Chapter 2
Tests of Magnetometer/Sun-Sensor Orbit Determination Using Flight Data*

2.1 Introduction

Orbit determination is an old topic in celestial mechanics and is an essential part of satellite navigation. Traditional ground-based tracking methods that use range and range-rate measurements can provide an orbit accuracy as good as a few centimeters[1]. Autonomous orbit determination using only onboard measurements can be a requirement of military satellites in order to guarantee independence from ground facilities[2]. The rapid increase in the number of satellites also increases the need for autonomous navigation because of bottlenecks in ground tracking facilities[3].

A filter that uses magnetometer measurements provides one possible means of doing autonomous orbit determination. This idea was first introduced by Psiaki and Martel[4] and has been tested by a number of researchers since then[3,5-9]. Magnetometers fly on most spacecraft (S/C) for attitude determination and control purposes. Therefore, successful autonomous orbit determination using magnetometer measurements can make the integration of attitude and orbit determination possible and lead to reduced mission costs. Although magnetometer-based autonomous orbit determination is unlikely to have better accuracy than ground-based tracking, a magnetometer-based system could be applied to a mission that does not need the accuracy of ground-station tracking. The Tropical Rainfall Measurement Mission

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(TRMM), for example, requires 40 km position accuracy. An integrated magnetometer-based attitude and orbit determination system can also be used as a backup system. Some of examples of backup operation are profile recovery of a mission when the spacecraft loses its orientation, antenna pointing in the case of loss of contact with the ground station, and checks of the primary orbit and attitude system to warn of abnormal behavior.

Ground-based tests of such systems that used real flight data reported position accuracies in the range from 8 km to 125 km[3,6-9]. Psiaki *et al*. conjectured that some of the inaccuracy was caused by uncertainty in the Earth's magnetic field[6]. To combat this source of uncertainty, batch filters have been developed that estimate field model corrections in addition to the orbit of the spacecraft[10,11]. Either 3-axis star sensor data[10] or sun-sensor data[11], in addition to the magnetometer data, was added in these studies in order to make the orbit and the field model corrections simultaneously observable. These studies used truth-model simulation to test their designs, and they predicted improvements in the accuracy of magnetometer-based orbit determination. Their predicted position accuracies ranged down to better than 1 km.

The main goal of this work is to test the efficiency of the field-model-correcting magnetometer-based orbit determination filter by using real flight data. This is the first such test. The particular filter that has been tested is based on the one that uses magnetometer and sun-sensor data, as in Ref. 11. This work also improves the orbit propagation dynamics model over what has been used in Refs 4-6 and 10-11, and it considers *a priori* variances for the field model corrections.

The rest of this chapter consists of three main sections plus conclusions. Section 2.2 explains how the batch filter of Ref. 11 has been improved for use in this study. Section 2.3 describes the spacecraft data that has been used in this study. The test results with real flight data are presented in Section 2.4.

2. 2 Modifications of Batch Filter

The field-model-correcting orbit determination batch filter of Ref. 11 is extensively used in this study. In order to deal with real flight data, the orbital dynamics model had to be improved. The cost function has also been changed to include *a priori* variances for the field model corrections, which allow the filter to decide whether it has enough data to accurately estimate these corrections. This section explains the orbital dynamics model improvements and gives a review of the batch filter that has been used in this study. The addition of *a priori* variances for the field model corrections is explained in the review of the batch filter.

It should be noted that the batch filter is not practical for autonomous orbit determination. The main focus of this study, however, is to prove the validity of the field-model-correction concept. Therefore, a batch filter that was already developed in Ref. 11 to use magnetometer and sun-sensor data is used extensively. Development of a Kalman filter version could be done in the future once the benefit of the field-model-correcting approach has been verified.

2.2.1 Update of Orbital Dynamics Model

The orbital dynamics model that was used in the Ref. 11 filter considered only secular J_2 effects and the effects of drag on altitude and mean motion. It ignored the periodic effects of J_2, higher order gravity effects, and the drag effects on eccentricity. A more accurate dynamics model is preferable for dealing with real flight data.

The new dynamics model used in this study considers the full J_2 effects, and it uses a drag model that is based on the 1976 U.S. standard atmosphere. The orbit is calculated via direct numerical integration of the following equation of motion, which is expressed in an Earth-centered Earth-fixed (ECEF) reference frame.

$$\ddot{\mathbf{r}} + 2\boldsymbol{\omega}\times\dot{\mathbf{r}} + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r}) = \mathbf{a}_{inertial} = -\frac{\mu_{\oplus}}{\|r\|^3}\mathbf{r} + \mathbf{a}_{\mathbf{J2}}(r) + \mathbf{a}_{\mathbf{d}}(r,\dot{\mathbf{r}}) \tag{2.1}$$

where ω is the Earth's rotation rate vector, $\mu_{\oplus}$ is the geocentric gravitational constant, r is the position vector of the S/C in the ECEF frame, $a_{J2}(r)$ is the gravitational acceleration due to Earth Oblateness, and $a_d(r,\dot{r})$ is the acceleration due to drag. Note that the ECEF reference frame is defined with $+x$ pointing through the equator at the Greenwich meridian and $+z$ pointing through the north pole. The standard 4th order Runge-Kutta method with a fixed integration step size is used in the integration of eq.(2.1). $a_{J2}(r)$ is[12]

$$a_{J2}(r) = \frac{9}{2}\frac{\mu_{\oplus}}{\|r\|^5} J_2 R_{\oplus}^2 \left[\left(\frac{z}{\|r\|}\right)^2 - \left(\frac{1}{3}\right)\right] r + 3\frac{\mu_{\oplus}}{\|r\|^7} J_2 R_{\oplus}^2 z \begin{bmatrix} zx \\ zy \\ -(x^2+y^2) \end{bmatrix} \tag{2.2}$$

where $R_{\oplus}$ is the Earth's equatorial radius, J_2 is the gravity field's lowest zonal harmonic coefficient, and *x, y,* and *z* are the Cartesian coordinates of the S/C in the ECEF frame, $r = [x, y, z]^T$. $a_d(r,\dot{r})$ is

$$a_d(r,\dot{r}) = -\frac{\bar{\beta}}{2}\rho(r)\|\dot{r}\|\dot{r} \tag{2.3}$$

where ρ is the atmospheric mass density and $\bar{\beta} = C_D S/m$ is the inverse of the spacecraft's ballistic coefficient – C_D is the spacecraft's drag coefficient, S is its aerodynamic reference area, and m is its mass. $\rho(r)$ is calculated from a cubic spline interpolation of the natural log of tabulated data from the 1976 U.S. standard atmosphere[13].

The accuracy of the filter's new dynamics model has been checked and compared to that of the old dynamics model. This has been done by using another batch filter to estimate the 6 initial conditions for this dynamic model plus $\bar{\beta}$ by minimizing the sum of the square errors between the measured position time history of an actual S/C and the modeled position based on propagation of eq.(2.1). The closeness of this fit gives an indication of the model's propagation accuracy.

This dynamics evaluation filter has been run on real flight data from DE-2 and MAGSAT. The maximum position error of this test when using the old dynamics model is 2.16 km for a 24-hour MAGSAT data set and 4.35-4.90 km for 48-hour DE-2 data sets. The new dynamics model has a smaller maximum position error than the old one: 1.6 km for a 24 hour MAGSAT data set and 3.50-4.33 km for 48 hour DE-2 data sets. Thus, the new dynamics model has a slightly better orbital propagation accuracy than the old one and is accurate enough for use in the magnetometer-based navigation filter.

It should be noted that the constraints in computing resources for autonomous operation are not severe considering the recent dramatic improvements in computer power. Therefore, the improved dynamics model is probably too simple to be used in a typical resource-rich computing environment. The goal of this study, however, is to verify the contribution of the field-model corrections to the accuracy improvement of magnetometer-based orbit determination using real flight data. Therefore, a very accurate dynamics model which includes higher order perturbations of the Earth's gravity field and perturbations from Sun or Moon has not been used in this study.

2.2.2 Estimation Vector

The batch filter's estimation vector with the new dynamics model is

$$\begin{aligned} p = [&x_o, y_o, z_o, \dot{x}_o, \dot{y}_o, \dot{z}_o, \overline{\beta}, b_x, b_y, b_z, q_1^0, q_1^1, s_1^1, \dot{q}_1^0, \dot{q}_1^1, \dot{s}_1^1, \dot{g}_1^0, \dot{g}_1^1, \dot{h}_1^1, \\ &\Delta g_1^0, \Delta g_1^1, \Delta h_1^1, \Delta g_2^0, \Delta g_2^1, \Delta h_2^1, \Delta g_2^2, \Delta h_2^2, \Delta g_3^0, \ldots\ldots\ldots, \Delta h_N^N]^T \end{aligned} \tag{2.4}$$

where the first 6 elements are the initial position and velocity of the S/C in the ECEF frame, the 7th element is the inverse of the ballistic coefficient, the 8th to 10th elements constitute a magnetometer bias vector, and the remaining elements are correction terms to coefficients of a spherical harmonic expansion of the Earth's magnetic field[10,11]. The field correction elements include an external ring current field model (the first 6 correction terms) and perturbations to the Earth's internal magnetic field model. Note that N is the maximum order and degree of the field model corrections.

2.2.3 Batch Filter

The batch filter used in this study is the same as the one used in Ref. 11 except for an improvement to the dynamics model and an addition to the cost function that models *a priori* variances of the field model corrections. It finds the p estimation vector that best approximates the measurement data based on the dynamics and measurement models.

Two pseudo measurements, the magnitude of the Earth's magnetic field and the cosine of the angle between the Earth's magnetic field vector and the sun direction vector, are used. These "measurements" are independent of the spacecraft's attitude. Let $B_{mes(k)}$ be the magnetic field vector that is measured by the magnetometer and let $\hat{s}_{mes(k)}$ be the sun unit direction vector that is measured by the sun-sensor, with both measurements occurring at sample time t_k. Both of these measurements are expressed in a common S/C-fixed coordinate system. Then the two pseudo measurements are given by

$$y_{1mes(k)}(p) = \sqrt{(B_{mes(k)} - b)^T (B_{mes(k)} - b)} \tag{2.5a}$$

$$y_{2mes(k)}(p) = \frac{\hat{s}^T_{mes(k)}(B_{mes(k)} - b)}{y_{1mes(k)}(p)} \tag{2.5b}$$

where $b = [b_x, b_y, b_z]^{\mathrm{T}}$ is the estimated magnetometer bias vector expressed in S/C coordinates.

The modeled values of the two pseudo measurements are

$$y_{1\,\mathrm{mod}}(t_k; p) = \sqrt{B^T_{sez} B_{sez}} \tag{2.6a}$$

$$y_{2\,\mathrm{mod}}(t_k; p) = \frac{\hat{s}^T_{ECIF}(t_k) \bullet (A_{ECIF/sez} \bullet B_{sez})}{y_{1\,\mathrm{mod}}(t_k; p)} \tag{2.6b}$$

where $B_{sez}[r(p,t_k),q_1^0,q_1^1,..........,\Delta h_N^N,t_k]$ is the modeled Earth's magnetic field vector in local south-east-zenith coordinates, $\hat{s}_{ECIF}(t_k)$ is the sun unit vector in an Earth-Centered Inertially Fixed reference frame (ECIF), and $A_{ECIF/sez}[r(p,t_k),t_k]$ is the transformation matrix from local south-east-zenith coordinates to ECIF coordinates. B_{sez} is calculated using a spherical harmonic expansion and includes the effects of the estimated field model corrections[10].

The nonlinear least squares cost function that gets minimized by the batch filter

$$J(\mathbf{p}) = \frac{1}{2}\sum_{k=1}^{M}\left[\frac{y_{1\,\mathrm{mod}}(t_k;\mathbf{p}) - y_{1mes(k)}(\mathbf{p})}{\sigma_B}\right]^2 + \frac{1}{2}\sum_{k=1}^{M}\left[\frac{y_{2\,\mathrm{mod}}(t_k;\mathbf{p}) - y_{2mes(k)}(\mathbf{p})}{\sigma_{y2(k)}}\right]^2$$
$$+\frac{1}{2}\left[\left(\frac{\Delta g_1^0}{\sigma_{g_1^0}}\right)^2 + \left(\frac{\Delta g_1^1}{\sigma_{g_1^1}}\right)^2 + \left(\frac{\Delta h_1^1}{\sigma_{h_1^1}}\right)^2 + \left(\frac{\Delta g_2^0}{\sigma_{g_2^0}}\right)^2 + \left(\frac{\Delta g_2^1}{\sigma_{g_2^1}}\right)^2 + \left(\frac{\Delta h_2^1}{\sigma_{h_2^1}}\right)^2 + \ldots\ldots\ldots + \left(\frac{\Delta h_N^N}{\sigma_{h_N^N}}\right)^2\right] \tag{2.7}$$

where M is the number of data samples, σ_B is the standard deviation of the field magnitude measurement, $\sigma_{y2(k)}$ is the standard deviation of the cosine pseudo measurement, which is described in Ref. 11, and the σ_g and σ_h quantities are *a priori* standard deviations of the spherical harmonic coefficients of the Earth's internal magnetic field model. The first and second terms are the ones that are used in Ref. 11, and the remaining terms are added in this study to model *a priori* knowledge of the field model coefficients. These additions help to ensure reasonable field model corrections. The second term of the cost function is calculated only for sample times when sun-sensor data is available.

The nominal σ_g and σ_h values of eq.(2.7) have been chosen as the average of two observed 5-year changes in the corresponding coefficients of the International Geomagnetic Reference Field (IGRF) model. One change was for the period 1980-1985, and the other change was for 1995-2000. Note, however, that a value of 0.5 nT has been used whenever the average change was less than this lower bound. A special 1980 Earth magnetic field model or the 1995 or 2000 IGRF model has been used as the *a priori* field model in each of this study's various tests. When the measurement date of real flight data is close to the starting date of an *a priori* magnetic field model, one tenth of the nominal σ_g and σ_h values have been used to allow only smaller variances in the field model corrections and to achieve a more reasonable predicted accuracy from a consider-covariance analysis. Nominal σ_g and σ_h values are used for measurement data that is measured far from the starting date of the *a priori* magnetic field model. Such cases need larger field model corrections. Table 2.1 shows the first few terms of the spherical harmonic coefficients of the 1995 IGRF model and the corresponding nominal σ_g or σ_h values. The coefficients of Table 2.1 are used with associated Legendre functions of degree n and order m that are normalized according to the convention of Schmidt[12].

The Gauss-Newton iterative numerical method is used to solve this nonlinear least-squares problem[11]. It requires the calculation of the partial derivatives of the measurement errors with respect to the p vector, which it uses to find search directions. They are also used to compute the filter's predicted position accuracy using consider-covariance analysis.

Consider-covariance takes into account the effects of unestimated parameters on the filter accuracy[15]. The systematic errors due to magnetic field model coefficients that do not get estimated are included in a consider-covariance analysis. Systematic errors of unmodeled perturbations to the dynamics model, such as unmodeled higher-order gravity terms and solar radiation effects, are not considered in the consider-covariance analysis in this study.

Table 2.1. Sample Coefficients of the 1995 IGRF Field Model and the Corresponding Nominal *a Priori* Variances (σ_g or σ_h) for the Field Model Corrections.

	g_1^0	g_1^1	h_1^1	g_2^0	g_2^1	h_2^1
Coefficients (nT) of 1995 IGRF model	-29682	-1789	5318	-2197	3074	-2356
Nominal σ_g or σ_h values (nT)	91.00	57.00	119.50	73.00	10.00	10.25

2.2.4 An Alternate Filter that does not Estimate Field Model Corrections

An alternate batch filter has been developed in order to provide a point of comparison for the field-model-correcting filter. It does not estimate field model corrections nor does it use sun-sensor measurements. It only considers the first term of the cost function in eq. (2.7). This filter is similar to the batch filter of Ref. 6, except that Ref. 6 used the poorer dynamics model mentioned above. This filter can be used to evaluate the advantages of field model corrections over the traditional magnetometer-based orbit determination without field model corrections.

2.2.5 Additional Filters

Two additional batch filters have been developed for test purposes. One is a field-model-correcting filter with an *a priori* drag coefficient estimate. It is assigned a relatively small a *priori* drag variance. This filter keeps its estimated $\bar{\beta}$ nearly fixed at the *a priori* value of 0.03 m^2/kg by adding an additional term to the cost function of eq. (2.7) that minimizes the weighted square error between $\bar{\beta}$ and its *a priori* value. This filter is useful for a higher altitude S/C (perigee altitude above the 500-600 km range) because $\bar{\beta}$ becomes nearly unobservable due to the minimal effects of drag at such altitudes.

The other filter is a field-model-correcting filter that does not estimate orbit. This filter uses tracked position instead of the estimated S/C orbit and only estimates field model correction terms. This filter can help to determine whether the accuracy of field model correction estimates is compromised by the simultaneous estimation of orbit.

2.3 Real Flight Data

Real flight data from three spacecraft, DE-2, MAGSAT, and Ørsted, has been used to test the batch filter. These spacecraft are described below. Data from a fourth spacecraft, the TRMM, also has been tested. Its magnetometer data, however, was uncalibrated, and the per-axis residual error standard deviations were over 300 nT. This poorly calibrated data led to large maximum position errors for the orbit determination filter, on the order of 50 km. Therefore, the results for TRMM are not included in this study because an accurately calibrated magnetometer is necessary if one wants to apply this technique.

2.3.1 DE-2

The DE-2 spacecraft was launched in August 1981 to study the coupling between the magnetosphere, the ionosphere, and the upper atmosphere. DE-2 had roughly a 290 km perigee altitude, a 810 km apogee altitude, and a high inclination, 89.9°. It operated for about two years. Magnetometer data, 3-axis attitude data derived from sensors, and position data derived from traditional range and range rate ground-based tracking are all available for DE-2. DE-2 is known to have a 28 nT 1-σ accuracy for the magnetometer data and better than 1 km accuracy for the position data. Sun-sensor data was not directly available. Therefore, it has been synthesized by transforming the known sun position in ECIF coordinates using DE-2's estimate of the attitude transformation matrix from ECIF coordinates to S/C coordinates. The attitude uncertainty of this transformation is about 0.3°. The position of the sun in the ECIF coordinate system is calculated using the algorithm in Ref. 16 which provides a 0.006° sun position accuracy. Eclipse by the Earth is also considered in synthesizing sun-sensor data.

Tests have been run using two sets of data extracted from a CD-ROM archive of the DE-2 data that is available from the National Space Science Data Center (NSSDC). One set was measured on November 1-2, 1981 (data set A), and the other set was measured on March 15-17, 1982 (data set B). Both data sets were measured when the activity index for solar magnetic storms was low. The sample period is 60 seconds, and both data sets are spread out over about 48 hours. There are, however, data gaps about every 2 or 3 hours, and some of these gaps are as long as 10 hours. Thus, the actual measurement coverage time is 13.9 hours and 6.7 hours for data sets A and B, respectively. Bad magnetometer data points have been removed before testing the batch filter. A bad point is considered to occur when the measured magnetometer data is off by over 1000 nT from the modeled magnetometer data based on the known S/C position from ground tracking. Two out of 835 data points in set A and 30 out of 433 data points in set B have been edited out.

The position tracking data of DE-2 is given in altitude, latitude, and longitude and is stated to be measured in geocentric coordinates assuming a spherical Earth. In the process of checking the accuracy of the filter's new dynamics model, however, it has been found that the position data must have been measured with respect to an ellipsoidal Earth, not with respect to a spherical Earth. Therefore, it is assumed that the tracking data of DE-2 has been measured with respect to the ellipsoidal World Geodetic Survey 1984 (WGS-84) model, which is used in describing the Earth as an ellipsoid throughout this study.

2.3.2 MAGSAT

The MAGSAT spacecraft was launched in October 1979. The objective of this mission was to obtain accurate vector measurements of the near-Earth geomagnetic field for field modeling purposes. The orbital properties were : 97° inclination, 330 km perigee height, and 500 km apogee height. This S/C had a cesium scalar magnetometer for calibration purposes and a vector fluxgate magnetometer. The accuracy of the vector magnetometer was determined to be 3 nT 1-σ after inflight calibration using the scalar magnetometer. The 3-axis magnetic field components and the tracked position of the S/C using range and range-rate data are available in the CD-ROM archive of the NSSDC. The accuracy of tracked position is 60 meters radially and 300 meters horizontally (along track). The magnetic field vector data is expressed in south, east, zenith local coordinates. Due to the imperfect calibration of the attitude determination system, frequent small jumps occur in the vector magnetometer data, but not in the magnitude of the measured magnetic field. Two MAGSAT data sets have been used. One set was measured on February 2, 1980 (data set A), and the other set was measured on March 12, 1980 (data set B). The sampling period of all three data sets was about 100 seconds without any gaps, and it was measured on magnetically quiet days. Each data set lasts 24 hours.

Sun-sensor data has been synthesized for data sets A and B. This is similar to what has been done for DE-2, except that the y_{2mes} dot product is computed in south, east, zenith local level coordinates instead of in S/C coordinates.

2.3.3 Ørsted

The Danish satellite Ørsted was launched in Febuary 1999. The main objective of this mission is to provide a precise global mapping of the Earth's magnetic field. The orbit of Ørsted had an apogee altitude of 865 km, a perigee altitude of 649 km, and an inclination of 96.48° as of Febuary 10, 2000. Ørsted has a scalar magnetometer and a fluxgate vector magnetometer, and both magnetometers are placed on an 8 m deployable boom. Absolute position data and vector magnetometer data in local south, east, zenith coordinates are available. The position of the S/C is determined by an on-board GPS receiver and has an accuracy on the order of 2-8 meter. The vector magnetometer data is calibrated using the scalar magnetometer for scale-factor, bias, and non-orthogonality of the axes and then is calibrated for the transformation to S/C coordinates which are the axes of the star imager. This alignment transformation is accurate to 0.003° after calibration. The accuracy of the vector magnetometer has been determined to be less than 3 nT 1-σ after in-flight calibration. The accuracy of the star imager is 0.003° perpendicular to its boresight and 0.014° about its boresight. The vector magnetometer data is transformed from S/C coordinates to local south, east, zenith coordinates using attitude data. Attitude information, however, is not explicitly available in the data archive of Ørsted. This is the same as for the MAGSAT data sets.

Four data sets which were measured on December 21-22, 1999 (data sets A and B) and January 18-19, 2000 (data sets C and D) have been tested. All data sets were measured when the solar storm activity level was low. The data was sampled at about 30 second intervals and has a maximum data gap of 57 minutes. Each data set lasts 24 hours, and the last three hours of data sets A and C overlap with the first 3 hours of data sets B and D, respectively. These overlapped data sets are used for orbit overlap tests that evaluate the precision of the filter's estimated position. Similar to MAGSAT, sun sensor measurements and the y_{2mes} dot product have been synthesized in local south, east, zenith coordinates.

2.4 Results of Filter Tests Using Flight Data

Two batch filters, one without field model corrections and the other with field model corrections, have been tested with DE-2, MAGSAT, and Ørsted flight data. The filter without field model corrections uses only magnetometer data and the one with field model corrections uses both magnetometer and sun-sensor data. The magnetometer bias vector is estimated in both filters only for DE-2 because this is the only S/C whose magnetic field vector data is given in S/C coordinates. It would not make physical sense to define a bias in the local south, east, zenith coordinate system in which the MAGSAT or Ørsted data is available. Tests of the filter without field model corrections can be compared with tests of the field-model-correcting filter in order to check whether there is an advantage to including field model corrections.

In order to use the batch filter that estimates field model corrections along with the orbit, sun-sensor data are used in addition to magnetometer data. When using mixed data types in a filter, it is important to use reasonable predictions of their accuracies in order to properly weight their errors as in eq.(2.7). The 1-σ accuracy of the magnetometer data and the sun-sensor data are assumed to be 15 nT and 0.15°, respectively, for all three S/C. With these σ values as inputs, the filter's optimal residual measurement errors show similar levels of accuracy. MAGSAT and Ørsted have attitude accuracies that are about 10 times better than this sun sensor σ, but this inconsistency is allowable because Ref. 11 indicates that the filter's performance is relatively insensitive to this σ value.

The performance of these filters has been evaluated by comparing their S/C position estimates with those that have been derived from ground tracking data or an on-board GPS sensor. The filters' predicted accuracies of their position estimates also have been calculated using the estimation vector's consider-covariance and linearized covariance propagation techniques[10, 15]. This predicted accuracy has been compared with the actual accuracy as a means of checking the validity of the consider analysis.

2.4.1 DE-2 Results

Langel and Estes's 80 model[14] is used as the *a priori* field model for the DE-2 data. Langel and Estes's 80 model was derived using the data from MAGSAT. The secular variation coefficients of this model are considered to be valid between 1978 and 1982, and the magnetic field in DE-2's 1981 – 1982 time frame is calculated by using this model of the secular variations. The Langel and Estes's 80 model only depends on measurements that were taken up to 1980. Therefore, Langel and Estes's 80 model would have been available during the operation period of DE-2 and thus is a reasonable model to be used in the test of an operational orbit determination filter. Since the measurement time of DE-2 data sets are not far from the starting date (1980) of the *a priori* field model, one tenth of the nominal σ_g and σ_h values of Table 2.1 are used in considering *a priori* variances of field model coefficients.

The DE-2 results for these batch filters are summarized in Table 2.2. Table 2.2 presents the results for data set A and data set B. Note that the table has 11 rows and 7 columns. The first column gives the value of N, the highest order and degree of the field model corrections that the filter estimates – review eq. (2.4). Thus, each row except the first row corresponds to a different maximum order and degree of the filter's estimated field model corrections, and the complexity of the corrections (and of the filter) increases as one moves from the upper rows to the lower rows. (Note that in all cases considered in this paper the filter's field model uses a 10^{th}-order/10^{th}-degree field model. It uses *a priori* coefficients for terms that do not get corrected by the filter's estimation process.) The first row corresponds to the filter that does not estimate field model corrections and that uses only magnetometer data. The 2^{nd} and 5^{th} columns tabulate the consider analysis' predictions of the maximum standard deviations of the position error magnitude for the two different cases. Unless otherwise noted, the term “maximum” refers to maximization of a given quantity over the entire batch interval. Columns 3 and 6 give the actual maximum position error magnitudes for these two cases as computed by differencing the estimated positions with the true positions as derived from ground-based tracking or GPS data. Columns 4 and 7 present the rms of the position error magnitudes.

Table 2.2. Position Error Metrics for Two Different DE-2 Data Sets

Maximal Order and Degree of Field Model Corrections	Data Set A			Data Set B		
	Predicted Maximum 1-σ Accuracy of Position (km)	Maximum Position Error Magnitude (km)	Rms Value of Position Error (km)	Predicted Maximum 1-σ Accuracy of Position (km)	Maximum Position Error Magnitude (km)	Rms Value of Position Error (km)
No corrections	-	5.29	3.94	-	5.87	3.68
1st	3.54	5.33	2.27	5.11	8.16	4.77
2nd	3.06	4.48	2.54	5.36	9.08	5.72
3rd	2.05	5.76	2.82	3.32	6.99	4.21
4th	1.36	6.58	3.11	3.07	8.19	4.23
5th	1.32	7.93	3.76	2.92	8.47	4.79
6th	1.30	8.17	3.85	2.87	7.61	4.20
7th	1.27	7.97	3.47	2.87	7.93	4.11
8th	1.25	8.08	3.54	2.87	7.77	4.13
9th	1.25	8.13	3.54	2.87	7.74	4.11
10th	1.25	8.17	3.54	2.87	7.76	4.13

The maximum position error magnitude without field model corrections is 5.29 km for data set A and 5.87 km for data set B. These results represent an improvement of a factor of 3 or more from the DE-2 results of Ref. 6. This improvement is thought to be the result of an improved calibration of the DE-2 data.

The position error magnitudes associated with data set A are smaller than the corresponding values for data set B for lower order/degree corrections. With 1st to 4th order/degree field model corrections, data set A yields maximum position error magnitudes from 4.48 km to 6.58 km, but data set B's maximum position error magnitude ranges from 6.99 km to 9.08 km. For higher order/degree field model corrections, maximum position error looks similar for both data sets, but the rms position errors show that data set A has better position accuracy than data set B.

The better position accuracy of data set A may come from the fact that the actual magnetometer data coverage time of data set A is two times longer than for data set B: 13.9 hours for data set A versus 6.7 hours for data set B. This effect is probably the reason why data set A yields lower predicted 1-σ position accuracies (compare columns 2 and 5 of Table 2.2).

Overall, the results with field model corrections are slightly worse than those without field model corrections. Even the best-case data set A results with field model corrections (maximum position error of 4.48 km) are only slightly better than the data set A results without these corrections (maximum position error of 5.29 km). The best case-B results with corrections (maximum position error of 6.99 km) are not as good as the results for the filter that does not estimate field model corrections (maximum position error of 5.87 km). These results may be caused by the low percentage of actual measurement coverage time for DE-2 over the 2-day span of each data set.

Note that the dynamic model used in this study can cause a maximum position error of about 4 km over the 48 hour duration of a DE-2 data set: 3.50 km for data set A and 4.33 km for data set B. Therefore, a maximum position error for magnetometer-based orbit determination as low as 4.48 km is very good. It is almost at the limit of the accuracy of the dynamics model.

The filter's predicted maximum position standard deviation improves as the order/degree of the corrections increases for lower order/degree corrections and keeps similar values for 6th or higher order/degree corrections. The actual maximum position error, however, does not get better with higher order/degree corrections.

Therefore, rough estimation of the actual position error of the batch filter based on its consider analysis position accuracy may not be good for DE-2 cases.

2.4.2 MAGSAT Results

All MAGSAT data used in this test was measured in 1980. Therefore, Langel and Estes' 80 model has been used for the MAGSAT data sets.

Two batch filters, one without field model corrections and the other one with field model corrections have been tried for data sets A and B. Table 2.3 summarizes the MAGSAT results for these filters. These results assume one tenth of the nominal σ_g and σ_h values as 1-σ accuracies of the field model coefficients.

Table 2.3. Position Error Metrics for Two Different MAGSAT Data Sets

Maximal Order and Degree of Field Model Corrections	Data Set A: Predicted Maximum 1-σ Accuracy of Position (km)	Data Set A: Maximum Position error Magnitude (km)	Data Set B: Predicted Maximum 1-σ Accuracy of Position (km)	Data Set B: Maximum Position Error Magnitude (km)
No corrections	-	7.02	-	4.56
1st	4.07	3.88	4.62	3.24
2nd	3.40	4.04	3.54	2.35
3rd	3.07	3.54	3.22	2.78
4th	1.45	3.15	1.56	3.29
5th	1.32	3.00	1.34	3.42
6th	1.23	2.86	1.26	3.38
7th	1.19	3.07	1.22	3.41
8th	1.18	2.97	1.18	3.28
9th	1.18	3.00	1.18	3.26
10th	1.18	3.00	1.17	3.23

The best maximum position error magnitudes for the field-model-correcting filter are 2.86 km for data set A with 6th order/degree field model corrections and 2.35 km for data set B with 2nd order/degree field model corrections. These results represent a factor of two or better improvement in the position accuracy for best-case field model corrections in comparison to the accuracy without field model corrections for both data sets. Thus, unlike DE-2, field model corrections can improve the performance of magnetometer based orbit determination for MAGSAT. It is also true that the results for all order/degree corrections for MAGSAT are better than the results when corrections are not used. The ratios of the maximum position error magnitudes to the consider analysis' predicted maximum position standard deviations range from 0.95 to 2.58 for data set A and from 0.66 to 2.80 for data set B. This shows that the statistical model of the consider analysis is reasonable. Therefore, the maximum position error of the batch filter can be inferred from the predicted position accuracy of the consider analysis.

2.4.3 Ørsted Results

There is a question of whether the DE-2 and MAGSAT results were helped by the fact that the data comes from a time period for which there is a very good *a priori* estimate of the Earth's magnetic field. This is true because the currently available field models for their flight time-frames are based on data that extends after the fact, or they are based on MAGSAT's accurate survey of the Earth. To clarify this issue, the orbit determination systems should be tested with recent magnetometer data that is measured when the only available Earth magnetic field model is one that is based only on past data of moderate accuracy. Ørsted data is useful for such a test because it carries an accurate magnetometer and is currently operational.

Unlike the DE-2 and MAGSAT cases, which have a much lower perigee altitude, $\overline{\beta}$ is not practically observable for Ørsted because the effects of drag on its orbit are minimal. Therefore, the filter that incorporates an *a priori* $\overline{\beta}$ estimate has been used on Ørsted data when orbit and, optionally, field model correction terms are estimated.

Tables 2.4 and 2.5 present the results for Ørsted data sets for different orders/degrees of the field model corrections. Results for the filter that does not correct the field model are also shown in the 1st row of both tables. Table 2.4 is for measurement data from December 1999 (data sets A and B), and the filter uses the 1995 IGRF model as its *a priori* field model. Table 2.5 is for measurement data from January 2000 (data sets C and D), and the filter's *a priori* field model is the 2000 IGRF model. The *a priori* field models have been calculated via propagation of the 1995 or 2000 IGRF model using secular terms. These models have been obtained from the NGDC[†].

Table 2.4. Position Error Metrics for Ørsted Data Sets (December 1999 Data) with *a priori* field model from 1995 IGRF.

Maximal Order and Degree of Field Model Corrections	Data Set A		Data Set B		Three Hour Overlap of Data Sets A and B	
	Predicted Maximum 1-σ Accuracy of Position (km)	Maximum Position Error Magnitude (km)	Predicted Maximum 1-σ Accuracy of Position (km)	Maximum Position Error Magnitude (km)	Maximum Position Difference Magnitude (km)	Rms Value of Position Difference (km)
No corrections	-	59.50	-	50.71	24.89	22.32
1st	34.51	58.72	36.55	54.63	17.34	16.38
2nd	41.97	29.14	38.80	26.48	10.29	9.31
3rd	17.95	17.55	16.38	19.07	7.81	6.69
4th	10.74	8.87	11.49	10.82	7.74	6.35
5th	4.28	5.64	4.44	7.39	6.63	5.14
6th	3.50	2.41	2.85	3.79	4.49	3.16
7th	2.35	2.48	1.86	3.21	3.56	2.62
8th	1.13	2.61	1.29	3.40	2.81	1.90
9th	1.11	2.46	1.16	3.85	3.10	2.18
10th	1.07	2.19	1.15	4.14	3.27	2.15

† http://www.ngdc.noaa.gov/seg/potfld/tab1igrf.shtml#IGRF95.html and ftp://www.ngdc.noaa.gov/Solid_Earth/Mainfld_Mag/Models

Table 2.5. Position Error Metrics for Ørsted Data Sets (January 2000 Data) with *a priori* field model from 2000 IGRF.

Maximal Order and Degree of Field Model Corrections	Data Set C		Data Set D		Three Hour Overlap of Data Sets C and D	
	Predicted Maximum 1-σ Accuracy of Position (km)	Maximum Position Error Magnitude (km)	Predicted Maximum 1-σ Accuracy of Position (km)	Maximum Position Error Magnitude (km)	Maximum Position Difference Magnitude (km)	Rms Value of Position Difference (km)
No corrections	-	9.09	-	7.52	12.63	8.58
1st	3.38	7.90	3.77	9.00	2.60	1.79
2nd	3.13	5.88	3.24	5.98	2.33	1.45
3rd	1.61	5.87	1.67	5.52	1.81	1.34
4th	1.16	5.69	1.10	5.08	2.09	1.30
5th	0.83	5.52	0.79	5.26	1.78	1.10
6th	0.82	5.87	0.78	5.40	2.22	1.39
7th	0.81	5.55	0.77	5.24	2.15	1.36
8th	0.81	5.38	0.77	5.22	2.02	1.27
9th	0.81	5.37	0.77	5.21	2.03	1.28
10th	0.81	5.36	0.77	5.21	2.00	1.26

Tables 2.4 and 2.5 show the predicted position accuracy from the consider analysis (2nd and 4th columns) and the maximum position error (3rd and 5th columns) for two data sets. The 6th column gives the maximum magnitude of the difference between the two data sets' position estimates during their 3-hour overlap period. The 7th column gives the rms of the position difference magnitude for the overlap period. The results in Table 2.4 used nominal σ_g and σ_h values, while the ones in Table 2.5 used one tenth of the nominal σ_g and σ_h values. The use of different σ_g and σ_h values is motivated by the fact that higher *a priori* variances of the field model corrections should be allowed for a very inaccurate *a priori* field model, such as the one used for the results of Table 2.4.

The maximum position error magnitude for the filter without field model corrections is 59.50 km for data set A, 50.71 km for data set B, 9.09 km for data set C, and 7.52 km for data set D. The large error for data sets A and B is not surprising because of the large residual error in the *a priori* field model. The rms residual field model error for data sets A and B from a comparison of raw magnetic field data with the 1995 IGRF *a priori* magnetic field model is 72.22 nT for data set A and 65.14 nT for data set B. These residual errors are far bigger than the corresponding errors for data sets C and D – 22.52 nT for data set C and 20.84 nT for data set D when compared with the 2000 IGRF *a priori* magnetic field model.

Table 2.4 shows the results for the case when an inaccurate *a priori* field model is applied. The overall trends of the results for data sets A and B in Table 2.4 are quite similar. The position estimation error of the field-model-correcting filter keeps improving as the complexity of the order/degree of the field model corrections increases. For data set A, the maximum position error keeps decreasing from 58.72 km at 1st order/degree field model corrections down to 2.19 km at 10th order/degree field model corrections. For data set B, the error in the estimated position reaches a minimum 3.21 km for 7th order/degree field model corrections. The results of Table 2.4 suggest that field model corrections of at least 6th or higher order/degree should be tried when an inaccurate *a priori* field model is used. Figure 2.1 shows the position error magnitude time histories for the best position estimate with field model corrections and for the one without field model corrections for data set A. The field model corrections cause a dramatic improvement of the accuracy of the estimated position. Therefore, the inaccuracy of the *a priori* Earth magnetic field model seems to be the driving factor in causing position estimation errors in the case of Ørsted data if one uses a filter that does not estimate corrections to the Earth's magnetic field.

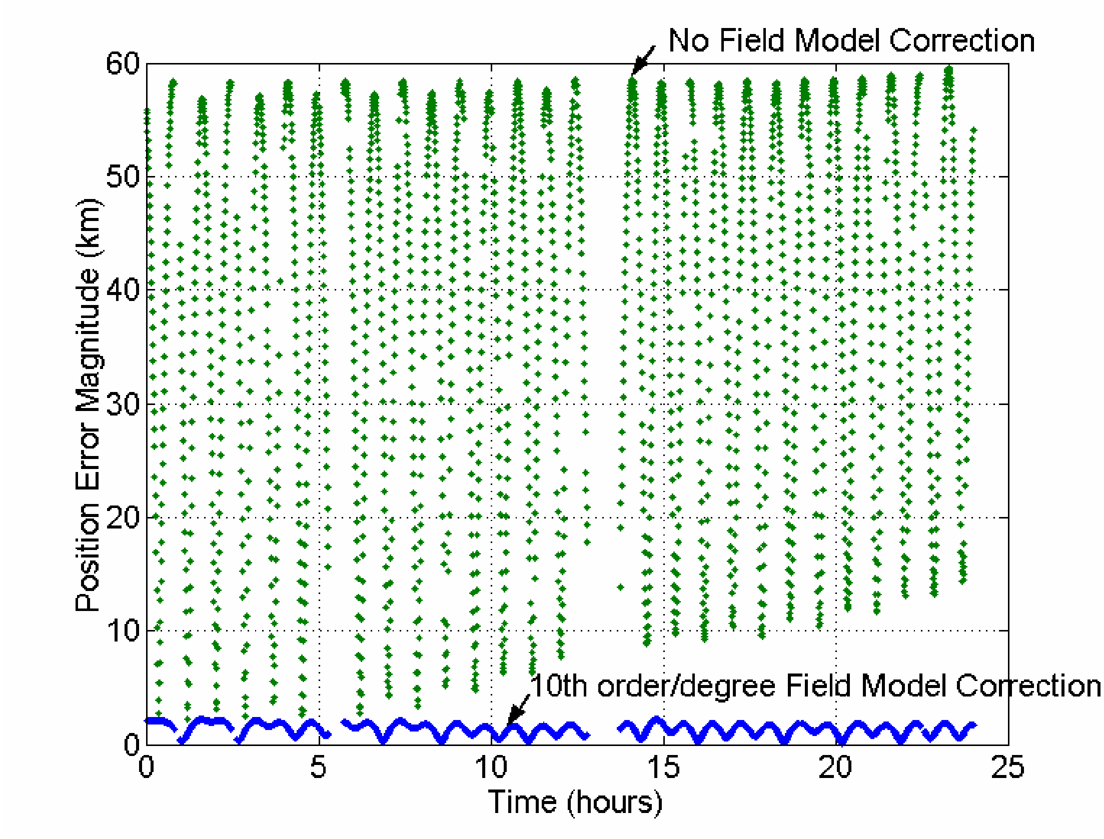

Figure. 2.1. Position error magnitude time histories for Ørsted data set A with two batch filters.

The results of Table 2.5 are less dramatic than the ones of Table 2.4 since a more accurate *a priori* field model is used for the results of Table 2.5. The best position error is 5.36 km with 10th order/degree corrections for data set C and 5.08 km with 4th order/degree corrections for data set D. The results for 3rd or higher order degree corrections remain almost the same for both data sets.

The Ørsted results confirm that the field-model-correcting batch filter can contribute to a substantial improvement in the orbit determination performance when the only available Earth magnetic field model is inaccurate due to the use of predicted secular variations over a long time scale (i.e., 5 years). This conclusion has been further confirmed by a set of erroneous runs that has been conducted in the course of this research. By mistake, Ørsted data set C was originally run using a propagation of the 2000 IGRF field model to 2010. This mistake caused very poor position estimation performance when no field model corrections were estimated or when only low-order corrections were estimated. When high order corrections were estimated, the position estimation errors become very small, comparable to those that have been obtained using the correct model. Thus, the field-model-correcting filter was able to correct the IGRF model dating bug.

Tables 2.4 and 2.5 also show that the maximum position error is consistent with the filter's predicted position accuracy and with the overlap test results. Compare column 2 with column 3 and column 4 with column 5 to see the consistency of the consider-based accuracy predictions with the actual errors. Compare columns 6 and 7 with columns 3 and 5 to see the consistency of overlap results with the actual errors. In a real autonomous application, maximum position error cannot be calculated by comparing estimated position with ground-station tracked position. The results of Tables 2.4 and 2.5 confirm that the combined use of predicted position accuracy from consider-covariance analysis and overlap test results can provide a solid means of estimating position accuracy autonomously and determining a reasonable order/degree to use for the field model corrections.

2.4.4 Tests of the Uniqueness and Reality of the Corrected Field Model

The following questions arise: Are the filter's field model corrections unique, and are they indicative of real variations in the Earth's magnetic field? These questions are answered through three tests of the field-model-correcting filter with Ørsted data set C, which was measured in January 2000. These tests compare various corrected and uncorrected field models. Note that the field model correction terms from the field-model-correcting filter include corrections to the Earth's internal magnetic field and external ring current effects. Therefore, when a corrected field model is compared to an *a priori* propagated IGRF field model, only internal terms are considered. If two different corrected field models are compared, then both internal and external terms are considered.

The first test is to check whether the corrected field is unique, which amounts to an observability test. Two different *a priori* field models, the 1995 IGRF model and the 2000 IGRF model, have been used in the filter, and 10^{th} order/degree corrections are estimated in each case. The *a priori* variance terms for the field model corrections are not included in the filter's cost function in this test because the *a priori* variances incline the filter to generate a corrected field that is close to the *a priori* field, which is not appropriate for an observability test. The rms value of the magnitude of the field vector error between the two resultant corrected field models has been computed along the Ørsted orbit. This value can be viewed as being a norm-type metric of the discrepancy between the two field models. It is 6.7×10^{-5} nT, which is very small. The high level of agreement between the two different corrected field models implies that the corrected field model is unique, and therefore simultaneously observable with orbit. While the small rms error of two resultant corrected fields implies observability, this is only a necessary condition for the corrected field model to capture actual field variations.

The second test checks whether the filter's field model corrections cause the field model to become more accurate. This test compares the magnetic field vector based on the propagated 2000 IGRF field model without corrections with one based on the propagated 1995 IGRF field model. The propagated 2000 IGRF model should be fairly accurate for the January 2000 time frame of data set C; thus, it is used as a sort of "truth" model for comparison purposes. In one comparison, the propagated 1995 IGRF model is uncorrected, and in the other case it is corrected by the filtering of data set C with nominal values of σ_g and σ_h. The rms value of the magnitude of the field vector error between the propagated 1995 IGRF model and the 2000 IGRF model is 95.87 nT along the orbit before corrections get applied to the 1995 model. This rms error decreases as higher and higher orders and degrees of corrections are estimated and applied to the 1995 model. It decreases to 31.72 nT for 10th order/degree corrections. This test shows that the field-model-correcting filter with higher order/degree corrections drives the inaccurately predicted January 2000 field of the propagated 1995 field model toward the more realistic 2000 IGRF field model. It also confirms that low order/degree corrections are not enough to predict the January 2000 field model from the 1995 model; for example, 1st order/degree corrections left a 93.23 nT rms error.

The third test considers whether the simultaneous estimation of orbit has a negative effect on the estimation of field model corrections. In this test, corrections to the 2000 IGRF field model are estimated for Ørsted data set C in two different ways. One way uses the field-model-correcting orbit determination filter. The other way uses the 2nd filter that is described in section II. E, the one that estimates only the corrections, not the orbit. Recall that this filter uses the tracking data to determine instantaneous spacecraft position. The corrected fields that have been estimated by these two techniques have been compared. The rms magnitude of their vector difference is 18.37 nT along the orbit. This small difference indicates that the simultaneous estimation of orbit does not have a large impact on the accuracy of the corrected field model.

The rms errors between the various field models show similar trends in the entire altitude range from 300 km to 900 km. The propagated 1995 IGRF model with estimated 10th order/degree corrections is 3 to 4 times closer to the uncorrected 2000 IGRF model than is the propagated, uncorrected 1995 IGRF model. Thus, the field model corrections from the Ørsted orbit determination filter are applicable to a broad range of altitude beyond that of the Ørsted trajectory.

These tests confirm that the field model corrections from the orbit determination filter are unique and that they provide reasonable estimates of the true magnetic field.

2.4.5 Summary Results

The performance of the field-model-correcting filter for different order/degree corrections is different for the three S/C. The maximum position error decreases a lot for data sets A and B of Ørsted as the order/degree of the field model corrections increases. The results for Ørsted data sets C and D and for both MAGSAT data sets improve modestly for lower order/degree corrections, but then do not vary much as the order/degree of the corrections increases further. They do, however, maintain low position estimation errors for higher order/degree corrections. DE-2 performance varies from Ørsted and MAGSAT and degrades slightly as the size of the order/degree of the corrections increases. Never the less, the position error are reasonably small for all cases that use 6^{th} order/degree or higher field model corrections. They are no larger than 8.2 km, which is sufficiently accurate for a number of missions.

The trends of the maximum position error of the field-model-correcting filter with different order/degree corrections may imply the following. Very high order/degree field model corrections improve the performance of filter significantly for an inaccurate magnetic field model like the one used for data sets A and B of Ørsted (propagation of the 1995 IGRF model to December, 1999). The 6^{th} or higher order/degree field model corrections are enough to assure good performance of the filter. If, however, the *a priori* model is more accurate (like the 2000 IGRF model used in data sets C and D of Ørsted and the 1980 IGRF model used for MAGSAT), then lower order/degree corrections are enough to achieve good accuracy. In this latter case, higher order/degree corrections will produce similar performance as with lower order/degree corrections because the higher order correction terms will not be very significant. In the DE-2 cases, however, the position estimation becomes a little worse with higher order/degree corrections, and the field-model-correcting filter does not necessarily improve position estimation accuracy. This may be due to DE-2's small measurement coverage time of magnetometer data with frequent and long data gaps.

Some facts about the magnetometer data used in this study should be noted. Firstly, all tests are done with magnetometer data on magnetically quiet days. The field-model-correcting filter may not be effective when solar storm activity is high. This system, however, might still be applied during magnetically active days by the propagation of estimated position from the data of magnetically quiet days. It might help to extend the magnetic field model to include the effect of solar winds, though this addition will make for a more complicated system. Secondly, the magnetometer data of the three S/C are very accurate, especially for MAGSAT and Ørsted which have their magnetometers mounted on separate booms. The field-model-correcting filter has not been tested on a mission with a less accurate magnetometer. In general, the magnetometer data should be "clean" with minimum spacecraft-generated magnetic fields. This might require the magnetometer to be mounted on a separate boom, away from main S/C body. It may also be necessary to discard magnetometer data that is taken when the local magnetic field changes abnormally due to interference from specific electric equipment. The failure of the filter with uncalibrated TRMM data also suggests that the magnetometer data should be well-calibrated. Calibration may include alignments, scale factors, biases and Torquer coupling matrices.

2.5 Conclusions

Several magnetometer-based orbit determination batch filters have been developed and tested using real flight data. The basic filter estimates a S/C's orbit along with magnetometer biases, and, optionally, correction terms to a spherical harmonic model of Earth's magnetic field. If field model corrections are estimated, then the filter requires sun-sensor data as well as magnetometer data. The enhancement to these filters as compared to previous work has been to use an improved orbital dynamics model, to consider *a priori* variances of field model corrections, and to allow the use of an *a priori* drag parameter estimate for a higher altitude S/C.

The filter performs well for flight data from the DE-2, MAGSAT, and Ørsted spacecraft. Using only magnetometer data without field model corrections, the maximum position error is 5.29 km and 5.87 km for two DE-2 data sets and 7.02 km and 4.56 km for two MAGSAT data sets. This filter's maximum position error for one Ørsted data set is 59.50 km. This large error is mainly due to errors in the *a priori* Earth magnetic field model. Another Ørsted data set yields a 9.09 km maximum position error largely because it was given a better *a priori* field model.

The performance of the batch filter with Earth's magnetic field model corrections varies slightly among the different S/C. This filter augments the magnetometer data with sun-sensor data in order to make the field model corrections observable. The addition of field model corrections does not make significant improvements in the case of DE-2, and the addition of high-order corrections slightly degrades the DE-2 position estimation accuracy. The performance with MAGSAT data improves by as much as a factor of 2 with lower order/degree corrections. Higher-order corrections also maintain low position errors for MAGSAT. Ørsted data shows the most benefit from the estimation of field model corrections. In one case, the peak position estimation errors are decreased from 59.50 km when no field model corrections are estimated down to 2.19 km when corrections up to 10th degree/order are estimated. In a case with a better *a priori* Earth magnetic field model, the improvements due to the estimation of field model corrections are less dramatic. The general trends of the MAGSAT and Ørsted results suggest that it would be wise to include field model corrections up to order and degree 6 if there is any significant uncertainty in the *a priori* Earth's magnetic field. The DE-2 results imply that the estimation of field model corrections will work less well if there are long gaps in the magnetometer or sun sensor data.

The predicted accuracy from the consider-covariance analysis and orbit overlap tests is consistent with the actual maximum position errors. Therefore, these metrics can be used to check the orbit solution accuracy autonomously and to determine a reasonable maximum order/degree of the field model corrections.

The uniqueness and realism of the corrected magnetic field model has been tested using an Ørsted data set measured on January 2000. The corrected field model is unique for 10th order/degree field model corrections regardless of the *a priori* field model. Estimation of corrections to a propagated 1995 IGRF field model using data from January 2000 reduces the rms value of the magnitude of field vector differences from the 2000 IGRF field model. The rms difference is 95.87 nT before the field corrections, but it is only 31.72 nT after one applies the filter's estimated field model corrections. This shows that the field-model-correcting orbit determination filter produces realistic field model perturbations.

2.6 Acknowledgements

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