BLIND CHANNEL IDENTIFICATION: METHODS FOR ESTIMATING EFFECTIVE CHANNEL ORDER

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BLIND CHANNEL IDENTIFICATION: METHODS FOR ESTIMATING EFFECTIVE CHANNEL ORDER

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ABSTRACT

The increasing prevalence of single input multiple output systems has created increased interest in equalization across multiple channels. An important step to creating an appropriate equalizer is to have a good estimate of the effective channel order. Traditionally, the selection criteria have used eigenvalues or singular values for estimation purposes. Using these values ignores important information that can be obtained from the nullspace of the least squares method's matrix. The nullspace has a structured basis that can be utilized to quantify the quality of channel coefficient estimates. These residual error measures cluster in the nullspace and spread in the signal space, when compared to the singular values. Those properties allow the residual errors to be substituted into information theoretic criteria for improved performance. Additionally, a simple exponential curve fit is used to further improve the estimate of the effective channel order. The new criterion outperforms other known criteria in a wide variety of scenarios at both high and low SNR.

BIOGRAPHICAL SKETCH

Joshua Daniel Gabet was born to Terry and Eva Gabet in Akron, Ohio, USA. From an early age, he had an aptitude for math and science. He also discovered that he not only loved to learn, but he also enjoyed helping others. His desire to help others started as early as kindergarten, for he would finish his work early and then help others. He continued this through high school, tutoring his peers in mathematics. High school engineering competitions taught him how to apply the techniques to real world problems. This led Joshua to nearby Case Western Reserve University, where he settled his focus on electrical engineering. There he was able to help others through a number of positions, including resident assistant, peer tutor, and Tau Beta Pi - member, Bookswap Chair, and leader. His studies were focused on signal processing. He continued his studies at Cornell University for a Master's degree, further refining his signal processing studies in the areas of blind channel identification and matrix computation. Also at Cornell, Joshua was a regular teaching assistant, a job in which he enjoyed and excelled.

Joshua is married to his closest friend, Mariel. They have two loving cats, Suzy and Pepper.

To my wife Mariel.

It is you who made my time in Ithaca enjoyable and meaningful.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

Intersymbol interference describes the corruption of a transmitted signal by delayed copies in communication channels due to the multipath phenomenon. The simplest intersymbol interference (ISI) model is a digital signal sent through a discrete channel. This model directly describes the effects of ISI by summing scaled, time shifted versions of an input signal. The channel in the model is relatively simple as it combines several aspects of a communications system, including digital-to-analog conversion, the physical layer, and analog-to-digital conversion, [1]. The number of interfering symbols with significant magnitude is assumed to be finite. This results in a finite channel order, called the effective channel order. A common goal is to recover the original input signal, which has been corrupted by the channel and a noise source. This symbol recovery occurs through a process call channel equalization. A single input multiple output (SIMO) system provides a framework for identifying the channel and can be used to create an equalizer.

Blind channel identification is channel identification with little or no prior knowledge of the source and channel parameters. This means there is no training signal for the receiver to use to identify the channel. This increases the complexity of the identification process. In order to fully identify a channel, both the channel coefficients must be identified as well as the effective channel order.

There is a vast amount of literature that addresses effective channel order estimation. Many works use eigenvalues to solve for the channel order, see for example [2–4]. According to [5,6], undermodeling or overmodeling the channel order can severely diminish equalizer performance. This issue is exacerbated when the noise eigenvalues are not sufficiently clustered [7]. In addition, using eigenvalues ignores

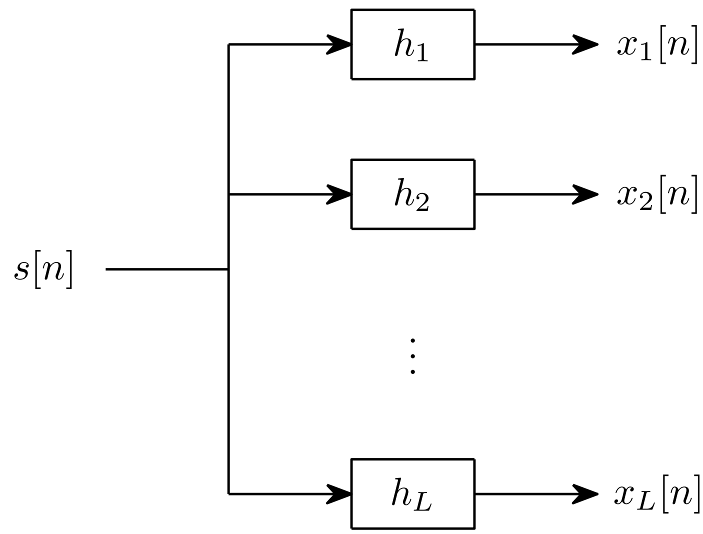

Figure 1.1: System with a single input and multiple outputs (SIMO).

extra information that can be garnered from the problem's structure. Thus, replacement of the eigenvalue measure by a residual error measure, which accounts for the problem's structure, is proposed. These residuals tend to closely cluster in the nullspace and separate well in the signal space. This makes estimation of the channel order more accurate and in turn improves the equalizer performance [8].

For data transmissions there are several possibilities to devise a SIMO channel. For example, several receivers can be used, see Figure 1.1. Each of the receivers would then forward channel samples onto a single node to be processed. For another example, several logical channels are generated by oversampling a single channel, provided there is adequate bandwidth excess. The received signal is divided into individual channels by time shifting and down sampling, [9].

All SIMO setups have the same resulting mathematical model. For the case where the channel does not vary with time and is linear, the model describing the output of L channels $x_i(n)$ from Figure 1.1 is as follows:

$$x_i(n) = \sum_{j=0}^{M} h_i(j)s(n-j) + w_i(n), \quad i = 1, \ldots, L. \tag{1.1}$$

Each channel has effective channel order M with coefficients $h_i(j)$, which are assumed to be slowly varying. The channel order is equivalent to the number of interfering symbols. The channel is further corrupted by noise $w_i(n)$. All channel and noise variables are assumed to be independent.

CHAPTER 2

PROBLEM STATEMENT

This chapter relies on the method described in [8], called the least squares method (LSM). The LSM exploits the commutative properties of convolution and provides a starting point for estimating the channel.

In the noiseless case, for each pair (i, j) of channels the following relation holds:

$$x_i(n) * h_j(n) = x_j(n) * h_i(n),$$

where $*$ denotes convolution.

If the channels are assumed to be of order m the problem can be recast into the matrix form:

$$\begin{bmatrix} X_i^{(m)} & -X_j^{(m)} \end{bmatrix} \begin{bmatrix} h_j^{(m)} \\ h_i^{(m)} \end{bmatrix} = 0, \tag{2.1}$$

where $h_l^{(m)} \equiv \begin{bmatrix} h_l(m) & \cdots & h_l(0) \end{bmatrix}^T$, $l = 1, \ldots, L$, and

$$X_l^{(m)} = \begin{bmatrix} x_l(0) & \cdots & x_l(m) \\ \vdots & \ddots & \vdots \\ x_l(N-1-m) & \cdots & x_l(N-1) \end{bmatrix}.$$

The l^{th} channel coefficients form the length $(m+1)$ vector $h_l^{(m)}$, while $X_l^{(m)}$ is the $(N-m) \times (m+1)$ Hankel matrix constructed from N received samples.

Instead of considering each channel pair independently, a single linear system involving all channels can be created. The solution of this combined system will provide the solution for all channels simultaneously, while relaxing the assumptions necessary to solve each individual system. This occurs since the number of linear equations grows quadratically with respect to the number of channels, while the number of parameters increases linearly.

Let $h^{(m)}$ be the length $L(m+1)$ vector that encompasses all the channel coefficients,

$$h^{(m)} \equiv \begin{bmatrix} h_1^{(m)T} & \cdots & h_L^{(m)T} \end{bmatrix}^T \tag{2.2}$$

and $X^{(m)}$ be the $\frac{1}{2}(L^2-L)(N-m) \times L(m+1)$ matrix,

$$X^{(m)} \equiv \begin{bmatrix} X_2^{(m)} & -X_1^{(m)} & 0 & \cdots & 0 & 0 \\ X_3^{(m)} & 0 & -X_1^{(m)} & \ddots & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \ddots \\ X_L^{(m)} & 0 & 0 & \cdots & 0 & -X_1^{(m)} \\ 0 & X_3^{(m)} & -X_2^{(m)} & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \ddots \\ 0 & 0 & 0 & \ddots & X_L^{(m)} & -X_{L-1}^{(m)} \end{bmatrix}.$$

The block rows of the matrix $X^{(m)}$ correspond to the individual systems (2.1). The combined system solution can be obtained by solving

$$X^{(m)}h^{(m)} = 0.$$

If $m = M$ and under mild assumptions, the nullspace of $X^{(m)}$ has dimension one, which allows the recovery of all channel coefficients in (2.2). A nontrivial $h^{(m)}$ is obtained by imposing the constraint $\left\|h^{(m)}\right\|_2 = 1$. This still results in a non-unique solution due to phase ambiguity.

Now assume the channel order is not known, but the maximum possible order $\bar{M}$ is. In this case $X^{(\bar{M})}$ has rank $(L-1)(\bar{M}+1)+M$. Thus the dimension of the nullspace of $X^{\bar{M}}$ is $\bar{M} - M + 1$. A simple basis for the nullspace can be created, by appropriately zero padding the $h^{(m)}$ vector [10].

The basis for the nullspace of $X^{(\bar{M})}$ can be described by the columns of the $L(\bar{M}+1) \times (\bar{M}-m+1)$ matrix $\mathcal{H}^{(\bar{M})}(h^{(m)})$,

$$\mathcal{H}^{(\bar{M})}(h^{(m)}) = \left[\left(H^{(\bar{M})}(h_1^{(m)})\right)^T \cdots \left(H^{(\bar{M})}(h_L^{(m)})\right)^T \right]^T, \tag{2.3}$$

where $H^{(\bar{M})}(h_l^{(m)})$ is the $(\bar{M}+1) \times (\bar{M}-m+1)$ convolution matrix of h_l,

$$H^{(\bar{M})}(h_l^{(m)}) = \begin{bmatrix} h_l(m) & 0 & \cdots & 0 \\ \vdots & h_l(m) & \ddots & \vdots \\ h_l(0) & \vdots & \ddots & 0 \\ 0 & h_l(0) & \ddots & h_l(m) \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \ddots & 0 & h_l(0) \end{bmatrix}.$$

This results in the system:

$$X^{(\bar{M})}\mathcal{H}^{(\bar{M})}(h^{(M)}) = 0. \qquad (2.4)$$

The structure (2.3) of the basis for the nullspace will be exploited in Chapter 4 to generate a residual error associated with channel coefficient estimates.

CHAPTER 3

REVIEW OF OTHER METHODS

This chapter is dedicated to a review of pertinent methods for estimating both the channel coefficients and the effective channel order for a SIMO system as described in 1.1. The first section will describe methods for estimating the channel coefficients. These estimations will play an important role in methods described in later chapters. The second section will describe major effective order estimation methods, which will be utilized for comparison purposes. The methods in this chapter require few restrictions on impulse responses, thus they are suitable for a wide variety of applications and possible systems.

3.1 Methods for Estimating Channel Coefficients

In what follows, first, it is assumed that the channel coefficients are identifiable and that there is sufficient information gathered to do so. Second, it is assumed that effective channel order is known.

3.1.1 Subspace Method

One successful method of estimating channel coefficients is the subspace method (SSM). This method was proposed by Moulines in [9]. It utilizes knowledge of the noise space and signal space of the sample autocorrelation matrix $R^{(W)}$,

$$R^{(W)} = \begin{bmatrix} X_1^{(W)} & \cdots & X_L^{(W)} \end{bmatrix}^H \begin{bmatrix} X_1^{(W)} & \cdots & X_L^{(W)} \end{bmatrix}.$$

Altering the windows size W allows for some smoothing effects of the results [11]. In order to ensure the SSM can compute the channel coefficients, the window size must be greater than or equal to the effective channel order, $W \geq M$. Thus, the windows size will always be set to the maximum possible channel order, $W = \bar{M}$. The eigenvectors associated with either the signal subspace or the noise subspace are recast to form a subspace matrix. Then depending on which subspace was chosen the eigenvector associated with the minimum/maximum eigenvalue of the subspace matrix comprises the channel coefficient estimates. In order to create this subspace matrix, the effective channel order M must be known. Numerous analysis of this method have shown that is robust to a wide variety of channel disturbances [12]. Its results generally are good estimates of the channel coefficients.

3.1.2 Least Squares Method

Another common method of estimating channel coefficients is the least squares method (LSM), described by Xu in [8]. This method like the subspace method requires knowledge of the effective channel order M. The channel coefficient estimates of this method are simply found from the eigenvector associated with the smallest eigenvalue of $\Phi^{(M)}$,

$$\Phi^{(M)} = \left(X^{(M)}\right)^H X^{(M)}.$$

This method is called least squares, because is finds the solution with the least sum-squared difference between the outputs of the channel pairs. Thus, the algorithm is also called a subchannel matching algorithm. Note that the solution to this least squares problem is associated with either the smallest singular value of $X^{(M)}$ or the smallest eigenvalue of $\Phi^{(M)}$. It is often convention to refer to the singular values of $X^{(\bar{M})}$ instead of the eigenvalues of $\Phi = \Phi^{(\bar{M})}$. This reduces the confusion with the eigenvalues of R. In practice Φ would be utilized, because it, like R, only relies on second order statistics.

3.1.3 Other Methods

Both the SSM and the LSM only require second order statistics (SOS) for computation. There are numerous other methods that require SOS or higher order statistics. Here SSM and LSM are described to provide the framework of a channel order estimation method. Numerous methods like the SSM require knowledge of the effective channel order to be able to separate the subspaces. These methods have varying dependencies on the effective channel order. However, they always perform well if it is correctly estimated. The SSM was chosen over other such methods in order to guarantee that the result is truly associated with effective channel order being assessed. Also, the SSM does not have issues with small leading channel coefficients [12].

An early method proposed by Tong, Xu, and Kailath [13] requires SOS and exploits cyclostationarity [12]. In order to avoid poor results when the channel order is incorrectly estimated, there are several linear prediction methods, including least square smoothing [14], multi-step linear prediction [15], and constrained minimum output energy [16]. These methods generally do well, but are not without their own flaws, but suffer from the aforementioned issue with leading coefficients. Another notable algorithm is Gazzah, Regalia, Delmas, and Abed-Meraim' method [11], which attempts to be robust to effective order overestimation. However, it performs generally poorly under most circumstances. There are also methods that utilized partial knowledge of the channel. This information can be used to augment just about any method including the SSM and LSM [17, 18]. These were not utilized in order to keep the algorithm generic, so it can be utilized when that information is not available. Many of these methods may be able to further improve performance of that algorithms presented in later chapters. However, they are not necessary for successful applications.

3.2 Methods for Estimating Effective Channel Order

Many of the channel coefficient estimators mentioned in the previous section require knowledge of the effective channel order. There are many existing estimators for the effective channel order. Here the GAP criterion is described as a motivational example for later chapters. The widely used information theoretic criteria are described. Also, for comparison purposes the canonical angle and combined identification/equalization methods are also described. In general, most of these effective channel order estimators are nothing more than a specialized rank estimation of either R or Φ.

3.2.1 Ratio Test

One early, simple test to find the boundary between the signal subspace and the noise subspace is a ratio test (GAP). Let $R = R^{(\bar{M})}$ and λ_i are its decreasingly ordered eigenvalues,

$$\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_p,$$

and $p = L(\bar{M}+1)$. With no noise present and under mild conditions, the eigenvalues of the noise subspace are zero and the eigenvalues of the signal subspace are non-zero. Thus, the ratio between the smallest signal subspace eigenvalue and the largest noise subspace eigenvalue is infinity. Further, in the high SNR regime when the noise only has small magnitude, there is still a large gap or ratio between consecutive eigenvalues at the boundary between subspaces. This leads to the simple

GAP test which estimates the effective channel order M as:

$$\hat{M}_{GAP(R)} = \hat{k}_{GAP(R)} - W,$$

where

$$\hat{k}_{GAP(R)} = \underset{k \in \{1,\ldots,p-1\}}{\arg\max} \frac{\lambda_k}{\lambda_{k+1}}.$$

Recall that the SSM's window size is fixed to $W = \bar{M}$ for this work.

Similarly, the test can be extended for finding the nullspace of the $\Phi = \Phi^{(\bar{M})}$ matrix, which uses the same assumption for maximum possible channel order as the autocorrelation matrix R, where $W = \bar{M}$. Often the singular values of $X^{(\bar{M})}$ are used instead of the eigenvalues of Φ. Thus, the estimate of the effective channel order becomes:

$$\hat{M}_{GAP(\Phi)} = p - 1 - \hat{k}_{GAP(\Phi)} + \bar{M}, \tag{3.1}$$

where

$$\hat{k}_{GAP(\Phi)} = \underset{k \in \{1,\ldots,p-1\}}{\arg\max} \frac{\sigma_k^2}{\sigma_{k+1}^2}$$

and σ_i is the ordered singular value of $X^{(\bar{M})}$.

The GAP test only relies on consecutive eigenvalues, so it does not provide a smooth shape with a clear maximum when the SNR is decreased. The ratio jumps around as the value of k is varied under noisy conditions. With lower SNR, the non-zero values of the noise subspace eigenvalues are mixed with the signal subspace ones. Also, due to a finite number of samples, the values in the noise subspace are not all equal. This causes the ratios in the nullspace to not be unity, as they would be in the expectation.

3.2.2 Information Theoretic Criteria

In order to analytically overcome the issues observed with the GAP test the Akaike information criterion (AIC) was developed [2] by Akaike. This method was produced by using information theoretic techniques to solve for a model that best fits the N observed data points [4]. The general form for computing the AIC is

$$\hat{k}_{AIC(R)} = \underset{k\in\{0,\ldots,p-1\}}{\arg\min} -2(p-k)(N-\bar{M})\ln\frac{\mathcal{G}(\lambda_{k+1},\ldots,\lambda_p)}{\mathcal{A}(\lambda_{k+1},\ldots,\lambda_p)} + 2k(2p-k),$$

where $\mathcal{G}$ and $\mathcal{A}$ are the geometric and arithmetic means, respectively. Following Akaike's work, independently Swartz [19] and Rissanen [3] developed the minimum description length (MDL) [4]. The result for the MDL is strikingly similar to the AIC and has the form

$$\hat{k}_{MDL(R)} = \underset{k\in\{0,\ldots,p-1\}}{\arg\min} -(p-k)(N-\bar{M})\ln\frac{\mathcal{G}(\lambda_{k+1},\ldots,\lambda_p)}{\mathcal{A}(\lambda_{k+1},\ldots,\lambda_p)} + \frac{1}{2}k(2p-k)\ln(N-\bar{M}).$$

The AIC and MDL estimate the effective channel order for both R and Φ matrices [12]. The extension and computation of the estimated effective channel order is done in the same manner as for the GAP.

These methods, while widely used, are flawed. They both make assumptions about the noise and data vectors that are too strong and thus lead to non-robust criteria. They are the optimal solution when the data vectors are i.i.d. zero-mean Gaussian and the noise is uncorrelated to the data and is white Gaussian [20]. The Hankel/Toeplitz structure of the matrices ensure the data vectors are correlated. Also, the noise is clearly colored due to the existence of leading or trailing coefficients that are too small to contribute to the effective channel order [20]. Since these assumptions do not hold, the information theoretic criteria are not robust. This is often shown by their tendency to overestimate the effective channel order under high SNR [7], due to non-clustered eigenvalues in the noise subspace.

3.2.3 Canonical Angle

The lack of a robust effective channel order estimator lead to the creation of a newer criteria, which estimates the canonical angle (CA) between subspaces [20]. This method does create a more robust criterion, however, its performance still lacks in many instances. This will be demonstrated in the simulation chapter. The criteria for the canonical angle is as follows,

$$\hat{k}_{CA(R)} = \underset{k \in \{1,\ldots,p-1\}}{\arg\min} \begin{cases} \frac{\lambda_{k+1}}{\lambda_k - 2\lambda_{k+1}}, & \lambda_{k+1} \leq \frac{\lambda_k}{3} \\ 1 & otherwise \end{cases} .$$

3.2.4 Combined Identification/Equalization

A more recent effective channel order criteria is the combined identification/equalization (CIE) [21]. This method linearly combines a normalized sum squared error from the identification using the LSM with a normalized sum squared error from the equalization using methods presented in [22]. This method shows some promise as a way to combine two techniques and fuse the results. However, numerical simulations presented in Chapter 5 indicate that this method suffers from some weaknesses. In reality, the main issue is the simple sum of the two normalized, but unique, measure is not an optimal solution. This can be seen in Figure 3.1, where the structure of the signal subspace of the identification measure has a sharp drop for $m = 3$. This leads the CIE to choose $\hat{M}_{CIE} = 3$. However, both the identification and equalization measures individually show that the effec-

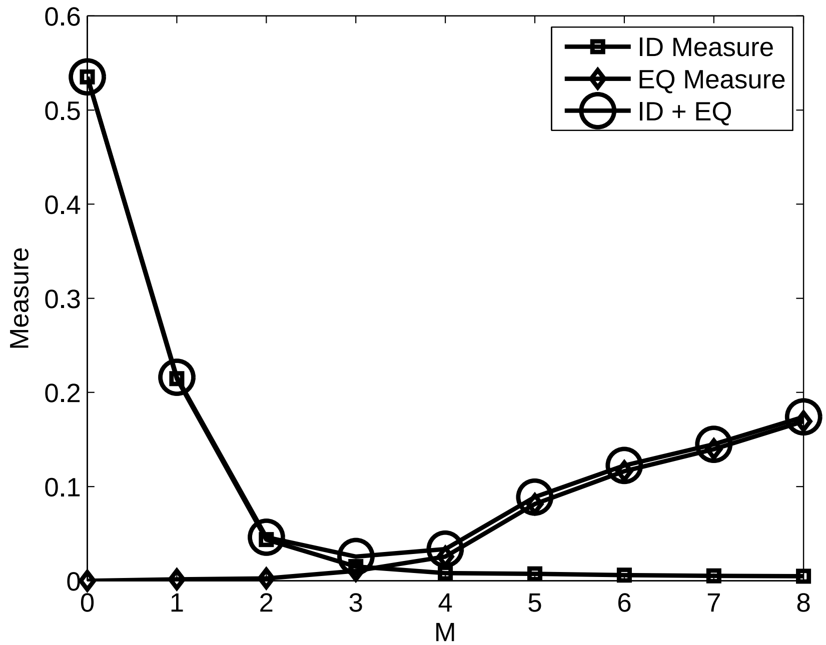

Figure 3.1: CIE identification and equalization measures for a single trial of Simulation 5.1 with SNR=20 dB and $M = 4$.

tive channel order should be 4. Hence, the simple sum combination is not optimal and leads to poor performance in the simulation.

CHAPTER 4

SOLUTION TO ESTIMATING THE EFFECTIVE CHANNEL ORDER[1]

Equalization, or the process of removing the ISI of the channel, typically maximizes performance when the computed order is equal to the effective channel order. With the order M unknown, estimates of the channel coefficients $\hat{h}^{(m)}$ for all possible channel orders $m = 0, \ldots, \bar{M}$ are computed. Next errors associated with each of the channel estimates are computed. From these errors, the channel order is determined. This process is shown in Figure 4.1.

4.1 Generating Channel Estimates

Since a basis for identifying channels was first outlined in [13] several competing methods have been proposed. One such method is the subspace method (SSM) developed in [9]. The SSM is used to compute channel estimates $\hat{h}^{(m)}$ for $m = 0, \ldots, \bar{M}$, while holding the window length steady at $\bar{M}+1$. The subspace method was chosen to estimate the channel coefficients since it performs well and does not rely on the source symbols being independently distributed [12]. Furthermore,

[1]Portions reproduced with permission from *IEEE Transactions on Signal Processing*, accepted for publication May 2010. ©2010 IEEE. Reprinted, with permission, from *IEEE Transactions on Signal Processing*, Effective channel order estimation based on nullspace structure and exponential fit, J. D. Gabet and A. W. Bojanczyk

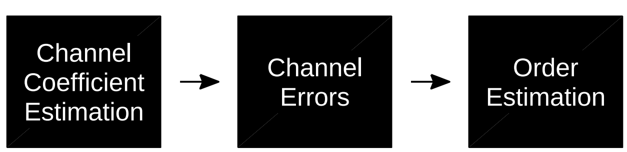

Figure 4.1: Process for estimating effective channel order.

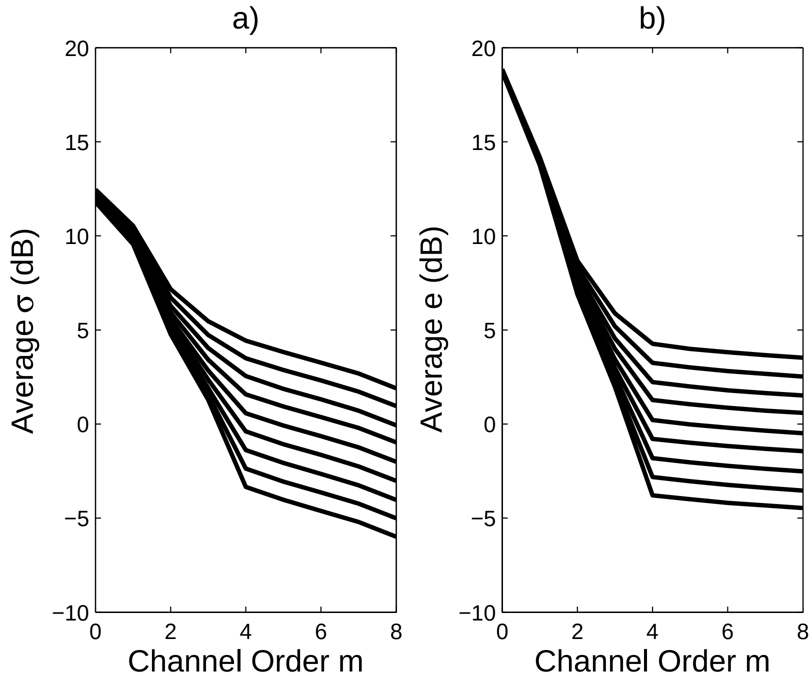

Figure 4.2: Comparison of singular values and errors. a) Average of singular values $\sigma_{p-\bar{M}+m}$. b) Average of $e(m)$ for Simulation 5.1, $M = 4$. For both a) and b) the SNR is swept from $16dB$ (top curve) to $24dB$ (bottom curve).

the residual errors associated with the SSM channel estimates tend to cluster the nullspace errors and spread the signal space errors. This clustering effect can be seen in Figure 4.2 as the flattening of the error values in the nullspace. This is important to the success of the algorithm, and does not occur when the LSM channel estimates are used. The clustering does not occur due to a strong correlation between the errors and the solution to the LSM for channel coefficients of all channel orders. This arises from the similar form of the matrices and the minimization problem that occurs. Note that for the SSM's coefficients to help smooth the residual errors, the number of channels L must be greater than 2. This is due to the solution to the LSM and SSM being equivalent for $L = 2$.

4.2 Generating the Errors

For a noisy channel, the singular values associated with the nullspace are no longer zero. Thus the traditional tactic of determining the rank of $X^{(\bar{M})}$ from its singular values is non-trivial [12]. It was noted in Chapter 3 that the AIC and MDL were designed to address this problem, but tend to overestimate the channel order when the nullspace eigenvalues are not clustered [7]. This lack of clustering can cause arbitrarily large ratios between eigenvalues in the nullspace, hence, creating false minima in the criteria.

In order to avoid order overestimation, the error measure was designed an error measure to have a higher level of clustering in the nullspace. First, each singular value is interpreted as a measure of the corresponding singular vector's the ability to null $X^{(\bar{M})}$. The singular value decomposition enforces that these vectors are orthogonal. That restriction ignores the structure of the basis derived earlier in equation (2.4). Thus, an error measure $e(m)$ that accounts for the structured basis $\mathcal{H}^{(\bar{M})}(\hat{h}^{(m)})$ is proposed as

$$e(m) = \frac{1}{\bar{M} - m + 1} \left\| X^{(\bar{M})} \mathcal{H}^{(\bar{M})}(\hat{h}^{(m)}) \right\|_F^2 \quad , \tag{4.1}$$

where $m = 0, \ldots, \bar{M}$. Working with the structured basis takes into account the characteristics of the SIMO channel model. The constraint imposed by the structured basis causes the nullspace errors to closely cluster and the signal space errors to separate, which improves channel order selection.

4.3 Estimating the Channel Order

There are many possible heuristics to estimate the channel order from the newly devised residual errors. Here two possibilities are described. The first substitutes the errors for the singular values in the information theoretic criteria. The residual errors were generated in order to alleviate the issue of non clustered values in the nullspace, which affected the information theoretic criteria. Thus, the substitution helps validate the success of the clustering solution. The second utilizes the idea behind the GAP test, but smooths its results in order to come up with a new criterion.

4.3.1 Modified Information Theoretic Criteria

Due to information theoretic criteria's wide use [20], the AIC and MDL are modified to utilize the errors. The modification is simple, just replace $\sigma^2_{p-\bar{M}+m}$ with $e(m)$. This substitution is rationalized by an assumption made in the derivation of the information theoretic criteria. That assumption is that the expectation singular values associated with the nullspace are equal to the square root of the noise variance. The method described here with the errors can be treated as a measure of the noise variance. If the channel coefficient estimate were perfect and the channel finite, then they would match to a scale factor. Also, it is claimed that the errors improve clustering in the nullspace. If true, then the performance of the AIC and MDL at high SNR should be improved. This direct substitution does break the strict analytical result of the AIC and MDL. However, note that both solutions are incredibly similar and only differ by a scale factor and offset. It is possible that a new derivation with the errors in mind could produce further improvements.

4.3.2 Exponential Fit Curve

Figure 4.2 illustrates the advantage of using the errors $e(m)$ over the singular values, as the dimension of the nullspace can be more easily identified for lower SNR values. Motivated by the graphs in Figure 4.2 and the GAP, a simple search for the nullity was designed by fitting a exponential curve between $e(0)$ and $e(\bar{M})$. The GAP compares two consecutive singular values, which fails in the low SNR case due to the noise power nearing the power of the smallest singular values of the signal space. Instead, the exponential fit curve (EFC) utilizes a common decay to eliminate the artificially large or small ratios that can occur in the GAP. For channel order m, the value of the curve is

$$f(m) = e(0)\left(\frac{e(0)}{e(\bar{M})}\right)^{-\frac{m}{\bar{M}}}.$$

Generally, to choose the channel order, the prevailing "bend" in $e(m)$ must be found. This is accomplished by maximizing the ratio $g(m) = \frac{f(m)}{e(m)}$. Thus the new criterion for selecting channel order $\hat{M}_{EFC}$ is

$$\hat{M}_{EFC} = \underset{m\in\{1,\ldots,\bar{M}\}}{\arg\max}\; g(m). \tag{4.2}$$

One motivation for the EFC is to overcome the deficiencies of the GAP criteria, which uses the instantaneous exponential rate of decay between successive errors. Instead of using instantaneous rates, EFC uses $f(m)$ which has a constant exponential rate of decay equal to average decay needed to go from $e(0)$ to $e(\bar{M})$. Comparing $e(m)$ to $f(m)$ instead of $e(m-1)$ smooths the effects of noise fluctu-

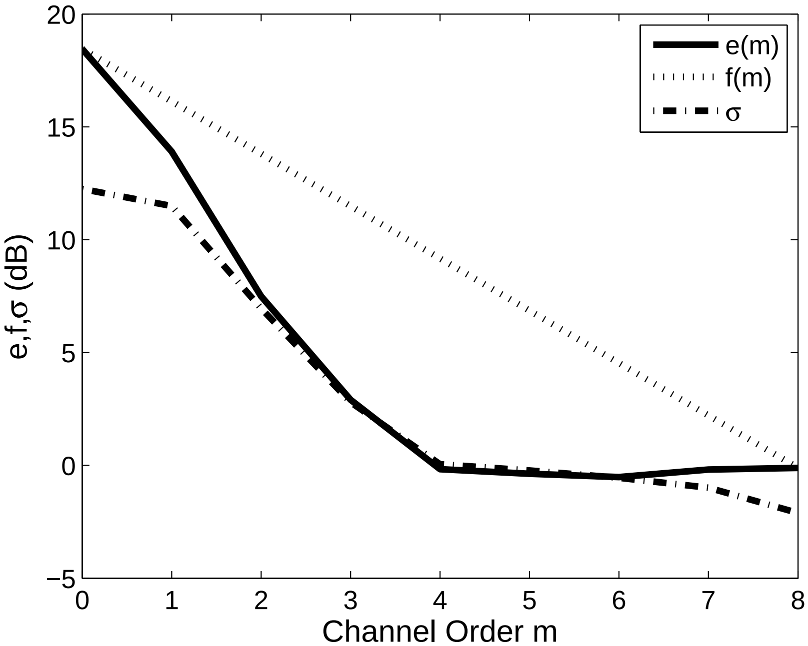

Figure 4.3: $e(m)$, $f(m)$, and $\sigma_{p-\bar{M}+m}$ for a single trial of Simulation 5.1 with SNR=20 dB and $M = 4$.

Table 4.1: Computational Costs of Method Steps

	Step Description	Comp. Cost
1.	Compute channel coefficients $\hat{h}^{(m)}$ for $m = 0, \ldots, \bar{M}$ using the subspace method, while holding the window equal to $\bar{M}$.	$\mathcal{O}(\bar{M}^3 L^3)$ + $\mathcal{O}(N\bar{M}L^2)$
2.	Compute e(m) for all $m = 0, \ldots, \bar{M}$.	$\mathcal{O}(\bar{M}^3 L^2)$
3.	Compute f(m) for all $m = 1, \ldots, \bar{M}$.	$\mathcal{O}(\bar{M})$
4.	Select the channel order $\hat{M}_{EFC}$.	$\mathcal{O}(\bar{M})$

ations, which makes determination of the channel order more robust, see Figure 4.3.

Due to division by 0, the EFC does not work in the noiseless case; however, in practice the noiseless case does not exist. If the smallest error is extremely small, then computational issues can be alleviated by adding a small regularization term. Despite the lack of rigorous analytical derivation, the simulation results shown in the next chapter indicate that the new criterion outperforms existing methods for low SNR levels. Thus, it provides a reference point for plausible performance of an analytically derived solution.

Major steps of the proposed method and the major computational costs are listed in Table 4.1.

CHAPTER 5

SIMULATIONS

In this chapter the criteria's performance was tested under multiple scenarios of 4-QAM systems. The received signals were corrupted by zero mean circularly symmetric additive white Gaussian noise $w_i(n)$ with variance γ^2. The SNR was calculated by the following formula,

$$SNR = 20\log_{10}\left(\frac{\|h^{(M)}\|_2}{\sqrt{L}\gamma}\right).$$

Each data point in the figures represents the results of 1000 Monte Carlo simulations. The performance measures used were the average μ of the estimated channel order and the mean squared error MSE_R of the approximated signal. See Table 5.1 for a list of simulation parameters.

Typically, the EFC will first be compared to the AIC(R), the MDL(R), the CA(R), and the CIE from [21]. The AIC(R), MDL(R), and CA(R) utilize the autocorrelation matrix R to select the channel order. The CIE method combines measures of identification and equalization for channel order selection. This criterion is computationally expensive and requires $\mathcal{O}(\bar{M}^4(L^3 + \bar{M}^3) + N\bar{M}L^2$ operations. The CIE requires solving for equalizers for all possible m. The equalizers provided by the CIE are utilized for signal recovery in the simulations. Thus the only difference between methods is the channel order selection criteria.

Next, the EFC is compared to the AIC(Φ), MDL(Φ), AIC(e), and MDL(e). The AIC(Φ) and MDL(Φ) utilize the eigenvalues of Φ in order to estimate the effective channel order. The AIC(e) and MDL(e) replaces the eigenvalues with the errors. This is done to gauge the success of clustering the nullspace errors to provide more reliable effective channel order estimates. Like the first comparisons,

Table 5.1: List of Simulation Parameters.

Simulation	L	M	Channel Set	N	$\bar{M}$	SNR
5.1	4	4	1	48	8	–
5.2	4	4	1	48	8	32
5.3	3	5	2	100	9	–
5.4	3	5	2	100	9	–
5.5	3	5	2	100	9	–
5.6	4	7	random	64	15	–

Table 5.2: Coefficients for Signal Set 1

h_1	h_2	h_3	h_4
$-0.171 + 0.061j$	$-0.087 - 0.054j$	$0.136 - 0.19j$	$-0.049 + 0.161j$
1	$0.189 - 0.208j$	$-0.284 - 0.524j$	$0.285 + 0.309j$
$-0.556 + 0.587j$	$0.921 - 0.194j$	1	$0.873 + 0.145j$
$0.482 - 0.569j$	1	$-0.199 + 0.918j$	1
$-0.049 + 0.359j$	$0.443 - 0.0364j$	$-0.211 - 0.322j$	$0.417 + 0.03j$

the equalizer used is the one created by the CIE method.

For comparing performance of an approximated signal, the mean squared error MSE_R is computed. MSE_R is defined as

$$MSE_R = 10\log_{10}\left(1 - \left|\frac{\tilde{s}^H s}{\|\tilde{s}\|_2\|s\|_2}\right|^2\right), \tag{5.1}$$

where s and $\tilde{s}$ represent 100 generated symbols and 100 recovered symbols, respectively. The measure (5.1) is similar to the one considered in [21], but was taken from [23]. This method measures how closely the output of the equalizer matches the sent symbols. It was chosen due to the likelihood of correctly choosing the transmitted symbol if the approximated one is near it.

5.1 Varying SNR for Channel Set 1

For Simulations 5.1 and 5.2 the signal was passed through the $L = 4$ channels of Channel Set 1 of order $M = 4$ from [9], see Table 5.2. The maximum possible

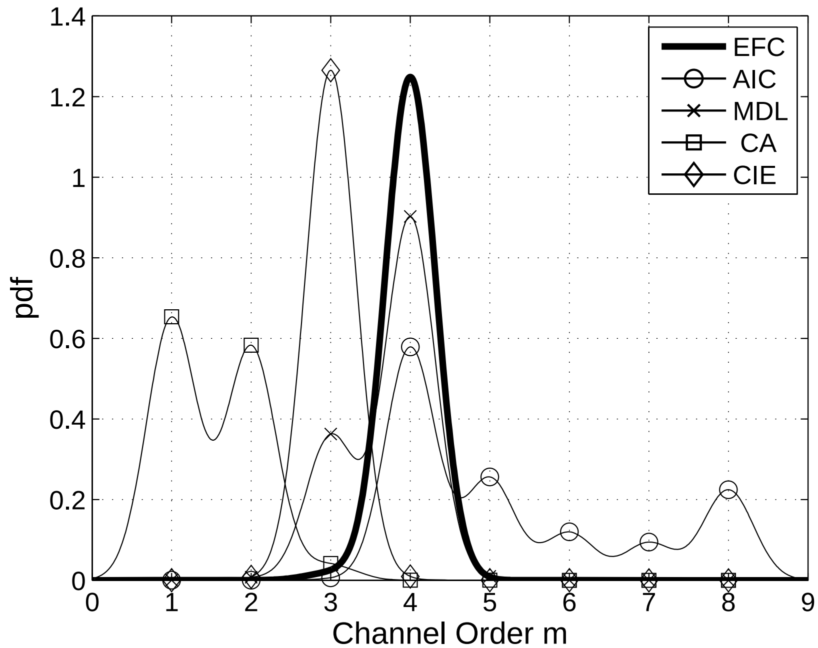

Figure 5.1: pdf of channel selection (using Parzen method) for Simulation 5.1 when the SNR = 20.

order was set at $\bar{M} = 8$ and the number of received samples was $N = 48$.

Figure 5.1 plots the pdf of the channel selection for the different methods. The Parzen method was used to make it continuous. The pdf for the EFC is the most consistent at choosing an appropriate channel order. The AIC(R) overestimates the order due to deficiencies in the algorithm. The MDL(R) and CA(R) underestimate due to low SNR. The CIE is choosing incorrectly most likely due to poor performance of the identification measure in its algorithm.

Figure 5.2 shows the average channel order μ chosen by each of the criterion. The EFC approaches the expected value $\mu = M = 4$ at lower SNR than all other methods. This translates to better signal approximation, see Figure 5.3, where the EFC is able to successfully improve performance over the existing methods at low SNR. This simulation reveals three trends that will hold throughout the

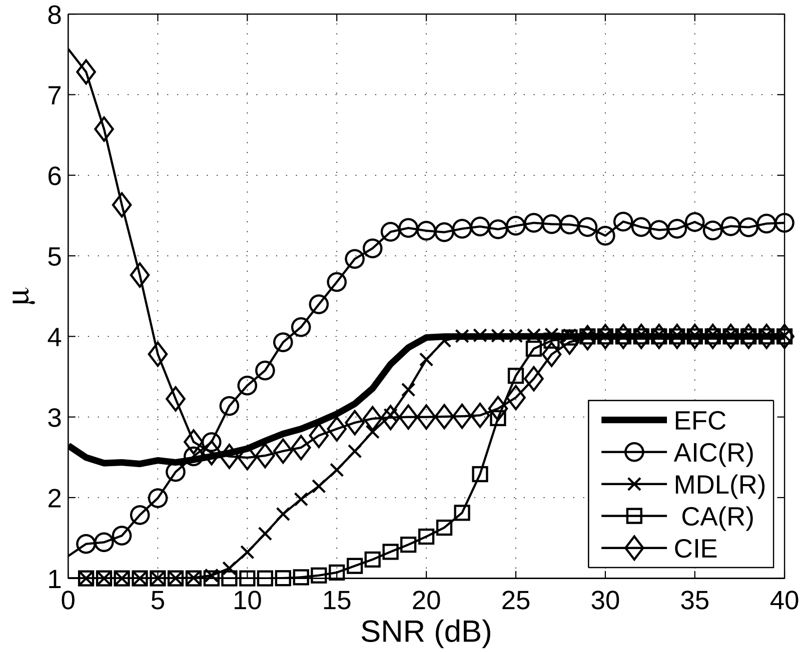

Figure 5.2: Simulation 5.1. Mean channel order selected μ vs. SNR. EFC performance for Channel Set 1.

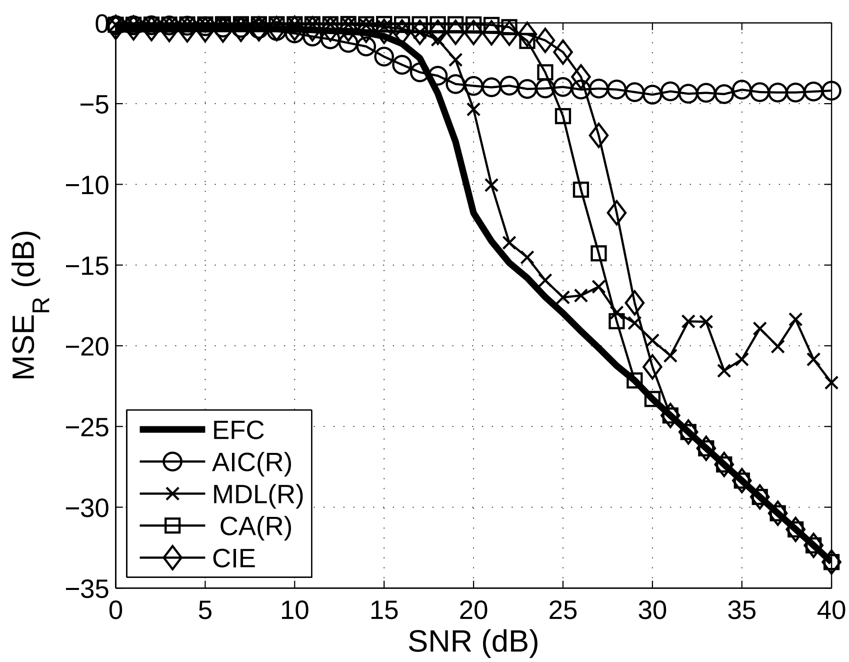

Figure 5.3: Simulation 5.1. MSE_R vs. SNR. EFC performance for Channel Set 1.

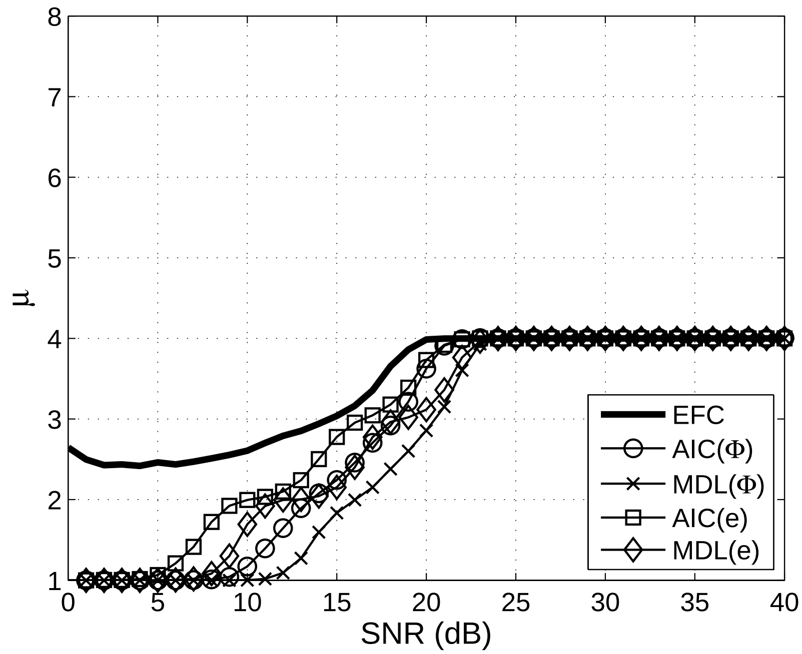

Figure 5.4: Simulation 5.1. Mean channel order selected μ vs. SNR. Error substituion performance for Channel Set 1.

rest of the simulations. The AIC(R) does not provide a reasonable channel order estimates; the MDL(R) is not consistent at high SNR; and the CA(R) provides good performance at high SNR. In the simulations, the performance of the CIE did not appear to be consistent when varying channel sets. Channel Set 1 seems to pose a problem for the algorithm. Further testing is needed to quantify the robustness of the CIE and whether the sum of the identification and equalization measures is the optimal amalgamation before putting it to field use.

The average channel order μ, shown in Figure 5.4, increases as the SNR increases to $\mu = 4$. However, the methods that utilize the errors instead of the eigenvalues tend to approach $\mu = 4$ more quickly. This faster rise turns into better performance, which can be seen in Figure 5.5. Figure 5.5 also illustrates the poor performance of the information criteria at high SNR, which also in Figure 5.3. However, the use of errors mitigates this problem and performance is restored.

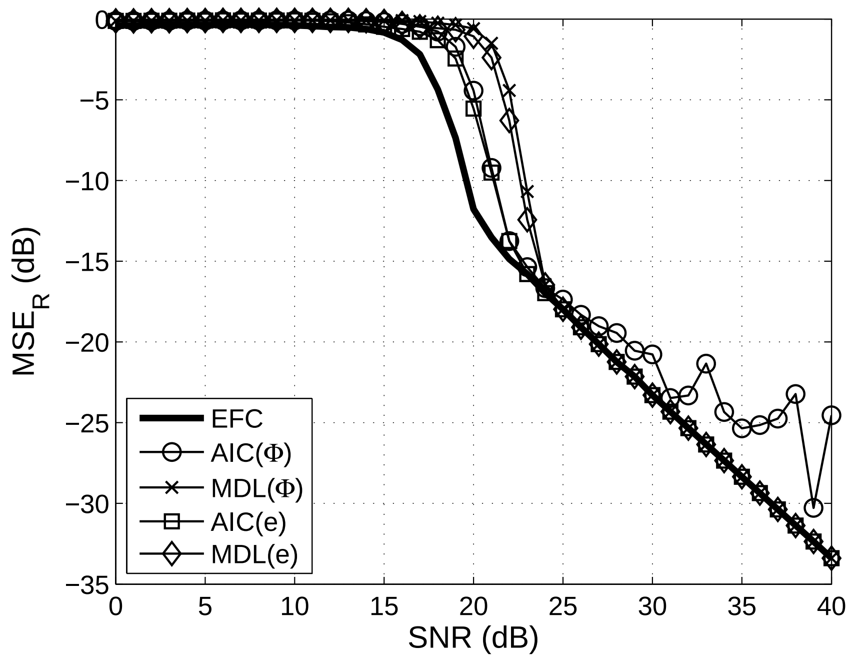

Figure 5.5: Simulation 5.1. MSE_R vs. SNR. Error substitution performance for Channel Set 1.

5.2 Varying $\bar{M}$ for Channel Set 1

Simulation 5.2 varies the maximum possible order $\bar{M}$ from 4 to 16. Here the SNR is held constant at 20 dB, while $N = 48$. The purpose of increasing $\bar{M}$ is to identify which algorithms are robust when the dimension of the nullspace is large.

This simulation reveals a potential weakness of the EFC, see Figure 5.6. The EFC requires that the maximum channel order be larger than the actual channel order for it to operate well at lower SNR values. This is due to $e(\bar{M})$ itself being used in the exponential fit curve. The EFC has a performance advantage at sufficiently large $\bar{M}$. The slight reduction in performance at high values of $\bar{M}$ is due to the increased difficulty to estimate the effective channel order on such few samples. Also, the equalizer performance is slightly reduced in this scenario as it must be able to perform for the largest possible effective channel order. The AIC

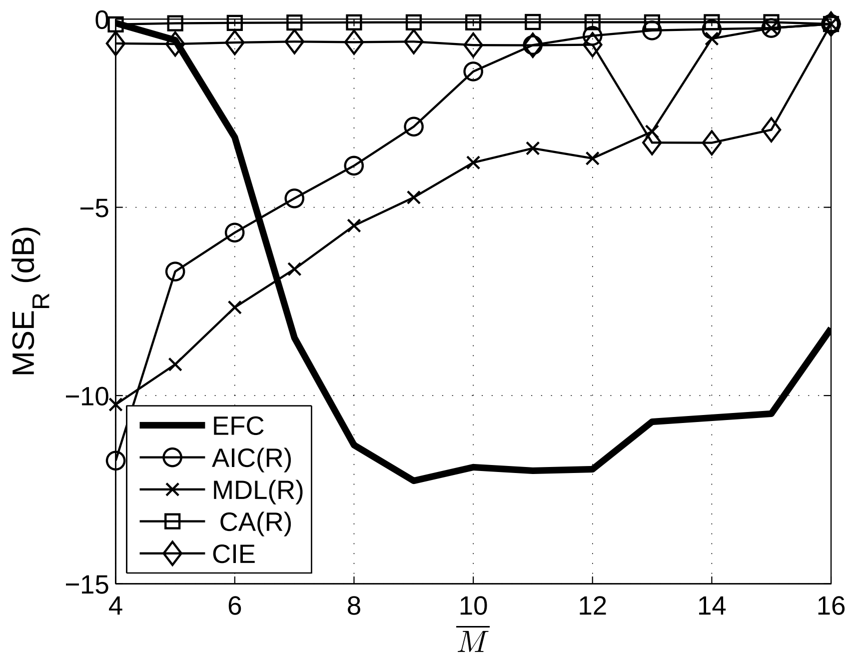

Figure 5.6: Simulation 5.2. MSE_R vs. $\bar{M}$. EFC performance.

and MDL do not handle a large nullspace well. Note that the CIE barely begins to recover performance at larger values of $\bar{M}$ in this simulation.

In order to further investigate the performance under this scenario, the SNR was increased and the simulations rerun. Figure 5.7 d), shows that under higher SNR the EFC does regains its performance for $\bar{M} \sim M$. The CIE recovers some performance at higher SNRs, but is not the best due to Channel Set 1. The CA, as expected, performs well when there is a large nullspace, but requires higher SNR.

The use of errors in place of the eigenvalues of Φ is shown to increase the performance of the information theoretic criteria, see Figure 5.8. Even with this performance increase, the overall effective channel order estimates of the information theoretic criteria are quite poor.

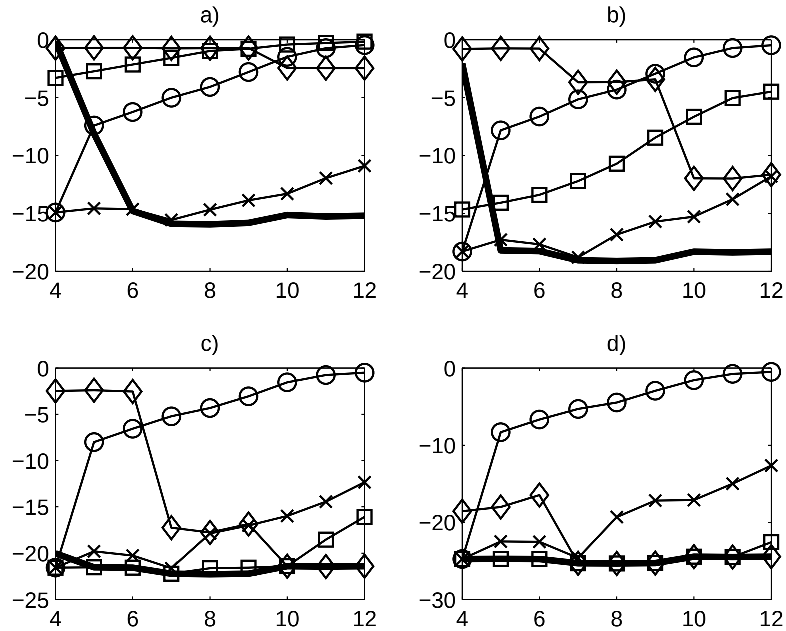

Figure 5.7: Simulation 5.2. MSE_R vs. $\bar{M}$. EFC performance with increased SNR. a) SNR=23 b) SNR=26 c) SNR=29 d) SNR =32. See Figure 5.6 for legend.

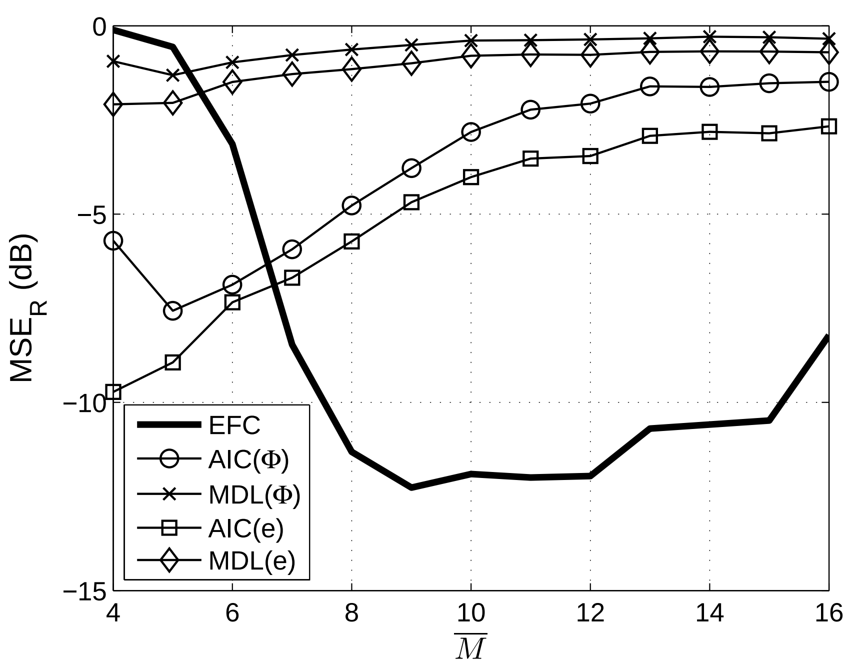

Figure 5.8: Simulation 5.2. MSE_R vs. $\bar{M}$. Error substitution performance.

Table 5.3: Coefficients for Signal Set 2

h_1	h_2	h_3
$1.7491 - 0.9173j$	$0.9323 - 0.7836j$	$1.0488 + 0.2484j$
$0.1326 - 1.1061j$	$1.1647 + 0.2133j$	$1.4886 + 0.0596j$
$0.3252 + 0.8106j$	$-2.0457 + 0.7879j$	$1.2705 + 1.3766j$
$-0.7938 + 0.6985j$	$-0.6444 + 0.8967j$	$-1.8561 - 1.0830j$
$0.3149 - 0.8176j$	$1.7411 - 0.1869j$	$2.1343 + 1.0354j$
$-0.5273 + 1.2688j$	$0.4868 + 1.0132j$	$1.4358 + 1.5854j$

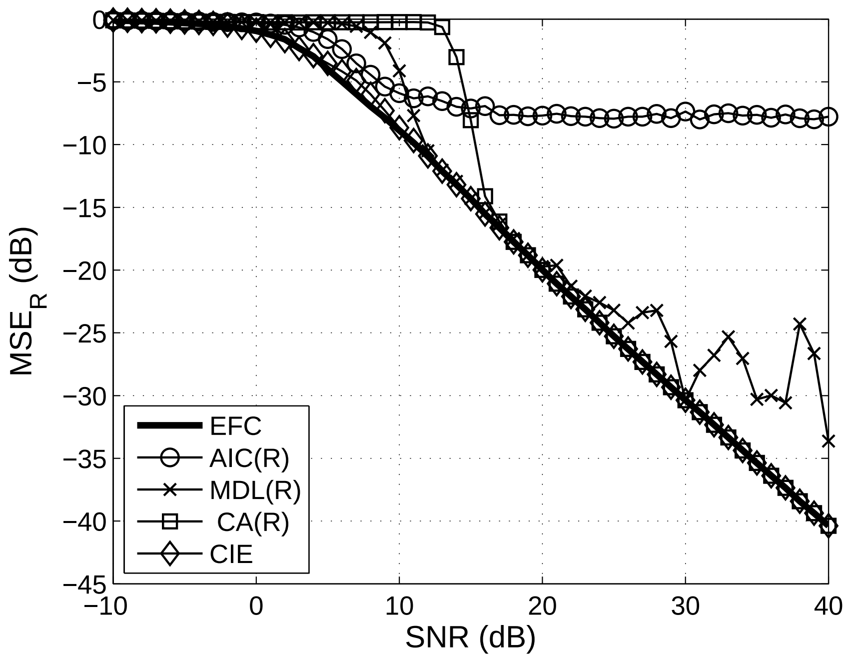

Figure 5.9: Simulation 5.3. MSE_R vs. SNR. EFC performance for Channel Set 2.

5.3 Varying SNR for Channel Set 2

For Simulations 5.3, 5.4, and 5.5 a 4-QAM signal was passed through the $L = 3$ channels of Channel Set 2 of order $M = 5$ from [21], see Table 5.3. For this simulation the number of received samples was $N = 100$ and the maximum channel order was $\bar{M} = 9$. The results of Simulations 5.3, 5.4, and 5.5 show the same trends as those in [21].

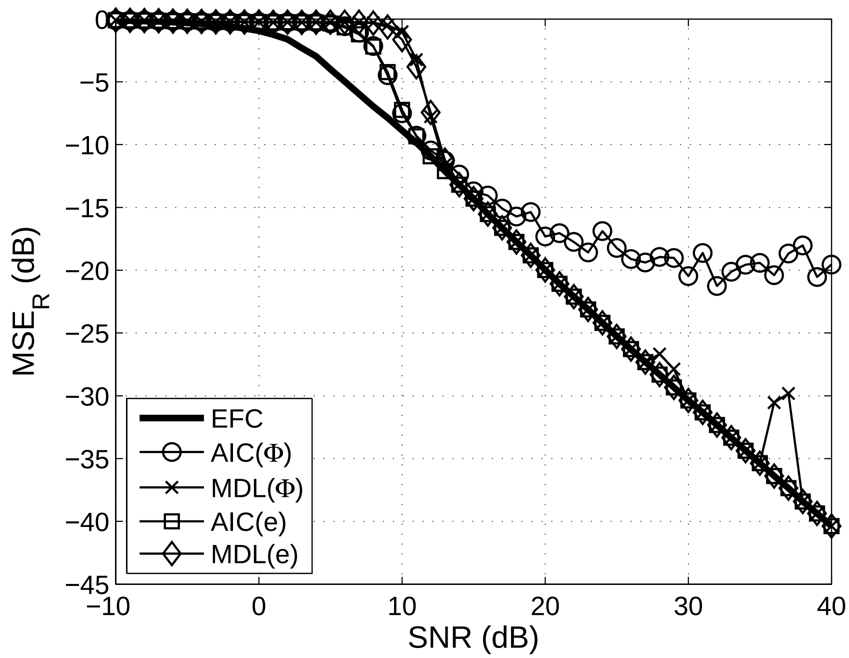

Figure 5.10: Simulation 5.3. MSE_R vs. SNR. Error substitution performance for Channel Set 2.

The CIE is able to show good performance for Channel Set 2, see Figure 5.9. The EFC slightly outperforms the CIE at low SNR, and the others perform as expected. In Figure 5.10, it can be seen that the utilization of the errors is able to drastically increase the performance of the AIC(Φ) and stabilize the performance of the MDL(Φ). It again appears that clustering the errors in the nullspace has a positive effect.

5.4 Varying SNR for Colored Inputs and Channel Set 2

In Simulation 5.4 the source signal was colored by convolution with the vector $[1, 1]$ as was done in [21]. This method was utilized to test performance under correlated signal transmissions. In this simulation the EFC fell slightly behind the CIE in

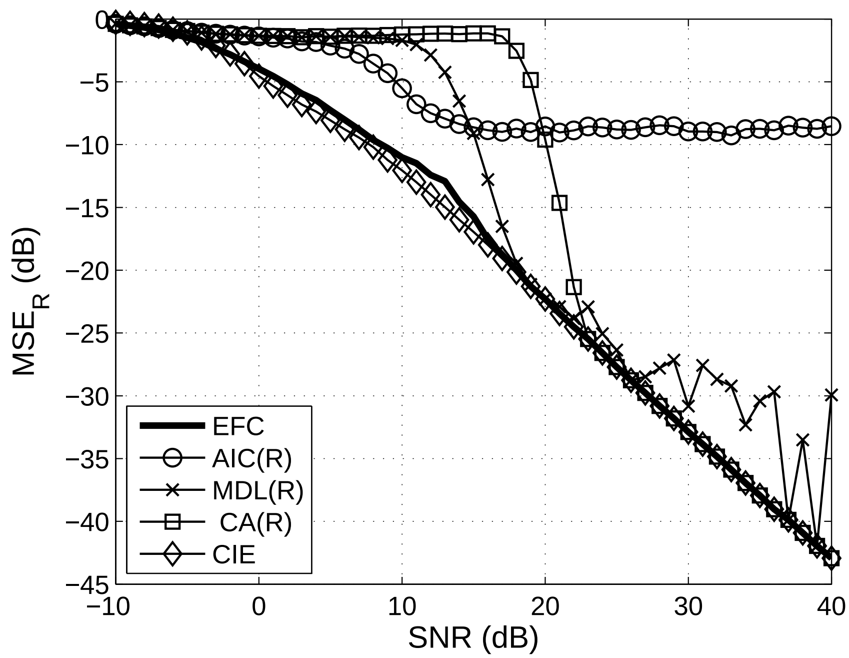

Figure 5.11: Simulation 5.4. MSE_R vs. SNR. EFC performance for colored inputs.

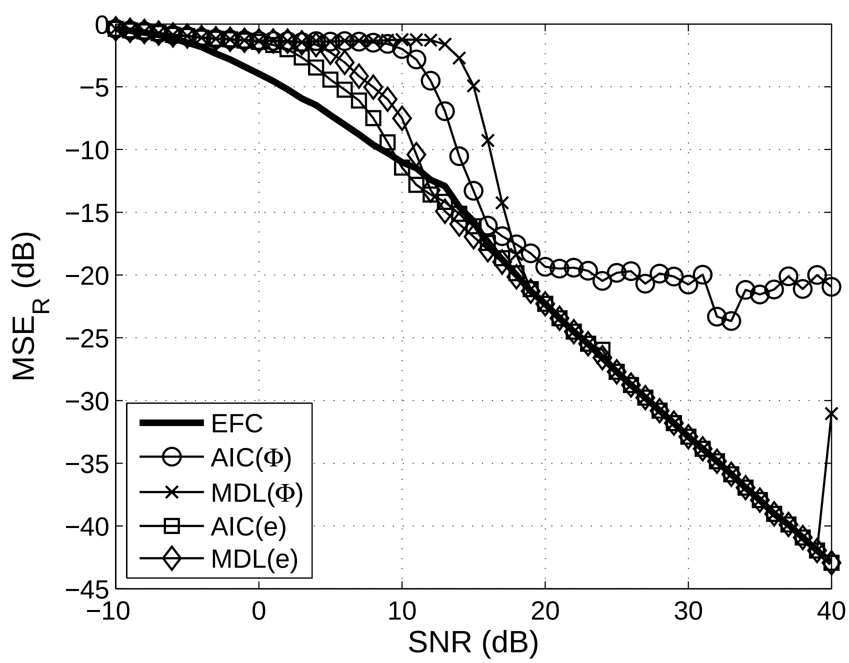

Figure 5.12: Simulation 5.4. MSE_R vs. SNR. Error substitution performance for colored inputs.

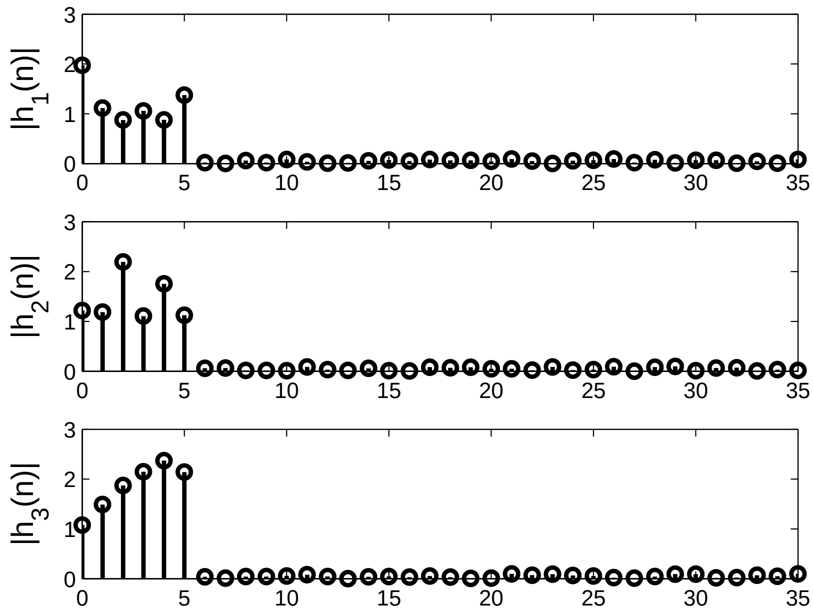

Figure 5.13: Simulation 5.5. Channel Set 2 with small trailing coefficients.

performance, and the rest AIC, MDL, and CA performed as expected, see Figure 5.11. Also, note that the errors improved the performance over the information theoretic criteria in Figure 5.12.

5.5 Varying SNR for Channel Set 2 with Trailing Terms

For the Simulation 5.5 another comparison to [21] was made by adding small trailing coefficients to the channel. Figure 5.13 plots the magnitude of the channel coefficients used. In Figure 5.14 it can be seen that the EFC slightly outperforms the CIE algorithm for low SNR. In Figure 5.15, the use of errors significantly increase performance of the information theoretic criteria at high SNRs. This particular simulation shows precisely the reason why the AIC and MDL cannot be utilized in SNR regimes. As extremely small coefficients become identifiable, it

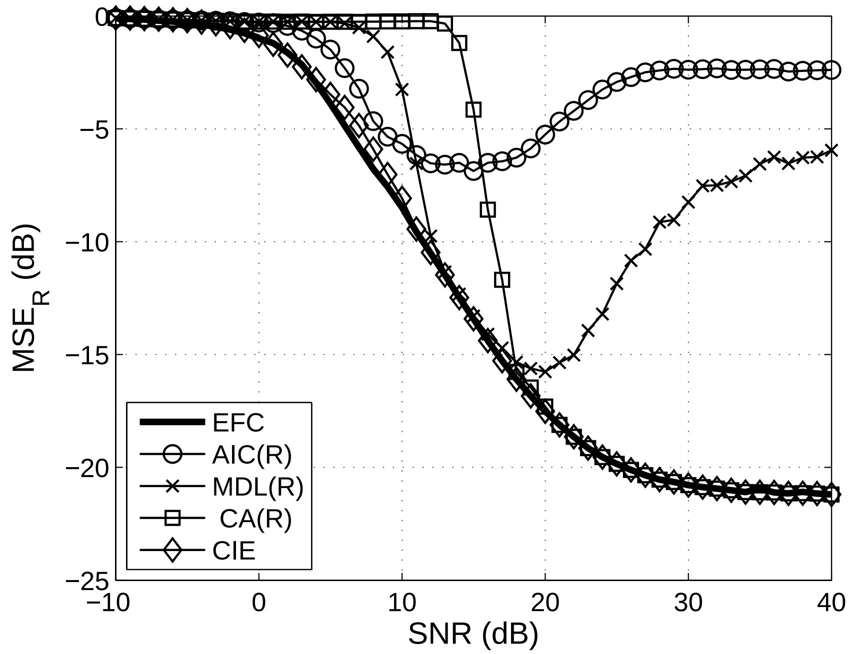

Figure 5.14: Simulation 5.5. MSE_R vs. SNR. EFC performance for channels with small trailing coefficients.

increases the estimated channel order. This can over complicate the equalizer and reduce performance.

5.6 Varying SNR for Random Channels

For Simulation 5.6, each Monte Carlo run was comprised of a unique random channel. The each channel was 8 coefficients long. Each coefficient was independent identically distributed circularly symmetric Gaussian. This means that on most runs the effective channel order was $M = 7$. There were $N = 64$ received symbols and $L = 4$ channels. The maximum possible channel order was $\bar{M}$. This test was designed to see how each each channel order estimation method would react to channels with different properties.

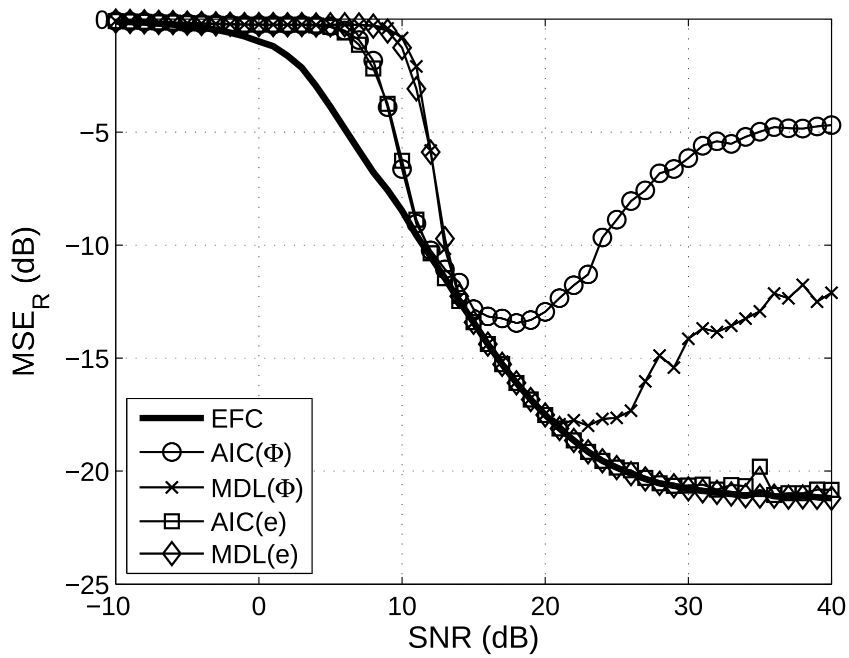

Figure 5.15: Simulation 5.5. MSE_R vs. SNR. Error substitution performance for channels with small trailing coefficients.

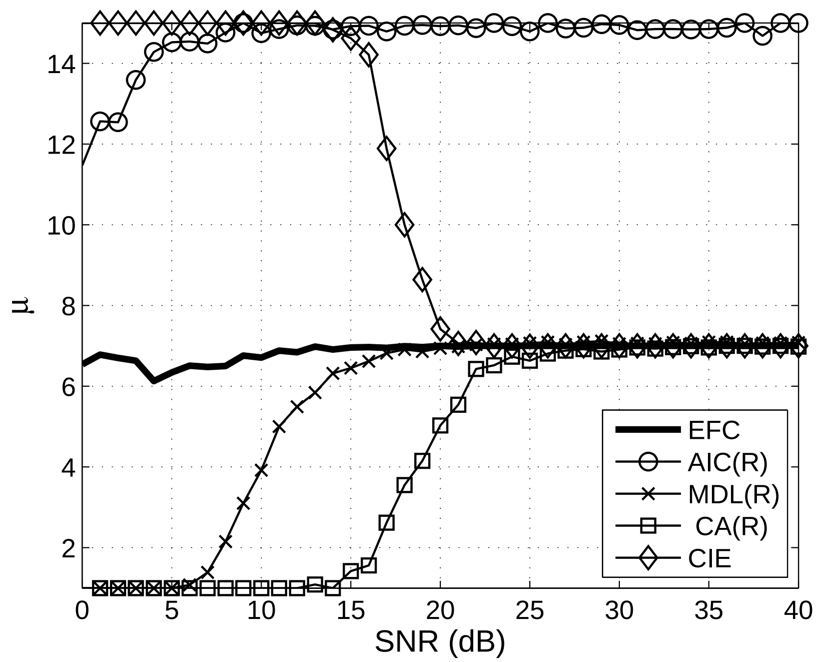

Figure 5.16: Simulation 5.6. Mean channel order selected μ vs. SNR. EFC performance for random channels.

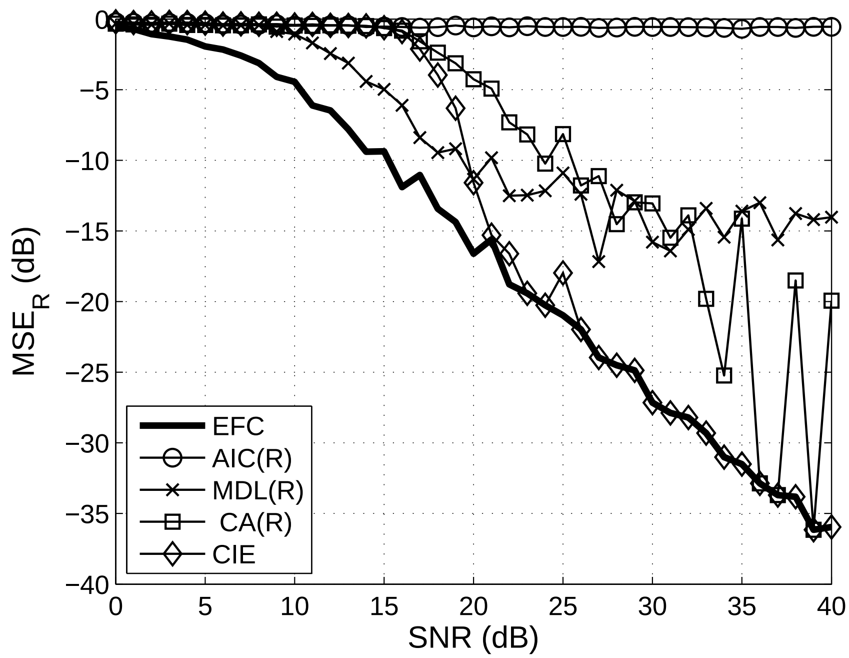

Figure 5.17: Simulation 5.6. MSE_R vs. SNR. EFC performance for random channels.

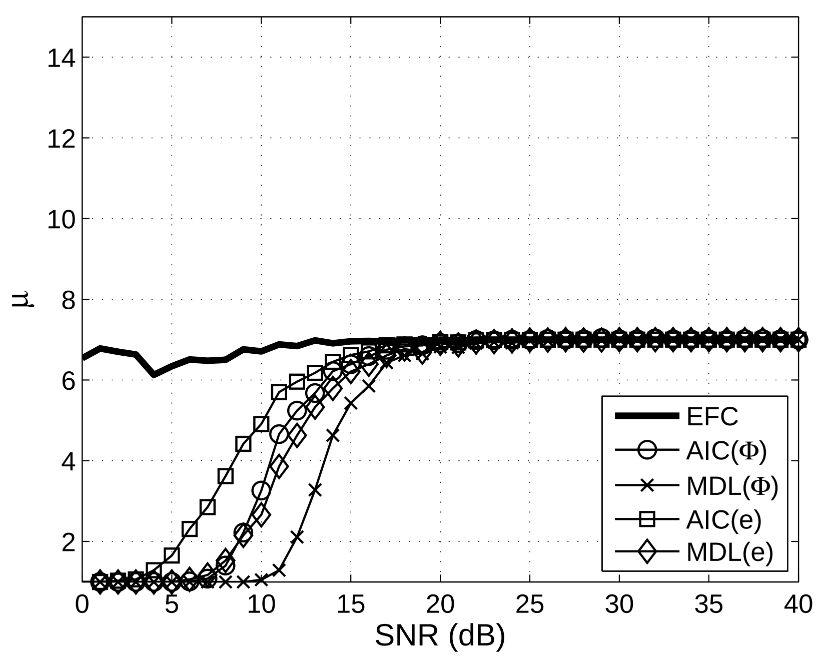

Figure 5.18: Simulation 5.6. Mean channel order selected μ vs. SNR. Error substitution performance for random channels.

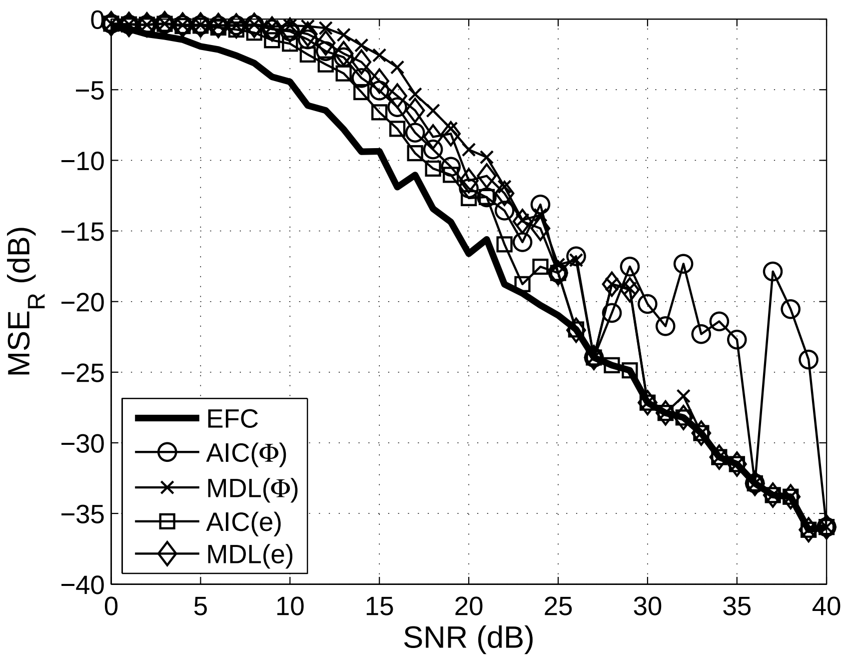

Figure 5.19: Simulation 5.6. MSE_R vs. SNR. Error substitution performance for random channels.

The AIC(R) and MDL(R) both tend under certain conditions to choose the effective channel order to be equal to the maximum possible order needlessly, as seen in Figure 5.16. It can be seen that the EFC is able to on average detect the effective channel order to be 7, even at low SNR. The CIE has reasonable performance if the SNR is high enough, see Figure 5.17. However, the CA(R) is correct on average, but picks inappropriately and the equalizer performance suffers.

Figure 5.18 shows the AIC(Φ), MDL(Φ), AIC(e), and MDL(e) performing appropriately as they all have reasonable estimations of the effective channel order at low SNR. The performance, however, is increase by replacing the singular values with the errors. The substitution appears to create criteria that is robust at high SNRs, see Figure 5.19.

CHAPTER 6

CONCLUSIONS

This paper has introduced three major ideas. The first is the use of residual error estimates (4.1) instead of the traditional singular values. The error estimates utilize knowledge of a structured basis of the nullspace. Simulations show these errors tend to cluster in the nullspace, when structured basis is computed by the subspace method and $L > 2$. This increases the robustness at high SNR. When the structured basis is computed by the LSM, clustering is often weaker than using the subspace method. Second, due to the increased clustering of error values in the nullspace, the errors can replace singular values in information theoretic criteria. This substitution increases performance and robustness, especially at high SNR. Third, a simple new criterion EFC (4.2) was proposed which is based on an exponential fit curve. Simulations show that the combination of the errors and the EFC increases performance under a wide variety of circumstances, especially at low SNR.

In general, the performance of the EFC provides a benchmark for comparison. Since it is not optimized analytically or computationally, it is likely that other algorithms would be able to outperform it in either category. However, it could be utilized in high or low SNR scenarios where accuracy is of the highest importance. Currently, only the CIE performed as well as the EFC for some examples, but the CIE is computationally inefficient in comparison. It costs $\mathcal{O}(\bar{M}^4(L^3+\bar{M}^3)+N\bar{M}L^2)$ for the CIE versus $\mathcal{O}(\bar{M}^3L^3 + N\bar{M}L^2)$ for EFC method. The ideas and methods proposed are purely heuristic. However, they do forecast opportunities to improve on the performance of existing criteria.

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