batch filter operates point-wise in time, like the QUEST algorithm and much like the standard code-range GPS navigation solution.

The batch filter’s estimated vector is
\[ \mathbf{p} = [q; e_c; e_s] \]  

(3.5)

where \( q \) is the attitude quaternion and \( e_c \) and \( e_s \) are the common-mode errors in \( x_c \) and \( x_s \), respectively. The number of independent variables of the \( \mathbf{p} \) estimation vector in equation (3.5) is five, since the four-element quaternion is subject to a normalization condition ( \( q^T q = 1 \)). Therefore, the tracking of three or more GPS satellites is necessary to estimate the attitude quaternion along with the common-mode errors because each tracked satellite yields two scalar attitude measurements.

The batch filter estimates the \( \mathbf{p} \) vector by minimizing the following nonlinear least-squares cost function

\[
J(\mathbf{p}) = \frac{1}{2} \sum_{j=1}^{N} \left( \frac{\omega_j r_a}{c} \right)^2 \left[ E_c^j, E_s^j \right] (P_{\omega}^j)^{-1} \left[ E_c^j, E_s^j \right] 
\]

(3.6)

subject to \( q^T q = 1 \), where \( j \) refers to the \( j \)-th GPS satellite, \( N \) is the total number of tracked GPS satellites, \( P_{\omega}^j \) is the measurement error covariance of \([x_c^j; x_s^j]\), and the errors \( E_c^j \) and \( E_s^j \) are defined as follows.

\[
E_c^j = [1,0,0] A(q) \hat{r}_{\text{ref}}^j - \frac{c}{\omega_j r_a} \{ x_c^j - e_c \} 
\]

(3.7a)

\[
E_s^j = [0,1,0] A(q) \hat{r}_{\text{ref}}^j - \frac{c}{\omega_j r_a} \{ x_s^j - e_s \} 
\]

(3.7b)

Recall that \( A(q) \) is the direction cosines matrix.

\( P_{\omega}^j \) in equation (3.6) is chosen to weight the measurement noise of each tracked satellite. It uses the covariance of \( x_c^j \) and \( x_s^j \) from the special PLL, whose tuning parameters and resulting covariance reflect the phase error of each satellite as induced by receiver thermal noise. This thermal noise covariance is added to the expected covariance of the multipath-induced errors to produce \( P_{\omega}^j \).

The cost function in equation (3.6) minimizes the errors in two components of the unit direction vector to each tracked GPS satellite. The error computations use \( \hat{r}_{\text{ref}} \).
transformed into table coordinates using the attitude quaternion estimate. Components of this vector constitute the first terms on the right-hand sides of equations (3.7a) and (3.7b). These components are compared with those calculated from the estimated $x_c$ and $x_s$ that are output by the special PLL. Common-mode errors are subtracted from $x_c$ and $x_s$ in these $\hat{r}_\text{table}$ component calculations. These “measured” vector components constitute the second terms on the right-hand sides of equations (3.7a) and (3.7b). $E_c'$ is the difference between the $x$ component of the two unit direction vectors, while $E_s'$ is the difference between the $y$ component of the two unit direction vectors. This batch filter uses only the first two components of the vector-type $\hat{r}_\text{table}$ “measurement” because they are the only actual measurements. In other words, the filter is effectively dealing with cosine-type measurements.

This batch estimation problem is a nonlinear least-squares problem with an equality constraint. It is solved using a Gauss-Newton method that has been modified to account for the quaternion normalization constraint [10]. It estimates the $p$ vector at each measurement time that minimizes the cost function in equation (3.6) subject to the equality constraint. The modified Gauss-Newton method solves a linear least-squares problem that includes a linearized version of the quaternion normalization constraint in order to determine a search direction $\Delta p$. Each search step includes an operation that preserves quaternion normalization while maintaining the convergence properties of the basic algorithm. An adaptive step-size algorithm is used to find a search step that guarantees a cost decrease on every Gauss-Newton step.

### 3.4.3 Batch Filter without Common-Mode Error Estimation

The batch filter without common-mode error estimation estimates only the attitude quaternion. It uses the cost function in equation (3.6), except that $e_c$ and $e_s$ are not subtracted from $x_c'$ and $x_s'$ in equations (3.7a) and (3.7b). This algorithm is similar to the batch filter with common-mode error estimation in all other respects.
Table 3.1. Summary of Three Turntable Experimental Data Sets

<table>
<thead>
<tr>
<th>Data Set</th>
<th>Measurement starting date and time (UTC)</th>
<th>Average rotation speed of turntable (rpm)</th>
<th>Number of tracked satellites above 30° elevation</th>
<th>GDOP of all tracked satellites</th>
<th>Carrier-to-noise densities (C/N₀) of tracked satellites in dB-Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>09/04/02 21h 26m 27s</td>
<td>997.6</td>
<td>5</td>
<td>7.7</td>
<td>40-44</td>
</tr>
<tr>
<td>B</td>
<td>09/18/02 00h 43m 40s</td>
<td>995.6</td>
<td>4 (one of them is not well tracked)</td>
<td>8.9</td>
<td>38-43</td>
</tr>
<tr>
<td>C</td>
<td>09/18/02 01h 02m 13s</td>
<td>1037.2</td>
<td>4</td>
<td>11.0</td>
<td>41-43</td>
</tr>
</tbody>
</table>

3.5 Experimental Results

3.5.1 Data Collection

Turntable experimental data has been collected on two different days; September 4, 2002 and September 18, 2002. Three data sets have been collected, and information about these data sets is summarized in Table 3.1. The rotation speed of the turntable is about 1000 rpm for all three data sets. The carrier-to-noise densities of the tracked satellites were in the range 38-44 dB-Hz, which indicates that the use of the re-radiating/re-receiving antenna pair in the experiment does not degrade the signal strengths more than about 4 dB. These data sets all lasted 3 seconds due to memory limits of the DAQ. The geometric distribution of the tracked satellites is very different between data sets A and B and is slightly different between data sets B and C.

3.5.2 Derivation of Attitude

The post-processing of data for attitude estimation is carried out in three steps. The first step calculates the time history of \( \hat{\mathbf{r}}_\text{ref}^j \), the unit direction vector to the \( j \)-th
GPS satellite in reference coordinates. Local south-east-zenith coordinates were the reference coordinate system in this experiment. \( \hat{\mathbf{r}}_{\text{ref}} \) is calculated using ephemeris and location data that were collected separately with auxiliary real-time receivers during the turntable experiment.

The second step estimates the time histories of \( x_c \) and \( x_s \) for each tracked satellite using the special PLL of [6]. The tuning parameters of the PLL are relatively slow. Slow tuning is chosen to reduce the effects of receiver thermal noise. Since the PLL converges slowly, and the data batch is only 3 s long, good initial state estimates and Kalman filter covariances are used to reduce the transients of the PLL’s Kalman filter and thereby achieve convergence within a short period of time. The nominal bandwidth of the PLL is 0.64 Hz for all estimated states, which are \( x_c, x_s \), the equivalent no-rotation carrier-phase, the equivalent no-rotation Doppler shift, and the equivalent no rotation rate-of-change of Doppler shift.

The third step applies an attitude determination algorithm. As noted earlier, three algorithms have been tested – the QUEST algorithm and the two batch filters. Both batch filters use the QUEST solution as the initial guess of the quaternion at each measurement time.

### 3.5.3 System Accuracy Evaluation Methods

The true attitude of this apparatus during the experiments is known for only two axes – roll and pitch. The turntable rotation axis is aligned with vertical using a spirit level. The accuracy of this alignment is 0.05°. Two different methods were used to evaluate the accuracy of the attitude determination system. The first measure of the attitude error is calculated in terms of vertical axis error, which gives the accuracy of the estimated turntable rotation axis direction. This evaluation method assumes that the true orientation is exactly vertical. The vertical axis error is the angular difference between vertical and the system’s estimate of the turntable rotation
axis direction. This angular error is a good indication of the system’s attitude estimation error because the table is well leveled.

The second indication of the system’s accuracy is the batch filter’s predicted standard deviation of the attitude error of a representative axis. This is calculated using the covariance of the estimated quaternion, $P_q$. This covariance is computed by inverting the projected Hessian matrix of the cost function in equation (3.6). The representative axis error is defined as

$$2 \sqrt{\frac{2}{3} \text{trace}(P_q)}$$  \hspace{1cm} (3.8)

where $P_q$ is the 4x4 quaternion estimation error covariance matrix, which has a rank of 3. The two-thirds factor is used in equation (3.8) to reflect the fact that two independent rotation errors contribute to the error in the attitude of any given axis. The factor of 2 comes from the definition of a quaternion in terms of half angles.

### 3.5.4 Accuracy of the Estimated Attitude

The accuracy of the estimated attitude from two batch filters – one without common-mode error estimation and the other with common-mode error estimation – is discussed in this section. The accuracy of the QUEST solution is also reviewed.

#### 3.5.4.1 Batch filter without common-mode error estimation

Figure 3.4 shows the time history of the vertical axis attitude error when processing data from all five tracked satellites of data set A. The transient response of the PLL is seen in the first second. Therefore, steady-state mean and maximum values have been calculated using only the last 2 s of data. The steady-state vertical axis error of the QUEST solution has a mean value of 0.61° and a maximum of 0.94°. The batch filter solution shows a significant improvement over the QUEST solution in vertical axis error; its mean steady-state value is 0.20°, and its maximum is 0.43°. The filter’s predicted standard deviation of the attitude error of a typical axis has a mean of
1.23°, which reflects primarily the expected multipath error. Further discussion of the multipath error is given in Section 3.5.4.3.

Figure 3.4. Time histories of attitude errors for data set A with five tracked satellites using the batch filter without common-mode error estimation.

Figure 3.5 shows additional time histories of the vertical axis error of the estimated attitude when using the non-common-mode error estimating batch filter. This figure uses three different data sets and various satellite combinations. All of its steady-state vertical axis errors are less than 1.9°. The use of more satellites in attitude estimation does not necessarily guarantee lower vertical axis error. For data set C, the vertical axis error with three satellites is about 0.3° lower than the case with four satellites. The deleted satellite in the test with four satellites is the one with the
lowest elevation (near 30°) among the four tracked satellites. A satellite with lower elevation might have higher multipath error, which could explain why the result with four satellites is worse than the one with three satellites for data set C.

![Graph showing vertical axis attitude estimation error time histories for different cases using the batch filter without common-mode error estimation.](image)

**Figure 3.5.** The vertical axis attitude estimation error time histories for four different cases using the batch filter without common-mode error estimation.

The vertical axis error of data set C with three satellites has a maximum of 1.54°, while that for data set B with three satellites has a maximum of only 0.87°. Both use the same set of satellites, but with a slightly different geometric distribution because of the 19 min difference between measurement times of the two data sets. This result suggests that the effects of different multipath errors due to different
geometric distribution of satellites may be important to the estimated attitude accuracy. The result with only two satellites of data set A is still reasonable, with a maximum error of only 1.90°. These results confirm that this attitude measurement system performs as predicted by [6].

The filter’s predicted standard deviation does not explain the differences in attitude accuracy among the four different cases of Figure 3.5. The filter’s predicted standard deviation of the attitude error of a typical axis is about 2° for the three cases that use three or fewer satellites, but is about 1.3° for the case that uses 4 satellites even though that case has nearly the worst performance. This suggests that multipath considerations can be as important as geometric dilution of precision (GDOP)-like considerations in determining the system’s accuracy. The predicted standard deviations, however, are all within 2.1°, and the actual peak errors are all less than 1.3 times the filter’s corresponding predicted standard deviation. This implies a reasonable estimation accuracy level for all four cases, and it implies that the filter’s covariance gives a reasonable indication of the accuracy.

Figure 3.4 shows that the attitude accuracy of the batch filter is significantly better than the accuracy of the QUEST solution. For the four cases in Figure 3.5, the batch filter also achieves better accuracy than the QUEST solution – about 0.2-0.5° better for all but one case. This suggests that the batch filter without common-mode error estimation is a better algorithm than QUEST for estimating attitude from this system’s data. Recall that the batch filter uses only the first two components of the vector-type \( \hat{r}_{\text{table}} \) “measurement” of this system, which constitute the actual measurements. QUEST, however, uses all three components of \( \hat{r}_{\text{table}} \), including the dependent third component. This makes QUEST suboptimal and thus less accurate in the present situation. This result shows that the availability of vector-type measurement data is not necessarily an advantage of this system, contrary to the claim
of [6]. Therefore, the direct use of two actual cosine-type measurements instead of a computed vector-type measurement is recommended for attitude estimation. Note, however, that use of QUEST is still recommended to generate a first guess for the batch Gauss-Newton algorithm.

This system can be used on a maneuvering vehicle if the attitude maneuver bandwidth lies within the bandwidth of the system’s PLL. If the PLL bandwidth is raised, then there is an increased thermal noise component in $x_c$ and $x_s$. This study has examined the trade-off between PLL bandwidth and thermal-noise effects on accuracy. The results shown in Figures 3.4 and 3.5 are based on tracking results with a PLL bandwidth of 0.64 Hz. The attitude estimation algorithm has also been tried with the outputs of a 3 Hz bandwidth PLL for the same cases as those depicted in Figures 3.4 and 3.5. The peak attitude error with this higher-bandwidth PLL lies between 1.1° and 2.9°. These results are slightly worse than those of Figures 3.4 and 3.5, but are still reasonable. Therefore, this system can be used as an attitude sensor on a maneuvering vehicle. Note also that the high-bandwidth accuracy may be improved through an increase in carrier-to-noise densities ($C/N_0$). It may be possible to increase $C/N_0$ by decreasing the distance between the re-radiating and re-receiving antennas.

3.5.4.2 Batch filter with common-mode error estimation

Figure 3.6 shows the vertical axis error of the attitude estimated with the common-mode error estimating batch filter when processing data from all five tracked satellites of data set A. The vertical axis error of the QUEST solution has a maximum value of 0.94°, but the steady-state attitude estimation error from the batch filter is above 10°. This batch filter result is poor, especially when one considers that the batch filter without common-mode error estimation for the same data set has a maximum steady-state vertical axis error of only 0.43° (Figure 3.4). The batch filter’s
predicted standard deviation is about 6.35°, which explains why the actual errors are so high. The common-mode error estimation batch filter tries to estimate too many things, and this degrades the attitude observability, which greatly increases its effective geometric dilution of precision (GDOP).

![Image](image-url)

**Figure 3.6.** Vertical axis attitude error time histories when using the batch filter with common-mode error estimation, data set A with five tracked satellites.

### 3.5.4.3 Effects of multipath error on estimated attitude accuracy

The large attitude errors of the batch filter with common-mode error estimation need further analysis. It has been hypothesized that the dominant error source of the turntable attitude sensor is multipath error [6]. Suppose that the effects of multipath
error on this system’s measurements, \( x_c \) and \( x_s \), are called as \( m_c \) and \( m_s \), respectively. These two values can be estimated by using the following relationship, which assumes that multipath error is the dominant source of measurement error:

\[
\hat{A}_{\text{true}} \cdot \hat{r}_{\text{ref}}^j = \hat{r}_{\text{table}}^j = \begin{bmatrix}
\frac{c}{(\omega_c r_\omega)} (x_c^j - m_c^j) \\
\frac{c}{(\omega_s r_\omega)} (x_s^j - m_s^j) \\
\sqrt{1 - \left[\frac{c}{(\omega_c r_\omega)}\right]^2 (x_c^j - m_c^j)^2 + (x_s^j - m_s^j)^2}
\end{bmatrix}
\]  

(3.9)

where \( \hat{A}_{\text{true}} \) is the direction cosine matrix that assumes a perfectly vertical table rotation axis. It is defined as

\[
\hat{A}_{\text{true}} = \begin{bmatrix}
\cos \hat{\theta}_{\text{true}} & \sin \hat{\theta}_{\text{true}} & 0 \\
-\sin \hat{\theta}_{\text{true}} & \cos \hat{\theta}_{\text{true}} & 0 \\
0 & 0 & 1
\end{bmatrix}
\]  

(3.10)

The angle \( \hat{\theta}_{\text{true}} \) is chosen to best fit the steady-state average of the table azimuth as determined from the batch filter attitude estimates that correspond to Figure 3.4. It should be noted that \( m_c^j \) and \( m_s^j \) in equation (3.9) look very similar to the common-mode errors \( (e_c, e_s) \) considered in equations (3.7a) and (3.7b) except that \( m_c^j \) and \( m_s^j \) are different for each satellite.

The derived \( m_c \) and \( m_s \) values for each satellite from equation (3.9) are plotted in Figure 3.7 for the five tracked satellites of data set A. The values are expressed in both degree and centimeter units. The \( m_c \) and \( m_s \) values in centimeter units correspond to the differential carrier phase ranging errors due to multipath errors between times when the antenna is on opposite sides of the turntable. The \( m_c \) and \( m_s \) values in degree units give their contributions to angular errors in the measurement of \( \hat{r}_{\text{table}}^j \). The curves for \( m_c \) and \( m_s \) are roughly constant for each satellite for the last 2 s of data, which is reasonable because multipath errors should be roughly constant over the 3 s data batch. The maximum value of these multipath effects is 2.54°, or 1.33 centimeters.
which agrees with typical carrier-phase multipath error levels. These results reinforce the assertion that carrier-phase multipath is the dominant source of errors in the estimated attitude.

When multipath error is dominant, as shown in Figure 3.7, the estimation of common-mode errors gives rise to large attitude errors. The common-mode error estimation algorithm amplifies multipath errors. The batch attitude estimation filter without common-mode error estimation, on the other hand, tends to attenuate multipath error through averaging and can estimate attitude with reasonable accuracy despite dominant multipath errors.

Figure 3.7. Multipath error time histories derived from $\hat{A}_m \cdot \hat{r}_{\text{ref}} = \hat{r}_{\text{true}}$.
3.6 Conclusions

A prototype of a new attitude sensor system using a single GPS antenna on a rotating turntable has been built. This system has been tested in conjunction with two batch filter algorithms – one that estimates common-mode errors in addition to attitude and another that estimates only attitude. This work has been carried out in order to validate this new GPS-based attitude determination concept experimentally. The batch filter without common-mode error estimation achieves reasonable attitude accuracy; a 3 s batch of experimental data at a turntable rotation speed of about 1000 rpm yields a peak attitude error of 0.4–1.9 deg, depending on the level of carrier-phase multipath error in the data and the geometry and number of tracked satellites. The algorithm with common-mode error estimation, however, amplifies multipath errors and does not achieve reasonable accuracy. This suggests that this type of system should be built to minimize common-mode errors instead of trying to estimate them.

Common-mode errors can be minimized by using the following guidelines. The turntable should rotate at a speed of 1000 rpm or more if it uses a poor receiver clock, such as a temperature-compensated crystal oscillator (TCXO). If the turntable rotates more slowly than 1000 rpm, it should employ a good receiver clock, such as an ovenized crystal oscillator, to minimize common-mode errors due to clock drift. The prototype that has been tested uses a TCXO, but minimizes the effects of clock drift by rotating at a speed of about 1000 rpm. The experimental results confirm that the proposed new system can be used to measure three-axis attitude to an accuracy of 1–2° or better if designed properly.

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3.8 References


