COSMIC STRINGS IN BRANE INFLATION AND SUPERSTRING THEORY

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Observable predictions of Superstring theories are rare and important. Recent theoretical advances and upcoming experimental measurements of cosmological physics make the testing of generic predictions of string theories possible. Brane anti-brane models of inflation within superstring theory are promising as string theory descriptions of the physics of the early universe, and while varied in their construction, they can have the generic and observable consequence that cosmic strings will be abundant in the early universe. This leads to possible detectable effects in the cosmic microwave background, gravitational wave physics, gravitational lensing and pulsar timing. The string theory physics involved in the production of these defects at the phase transition at the end of inflation is reviewed herein. Detailed calculations of cosmic string interactions within string theory are also presented, in order to distinguish these cosmic strings from those in more conventional theories. It is found that cosmic strings are stable and have tensions compatible with current upper bounds and which are detectable in upcoming experiments. Interaction probabilities of these strings are found to be very different from conventional strings, providing the possibility of experimental tests of string theory.

BIOGRAPHICAL SKETCH

Nicholas T. Jones was born in Melbourne, Australia on the 2nd of February, 1975. A sickly and feeble child, he somehow flourished in the rough-and-tumble world of suburban Melbourne. Rising quickly to the top of his small class of five in Pinewood Primary School, he went on to attend Haileybury College, from which he graduated in 1992, having gained a solid secondary education, but already specialising in the Mathematical Sciences. Attending Monash University in Clayton, Melbourne, between 1993 and 1998, with an unexpected hiatus in South America in 1996, he achieved the titles of Bachelor of Arts and Bachelor of Science, with Honours in Theoretical Physics. This pleased him, and he deemed that he should continue along this vein of study. Upon being accepted to the graduate program in physics at Cornell University, thence he forthwith went, to study String Theory. This document is the outcome of that course of action.

To my ever-loving parents. Although they shall never read it but shall probably show it to the butcher, without them this thesis would not be possible.

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CHAPTER 1

INTRODUCTION

1.1 String Theory and Observation

String theory is not surprisingly a theory in which the fundamental excitations are oscillating strings. Strings with different numbers of oscillation quanta are, when viewed on a large scale, particles of different mass, spin and other conserved quantum numbers. Strings with oscillations that are anisostropic in the spacial directions, when viewed on large scales, behave just as particles with spacetime indices - vector bosons, tensor particles and their generalisations. The gauge symmetries of vector bosons, for instance, arise in string theory as spurious, non-physical states. Fermions arise from fermionic excitations of the worldsheet of the string, and by making projections onto a particular particle content of the theory, the corresponding theory of spacetime particles can be supersymmetric.

Although not its original *raison d'être*, string theory can to be understood as a quantum theory of gravity. Prior to its identification as the graviton, string theorists were troubled by the presence of a spin-2 massless particle in the closed string spectrum. In 1974, Yoneya and Scherk and Schwarz proposed that this particle identifies string theory as an excellent candidate for a theory of quantum gravity [1,2]. In fact, it was later realised [3,4] that if strings were quantised in a background of such particles, consistency of the theory on the string worldvolume was shown to force the background to satisfy Einstein's equations. Being a fundamentally quantum theory, string theory was then known to be a quantum theory of gravity. Recent developments confirm this fact, especially in that string theories exhibit holographic behaviour [5,6,7], and string black-holes have temperature and entropy consistent with Hawking's arguments on black-hole radiation [8].

Other crucial ingredients of string theory are D-branes [9]. D-branes are dynamical surfaces of various dimensions and can be understood from two basic points of view. From an open string perspective, D-branes are surfaces on which strings can end, and their degrees of freedom are described by the open strings ending on them; for instance the position of a D-brane and fluctuations about that position are described by certain open string states which are represented by massless scalar fields existing on the D-brane worldvolume. Also when more than one D-brane is coincident, the massless open strings on their collective worldvolume describe a non-Abelian gauge theory. From this perspective gauge theories can be *geometrically* constructed within string theory. One may also understand D-branes from a closed string perspective: the massless closed strings describe 10D gravitational theories, which have black hole, and black *brane* solutions. These are the higher dimensional generalisations of 4D black holes, and in the supersymmetric string theories, are charged under the higher dimensional generalisations of the 1-form vector potential. D-branes are understood, from this point of view, as the generalised charged black holes of string theory[1]. The equivalence of these two drastically different representations of D-branes has led to some quite remarkable insights recent [7, 8].

At present, superstring theory is the only known framework in which the quantum gauge interactions of the standard model can be successfully incorporated in a quantum theory of gravity. The minor problem that superstring theories are only mathematically consistent in 10 spacetime dimensions can be used to the advantage of the theory, in that extra-dimensional models can provide apparent

[1]When D-branes have charges, there exist anti-D-branes with the opposite charges which can annihilate with a D-brane of the same dimension.

solutions to some of the contemporary problems of physics. Brane anti-brane ($D\bar{D}$) inflation described in this work is such an example.

One of the major criticisms of string theory as a *physical* theory is that it has in its history few, if any, definitive predictions that may be experimentally tested. This embarrassing fact is ironically due to the fact that it is a quantum theory of gravity, so naïvely the characteristic mass scale of the theory, m_s is close to the Planck scale $M_{\rm Pl} \sim 10^{19}$GeV. Traditionally, since the physics which is distinctive of string theory - an infinite tower of massive particles, effects due to the necessity of having extra-dimensions compactified such as Kaluza-Klein modes, *etc.* - emerges at this scale, this "stringy" physics is inaccessible to any conceivable laboratory measurement.

In recent years, this lore has been somewhat subject to review, by creative understanding of physics in extra-dimensions. String theory inspired work on large extra-dimensions with branes [10], and models with warped compactifications [11, 12] can lead to a revision of the logic above, because these structures allow either the scale at which extra-dimensional effects begin or the fundamental scale of gravity and the string scale to be lower than it appears in low energy experiments; the true $M_{\rm Pl}$ could be lowered down to near the TeV scale, in these models. One concern with such ideas has been a lack of a UV completion, or a formal embedding within string theory. This is not assumed insurmountable, and if such an embedding exists (as recent work provides a step toward [13, 14, 15]) there is the possibility in these scenarios of the true Planck scale being low enough that black hole production in particle accelerators is feasible [16]. In this work another proposed window into microscopic physics is described - the imprint on cosmology.

1.2 Cosmological Inflation

In response to certain philosophical problems with the hot big bang model of cosmology, cosmological inflation was born[2]. These problems are related to "naturalness" of the initial conditions for the hot big bang; the big bang seemed to imply that the very early universe needed to be tuned to be exceptionally flat and homogeneous, with vanishing initial density of relic particles (such as monopoles, cosmic strings or domain walls, or gravitinos from a supergravity breaking phase transition). These chagrins are literally blown away by the paradigm of scalar field inflation, which is effectively a physical mechanism that dilutes initial conditions.

Inflation is based on the curved spacetime dynamics of a scalar field. Although the simplest field type to study, no fundamental scalar field has ever been detected. Given that scalar fields are ubiquitous in high energy theories, the explanation for their lack of existence in the "low energy" (below the GeV scale) world is that since they are protected against developing masses by no symmetries, they do so at a high scale. The scale of inflation is a very high scale, and can be close to the Planck scale. Inflation relies on the simple property of a scalar field that it may have negative pressure, and so is an exotic type of matter. A homogeneous, evolving scalar field, ϕ with potential $V(\phi)$ has energy density and pressure

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi), \qquad\qquad P = \frac{1}{2}\dot{\phi}^2 - V(\phi).$$

If the potential dominates the kinetic energy, the pressure in such a field will be negative. A scalar field clearly admits the possibility of having $-\rho < P < \rho$. A fluid of negative pressure has the important feature that its energy density dilutes with the size of the expanding universe, $a(t)$, less than does that of ordinary

[2]See [17] for a complete review and references.

matter. Further if $P < -\frac{1}{3}\rho$ (the bound is satisfied by a fluid of cosmic strings), Einstein's equations dictate that the expansion of the universe will accelerate. The early universe being dominated by matter with $P < -\frac{1}{3}\rho$, and the consequent cosmological acceleration, defines inflation.

That such inflation solves the problems with initial conditions of the hot big bang has been well reviewed [17]; basically a sufficient period of inflation expands a minuscule part of the early universe to the current observable universe. The original size of the current Hubble volume was once so small to appear effectively flat and devoid of relics. Isotropy follows because the comoving Hubble radius shrinks during inflation, so regions which were once within causal contact fall through each other's Hubble horizons, as comoving scales leave the Hubble horizon.

It is the large negative pressure of the scalar field that generates cosmic acceleration, and at some stage this must end. When it does so, as the scalar field reaches the minimum of its potential, the universe has been diluted so extensively by the expansion that it is cold and empty, and there must be means of *re*-heating it. A complete model which embeds the inflationary paradigm must include some mechanism for transferring the residual energy into standard model excitations; in particular, the universe must enter a phase dominated by a radiation gas of temperature greater than around the scale of the electroweak phase transition.

For high energy theory, cosmological inflation is both a boon and a bane. Naïvely, the fact that tiny scales are blown up to cosmological scale could provide a window into the microscopic physics of the early universe. However, it is difficult to find some physics that survives dilution by inflation to late times. It seems only post-inflation effects can be observed, and these effects must be determined by the explicit details of inflation.

1.3 Short Distance Physics and Cosmological Physics

Inflationary cosmology provides theoretical challenges and enormous opportunity. The main challenge is to provide an explicit embedding of inflation within string theory[3]. This goal seems to be drawing closer, in light of work described in Chapter 2. The value in this for cosmology is rather philosophical, in that conventional models of slow-roll inflation do not dictate an inflationary potential, they are rather plucked from a pool of multitudinous scalar field potentials which satisfy the slow-roll conditions.

A true unified theory of the universe should *dictate* an inflaton potential. Although this is true in string theory, one must first choose the vacuum - the geometry and brane content - and it could be argued this is akin to choosing an inflaton potential. However, in the most developed inflationary models in string theory, generic predictions can be made, irrespective of the precise details of the geometry and the inflaton potential. These shall be described in this work. The generic predictions come from the broad structure of models of inflation in string theory, rather than explicit details. These models are invariably instantiations of *hybrid inflation* [18], and which predict the generation of cosmic strings after inflation.

In some sense, it is the rigour of string theory that allows physics of the string scale to survive being washed out by inflation, and to be manifest in our universe on an astrophysical scale. Brane inflation appears to be a most natural embedding of inflation in string theory, and the structure of the theory and causality dictate that generically in brane inflation, cosmic strings will be formed. Cosmic strings, of tensions which are likely to be produced, generate many observable effects, and those arising in string theory can have a distinctive signature.

[3] ...or one's favourite U.V. complete quantum theory of gravity.

Cosmological physics is currently entering an exciting epoch. Experiments present and future aim to probe many of the corners of the now standard cosmological model. With the WMAP data [19], the CMB is measured to astounding accuracy and measurements of CMB polarisation will soon rule out many models of inflation and perhaps detect exciting new physics. Observatories are being planned and built which will detect gravitational waves which have thus far eluded all attempts at measurement and which are generated by astrophysical events and have a relic background from the early universe. This is a particularly apt time to determine whether any physics from beyond the GeV scale can be written in the sky.

CHAPTER 2

OVERVIEW OF BRANE INFLATION

2.1 Why Brane Inflation

Much of the appeal of string theory is æsthetic. String theory often allows the visualisation of complicated phenomena in theoretical physics in geometric terms. A good example is the study of classical and quantum gauge theory using branes (reviewed in [20]), in which quantum corrections in the lower dimensional field theory are represented and calculated by classical corrections to the brane shape in the higher dimensional theory. Brane inflation is another such example, in which the inflaton is geometrically interpreted as the position of one brane with respect to some other object, which may be an anti-brane, a brane of another dimensionality, or a singularity of the geometry. In particular, it is an attempt to realise slow-roll scalar field inflation in string theory, with the inflaton potential dictated by the rigours of string theory, such that no fine tuning of its parameters is necessary or possible. There are other models which attempt to use brane physics to generate inflation, in which the inflaton is another field, or in which inflation is not slow-roll scalar field driven; these shall not be covered in this work. Different attempts within string theory and extra-dimensional models for inflation are reviewed in [21].

Aside from the intuitive appeal of this picture, there are excellent reasons to seek situations in which brane position acts like the inflaton. These were first outlined in [22] which is the first instance of this idea. Their motivating reasons were that the brane position is weakly coupled to bulk modes, and remains so even at the end of inflation; at that time it couples strongly to other brane modes which conceivably could provide efficient means of reheating the universe. This leaves the bulk cool, and thus avoids generating a population of Kaluza-Klein graviton modes which could overclose the universe. Also, the inter-brane potential was thought to be sufficiently flat, arising from some supersymmetry breaking on the branes. Although this intuitive picture is appealing, as shall be discussed, many of these issues are yet to be definitively resolved in a rigourous way.

2.1.1 Early Models

In [22] brane separation was used as the inflaton, and breaking of supersymmetry on the brane was thought to produce a sufficiently flat potential. The picture therein was not precise enough however to enable a specific inflaton potential to be calculated. In [23] the first brane anti-brane model using the separation as the inflaton was constructed, allowing the explicit brane anti-brane potential of [24] to be used as the inflaton potential[1]. The brane anti-brane potential is obtained by the standard method of the one-loop open string partition function, and in the large distance limit it reduces to the exchange of massless closed string states. Since an anti-brane is merely a brane with reversed RR charge, the NS-NS and RR forces add instead of cancel for the BPS brane brane system, resulting in a long-distance ($r \gg l_s$) potential for branes of dimension $p+1$

$$V_p(r) = 2\tau_p \left[1 - \frac{2g_s(4\pi)^{\frac{5-p}{2}} \alpha'^{\frac{7-p}{2}} \Gamma(\frac{7-p}{2})}{r^{7-p}} \right], \qquad \tau_p = \frac{1}{(2\pi)^p \alpha'^{\frac{p+1}{2}} g_s}. \tag{2.1}$$

This behaviour is merely the potential for exchange of massless particles between co-dimension $9-p$ objects in 10 dimensions. It can be immediately seen that this potential cannot produce slow-roll inflation. For slow roll inflation, the parameters

[1]Note that earlier [25] used a brane anti-brane model to motivate topological inflation.

for the inflaton potential [17],

$$\epsilon \equiv \frac{M_{\rm Pl}^2}{2}\left(\frac{V'}{V}\right)^2, \qquad\qquad \eta \equiv M_{\rm Pl}^2\frac{V''}{V}, \tag{2.2}$$

are required to have small magnitude. To obtain a finite 4D Plank mass, the extra dimensions must be compactified. Assuming the branes involved have spacial dimension $p \geq 3$, and compactifying the $p-3$ extra brane dimensions on a volume $V_{\|}$ and the $9-p$ bulk dimensions on a torus of length $l_{\perp}$ and volume $V_{\perp} = l_{\perp}^{9-p}$, the 4D Planck mass is $M_{\rm Pl}^2 = M_{10}^8 V_{\perp} V_{\|}$. However from the potential for the canonically normalised inflaton from (2.1) for the inflationary period when the branes are separated by $\sim r$ [23],

$$\eta = \left(\frac{l_{\perp}}{r}\right)^{9-p}\left[\frac{(7-p)(8-p)\Gamma(\frac{7-p}{2})}{2\pi^{(9-p)/2}}\right]. \tag{2.3}$$

Since by definition $r < l_{\perp}$, $\eta \gtrsim 1$ and slow roll inflation cannot be achieved.

The work of [23, 26, 27] are attempts to correct this flaw of the simplest framework, but using the correct Green's function for the inflaton potential on the compactification manifold. In the simple case of toroidal compactification, the complete inter-brane potential can be viewed as the potential obtained by summing over a source for the NS-NS and RR fields and all their images. This process is however ambiguous, because the series obtained are non-convergent, and must be regularised in some way; the result will be regularisation dependent. The works [23, 26, 27] apply the most intuitive regularisation, but unfortunately this fails because it does not correctly account for the necessary background charge. As explained in [28] and later expounded in [29], the Green's function for the potential due to the exchange of massless modes on a compact manifold must contain the contribution of the *Jellium* term which is the minimal change necessary in order that integrating the equation for motion for the potential over the compact space is consistent. In particular, if the D$\bar{\text{D}}$ are again of spacial dimension p with $p-3$ directions wrapped on a torus of length $l_{\|}$, and the remaining compact directions have volume $V_{\perp}$, then the inter-brane potential satisfies

$$\nabla^2 V(\boldsymbol{x}) = \sum_{\text{sources},i} q_i \left[\delta^{(9-p)}(\boldsymbol{x} - \boldsymbol{x_i}) - \frac{1}{V_{\perp}}\right]. \tag{2.4}$$

Noting that the canonically normalised 4D scalar inflaton field, ϕ, is related to the brane separation, X, via $X = 2\pi\alpha'\phi$, and that (2.4) implies the there is at least one direction in which the potential has derivative $V'' \leq -q/(9-p)V_{\perp}$. Then, for a single D$\bar{\text{D}}p$ system, with the brane charge under the bulk closed string fields $q = 4\kappa^2\tau_p/(2\pi\alpha')^2$, the slow roll parameter is [28]

$$\eta < -\frac{4}{9-p},$$

so slow roll inflation cannot be achieved. The denominator in η in (2.2) is the value of the potential during inflation and is in this case the total D$\bar{\text{D}}$p tension, $2\tau_p l_{\|}^{p-3}$.

This obstacle for brane inflation is solved in the literature in two ways, both of which moderate the strength of the potential; by altering the brane configuration so that the system is less attractive - *i.e.* by understanding the D$\bar{\text{D}}$ system as the extreme limit of a system of branes at angles, and then reducing the angle; or by embedding the D$\bar{\text{D}}$ pair in a warped compact space, such that the curved space inter-brane potential is less attractive than its flat space analogue. These possibilities are discussed in § 2.1.2 and § 2.2.3.

2.1.2 Branes at Angles

As just mentioned, the inflaton potential for the flat space D$\bar{\text{D}}$ system is too steep for slow-roll inflation. This potential can be softened, by considering other non-BPS multi-brane systems. In particular, the brane anti-brane system is the extreme limit of branes at angles; systems of branes aligned at angles less than π, interpolate between the BPS parallel brane configuration, and the extreme brane anti-brane.

Brane at angles were originally studied in [30] and [31]. Such systems have been used extensively in the search for constructions of the standard model within string theory (a recent review is [32]), since there are stable, tachyon free configurations of angled branes, with massless chiral fermions localised to the intersection points. The systems of interest to cosmology are of course, those non-BPS configurations which at zero separation have a localised tachyon at the intersection point. As is now understood, the rolling of this tachyon describes the dynamics of the D-branes recombining (see for instance [33, 34, 35]).

The supergravity potential between the angled branes was first calculated using the open string one-loop diagram, and taking its modular transformation in [31]. The Coulombic piece due to the long distance exchange of massless fields is

$$V_\theta(r) \sim 4\tan(\tfrac{\theta}{2})\sin^2(\tfrac{\theta}{2})(4\pi^2\alpha')^{3-p}\frac{(2\pi^2\alpha')^{(6-p)/2}\,\Gamma\left(\frac{6-p}{2}\right)}{r^{6-p}}.$$

The small angle scaling of this piece, and hence of the relative charge between angled branes is $\sim \theta^3$. The constant part of the potential, however will have different scaling with small θ: this is because in a realistic model there will be a cancellation of most of the vacuum energy post-inflation. After inflation when the branes have recombined, the vacuum energy that is canceled (by orientifold planes for instance) will be $2\tau_p l_\|$. In the simplest model for inflation, the branes are D4-branes and have $(1, \pm 1)$ wrapping on the 2 1-cycles of a 2-torus [26, 27, 29] with lengths $l_\|$ and $l_\| \frac{\theta}{2}$ respectively

$$V_{0,\,\theta} = 2\tau_p\left[\sqrt{l_\|^2 + l_\|^2\frac{\theta^2}{4}} - l_\|^2\right] \simeq \tau_p l_\| \frac{\theta^2}{4}.$$

Finally, since $M_{\rm Pl}^2 \sim \theta l_\parallel^2 \ldots$, the slow-roll parameter can be calculated to be

$$\eta \simeq -\frac{\theta^2}{4},$$

rendering slow-roll inflation possible with $\theta \lesssim 0.25$.

Although this is encouraging, there are many unsolved problems with this "scenario," the main one being that moduli were assumed stabilised by some mechanism, and this mechanism must not back-react to disturb any of the physics determining the brane potential. This is discussed in the following section.

2.2 Moduli and Inflation

2.2.1 The Moduli Problem

Although promising, the models described above have serious deficiencies. Although they progress some way to achieving the goal of a string theoretic embedding of slow-roll inflation, with a rigourously determined inflaton potential, they are not particularly mature. A basic assumption underlying the constructions [23, 26, 27], is that all the closed string moduli are somehow stabilised by mechanisms unknown. This assumption is a common one in the string theory literature mainly because the moduli problem is so difficult. This embarrassing fact is briefly outlined in this section, as is a recent resolution [13].

Moduli are massless fields with exactly flat potentials. The dilaton is one such example, which in many models has no perturbative potential. This massless field can therefore roll to indefinite values, which is troublesome given its physical rôle as the string coupling constant. Further, since strongly coupled string theories are S-dual to weak ones, the dilaton cannot be stabilised at strong coupling. It is therefore generally assumed that dilaton must be stabilised at $\Phi \sim 0$ by non-perturbative effects where both the string theory and its S-dual have coupling of order unity. Another important closed string modulus is the radion. The radion for a compactified direction is the diagonal component of the metric for that direction, and for general compactifications will be a modulus.

Without stabilisation however, these volume moduli will become problematic; [28] point out the very simple fact that the inflaton potentials in these models always contain these volume moduli which are assumed to be stabilised; in the absence of an explicit mechanism to do so though, the inflaton potential will be potential for the volume "moduli," leading to rapid volume expansion. The method *de jour* for solving the moduli problem, flux stabilisation, is outlined in the next section.

2.2.2 Flux Stabilisation

Motivated in part by the models of Randall and Sundrum [11, 12] which provided an 5D effective field theory description of the large separation of electroweak and Planck scales, GKP [13], following ideas of Verlinde [36] attempted to embed RS-like models in string theory compactifications. In their work, they were able to stabilise all moduli of the theory, save the Kähler moduli of the compact CY using NS-NS and RR fluxes. In order to provide a solution to the closed string equations of motion, it is necessary to include negative tension objects in the bulk - orientifold planes - which evades no-go theorems [37, 38] that obstructed previous attempts. The outcome of this work was to show that AdS-like warped throats could be embedded consistently in a CY compactification of Type II string theory, where the conifold singularity at its tip is deformed by blowing up the $\mathbb{S}^3$ with RR 3-form flux; the volume of the $\mathbb{S}^3$ at the tip of the throat goes like $(g_s M)^{3/2}$ if the RR flux is quantised on the $\mathbb{S}^3$ as M. The length of the throat is stabilised by this NS-NS and RR 3-form flux; the warp factor is a measure of the relative size of the $\mathbb{S}^3$ at the tip and the remaining part of the CY in which the throat is embedded [13].

The construction of [13] is still a supersymmetric solution of supergravity, and as such the effective 4D theory has a negative cosmological constant. Subsequently, KKLT [14] added to the model a $\bar{\text{D}}3$ brane, which is non BPS with respect to the rest of the geometry, and so breaks supersymmetry. They argue that this lifts the AdS vacuum to a de-Sitter space, and further they claim that non-perturbative effects provide a potential for the Kähler moduli leaving a completely stabilised compactification. The $\bar{\text{D}}3$ brane is want to sit at its potential minimum at the tip of the warped throat, attracted there both gravitationally and by RR 5-form flux, and is unstable to decay into (negative) RR 3-form flux mediated by an NS5-brane instanton [39]. However, it is argued that the decay rate is extremely slow, even on cosmological time-scales. Explicitly, if p $\bar{\text{D}}3$-branes are added to a system with M units of RR flux on the $\mathbb{S}^3$, [39] show that the decay rate goes like $\exp(-c g_s M^6/p^3)$ (c is a constant of $\mathcal{O}(10)$), which can be insignificant for small p/M, given that $g_s M \gg 1$ for the supergravity approximation to remain valid.

2.2.3 Flux Stabilised Brane Inflation

Finally KKLMMT [28] embedded the $\text{D}\bar{\text{D}}$ inflation ideas of [22] into KKLT geometry to provide a mature string theory embedding of inflation with an explicit determined inflaton potential. With the supersymmetry breaking $\bar{\text{D}}3$-brane(s) at the tip of the *inflationary* throat, a mobile D3-brane is added to the base of the CY. Without the supersymmetry breaking $\bar{\text{D}}3$-brane(s), the position of the D3-brane would be a modulus, however attraction to the $\bar{\text{D}}$3-brane(s) provides a weak potential. The potential is softened to be a slow-roll potential in comparison to the flat space D$\bar{\text{D}}$ potential because of the warped geometry. The KKLMMT inflationary model is represented in Figure 2.1.

In a more sophisticated model, the standard model needs also to be embedded into the KKLT geometry. Progress has already been made in doing so [15]. In general though, there are three broad categories of KKLMMT models distinguished by their standard model embeddings:

- The standard model branes are $\bar{\text{D}}$3-branes in the inflationary throat; they provide both the matter content of the standard model, and the breaking of the supersymmetry of the GKP compactification to lift the AdS vacuum.

- The standard model branes are D3 or $\bar{\text{D}}$3-branes located in a different throat. This is not unlikely, given that recent work suggest that warped throats are statistically common [40, 41].

- The standard model branes are D3 or $\bar{\text{D}}$3-branes in a more general F-theory construction, which necessarily includes D7-branes extended throughout the Calabi-Yai manifold. Although the D3-branes need not necessarily be in the inflationary throat, some D7-branes will pass through the bottom of the throat [42].

The way in which the KKLMMT models solves the slow-roll problem of § 2.1.1 is due to the fact that the $\bar{\text{D}}$3-brane which attracts the mobile D3-brane is fixed at the end of the warped throat. There its tension is red-shifted by the warp factor, so the effective attraction on the D3-brane is appropriately reduced. When the D3-brane is in the warped throat, the space can be approximated by $\text{AdS}_5 \times X^5$, with X^5 some compact 5-manifold, with metric

$$ds^2 = h^{-\frac{1}{2}}(r)\eta_{\mu\nu}dx^\mu dx^\nu + h^{\frac{1}{2}}(r)\left(dr^2 + g_{ab}dy^a dy^b\right), \qquad h(r) = \frac{R^4}{r^4}.$$

R is the radius of AdS, expressed in terms of the 5-form flux supporting it by $R^4 = 4\pi c g_s \alpha'^2 N$ (c is an $\mathcal{O}(1)$ constant determined by X^5; $c = 1$ for $X^5 = \mathbb{S}^5$ for instance). Perturbing this background by the addition of the $\bar{\text{D}}$3-brane[2] at $r = r_{\bar{\text{D}}}$, the potential for the D3-brane at position $r_{\text{D}} \gg r_{\bar{\text{D}}}$ is then given by [28]

$$V = 2\tau_3 \left(\frac{r_{\bar{\text{D}}}}{R}\right)^4 \left[1 - \frac{1}{N}\left(\frac{r_{\bar{\text{D}}}}{r_{\text{D}}}\right)^4\right].$$

The warp factor $r_{\bar{\text{D}}}^4/R^4$ is the factor by which the effective $\bar{\text{D}}$3-brane tension is reduced, and is $\mathcal{O}(10^{-4})$. In fact, this quantity is exponentially suppressed in the RR and NS-NS fluxes:

$$\left(\frac{r_{\bar{\text{D}}}}{R}\right)^4 = e^{-8\pi K/3g_s M}, \qquad \frac{1}{2\pi\alpha'}\oint_A F^{(3)} = 2\pi M, \qquad \frac{1}{2\pi\alpha'}\oint_B H^{(3)} = 2\pi K,$$

and $N = KM$.

This model, along with others built upon these foundations [15,43,44,45,46,47, 48], is the most developed string theory embedding of inflation. Still not without its problems, both philosophical and computational, these are assumed in this work not insurmountable, and in the following chapters the generic consequences of inflation being driven by D$\bar{\text{D}}$ movement are drawn and studied.

[2]Because the throat of finite length is approximated by AdS space, the end of the throat is approximated by a cut-off in the radial coordinate at $r = r_{\bar{\text{D}}}$.

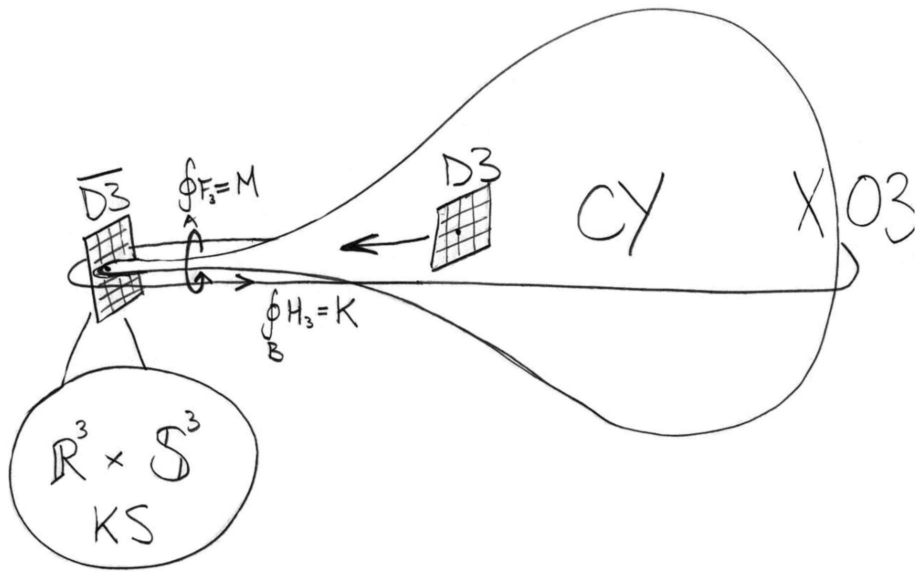

Figure 2.1: Schematic diagram of the KKLMMT model of D$\bar{\text{D}}$ inflation; the compact space is a CY 3-fold with moduli stabilised by flux, a warped inflationary throat which at its tip approximated a KS throat where the conifold singularity is deformed by RR flux, giving the $\mathbb{S}^3$ finite size. There are $\bar{\text{D}}$3-branes at the minimum of their potential at the end of the throat, to which the mobile D3-branes are attracted, generating an inflaton potential.

CHAPTER 3

ENDING INFLATION AND COSMIC STRING PRODUCTION

The models described in the previous sections are varied, and the details of inflation shall in general be model dependent. Further, in general in the realistic - or moduli fixed - examples, it has been shown that there persists a tuning of the fluxes in the geometry in order to produce realistic 4D inflation. Thus it appears a tuning of the inflaton potential has been traded for tunings of the geometry. Although there is now an simple geometric origin to inflation, the achievements for cosmology are arguable (it is certainly a grand triumph for the string theorist though, at least at the level of an existence proof). Further, if this entire framework is unable to make predictions beyond some tuned 4D scalar field inflationary model, then it is useless as a physical theory.

Pessimism aside, it is possible to make generic predictions, in most of the models described above, based only on the assumption that brane anti-brane (or branes at angles) motion provides inflation and that this process ends in brane anti-brane annihilation. As described in the previous chapter, this phenomenon can be embedded in various string vacua. To understand the end of inflation conceived thus, some of the developments in understanding regarding non-BPS brane systems of the last few years must be studied.

3.1 Open String Tachyons in Superstring Theories

The presence of the ubiquitous tachyons in string theories has for a long while been an annoyance. The most simple states to arise in the Bosonic theories and the (pre-GSO projected) theories with worldsheet supersymmetries are spacetime tachyons.

For a long time these theories were ignored as unstable, until Sen[1] realised the importance of such systems: the presence of tachyons indicated an instability in the system, and self-interactions of the tachyons would lead to interesting, and non-pathological behaviour, just like the Higgs field, which is a "tachyon" at the unstable point before spontaneous symmetry breaking, but which has higher stabilising interactions so the theory is physically well-behaved. It is then imperative to calculate these higher self-interactions of the tachyons, which is the domain of the string field theories.

Open string tachyons turned out be be easily interpretable in terms of the instabilities of the branes to which the strings were attached. The real open string tachyon of the Bosonic D-brane can decay to the closed string vacuum [49], and importantly, kink solitons that can arise during the decay can be shown to be lower dimensional D-branes. In this way all D-branes can be constructed as solitons of the tachyon on D26-branes. In type II superstring theories the brane anti-brane tachyons are complex, being composed of strings stretching both ways between the branes. The resulting solitons can have only even codimension; tachyon condensation can result in stable branes of either even or odd spacial dimensionality in type II A or B theories respectively.

As described in the previous chapter, the most important non-BPS brane system relating to inflation is that of the brane anti-brane and its variants. It has been discussed that an anti-brane is a brane with reversed RR charge. By flipping the RR contribution of the open string one-loop diagram, it is easily seen that the resulting open string spectrum between the brane and anti-brane is the NS- spectrum, which is usually GSO projected out and includes a complex tachyon.

[1] See [49] for a review and references.

This is more readily seen by considering an anti-brane as a brane rotated by π in some direction. This process is reviewed here. Note that a non-BPS brane system can be obtained from a spacetime $(-1)^{F_L}$ projection [49] on the brane anti-brane system, so it shall not be treated separately.

3.1.1 Spectrum of Angled Branes

Here the spectrum of strings stretching between two D-branes of the same dimension intersecting at an arbitrary angle [30, 31, 50] is reviewed. For simplicity two D8-branes in IIA theory angled in the 8-9 plane are considered. Other configurations are easily obtained via T-duality. Also, the spectrum described in these notes is only half of the complete spectrum stretching between the branes, because there is a copy of this coming from the other Chan-Paton sector; these correspond to the off-diagonal entries in the 2×2 Chan-Paton matrix for the two brane system. Here the worldsheet is represented by the strip parameterised by (τ, σ) with boundaries located at $\sigma = 0, \pi$. Transforming to complex coordinates in the 8-9 directions, the worldsheet bosons and NSR fermions are $Z = 2^{-\frac{1}{2}}[X^8(z,\bar{z}) + iX^9(z,\bar{z})]$, $\Psi(z) = 2^{-\frac{1}{2}}[\psi^8(z) + i\psi^9(z)]$. With one brane lying along the X^8 direction and the other tilted by an angle $\phi \equiv \nu\pi$, the boundary conditions on Z for a string stretching between the branes are

$$\text{at} \quad \sigma = 0 \quad \begin{cases} \partial_\sigma \mathrm{Re}\, Z = 0, \\ \mathrm{Im}\, Z = 0, \end{cases} \qquad \text{at} \quad \sigma = \pi \quad \begin{cases} \partial_\sigma \mathrm{Re}\, (e^{i\pi\nu} Z) = 0, \\ \mathrm{Im}\, (e^{i\pi\nu} Z) = 0, \end{cases}$$

and that for the worldsheet fermion Ψ is

$$\Psi(\tau, 0) = e^{-2\pi i \nu'} \Psi(\tau, 2\pi).$$

The standard doubling trick has been used to extend the fermions on $\sigma = [0, \pi]$ to $\sigma = [0, 2\pi)$. Note that $0 \leq \nu \leq 1$, and the configuration corresponding to $\nu = 0$ being conventionally the brane anti-brane system and $\nu = 1$ corresponding to the brane brane system. The twist ν' is $\nu + \frac{1}{2}$ for NS sector fermions and ν for Ramond sector fermions.

Transforming to the upper half plane (UHP) through $z = e^{i\sigma+\tau}$, with the boundary now being along the real axis, the mode expansions for the free string become

$$\begin{aligned} Z(z,\bar{z}) &= i\left(\frac{\alpha'}{2}\right)^{\frac{1}{2}} \sum_{n\in\mathbb{Z}} \left[\frac{\alpha^{\dagger}_{n-\nu}}{(n-\nu)z^{n-\nu}} + \frac{\alpha_{n+\nu}}{(n+\nu)\bar{z}^{n+\nu}}\right], \quad \Psi(z) = \sum_{r\in\mathbb{Z}-\nu'} \frac{\psi_r}{z^{r+\frac{1}{2}}}, \\ Z^{\dagger}(z,\bar{z}) &= i\left(\frac{\alpha'}{2}\right)^{\frac{1}{2}} \sum_{n\in\mathbb{Z}} \left[\frac{\alpha_{n+\nu}}{(n+\nu)z^{n+\nu}} + \frac{\alpha^{\dagger}_{n-\nu}}{(n-\nu)\bar{z}^{n-\nu}}\right], \quad \Psi^{\dagger}(z) = \sum_{s\in\mathbb{Z}+\nu'} \frac{\psi^{\dagger}_s}{z^{s+\frac{1}{2}}}. \end{aligned} \tag{3.1}$$

For these expansions, the negative real axis has one boundary condition, and the positive real axis the other; hence there is a flip in boundary conditions at $z = 0$ and $z = \infty$. Quantisation of the string leads to the oscillator algebra

$$\left[\alpha_{m+\nu}, \alpha^{\dagger}_{n-\nu}\right] = (m+\nu)\delta_{n+m=0}, \qquad \left\{\psi_r, \psi^{\dagger}_s\right\} = \delta_{r+s=0}.$$

The important commutation relations are

$$\begin{aligned} \left[\alpha^{\dagger}_{n-\nu}, L_0\right] &= (n-\nu)\alpha^{\dagger}_{n-\nu}, & \left[\alpha_{n+\nu}, L_0\right] &= (n+\nu)\alpha_{n+\nu}, \\ \left[\psi_r, L_0\right] &= r\psi_r, & \left[\psi^{\dagger}_s, L_0\right] &= s\psi^{\dagger}_s, \end{aligned}$$

and the Hamiltonian for this subsystem is

$$L_0 = \left(\sum_{n\in\mathbb{Z}} :\alpha^{\dagger}_{n-\nu}\alpha_{-n+\nu}: + \sum_{r\in\mathbb{Z}-\nu'} r :\psi^{\dagger}_{-r}\psi_r:\right) + \begin{cases} \frac{1}{2}(\nu-1) & \text{(NS sector);} \\ 0 & \text{(R sector).} \end{cases}$$

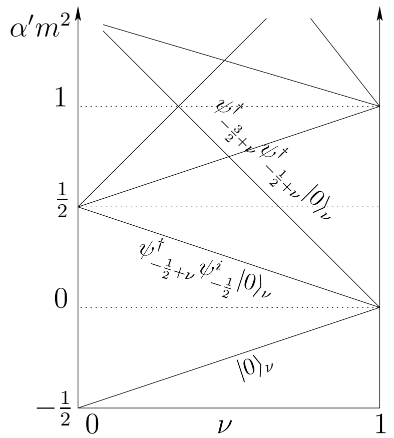

Figure 3.1.a: The NS+ spectrum of the states stretched between a brane pair at an angle $\phi = \pi\nu$. From the brane anti-brane system $\nu = 0$, the spectrum flows to the NS+ sector for the brane brane system at $\nu = 1$. $\psi^i_{-\frac{1}{2}}$ are the fermionic raising operators in the transverse directions in which the branes are parallel.

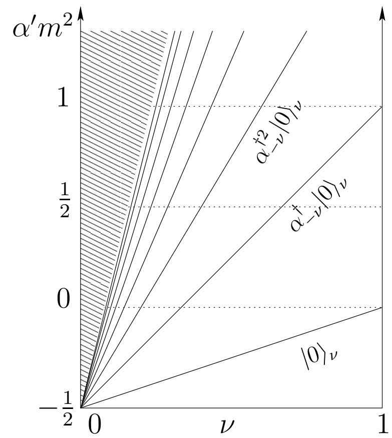

Figure 3.1.b: The bosonic excitations of the tachyonic NS vacuum which become momentum modes when $\nu = 0$; the states of different masses correspond to KK-like excitations of the lowest state, and the spectrum is indicative of an expansion in states localised about the intersection.

The twisted vacuum $|0\rangle_\nu$ is defined by

$$\alpha^\dagger_{n-\nu}|0\rangle_\nu = 0, \quad n \geq 1, \qquad\qquad \psi_{n-\nu'}|0\rangle_\nu = 0, \quad n \geq 1,$$

$$\alpha_{m+\nu}|0\rangle_\nu = 0, \quad m \geq 0, \qquad\qquad \psi^\dagger_{m+\nu'}|0\rangle_\nu = 0, \quad m \geq 0.$$

Note that flowing from $\nu = 0$ through $\nu = \frac{1}{2}$, $|0\rangle_\nu$ becomes the first excited state in the NS sector, and $\psi_{-\frac{1}{2}+\nu}|0\rangle_\nu$ becomes the state of lowest energy.

The spectrum starting with the brane anti-brane (NS−) spectrum at $\phi = \pi\nu = 0$ which flows to the brane brane (NS+) spectrum at $\phi = \pi\nu = \pi$. For $\nu > 0$ there is a splitting of the spectrum since the ψ oscillators in the different directions have different weights. The NS spectrum can be easily computed from the results above and is described in Fig. 3.1.a.

It is important to note also that the bosonic oscillators, $\alpha^{\dagger}_{-\nu}$ and $\alpha_{-1+\nu}$ are momentum oscillators at $\nu = 0$ and $\nu = 1$ respectively. Close to the brane anti-brane ($\nu = 0$) the modes created by $\alpha^{\dagger}_{-\nu}$ acting on the lowest tachyonic mode can still be tachyonic, as shown in Fig. 3.1.b. As the angle is decreased, additional tachyonic states appear at discrete intervals of the angle given by $\nu = 1/(2n+1)$ with $n = 0, 1, 2, \ldots$ For $\nu \neq 0$ or 1 these states are localised near the intersection point of the branes. Their masses decrease with ν as shown in Fig. 3.1.b until they all become degenerate with the lowest lying tachyon at $\nu = 0$. Hence, there is have an infinite tower of tachyonic states collapsing on top of each other at $\nu = 0$, reminiscent of decompactification in Kaluza-Klein theories. In this case, the process is similar to decompactification along the brane worldvolume: when the branes become anti-parallel at $\nu = 0$, the tachyon is no more localised at an intersection point and can travel along the worldvolume. Finally, note also that a mirror picture takes hold on the other side at $\nu = 1$. In that region, a infinite number of massive states collapse to zero mass comprising the modes of (gauge) fields localised at the intersection point.

3.1.2 The Brane Anti-Brane System in Boundary Superstring Field Theory

Boundary String Field Theory (BSFT) is a formalism that rose to power in verifying Sen's conjectures in the early 21$^{\text{st}}$ century. Although not really a "String Field Theory" by a purist's definition because of its inability to treat massive modes [51], it is extremely useful to understand the behaviour of tachyons and massless particles. For instance, the DBI can be derived by BSFT, which is basically a formalisation of the sigma-model approach to string theory (for massless fields at least) [52]. Already reviewed in many places (see [51, 53] for instance), here shall be given only a heuristic description of the formalism.

The idea is simple: the usual way in which the string theorist calculates an effective action for a spacetime mode is to calculate various tree-level scattering amplitudes, extract the corresponding Feynman rules, and reconstruct the Lagrangian. The sigma-model approach for massless open string modes is to do this infinitely many times all at once by evaluating the disc path integral with insertion of appropriate operators on the boundary of the disc. The expansion of the exponential of local boundary operators is akin to summing over all scattering amplitudes. This process is only useful when the path integral with the boundary insertion can be evaluated: in the special cases in which the worldsheet theory remains free, or the worldsheet is some interacting CFT. BSFT essentially extends the sigma-model approach to string theory, in that (under certain conditions [54, 55]) the disc world-sheet partition function with appropriate boundary insertions gives the classical spacetime action. For the Bosonic theory there are complications when the boundary insertion is non-conformal and the string field is not on-shell; then the β-functions of the boundary operators must be taken into account. These concerns evaporate for the superstring version [56,57,58], and there is the simple correspondence between the spacetime action of the string field and the perturbed worldsheet partition function.

The brane anti-brane effective action was calculated in [59, 60], following work on brane decay in Bosonic string theory [55, 61] and non-BPS branes in type II superstring theories [56]. Here, attention shall be restricted to D9-branes in type IIB theory; generalisations can be made using by T-dualing this configuration. In the NS sector the spacetime action is

$$S_{\text{spacetime}} = -\int \mathcal{D}X\mathcal{D}\psi\mathcal{D}\tilde{\psi}\; e^{-S_\Sigma - S_{\partial\Sigma}}. \tag{3.2}$$

where Σ is the worldsheet disc and $\partial\Sigma$ is its boundary. The worldsheet bulk disc action is the usual one

$$\begin{aligned} S_\Sigma &= \frac{1}{2\pi\alpha'}\int d^2z\, \partial X^\mu \bar{\partial} X_\mu + \frac{1}{4\pi}\int d^2z \left(\psi^\mu \bar{\partial}\psi_\mu + \tilde{\psi}^\mu \partial \tilde{\psi}_\mu\right) \\ &= \frac{1}{2}\sum_{n=1}^{\infty} n X^\mu_{-n} X_{n\,\mu} + i\sum_{r=\frac{1}{2}}^{\infty} \psi^\mu_{-r}\psi_{r\,\mu}, \end{aligned}$$

after expanding the fields in the standard modes. To reproduce the DBI action for a single brane, the appropriate boundary insertion is the boundary pullback of the $U(1)$ gauge superfield to which the open string ends couple; for the N brane M anti-brane system, the string ends couple to the superconnection [62,63], hence the boundary insertion should be

$$\begin{aligned} e^{-S_{\partial\Sigma}} &= \mathrm{Tr}\, \boldsymbol{P} \exp\left[\int d\tau d\theta \mathcal{M}(\boldsymbol{X})\right], \\ \mathcal{M}(\boldsymbol{X}) &= \begin{pmatrix} iA^1_\mu(\boldsymbol{X}) D\boldsymbol{X}^\mu & \sqrt{\alpha'}T^\dagger(\boldsymbol{X}) \\ \sqrt{\alpha'}T(\boldsymbol{X}) & iA^2_\mu(\boldsymbol{X}) D\boldsymbol{X}^\mu \end{pmatrix}, \end{aligned} \tag{3.3}$$

where bold quantities indicate superfields. The complex tachyons appear in the off-diagonal Chan-Paton sector as noted in § 3.1.1, because they are the string modes stretching between the branes and anti-branes. The insertion above must be supersymmetrically path ordered to preserve worldsheet supersymmetry and gauge invariance. $A^{1,2}$ are the $U(N)$ and $U(M)$ connections, and T is the tachyon matrix transforming in the $(N, \bar{M})$ of $U(N)\times U(M)$. The lowest component of $\mathcal{M}$ is proportional to the superconnection. To proceed, it is simplest to perform the path-ordered trace by introducing boundary fermion superfields [64]; the reader should refer to [59] for details. The insertion (3.3) can then be simplified to be the path ordered trace of

$$\exp\left[i\alpha'\int d\tau\begin{pmatrix} F^1_{\mu\nu}\psi^\mu\psi^\nu + iT^\dagger T + \frac{1}{\alpha'}A^1_\mu\dot{X}^\mu & -iD_\mu T^\dagger\psi^\mu \\ -iD_\mu T\psi^\mu & F^2_{\mu\nu}\psi^\mu\psi^\nu + iTT^\dagger + \frac{1}{\alpha'}A^2_\mu\dot{X}^\mu \end{pmatrix}\right], \tag{3.4}$$

where the tachyon covariant derivatives are

$$D_\mu T = \partial_\mu T + iA^1_\mu T - iTA^2_\mu. \tag{3.5}$$

This expression reproduces the expected results when the tachyon and its derivatives vanish: the only open string excitations will be the gauge fields on the branes and the anti-branes, for each of which the action is the standard DBI action (although for N or $M > 1$, this will be the non-Abelian generalisation of the DBI action, the full form of which is unknown). When $N = M = 1$, $DT = T = 0$, $F^{1,2}$ =const, the partition function (3.2) with the insertion (3.4) leads to

$$S_{\mathrm{D\bar{D}}} = -\tau_9\int d^{10}x\left[\sqrt{-\det(g + 2\pi\alpha' F^1)} + \sqrt{-\det(g + 2\pi\alpha' F^2)}\right]. \tag{3.6}$$

Note that this derivation also makes clear the oft-stated fact that the DBI receives corrections from non-constant field-strengths; in such cases there are corrections to (3.6), and the exact expression cannot be obtained because the worldsheet theory is interacting.

The measure in (3.2) was defined to reproduce the correct tension for the D9-branes, $\tau_9 = 1/[(2\pi)^9 g_s {\alpha'}^5]^2$. Unfortunately (3.4) cannot in general be simplified, but for a single brane anti-brane pair, $N = M = 1$, demanding that the gauge

[2]Throughout this section the dilaton is assumed stabilised to give an effective string coupling $e^\phi = g_s$.

field to which the tachyon couples vanishes, $A^- \equiv A^1 - A^2 = 0$, the path-ordered trace can be performed. Writing $A^+ = A^1 + A^2$ [59],

$$S_{\partial\Sigma} = -\int d\tau \left[\alpha' T\bar{T} + \alpha'^2(\psi^\mu \partial_\mu T)\frac{1}{\partial_\tau}(\psi^\nu \partial_\nu \bar{T}) + \frac{i}{2}\left(\dot{X}^\mu A^+_\mu + \frac{1}{2}\alpha' F^+_{\mu\nu}\psi^\mu\psi^\nu\right)\right]. \tag{3.7}$$

The operator $1/\partial_\tau$ acting on a function $f(\tau)$ is defined to be the convolution of f with $\text{sgn}(\tau)$ over the worldsheet boundary. For linear tachyon profiles, gauge and spacetime rotations allows the tachyon profile to be written as $T = u_1 X^1 + iu_2 X^2$, and (3.2) can be calculated, since the functional integrals are all Gaussian. The result when $A^+ = 0$ is derived in [59, 60]:

$$S_{\text{D}\bar{\text{D}}} = -2\tau_9 \int d^{10}X_0 \;\exp\left[-2\pi\alpha'[(u_1 X_0^1)^2 + (u_2 X_0^2)^2]\right] \mathcal{F}(4\pi\alpha'^2 u_1^2)\, \mathcal{F}(4\pi\alpha'^2 u_2^2). \tag{3.8}$$

where the function $\mathcal{F}(x)$ is given by [56]

$$\mathcal{F}(x) = \frac{4^x x\Gamma(x)^2}{2\Gamma(2x)} = \frac{\sqrt{\pi}\Gamma(1+x)}{\Gamma(\frac{1}{2}+x)}. \tag{3.9}$$

Note that $\mathcal{F}(x) = 0$ at $x = -1/2$, and

$$\mathcal{F}(x) = \begin{cases} 1 + (2\ln 2)x + \left[2(\ln 2)^2 - \frac{\pi^2}{6}\right] x^2 + \mathcal{O}(x^3), & 0 < x \ll 1, \\ \sqrt{\pi x}\left[1 + \frac{1}{8x} + \mathcal{O}(\frac{1}{x^2})\right], & x \gg 1, \\ -\frac{1}{1+x}, & x \to -1. \end{cases} \tag{3.10}$$

This action exhibits all the intricate properties of the D$\bar{\text{D}}$ system expected from Sen's conjectures: the tachyon potential at its minimum $T \to \infty$ completely cancels the brane tensions; even codimension solitons can appear on the D9-brane worldvolume, with exactly the correct tension to be lower dimensional D-branes; odd codimension solitons on which tachyonic fields reside can appear, with exactly the tension of the unstable non-BPS branes of type II string theories [49].

BSFT can also give the analogue of the D-brane Chern-Simons action for the D$\bar{\text{D}}$ system, defined similarly to (3.2), but with all fermions in the Ramond sector. The bulk contribution to the partition sum can be written as the wave-functional [59, 60]

$$\Psi^{RR}_{\text{bulk}} \propto \exp\left[-\frac{1}{2}\sum_{n=1}^{\infty} nX^{\mu}_{-n}X_{n\,\mu} - i\sum_{n=1}^{\infty}\psi^{\mu}_{-n}\psi_{n\,\mu}\right] C,$$
$$C = \sum_{\text{odd }p} \frac{(-i)^{\frac{9-p}{2}}}{(p+1)!} C_{\mu_0\cdots\mu_p}\psi_0^{\mu_0}\cdots\psi_0^{\mu_p}.$$

The ψ^{μ}_0 are the zero modes of the Ramond sector fermions, and $C_{\mu_0\cdots\mu_p}$ are the even RR forms of IIB string theory. The normalisation of Ψ can be set later by demanding that the correct brane charge is reproduced. The Chern-Simons action is then defined by

$$S_{\text{CS}} = \int \mathcal{D}X\mathcal{D}\psi\ \Psi^{RR}_{\text{bulk}}\text{Tr}^*\ Pe^{-S_{\partial\Sigma}},$$

in which the trace given by

$$\text{Tr}^*\ O \equiv \text{Tr}\ \left[\begin{pmatrix} \mathbb{1}_{N\times N} & 0 \\ 0 & -\mathbb{1}_{M\times M}\end{pmatrix} O\right]$$

results from the periodicity of the worldsheet fermion superfield which was necessary to implement to the supersymmetric path ordering. Again $e^{-S_{\partial\Sigma}}$ can be written as (3.4), with Ramond sector fermions. This expression can be viewed as a one dimensional supersymmetric partition function on $\mathbb{S}^1$, and because the Ramond sector fermions are periodic, this is equivalent to Tr $(-1)^F e^{-\beta H}$. By Witten's argument [65], only the zero modes contribute to the partition sum, giving [59, 60, 66]

$$S_{\rm CS} = \tau_9 g_s \int C \wedge {\rm Tr}^* \, e^{2\pi\alpha' i\mathcal{F}}, \tag{3.11}$$

$$\mathcal{F} = \begin{pmatrix} F^1 + iT^\dagger T & -i(DT)^\dagger \\ -iDT & F^2 + iTT^\dagger \end{pmatrix}$$

$\mathcal{F}$ is the curvature of the superconnection, and as usual, the fermion zero modes form the basis for the dual vector space and all forms above are written with $\psi_0^\mu \to dx^\mu$. This expression is exact[3] and although it was derived for 2^{m-1} brane anti-brane pairs in [59, 60] it can be shown to be valid for the general N brane M anti-brane case. As for the action (3.8), this result affirms Sen's conjectures in that it exhibits appropriate coupling to the RR 10-form potential, and the even codimension solitons have the correct couplings to the relevant RR forms to be identified as lower dimensional branes.

As written, the action (3.8) for a single brane anti-brane pair does not manifest the necessary gauge covariance, and this form of the action is valid only for linear tachyon profiles[4]. The theorist deriving the DBI action confronts the same problem, because that derivation relies on choosing a spacetime gauge, performing the path-integral, and restoring gauge covariance by hand in the result. To perform this process for the brane anti-brane tachyon action, note first that there are precisely two independent Lorentz and $U(1)$ invariant expressions in terms of first derivatives of the complex tachyon T [60, 67],

$$\mathcal{X} \equiv 2\pi{\alpha'}^2 g^{\mu\nu}\partial_\mu T \partial_\nu \bar{T}, \qquad \mathcal{Y} \equiv \left(2\pi{\alpha'}^2\right)^2 \left(g^{\mu\nu}\partial_\mu T\partial_\nu T\right)\left(g^{\alpha\beta}\partial_\alpha \bar{T}\partial_\beta \bar{T}\right),$$

(with normalisations chosen for convenience). For the linear profile $T = u_1 x_1 +$

[3] As discussed in [59], this action is exact in T and $A^\pm$ and their derivatives, but has corrections for non-constant RR forms.

[4] A covariant perturbative action was derived in [60] to order ${\alpha'}^2$.

iu_2x_2, the only translationally invariant way to re-express $u_{1,2}$ is as $u_{1,2} = \partial_{1,2}T^{1,2}$; then with $g^{\mu\nu} = \eta^{\mu\nu}$, $\mathcal{X}$ and $\mathcal{Y}$ can be calculated

$$\mathcal{X} = 2\pi{\alpha'}^2(u_1^2 + u_2^2),$$
$$\mathcal{Y} = \left(2\pi{\alpha'}^2\right)^2(u_1^2 - u_2^2)^2,$$

and so the arguments of $\mathcal{F}$ in (3.8) must be written as

$$4\pi{\alpha'}^2u_1^2 = \mathcal{X} + \sqrt{\mathcal{Y}},$$
$$4\pi{\alpha'}^2u_2^2 = \mathcal{X} - \sqrt{\mathcal{Y}}.$$

This provides a unique way to covariantise (3.8) as

$$S_{\mathrm{D}\bar{\mathrm{D}}} = -2\tau_9\int d^{10}x\sqrt{-g}\; e^{-2\pi\alpha' T\bar{T}}\,\mathcal{F}(\mathcal{X} + \sqrt{\mathcal{Y}})\,\mathcal{F}(\mathcal{X} - \sqrt{\mathcal{Y}}). \tag{3.12}$$

Restoring the spherical and gauge symmetry in this expression shall later be seen to allow construction of multiple codimension-2 BPS solitons as expected from K-theory arguments [63].

Further, the A^+ dependence of the action can be restored, since (3.7) remains quadratic when $A^+ \neq 0$ if F^+ is constant and the partition function (3.2) will be Gaussian. A similar calculation was performed for the non-BPS brane action [68], and borrowing that result gives the extended tachyon and gauge field action

$$S_{\mathrm{D}\bar{\mathrm{D}}} = -2\tau_9\int d^{10}x\; e^{-2\pi\alpha' T\bar{T}}\sqrt{-G}\;\; \mathcal{F}(\mathcal{X} + \sqrt{\mathcal{Y}})\,\mathcal{F}(\mathcal{X} - \sqrt{\mathcal{Y}}), \tag{3.13}$$

where now $G_{\mu\nu} = g_{\mu\nu} + \pi\alpha' F^+_{\mu\nu}$ forms the effective metric for the tachyon, as is usual for open string states in the presence of a gauge connection [69]:

$$\mathcal{X} \equiv 2\pi{\alpha'}^2 G^{\{\mu\nu\}}\partial_\mu T\partial_\nu\bar{T} \qquad\qquad \mathcal{Y} \equiv \left|2\pi{\alpha'}^2 G^{\mu\nu}\partial_\mu T\partial_\nu T\right|^2.$$

Indices are raised and lowered with respect to G: $G^{\mu\nu}G_{\nu\alpha} = \delta^{\mu}_{\alpha}$, and $G^{\{\mu\nu\}}$ indicates the symmetric part of G; this symmetrisation is necessary to obtain a real action[5]. This coupling to F^+ can be confirmed considering that the D$\bar{\text{D}}$ system reduces to the non-BPS brane system under the spacetime IIA $\leftrightarrow$ IIB quotient $(-1)^{F_L}$ [49], which in this system is applied by setting $\bar{T} = T$, $F^1 = F^2$:

$$S_{\mathrm{D\bar{D}}} \xrightarrow[A^1=A^2]{\bar{T}=T} -2\tau_9 \int d^{10}x\, e^{-2\pi\alpha' T^2} \sqrt{-G}\; \mathcal{F}(2\mathcal{X})\, \mathcal{F}(0) = \sqrt{2} S_{\mathrm{nBPS}}.$$

The overall normalisation of the action must be divided by $\sqrt{2}$ to compensate for the extra boundary fermion in the D$\bar{\text{D}}$ system which was integrated over, which is superfluous in the non-BPS brane system.

The action (3.13) is still incomplete in that $A^- = A^1 - A^2$, the $U(1)$ connection to which the tachyon couples, was set to zero in its derivation. The extension to $A^- \neq 0$ can be conjectured based on the following information:

- Gauge covariance demands that all tachyon derivatives must be replaced by covariant derivatives. A^- cannot appear outside a covariant derivative, so (3.13) with $\partial T \to DT$ can only suffer corrections for non-constant A^- (and of course, the higher T and A^+ derivative corrections).

- (3.6) should be reproduced for $T = DT = 0$.

- The gauge connections are expected to appear in the matrix form

$$\begin{pmatrix} \frac{1}{2}F^+ & 0 \\ 0 & \frac{1}{2}F^+ \end{pmatrix} \to \begin{pmatrix} F^1 & 0 \\ 0 & F^2 \end{pmatrix},$$

[5]It is also possible to include a term in $\mathcal{X}$ proportional to the anti-symmetric part of $G^{\mu\nu}$, which must have an imaginary coefficient for the sake of reality. The coefficient of such a term is undetermined by these arguments, and shall be unimportant in this analysis of the action.

when $F^- = F^1 - F^2 \neq 0$ is restored. This can be inserted into the action (3.13) and $U(2)$ indices can be traced over.

This leads to the next improvement to (3.13),

$$S_{\mathrm{D\bar{D}}} = -\tau_9 \int d^{10}x \; e^{-2\pi\alpha' T\bar{T}} \left[\begin{array}{l} \sqrt{-\det[G_1]}\, \mathcal{F}(\mathcal{X}_1 + \sqrt{\mathcal{Y}}_1)\, \mathcal{F}(\mathcal{X}_1 - \sqrt{\mathcal{Y}}_1) \\ +\sqrt{-\det[G_2]}\, \mathcal{F}(\mathcal{X}_2 + \sqrt{\mathcal{Y}}_2)\, \mathcal{F}(\mathcal{X}_2 - \sqrt{\mathcal{Y}}_2) \end{array} \right], \tag{3.14}$$

$$(G_{\mu\nu})_{1,2} \equiv \left(g_{\mu\nu} + 2\pi\alpha' F^{1,2}_{\mu\nu}\right), \qquad \begin{aligned} \mathcal{X}_{1,2} &\equiv 2\pi\alpha'^2 G^{\{\mu\nu\}}_{1,2} D_\mu T D_\nu \bar{T}, \\ \mathcal{Y}_{1,2} &\equiv \left|2\pi\alpha'^2 G^{\mu\nu}_{1,2} D_\mu T D_\nu T\right|^2, \end{aligned}$$

The tachyon is charged only under A^-:

$$D_\mu T = \partial_\mu T + i A^-_\mu T, \qquad A^\pm_\mu = A^1_\mu \pm A^2_\mu,$$

and the function $\mathcal{F}(x)$ is defined in (3.9). This is the effective action which shall be studied in this work. Corrections to this action will include higher derivative terms in T and $F^\pm$. Possible terms like $(F^-)^n T\bar{T}$ may be included in higher tachyon derivatives since $[D_\mu, D_\nu] = iF_{\mu\nu}$. Being non-supersymmetric, there will be quantum corrections to the action as well.

3.1.3 Brane Anti-Brane Defects

In this section, the brane anti-brane action (3.14, 3.11) is analysed, and codimension two solitons are constructed. These vortices are shown to have the correct tension and charge to be identified as D7-branes, and the covariant D$\bar{\mathrm{D}}$ action allows construction of multiple parallel solitons. Let $z = x^1 + ix^2$ be the coordinate in the complex plane transverse to the D7-branes. Since T is uncharged under $A^+ = A^1 + A^2$ only solutions with $A^+ = 0$ are studied. For the *ansatz* considered the energy density $\varepsilon_7 = (n+m)\tau_7$ while the total RR charge is $\mu_7 = (n-m)\tau_7 g_s$. The solution for multiple D7-branes without gauge flux was first studied by worldsheet methods in [70]; the tensions for multi-kink solitons on non-BPS brane worldvolumes were calculated in [71], and the multiple vortex solutions were found in [72].

More insight into the form of the solution giving multi-soliton branes can be gained by looking at the Chern-Simons action (3.11). Multi-soliton solutions can be constructed with trivial gauge fields, just as in the single soliton case. In fact, for soliton solutions the RR charge is completely independent of the gauge field to which the tachyon couples [72].

Starting with Eq. (3.11), for D7-brane solitons on a single brane anti-brane pair, only nonzero RR field C_8 is considered, and the gauge field under which T is inert is set to zero, $A^+ = 0$, $F^+ = 0$:

$$S_{\text{CS}} = \tau_9 g_s \int e^{-2\pi\alpha' T\bar{T}}(-iC_8) \wedge \left[2\pi\alpha' iF^- - (2\pi\alpha')^2 DT \wedge D\bar{T}\right]. \tag{3.15}$$

The coupling to the field strength, F^-, is the standard one, giving the unstable 9-brane system coupling to 7-branes. The second term gives the soliton coupling, and the system can decay to solitons with trivial gauge fields. For brevity, it is useful to write the RR charge μ_7 of the soliton under a C_8 which is constant in the plane in which T condenses as

$$\mu_7 = -i\frac{\tau_7 g_s}{2\pi}\int_{\mathbb{R}^2} e^{-2\pi\alpha' T\bar{T}}\left[iF^- - (2\pi\alpha')DT \wedge D\bar{T}\right], \qquad \tau_7 = 4\pi^2\alpha'\tau_9. \tag{3.16}$$

A single D7-brane solution [59, 60], can be written in polar coordinates on $\mathbb{R}^2$ as $A^{\pm} = 0$, $F^{1,2} = 0$, $T = uz = ure^{i\theta}$:

$$\begin{aligned}\mu_7 &= i\frac{\tau_7 g_s}{2\pi}\int e^{-2\pi\alpha' u^2 r^2}(2\pi\alpha')u^2(-2ir)dr \wedge d\theta \\ &= \tau_7 g_s.\end{aligned}$$

Here u can take any value without altering the RR charge of the soliton.

Multi-centered solutions were constructed in [72] where it was proved that μ_7 is independent of the gauge field winding about the vortex centres. With weak assumptions that are sensible for vortices, (3.16) can be shown to reduce to

$$\mu_7 = -i\frac{\tau_7 g_s}{2\pi}\int\limits_{\mathbb{C}} \frac{1}{2} e^{-2\pi\alpha' T\bar{T}} d\left[\frac{dT}{T} - \frac{d\bar{T}}{\bar{T}}\right]. \tag{3.17}$$

This expression is independent of the gauge field and essentially counts the zeros and poles of T. Although this expression might not appear to be gauge invariant, the number of zeros and singularities of T is a manifestly gauge invariant quantity. Hence μ_7 is not only gauge invariant, but completely independent of A^- and its curvature. As an example, if a solution is constructed with n holomorphic and m anti-holomorphic zeros

$$T = u\prod_{i=1}^{n}(z - z_i)\prod_{j=1}^{m}(\bar{z} - \bar{z}'_j),$$

then the total RR D7-brane charge is

$$\mu_7 = \tau_7 g_s \int\limits_{\mathbb{C}} e^{-2\pi\alpha' T\bar{T}} \delta^{(2)}(T,\bar{T}) dT \wedge d\bar{T} = (n-m)\tau_7 g_s. \tag{3.18}$$

Physically, every soliton contributes one topological unit to the total RR charge of the solution as is expected.

These calculations reveal that (3.17) behaves in an intuitive manner; holomorphic or "positively wound" zeros of T correspond to D7-branes, and contribute one topological unit to μ_7, whereas anti-holomorphic or "negatively wound" zeros of T represent anti-D7-branes, and contribute oppositely to the RR charge, the sign arising from the antisymmetry of the volume element. As for the single soliton case, it is not necessary to take $u \to \infty$ to get the exact answer; this is not so when the D$\bar{\text{D}}$ action is considered.

The energy density of the solitons is calculated by first setting F^+ to zero in (3.14). To obtain the lowest energy solution it is necessary to take ∂T to be a constant, which is taken to ∞. On the worldsheet, this limit corresponds to the infrared conformal limit, or equivalently to on-shell physics. Also on the worldsheet, this amounts to localising the Bosons at the ends of the strings in the $1, 2$ directions; *i.e.* to imposing Dirichlet boundary conditions in those direction [73]. In the effective theory, the limit allows the tension to be calculated exactly, and since the solitons are to be interpreted as classical D-branes which have zero width, the regions in the plane at which $V(T\bar{T}) = 1$ must be points with all other regions having $V(T\bar{T}) = 0$. Since $V(T\bar{T}) = \exp[-2\pi\alpha' T\bar{T}]$, the potential will be maximal at the zeros of T, and shall vanish elsewhere when $u \to \infty$.

The energy per 7-volume of the action (3.14) is

$$\varepsilon_7 \equiv \int d^2x \frac{2}{\sqrt{-g}} \frac{\delta S_{\mathrm{D\bar{D}}}}{\delta g^{0\nu}} g^{\nu 0} = 2\tau_9 \int d^2x \, e^{-2\pi\alpha' T\bar{T}} \left[\mathcal{F}(\mathcal{X} + \sqrt{\mathcal{Y}}) \, \mathcal{F}(\mathcal{X} - \sqrt{\mathcal{Y}}) \right], \tag{3.19}$$

when $F^{\pm} = 0$, all fields are time independent, and spacetime is flat. $A^- = 0$ is imposed as a simplifying constraint, although more general solutions can be found [72]. Assuming T is a holomorphic function with n zeros at the points $\{z_j\}$, $T = \lim_{u\to\infty} u \prod_{j=1}^{n}(z - z_j)$, T represents n separated D7-branes, although the result is identical when some D7-brane locations coincide. The gauge field is trivial, hence $G^{z\bar{z}} = 2$ and the tension (3.19) becomes (after taking $u \to \infty$)

$$\begin{aligned} \varepsilon_7 &= 2\tau_9 \int e^{-2\pi\alpha' T\bar{T}} 4\pi^2 \alpha'^2 \, |\partial_z T \partial_{\bar{z}} \bar{T}| \, \frac{i}{2} dz \wedge d\bar{z}, \\ &= i\tau_9 \int e^{-2\pi\alpha' T\bar{T}} (2\pi\alpha')^2 dT \wedge d\bar{T}. \end{aligned}$$

Since $\partial_z T \partial_{\bar{z}} \bar{T}$ is always positive, the absolute value could be ignored. This is identical (up to the factor of g_s) to the D7-brane charge under the RR 8-form field (3.15); it was shown above that this is always equal to $n\tau_7 g_s$, hence the multi-solitons have the correct tension ($\varepsilon_7 = n\tau_7$) and exhibit BPS properties. Further, solutions can be formulated with vortices moving at constant velocities, $z_j = z_{j,0} + v_j t$. Of course the (3.19) is no longer valid when the solution has time dependence, so must be appropriately modified. The velocity dependence leads to the special relativistic γ-factors in the energy density of the resultant solution,

$$\varepsilon_7 = \tau_7 \sum_{j=1}^{n} \frac{1}{\sqrt{1-|v_j|^2}}. \tag{3.20}$$

Finally, a general configuration of D7-branes and anti-branes can be represented by the configuration $T = \lim_{u\to\infty} u \prod_{i=1}^{n}(z-z_i)\prod_{j=1}^{m}(\bar{z}-\bar{z}'_j)$. This has n D7-branes and m anti-branes (all parallel), and assuming no brane and anti-brane position coincides $z_i \neq z'_j, \forall\{i,j\}$, the tension is

$$\varepsilon_7 = 2\tau_9 \int e^{-2\pi\alpha' T\bar{T}} (2\pi\alpha')^2 T\bar{T} \left| \sum_{i,k=1}^{n} \frac{1}{z-z_i}\frac{1}{\bar{z}-\bar{z}_k} - \sum_{j,l=1}^{m} \frac{1}{z-z'_j}\frac{1}{\bar{z}-\bar{z}'_l} \right| \frac{i}{2} dz \wedge d\bar{z}.$$

In regions about each z_i or z'_j (D7-brane or D7-anti-brane) the term in absolute values is positive or negative respectively. Denoting these regions by Γ_i and Γ'_j the tension is

$$\varepsilon_7 = i\tau_9 \left(\sum_{i=1}^{n} \int_{\Gamma_i} - \sum_{j=1}^{m} \int_{\Gamma'_j} \right) e^{-2\pi\alpha' T\bar{T}} (2\pi\alpha')^2 dT \wedge d\bar{T}.$$

Each integral precisely resembles (3.15) and so the value of the integral is proportional to $+1$ for each holomorphic zero and -1 for each anti-holomorphic zero of T in the region. The boundary terms again vanish because $u \to \infty$, giving the expected result

$$\varepsilon_7 = (n+m)\tau_7, \qquad \mu_7 = (n-m)\tau_7 g_s,$$

where the result of the RR charge calculated earlier has been included for comparison.

More complicated defect configurations have been constructed, based on and agreeing with the predictions of K-theory [63]. For instance, on 2^{k-1} pairs of Dp-branes and anti-branes, codimension-$(2k)$ vortices can form and have the RR charge of D$(p-2k)$-branes [59, 60] in a process known in the K-theory literature as the Atiyah-Bott-Shapiro construction.

Finally if the brane pair are D3-branes, then the vortices that can be produced are D1-branes, or D-strings. It is trivial to show that these D-strings can be formed with electric flux along their world-volume; this follows because for the vortices to be interpretable as D-strings, they must admit electric flux, and this flux just alters the charges and tension of the vortex (whether these string types will be produced in a particular dynamical situation is another matter). By keeping the F^+ dependence in the brane anti-brane action, a q-wound vortex has action

$$S_{\mathrm{D1}} = -\tau_1 \int d^2x \mathrm{Tr}_{\,q\times q} \sqrt{-\det\left[P[g] + \pi\alpha' F^+\right]} + \mu_1 \int \left\{ C^{(2)} + C^{(0)} \wedge e^{\pi\alpha' F^+ + B} \right\}$$

The electric worldvolume flux F^+ can be seen to be quantised by the usual arguments of Dirac quantisation. These objects are the (p, q)-strings, bound states of D-strings and fundamental (F-)strings, which shall play an important rôle later, and shall be described more fully in § 5.1.

3.1.4 Separated Brane Anti-Branes

There is one final crucial property of brane anti-brane systems that must be emphasised: since the masses squared of all strings between two branes increases with brane separation squared, there is only a tachyon between a brane and anti-brane at separation of order the string scale. First discussed in [24], this is merely due to the fact that the stretching of these strings increases their energy. In fact, using T-duality with the results of the previous sections, the separation-dependent tachyon potential was calculated [72], where the procedure of [74] was closely followed. The T-duality properties of the various fields in the action are well known; the gauge fields in the T-dual directions transform into the adjoint scalars, the metric and Kalb-Ramond field mix, the string coupling scales. Being an open string scalar state, the tachyon is inert under T-duality, as can be easily verified by observing the action of T-duality on its vertex operator. Under T-duality in directions labeled by uppercase Latin indices, (lowercase Latin indices labeling unaffected directions on the brane), the fields transform as [74]

$$\begin{aligned} &T \to T, && A_a \to A_a, && A_I \to \frac{\Phi^I}{2\pi\alpha'}, \\ &E_{\mu\nu} \equiv g_{\mu\nu} + B_{\mu\nu}, && e^{2\phi} \to e^{2\phi}\det E^{IJ}, && E_{IJ} \to E^{IJ} \\ &E_{ab} \to E_{ab} - E_{aI}E^{IJ}E_{Jb}, && E_{aI} \to E_{aK}E^{KI}, && E_{Jb} \to -E^{JK}E_{Kb}, \end{aligned}$$

where E^{IJ} is the matrix inverse to E_{IJ}. The result of T-dualing $9-p$ dimensions can be written most simply by defining the pull-back in normal coordinates as:

$$\begin{aligned} P[E_{ab}]^{1,2} &\equiv E_{ab} + E_{I\{a}\partial_{b\}}\Phi^{I\,1,2} + E_{IJ}\left(\partial_a\Phi^I\partial_b\Phi^J\right)^{1,2}, \\ P[E_{aI}]^{1,2} &\equiv E_{aI} + E_{JI}\partial_a\Phi^{J\,1,2}. \end{aligned}$$

Care must be taken because there are two sets of scalars which describe the position of each brane; they are denoted herein by $\Phi^{I\,1,2}$ and their difference as $\varphi^I \equiv \Phi^{I\,1} - \Phi^{I\,2}$ which is the scalar representing the D$\bar{\text{D}}p$ separation. In calculating the pull-back of any quantity only the indices corresponding to directions along the brane are affected. After T-dualing the fields in the D$\bar{\text{D}}$ action as above and performing manipulations similar to those in [74], the action for a Dp brane anti-brane pair is:

$$S_{\mathrm{D\bar{D}}p} = -\tau_p \int d^{p+1}x\, e^{-2\pi\alpha' T\bar{T}} \Big[\sqrt{-\det[G_1]}\, \mathcal{F}(\mathcal{X}_1 + \sqrt{\mathcal{Y}}_1)\, \mathcal{F}(\mathcal{X}_1 - \sqrt{\mathcal{Y}}_1) + \sqrt{-\det[G_2]}\, \mathcal{F}(\mathcal{X}_2 + \sqrt{\mathcal{Y}}_2)\, \mathcal{F}(\mathcal{X}_2 - \sqrt{\mathcal{Y}}_2)\Big] \quad (3.21)$$

where now the effective metric contains the spacetime metric pulled-back to the brane worldvolume (and includes any non-zero NS-NS B field)

$$G^{1,2}_{ab} \equiv P[E_{ab}]^{1,2} + 2\pi\alpha' F^{1,2}_{ab},$$

and the covariant derivative dependence of $\mathcal{X}$ and $\mathcal{Y}$ in (3.14) leads to Φ dependence in the T-dual action. The complete expressions for $\mathcal{X}$ and $\mathcal{Y}$ are

$$\mathcal{X}_{1,2} = 2\pi\alpha'^2 \left[G^{\{ab\}}_{1,2} D_a T D_b \bar{T} + \tfrac{1}{(2\pi\alpha')^2}\varphi^I\varphi^J T\bar{T}\left(E_{\{IJ\}} - G^{ab}_{1,2} P[E_{\{Ia}E_{bJ\}}]^{1,2}\right) + \tfrac{i}{2\pi\alpha'}\left(G^{ab}_{1,2}P[E_{bI}]^{1,2} - G^{ba}_{1,2}P[E_{Ib}]^{1,2}\right)\left(TD_a\bar{T} - \bar{T}D_aT\right)\varphi^I \right]$$

$$\mathcal{Y}_{1,2} = \left(2\pi\alpha'^2\right)^2 \left| G^{ab}_{1,2} D_a T D_b T + \tfrac{i}{2\pi\alpha'} G^{ab}_{1,2}\left(P[E_{bI}]^{1,2} D_a T - P[E_{Ia}]^{1,2} D_b T\right) T\varphi^I - \tfrac{1}{(2\pi\alpha')^2}\varphi^I\varphi^J T^2 \left(E_{IJ} - G^{ab}_{1,2} P[E_{Ia}E_{bJ}]^{1,2}\right) \right|^2 .$$

Some important properties of the separation dependent tachyon potential can be verified. The potential is equal to that part of the Lagrangian which is independent of gauge fields and derivatives,

$$V(T,\varphi) = 2\tau_p e^{-2\pi\alpha' T\bar{T}}\, \mathcal{F}\left(\frac{1}{\pi}|\varphi|^2 T\bar{T}\right), \qquad |\varphi|^2 \equiv E_{IJ}\varphi^I\varphi^J, \quad (3.22)$$

which gives as the position dependent mass of the tachyon

$$m_T^2 = \frac{1}{2\alpha'}\left(\frac{|\varphi|^2}{2\pi^2\alpha'} - \frac{1}{2\ln 2}\right).$$

Apart from the discrepancy by $2\ln 2$ which appears in the BSFT calculations of the tachyon mass, this is consistent with the familiar result that as a parallel

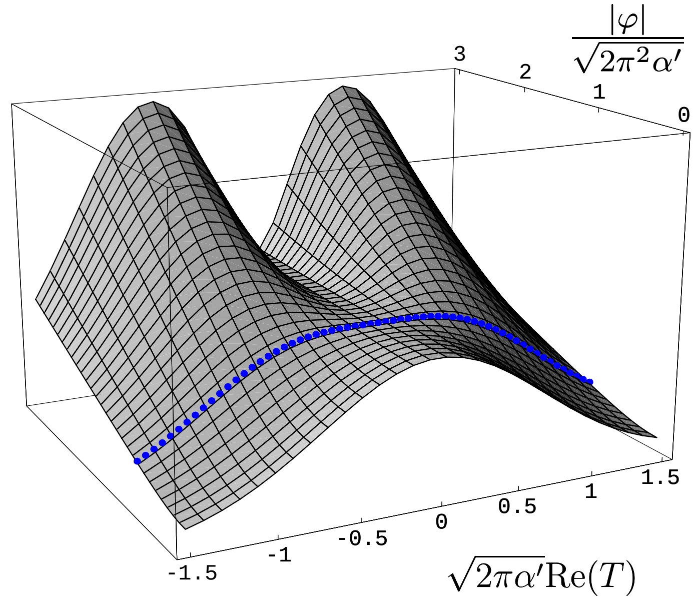

Figure 3.2: The potential for the brane anti-brane tachyon as a function of the inter-brane separation. Beyond the critical separation (dotted line), $\phi_{\rm crit} = 2\pi^2\alpha'/(2\ln 2)$, the tachyon is classically stable, and the barrier increases linearly with separation.

Dp-brane and Dp-anti-brane are moved toward each other, the lowest open string scalar mode becomes tachyonic at separations $|\varphi|^2 < 2\pi^2\alpha'$ [24].

That the separated D$\bar{\rm D}p$ system will annihilate via quantum mechanical tunneling has been well studied [75, 76]. The physical description of the instanton connecting the false and real vacua is the formation of a throat between the brane and anti-brane, which can then decay classically [75, 77] to the closed string vacuum. The separation-dependent tachyon potential (3.22) can be seen to have appropriate behaviour to represent this throat formation, because at large separations, the height of the potential barrier increases linearly with separation, as does the energy required to form a throat. The process has been studied rigorously in [72], where the above effective action was used to find the decay rate and check the validity of the thin wall approximation, including the contribution from all kinetic terms. The calculation is tractable since the tachyon decays in one direction in field space only, $T = \bar{T}$ by a choice of the diagonal $U(1)$ gauge. The value of the tachyon potential at the false vacuum, $T = 0$, is set to be zero, and the resulting Euclidean Lagrangian for the bounce [78] becomes

$$\mathcal{L}_E = 2\tau_p \sqrt{g_E}\ \left[e^{-2\pi\alpha' T^2}\, \mathcal{F}(\frac{1}{\pi}|\varphi|^2 T^2)\, \mathcal{F}(4\pi\alpha'^2 \partial_\mu T \partial^\mu T) - 1 \right].$$

The tunneling rate can be computed numerically by the standard instanton methods, where the probability of tunneling is

$$\mathcal{P} \sim K(\varphi) e^{-S_E(\varphi)},$$

$S_E(\varphi)$ being the Euclidean action of the instanton and the factor $K(\varphi)$ is due to both the quantum fluctuations about the instanton transition and to solutions of higher action which shall in general depend on the separation, φ. Calculating the bounce solution and integrating it numerically gives, to a good approximation [72],

$$S_E(\varphi) \simeq 4\pi c_1 c_2^{p+1} \left[\frac{p^p}{(p+1) g_s} \frac{2\pi^{\frac{p+1}{2}}}{\Gamma\left(\frac{p+1}{2}\right)} \right] \left(\frac{|\varphi| - |\varphi_c|}{\sqrt{\alpha'}} \right)^{\frac{p+1}{2}}, \qquad \begin{array}{l} c_1 \sim 1.5, \\ c_2 \sim 0.29, \end{array} \tag{3.23}$$

when $|\varphi| > |\varphi_c|$. S_E is expressed in this form to most easily compare to the expression for the thin wall approximation [78],

$$S_E(\varphi) \simeq \left[\frac{p^p}{(p+1)} \frac{2\pi^{\frac{p+1}{2}}}{\Gamma\left(\frac{p+1}{2}\right)} \right] \left(\frac{S_1}{\epsilon_p} \right)^{p+1} \epsilon_p$$

where S_1 is the action for the one-dimensional instanton and $\epsilon_p = 2\tau_p$. This implies that the thin wall bounce has the form

$$\frac{S_1}{\sqrt{\alpha'}\epsilon_p} = 2\pi c_2 \left(\frac{|\varphi| - |\varphi_c|}{\sqrt{\alpha'}} \right)^{\frac{1}{2}}.$$

In the thin wall approximation $c_1 = 1$, and comparison with the numerical result shows that $c_2 \simeq 0.29$. The thin wall approximation is expected to be valid when

φ becomes large. Note that S_1 differs from that in [76] (in which S_1 is linear in φ) because that calculation was performed with truncated kinetic and potential terms.

Classically, for large enough separation, when $m_T^2 > 0$, the ground state is $T = 0$, and $V(0, \varphi) = 1$. This implies that there is no force in the D$\bar{\text{D}}p$ system. However, since the system is non-supersymmetric, quantum corrections are clearly present. It is known that the one-loop open string contribution is dual to the closed string exchange. For large separation, this is dominated by the exchanges of the graviton, dilaton and RR field $C^{(p+1)}$ between the Dp and the anti-Dp-brane and has been calculated. These one-loop open string corrections can be included by inserting the classical closed string background produced by a Dp and a $\bar{\text{D}}p$ brane into the D$\bar{\text{D}}p$ action. The supergravity solution is well known [79].

$$\begin{aligned}
ds^2 &= h(r)^{\frac{1}{2}} \left(-dt^2 + \sum_{i=1}^{p} dx^i dx^i \right) + h(r)^{-\frac{3}{2} - \frac{5-p}{7-p}} dr^2 + r^2 h(r)^{\frac{1}{2} - \frac{5-p}{7-p}} d\Omega_{8-p}^2, \\
e^{-2\phi} &= g_s^{-2} h(r)^{-\frac{p-3}{2}}, \qquad\qquad h(r) = 1 - \frac{g_s \beta}{r^{7-p}} \\
C^{(p+1)}_{a_1 \ldots a_{p+1}} &= \frac{\beta}{r^{7-p}} \epsilon_{a_1 \ldots a_{p+1}}, \qquad\qquad \beta \equiv (4\pi)^{\frac{5-p}{2}} \alpha'^{\frac{7-p}{2}} \Gamma\left(\frac{7-p}{2}\right).
\end{aligned}$$

This classical closed string background of a brane shall be "felt," in a probe-brane approximation, by the anti-brane so this should be inserted into that part of the action corresponding to an anti-brane and the similar background into the brane action. On the brane worldvolumes the separation, r, is represented by the scalar field $|\varphi|$. The result of performing these steps is that when $|\varphi|^2 \gg \alpha'$, the total tension of the system is renormalised:

$$S = S_{\mathrm{D\bar{D}}p}(\varphi) + S_{\mathrm{CS}}(\varphi),$$

$$\tau_p \to \tau_p(\varphi) = \tau_p\left(\overbrace{1 - \frac{g_s\beta}{|\varphi|^{7-p}}}^{\text{NS}-\text{NS}} \underbrace{- \frac{g_s\beta}{|\varphi|^{7-p}}}_{\text{RR}}\right).$$

Clearly for a brane brane system, the sign of the RR contribution is reversed and the tension is unrenormalised. The renormalised tension then gives a potential for the scalar representing the separation, and there is an attractive force between the brane and anti-brane. When the brane separation decreases, massive closed string modes start to contribute to $\tau_p(\varphi)$. Their contributions are easy to include, except when the brane separation becomes so small that m_T^2 becomes negative; in such a case, the potential for ϕ naïvely diverges and is more difficult to calculate, but has been done in [26, 80]. When the tachyon appears, $\tau_p(\varphi)$ becomes complex; the imaginary piece indicates the instability of this vacuum, and the real part of $\tau_p(\varphi)$ remains finite. However, because the tachyon rolling occurs so rapidly, the precise form of $\tau_p(\varphi)$ at short distance becomes phenomenologically unimportant [81].

3.2 The Formation of Defects in a Cosmological Brane Model

The previous section describes the allowable defects on a brane anti-brane system, and as was discussed, when the brane pair becomes close enough, a pair of scalars becomes tachyonic and the rolling of this tachyon can form vortices. However, these vortices are higher energy configurations than the closed string vacuum that remains in the absence of defects, and so a dynamical reason is necessary to argue that these defects will be formed. This section departs from the rigor of the previous section, and relies on more conceptual arguments because the dynamics is so complicated (consequently the results are more contentious). However, many of these rudimentary arguments have been backed up with simulations of defect formation and evolution in the early universe.

3.2.1 The Kibble Mechanism and Dangerous Defects

The first concern that arises with the cosmological models of D$\bar{\text{D}}$ systems relates to overproduction of defects. One of the triumphs of inflationary cosmology is its ability to solve the monopole problem: presence of monopoles in the early universe from some spontaneous symmetry breaking transition in quantities sufficient to overclose the universe. Inflation solves this problem by inflating any monopole density which may have been produced before inflation to an insignificant level. In D$\bar{\text{D}}$ models of inflation, the symmetry-breaking phase transition occurs post-inflation, after the branes have become close enough for the tachyon to destabilise, so overclosure becomes a concern again.

The Kibble mechanism [82], reviewed in [83], is a lower bound on the production of defects during a second order phase transition in an expanding space-time. The idea is simply that at the time of the phase transition, the correlation length of the tachyon field, which in flat space should tend to infinity (being second order), in the expanding space should be limited by the Hubble volume. Neighbouring Hubble volumes, not being in causal contact shall see the tachyon roll in different directions, and a defect *can* form at the intersection of three regions of different tachyon phase. Note here that the temperature of the universe does not control the phase transition, since all thermal excitations have been inflated away and the universe is yet to reheat; rather, the inflaton field (brane separation) itself is the control parameter, and $\langle T \rangle$ is the order parameter.

This reasoning gives a typical initial scale to the string network which results: roughly one per Hubble volume at the time of the phase transition - in the D$\bar{\text{D}}$ case, the end of inflation. Although this tends to slightly overestimate the defects formed in this way because the probability of such junctions forming defects is less than unity [83], it is a good lower bound. A higher string density can result in more detailed studies because of quantum fluctuations of the tachyon field as it rolls down its potential; for certain separations below ϕ_{crit}, fluctuations of the tachyon field near the symmetry point can push the field locally over the maximum to the other side of the potential, resulting in a increase in the local defect density.

The problem of overclosure thus seems to reemerge. It shall be described in Chapter 5 why production of string-like defects is not problematic, as long as they possess self-interactions and do not dominate the energy density of the universe; however if domain wall or monopole defects can be produced by such reckoning, they will present the usual problems. Here, the details of the tachyon dynamics rescues the situation. Note first that the tachyon field is always complex, and so any stable defects will be of even codimension. This fact immediately solves the problem for all models which involve only D$\bar{\text{D}}$ branes of 3 spacial dimensions like the flux stabilised models of [28], since only strings, D1-branes and F-strings, can be produced. For other models, since they must produce a 4D world, the extra 6 dimensions must be compactified. In order to avoid the dangerous defects, the scales of compactification and the mass-scale of inflation at the time of tachyon condensation must be such that the Kibble mechanism cannot apply in the compact directions. If this were not satisfied, the tachyon field could for instance wind partially in a compact and partially in a non-compact direction, leading to a string-defect partially wrapping the compact manifold, giving a 4D monopole. In other situations domain walls could be produced.

Conversely, if the Hubble length at the end of inflation, $\frac{1}{H}$, is much greater than the compactification length, $l_{\rm comp}$, all regions in a compact direction will be in causal contact and because the correlation length of $\langle T \rangle$ should be large and extend throughout $l_{\rm comp}$, production of monopoles and domain walls would therefore be suppressed. This again relies critically on the string theoretic constraint that $V(T)$ admits only even codimension defects. That the Hubble length at the tachyon phase transition greatly exceeds the compactification scale can be easily checked for the toroidal orientifold models [23, 26, 27, 84], in which the Hubble scale at the end of inflation is $\frac{1}{H} \gtrsim (M_{\rm Pl}/M_{\rm GUT})l_s$, and the directions along which the branes must be wrapped have radii of order the string length, l_s (or at most an order of magnitude larger).

That the Kibble mechanism produces such defects has been challenged in [85] in which the authors argue that in order to produce D- or F-strings as defects on a brane anti-brane system, because these strings couple to bulk RR and NS-NS fields respectively, the defect formation mechanism must excite these bulk fields. Since the Kibble mechanism can only take place on the branes which are inflating, they argue that string types which couple to bulk fields cannot be Kibble produced. However, this argument overlooks the fact that the order parameter for the phase transition is the tachyon field, which exists only on the inflating branes. The Kibble mechanism relies only on causality and fluctuations of the order parameter, which necessarily leads to defects. The generation of bulk 2-form potentials is a question of dynamics, but cannot affect the lower bound of one defect per Hubble volume placed by causality on the system[6].

3.2.2 Thermal Defect Formation

As shall be described in Chapter 4 with specific examples, if unstable systems can survive beyond the end of inflation until such time as the universe has reheated, defects can be thermally produced. This cannot be the main mechanism for defect production, since the main phase transition occurs at essentially zero temperature; thermal production requires a phase transition at high temperature, which in the brane picture, requires an unstable brane to survive until after reheating. In order to survive and for the phase transition to occur at this late stage, the brane must firstly be a fundamentally stable object, but unstable with respect to the remaining branes of the system; it must then be temporarily stabilised by being separated from the standard model brane stack. Similar arguments as in the previous section apply limiting the thermal defects produced to be string-like. Because of all these conditions, no generic statements can be made about thermal defect production, but it will not be a significant part of the story of cosmic strings in string theory.

3.3 An Unsolved Problem: Reheating versus Tachyon Matter

There is a significant unsolved issue with any brane anti-brane model of inflation, concerning the details of reheating. In any of these models, the D$\bar{\text{D}}$ tachyon potential is a runaway potential; BSFT with a constant or linear tachyon profile gives $V(T\bar{T}) \sim \exp(-2\pi\alpha' T\bar{T})$, whereas the exact time-dependent CFT rolling

[6]Saswat Sarangi aided in refining this argument.

tachyon solution of Sen [86] is compatible with an effective action with $V(T\bar{T}) \sim 1/\cosh(|T|/\sqrt{2})$ [87, 88]. Naïvely, this would be disastrous for cosmology with inflation ending in the rolling of this tachyon, since the tachyon would continue to roll, and never decay to matter and reheat the standard model.

In recent years there has been significant attention given to the eventual state of a decaying brane anti-brane system. A precise BCFT description of the rolling D$\bar{\text{D}}$ tachyon was found [86]; *i.e.* the BCFT is an interacting but conformal worldsheet theory with non-trivial time dependence; as such, according to the mantra of string-theory, the CFT is an exact solution to classical string theory, which has a natural interpretation only as a decaying D$\bar{\text{D}}$ system. After this development followed analyses of tachyon effective actions with runaway potentials and DBI-like kinetic terms for the tachyon [87][7]. The result of both these efforts showed that as the tachyon rolls, the energy density on the brane remains constant, whereas the pressure of the "tachyon matter" vanishes. The final state of the D$\bar{\text{D}}$ system is then a pressureless dust, with the tachyon rolling approaching $\dot{T} \to 1$, and the tachyon never reaching the minimum of the potential, which for this field definition is at $T \to \infty$.

This interpretation alone is disastrous for cosmology, since the D$\bar{\text{D}}$ energy is dumped into rolling "tachyon matter", a pressureless dust, and the standard model never reheats. However, further developments, mainly in [89] interpreting the results of [88], give hope for D$\bar{\text{D}}$ cosmology. In the work [88], it is shown that Sen's interacting, time-dependent, rolling tachyon boundary state is responsible for the production of massive and spinless closed string states; this is merely particle creation in a time-dependent background. A related calculation [80] achieves the

[7]Although the BSFT D$\bar{\text{D}}$ tachyon action is not exactly DBI-like, it is very similar for some values of ∂T [59, 60].

same result by a very different method: by evaluating the imaginary part of the closed string exchange (open string one-loop) amplitude between two non-BPS D-branes. In fact, in both calculations, it is shown that the entire energy of the D$\bar{\text{D}}$ system is put into massive and almost motionless closed string modes. In [89], the tachyon matter was conjectured to be an open string description of the massive closed strings, both exhibiting the same properties of their energy momentum tensors.

This reinterpretation is promising for two reasons: there is nothing to prevent the massive closed strings from decaying, and being an intermediary step in reheating the standard model branes[8]; and all calculations thus far have been performed in the presence of no residual branes. It is conceivable that the rolling D$\bar{\text{D}}$ tachyon can directly excite fields on a nearby brane, thermally exciting the open string modes on it. This calculation is a one-loop open string calculation and looks to go like $\mathcal{O}(g_s^1)$, whereas the closed string process is $\mathcal{O}(g_s^0)$. The details of this calculation could give an understanding as to the feasibility of reheating in D$\bar{\text{D}}$ inflation.

[8] In personal communication with Ira Wasserman it was pointed out that this reheating via a bulk intermediate stage does not need to be completely efficient.

CHAPTER 4

COSMIC STRING STABILITY

In Chapter 2 it was shown that there are realistic models of inflation involving the relative movement of branes and anti-branes, and in Chapter 3 it was shown that on brane anti-brane systems there are a variety of defects which can be produced, and in physical models, only string-type defects shall be produced at the end of inflation. Here, the stability and production of these cosmic strings is discussed, and it is argued again that there are generic classes of brane inflation models in which cosmic strings can be stable or meta-stable and long-lived on cosmological timescales.

4.1 Models with Unwarped Compactifications and High Scale Supersymmetric Cosmic Strings

Here [90] is reviewed in which various conditions under which string-like vortices are stable and produced after brane inflation. Largely, only supersymmetric configurations of branes and string-defects are cataloged, and the details of inflation - whether the bulk spacetime is curved and how the moduli are fixed - is unimportant for the catalog. Given that such models exist, their cosmic string types are listed. In these models, cosmic strings are Dp-branes with $(p-1)$ dimensions compactified. The CMB radiation data fixes the superstring scale to be close to the grand unified (GUT) scale, which then determines the cosmic string tensions. The cosmic strings appear as defects of the tachyon condensation and can be D1 branes or Dp-branes wrapping a $(p-1)$-dimensional compact manifold, which yield a spectrum of cosmic string tensions including Kaluza-Klein modes. It is important to note that the analysis of this section makes many assumptions to understand the high-scale stability of cosmic strings; for instance moduli are assumed stabilised, RR and NS-NS fields to which strings couple are assumed to be projected out at a lower scale. Some of these more detailed issues of stability are discussed in the next section.

4.1.1 Various Supersymmetric Brane Inflation Scenarios

The brane inflationary scenarios of interest have the string scale close to the GUT scale so only brane world models which are supersymmetric (post-inflation) at the GUT scale are considered in this section. (Supersymmetry is expected to be broken at the TeV scale, which is negligible for the physics of interest here). In the 10-dimensional superstring theory, the cosmic strings in 4-dimensional space-time shall be D-branes with one spatial dimension lying along the 3 large spatial dimensions representing the universe. The possible stable configurations of branes of different dimensionality in 10 dimensions, compactified on a six manifold are enumerated. To be specific, consider a typical Type IIB orientifold model compactified on $(T^2 \times T^2 \times T^2)/\mathbb{Z}_N$ or some of its variations (see for instance [91,92,93,94,95,96,97,98]). The model has $\mathcal{N} = 1$ spacetime supersymmetry. Although Type IIB orientifolds are focused on, the underlying picture is clearly more general. The stable configurations of branes which remain after inflation are categorised, which give rise to stable cosmic strings in the universe. "Stable" is meant to indicated that some fraction of the cosmic strings produced is required to persist until at least the epoch of nucleosynthesis in order for observable effects to be generated.

In supersymmetric Type IIB string theory with branes and orientifold planes, it is well known that only odd spacial dimensional branes are stable. The conditions for stable brane configurations are simple given the compactification manifold; branes must differ by only 0, 4 or 8 in dimension, and branes of the same dimension can be angled at right angles in two orthogonal directions [30, 99]. In the generic case where the second homotopy class of the compactification manifold is $\pi_2 = \mathbb{Z}^3$, branes will be stable when wrapping 2-cycles in the compact manifold. From these conditions, Table 4.1 of branes from which are built simple models of post-inflation cosmology is formulated.

Table 4.1: Stable configurations of D-branes. The labels on the the D-branes indicate which of the three 2-cycles they wrap in the compactification dimensions and an empty spot indicates no wrapping/presence. For simplicity the cosmic strings are placed along the 1-direction.

	Dimension					
Stable Branes	01	23	45	67	89	
D9	✓	✓	✓	✓	✓	
$D5_1$	✓	✓	✓			} $\mathbb{R}^{3,1}$ branes
$D5_2$	✓	✓		✓		
$D5_3$	✓	✓			✓	
$D5_{1,2}$	✓		✓	✓		
$D5_{1,3}$	✓		✓		✓	} cosmic strings
$D5_{2,3}$	✓			✓	✓	
$D1_0$	✓					

In cosmological situations, branes which are non-BPS relative to the others can be present. Naïvely the non-BPS configurations will decay, and the decay products of many are well known. For instance a Dp-D$(p-2)$ brane combination will form a bound state of a Dp brane with an appropriate amount of magnetic flux [100][1]. This process is best understood as the delocalisation or "smearing out" of the D$(p-2)$ brane within the Dp brane. In the Dp-D$(p-2)$ brane system this is described by the presence of a tachyon field, an open string that stretches between them. This tachyon condenses as the D$(p-2)$ brane decays and leads to a singular magnetic flux on the Dp brane; this magnetic flux then spreads out across the Dp brane and diminishes, leaving the total flux conserved. In an uncompactified theory, the residual magnetic field strength then vanishes. Since the tachyon in the Dp-D$(p-2)$ brane combination is a complex scalar field inside the D$(p-2)$ brane world volume, its rolling/condensation allows the formation of D$(p-4)$-branes as defects. (The actual formation/production of D$(p-4)$-branes may require the dissolution of a D$\bar{\text{D}}(p-2)$ pair inside the Dp-brane.)

Another important set of non-BPS brane configurations which will be generated in early universe brane-world cosmology are branes of the same dimension oriented at general angles, which will also decay into branes with magnetic flux, as described above. There are also special cases of non-BPS configurations which will not decay; between a $\text{D}3_3$ and $\text{D}5_1$ brane (or its T-dual equivalents, for instance a D1 and D7 brane) there is a repulsive force as seen in the total inter-brane potential, which includes all gravitational and RR forces, between a Dp and a Dp'-brane ($p' < p$) (in terms of the separation distance r when $r \gg M_s^{-1}$)

$$V(r) \sim -\frac{4-(p-p'+2a)}{r^{p-7+a}}, \tag{4.1}$$

where a is the number of directions in which the branes are orthogonal [99]. This

[1]There are however some arguments to suggest that a D$(p-2)$-brane becomes a non-BPS string-like object which is stable within the Dp-brane when taking into account the RR-field dynamics as well [101], in compact cases. In this section, only BPS configurations are considered.

potential also makes clear that there is no force between the BPS configurations of branes described above - those which differ in dimension by 4 and those of the same dimension which are angled in two orthogonal directions.

Brane world models of inflation require brane anti-brane pairs (or branes oriented at non-BPS angles) [23, 26, 27, 21]; the inflaton field is described by the separation between the branes, and its potential can be organised to give slow roll inflation. To describe the Standard Model, a chiral post-inflation brane-world is necessary, which requires that the branes which form the universe are angled in some dimension; sets of $D5_1$ and $D5_2$ branes will give a stable chiral low energy effective theory, for instance.

After the compactification to 4-dimensional spacetime, the Planck mass $M_P = (8\pi G)^{-1/2} = 2.4 \times 10^{18}$ GeV is given by

$$g_s^2 M_P^2 = M_s^2 (M_s r_1)^2 (M_s r_2)^2 (M_s r_3)^2 / \pi \tag{4.2}$$

where M_s is the superstring scale and the effective compactification volumes (of the (45)-, (67)- and (89)-directions) are $V_i = l_i^2 = (2\pi r_i)^2$ for $i = 1$, 2 and 3 respectively. Also $M_s r_i \geq 1$. The string coupling g_s should be large enough for non-perturbative dynamics to stabilise the radion and the dilaton modes (but not too large that a dual version of the model has a weak coupling). The string coupling is expected generically in these models to be $g_s \gtrsim 1$. To obtain a theory with a weakly coupled sector in the low energy effective field theory (*i.e.* the standard model of strong and electroweak interactions with weak gauge coupling constant), it then seems necessary to have the brane world picture [102]. Suppose the $D5_1$-branes contain the standard model open string modes, then

$$g_s \simeq \alpha_{\mathrm{GUT}} (M_s r_1)^2 \tag{4.3}$$

where $\alpha_{\rm GUT} \simeq 1/25$ is the standard model coupling at the GUT scale, which is close to the superstring scale M_s. This implies that $(M_s r_1)^2 \sim 30$. If some standard model modes come from $D5_2$-branes, or from open strings stretching between $D5_1$- and $D5_2$-branes, then $(M_s r_2)^2 \sim 30$.

In the early universe, additional branes (and anti-branes) may be present. Additional branes must come in pairs of brane anti-brane (or at angles), so that the total (conserved) RR charge in the compactified volume remains zero (or cancels the background fluxes and those due to any orientifold planes as in [13]). Any even dimensional D-branes are non-BPS and so decay rapidly. The Hubble constant during inflation is roughly

$$H^2 \simeq M_s^4/M_P^2 \tag{4.4}$$

In Table 4.2, the various brane anti-brane pairs which provide inflation are cataloged (assuming their potentials have been arranged to provide slow-roll inflation) which can inflate the 4 dimensional Minkowski brane world volume of the pertinent example of $D5_1$- and $D5_2$ standard model branes. As described in Chapter 3, *a priori*, the defects (only cosmic strings here) which are allowable under the rules of K-theory [63] may be produced immediately after inflation, when the tachyon field starts rolling down. The cosmological production of these defects toward the end of inflation are referred to as "Kibble" in Table 4.2. During this epoch, the universe is essentially cold and so no thermal production of any defect is possible. Generically, codimension-one non-BPS defects may also be produced, however, these decay rapidly and can be ignored.

If a non-trivial 3-cycle is present in the orientifold model, a D5-brane wrapping such a cycle can appear as a domain wall, while a D3-brane wrapping it can appear as a monopole. However, such a 3-cycle will be in the (468) (or an equivalent)

Table 4.2: Various inflatons and the cosmic strings to which they decay for a brane world built of sets of $D5_1$ and $D5_2$ branes. The cosmic string types allowed are determined by K-theoretic analysis of the non-BPS systems. Since the Hubble size is greater than the compactification radii, the Kibble mechanism is capable of producing only defects localised in the three large spacial dimensions. Cosmic strings can be thermally produced if unstable states are able to persist until reheating.

	Inflation	Cosmic String Types		
Inflaton	Possible	Allowed	Kibble	Thermal
$D(9-\bar{9})$	×			
$D(7-\bar{7})_{1,2}$	✓	1_0,3_1,3_2,$5_{1,2}$	$5_{1,2}$	-
$D(7-\bar{7})_{1,3}$	✓	1_0,3_1,3_3,$5_{1,3}$	$5_{1,3}$	-
$D(5-\bar{5})_1$	✓	1_0,3_1	3_1	1_0
$D(5-\bar{5})_3$	✓	1_0,3_3	3_3	-
$D(3-\bar{3})_0$	✓	1_0	1_0	-
$D(1-\bar{1})$	×			

directions. Since none of the $D\bar{D}p$ pair that can generate inflation wrap all these 3 directions, such defects are not produced.

The various possibilities in Table 4.2 are:

- $D\bar{D}9$ pair. In this case, the tachyon field is always present and the annihilation happens rapidly. Also since the branes are coincident, there is no inflaton.

- $D\bar{D}1$ pair. Since they do not span the 3 uncompactified dimensions, they do not provide the necessary inflation. In the presence of inflation (generated by other pairs), a density of these D1-branes will be inflated away.

- $D\bar{D}3_0$ pair. They span the 3 uncompactified dimensions and move toward each other inside the volume of the 6 compactified dimensions during inflation. At the end of inflation, their collision heats the universe and yields $D1_0$-branes as vortex-like solitons. These $D1_0$-branes appear as cosmic strings, and the formation of other defects is unambiguous - they are not topologically allowed to form. The strings form a gas (with strings at all possible orientations in the 3-dimensional uncompactified space). The $D3_0$-branes are unstable in the presence of the $D5_1$ and $D5_2$ branes. It is possible that during inflation, the $D3_0$-brane can simply move toward a D5-brane and then dissolve into it. The $\bar{D}3_0$-brane can either hit the same D5-brane ending inflation, producing $D1_0$-branes as cosmic strings, or it can collide with another D5-brane. This D5-brane shall no longer be BPS with respect to the other D5-branes and more inflation may result from their interactions. Toward the end of inflation these D5-branes collide with the BPS D5-branes. $D1_0$-branes are expected to be produced as defects in this scenario.

- $D\bar{D}5_1$ pair. This $D5_1$ brane is indistinguishable from the other $D5_1$-branes that are present. They span the 3 uncompactified dimensions and move toward each other inside the volume of the 4 compactified dimensions during inflation. Toward the end of inflation, a tachyon field appears and its rolling produces $D3_1$ branes as cosmic strings. However such $D3_1$ branes are unstable and eventually a tachyon field (an open string mode between the D3- and the D5-branes) will emerge. Its rolling signifies the dissolution of the D3-brane into the $D5_1$ branes. Generically, by the time these D3-branes start dissolving, reheating of the universe should have taken place, so the tachyon rolling can thermally produce $D1_0$-branes as cosmic strings.

- $D\bar{D}5_3$ pair. They may generate inflation directly, and being mutually BPS with the $D5_1$ and $D5_2$ branes shall not be subject to more complicated interactions. After inflation, $D3_3$-branes as cosmic strings will be produced. Although they are not BPS with respect to the D5-branes, the interaction is repulsive (with $p=5$, $p'=3$ and $a=2$ in (4.1)), so they are expected to move away from the $D5_1$-branes in the (67) directions (to the antipodal point) and from the $D5_2$-branes in the (45) directions. These $D3_3$-branes shall mostly survive and evolve into a cosmic string network. However, some of the $D3_3$-branes will scatter with the D5-branes in the thermal bath. This may also result in the production of some $D1_0$-branes as cosmic strings.

- $D\bar{D}7_{1,3}$ pair. To provide the needed inflation, these pairs wrap 4 of the 6 compactified dimensions and move toward each other in the remaining 2 compactified dimensions during the inflationary epoch. Their collision heats the universe and yields $D5_{1,3}$-branes as cosmic strings. The D5-branes that wrap only 2 of the 4 wrapped dimensions of the D7 branes may appear to simply span all 3 uncompactified dimensions. However, the production of these objects is severely suppressed since the Hubble size is much bigger than the typical compactification sizes. While the tachyon is falling down, the universe is still cold, so no thermal production is possible either. As a result, only $D5_{1,3}$-branes that appear as cosmic strings are produced.

  It is possible for the $D5_1$-branes to dissolve into magnetic flux on the D7-brane during inflation. After the annihilation of the $D\bar{D}7_{1,3}$ pair, this flux shall reemerge as D5-branes, together with any additional D5-branes solitons as cosmic strings.

- $\mathrm{D\bar{D}7}_{1,2}$ pair. This case is similar to the above case, except both sets of D5-branes may dissolve into the D7 pair during inflation.

Only type IIB theory has been considered here, with two sets of D5-branes providing the standard model fields. Under T-duality, the branes become D9-D5-branes, or D7-D3-branes in a IIB orientifold theory, with corresponding descriptions. Generalising the above analysis to the branes-at-angle scenario [26] should be interesting. It is also possible to describe similar inflationary models with cosmic strings in Type IIA theory, in which even dimensional branes are stable. In this case, one simply adds additional brane anti-brane pairs to the $\mathcal{N}=1$ spacetime supersymmetric IIA orientifold models [103].

4.1.2 The Spectrum of the Cosmic Strings in Unwarped Models

The cosmic string tension μ is estimated for a number of brane inflationary scenarios in which the bulk is not highly warped in [27, 84, 90]. The value μ is quite sensitive to the specific scenario, and here order of magnitude estimates are provided.

The density perturbation generated by the quantum fluctuation of the inflaton field is [23, 27]

$$\delta_H \simeq \frac{8}{5\pi^2}\frac{N_e^{3/2}}{M_P r_\perp}$$

Using COBE's value $\delta_H \simeq 1.9\times 10^{-5}$ [104, 105],

$$M_p r_\perp \simeq 3\times 10^6. \tag{4.5}$$

This still leaves M_s unfixed. To estimate M_s and the cosmic string tension μ, let us consider a couple of scenarios. Consider $\mathrm{D\bar{D}5}_1$ brane inflation. With $(M_s r_1)^2 \sim 30$

and $r_2 = r_3 = r_\perp$, (4.2) and (4.5) then imply that $M_s \sim 10^{14}$ GeV. If the cosmic strings are D1-branes, the cosmic string tension μ_1 is simply the D1-brane tension τ_1:

$$\mu_1 = \tau_1 = \frac{M_s^2}{2\pi g_s}.$$

This implies that $G\mu \simeq 6 \times 10^{-12}$. Now the D1-brane may have discrete momenta in the compactified dimensions. These Kaluza-Klein modes give a spectrum of the cosmic string tension,

$$\mu \rightarrow \mu + \frac{e_1}{r_1^2} + \frac{e_2}{r_2^2} + \frac{e_3}{r_3^2},$$

where e_i (i =1, 2, 3) are respectively the discrete eigenvalues of the Laplacians on the (45), (67) and (89) compactification cycles. An order of magnitude estimate sees the lowest excitation raise the tension by a few percent.

For D$\bar{\text{D}}7_{1,2}$ pair inflation, and $(M_s r_1)^2 \simeq (M_s r_2)^2 \sim$30, $r_3 = r_\perp$. In this case, $M_s \sim 4 \times 10^{14}$ GeV, with D$5_{1,2}$-branes as cosmic strings. Noting that a Dp-brane has tension $\tau_p = M_s^{p+1}/(2\pi)^p g_s$, the tension of such cosmic strings is

$$\mu_5 = \frac{(M_s r_1)^2 (M_s r_2)^2 M_s^2}{2\pi g_s},$$

which yields $G\mu \sim 10^{-8}$. This tension is bigger than that of D1-branes. Depending on the particular inflationary scenario, this value may vary by an order of magnitude. For D$\bar{\text{D}}$ inflation, roughly [84]

$$10^{-12} \lesssim G\mu \lesssim 10^{-7}.$$

Higher values of $G\mu$ are possible wen branes at small angles provide the inflaton.

The interesting feature of this type of cosmic strings is that there is a spectrum of cosmic string tension. The branes can wrap the compactified (4567)-dimensions more than once, which gives

$$\mu \sim nw\mu_5$$

where n is the defect winding number, and w is the wrapping number (*i.e.* the number of times it wraps the compactified volume) inside the brane, so nw is equivalent to the number of cosmic strings. Moreover, there can be Kaluza-Klein excitations of the branes propagating in these compactified directions. All these result in quite an intricate spectrum of cosmic string tensions. For $n = w = 1$,

$$\mu \sim \mu_5 \left(1 + \frac{p_1}{(M_s r_1)^2} + \frac{p_2}{(M_s r_2)^2}\right) + \frac{e_3}{r_3^2}$$

where p_1 and p_2 are discrete momentum excitation modes depending on the geometry of the (45) and the (67) directions. Using $(M_s r_1)^2 \sim (M_s r_2)^2 \sim 30$, each momentum excitation typically raises the cosmic string tension roughly by a few percent.

The cosmic string tension can have a rich spectrum. This is very different from the field theory case, where the cosmic string always appear with the same tension, up to the vorticity number n. As explained in [42] and reiterated in Chapter 5, a spectrum of string tensions can also arise in theories in which (p, q)-strings are stable, and more than one string type is produced, adding to the richness of string theoretic cosmic string physics.

4.2 Cosmic String Stability in Flux Stabilised Models

The recent work [42] expounds on the possibilities of cosmic strings in the flux stabilised, warped models of [28]. The sophistication of the model introduces a number of possible instabilities into the cosmic string physics; some are shown to be absent or projected out, and others are shown to be highly suppressed in the geometry, rendering cosmic strings meta-stable with so drastically suppressed decay times that they are effectively stable. Here, the various instabilities are discussed as [42] is reviewed. Conflicts between [42] and the more optimistic work [101] are also discussed.

4.2.1 String Breaking on Branes

F-Strings

The original definition of D-branes as worldsheet boundary conditions [106] implies that fundamental strings can end and break on D-branes. Therefore, if F-strings are to be cosmic strings, they must not be coincident with any D-branes. There is a subtlety, however, in that because NS-NS 2-form charge must be conserved, and F-strings couple to this 2-form, electric flux of the $U(1)$ of the D-brane provides this charge when an F-string breaks on a D-brane [50]. If the D-branes are D7-branes, this $U(1)$ may not be present as a local field in F-theory compactifications [107], in which case, F-strings would be stable to breakage on these D7-branes.

In KKLMMT type models, there are two classes of D-branes which are present and fill the 4D universe: "Standard Model" D-branes on which the fields of the standard model are localised as the light open string excitations [102]; and supersymmetry breaking D-branes, which are required in models with KKLT geometry [14] to lift the anti de-Sitter supersymmetric vacuum to a de-Sitter, post-inflationary universe. The D-branes are generally either D3-branes located at the end of a warped Klebanov Strassler throat [108] or D7-branes wrapping some 4-cycle of the Calabi-Yau 4-fold in an F-theory compactification [28]. The D7-branes will be extended objects in the compact CY, and excepting the case in which the

$U(1)$ is confining mentioned above, the D7-branes can extend down the inflationary throat and lead to F-string decay. If D3-branes are present in the inflationary throat, these rudimentary arguments indicate that F-strings cannot be stable. Then:

> *F-strings are stable when they are localised to a throat in which there are no D3-branes, or D7-branes which have a locally free $U(1)$ field.*

D & (p, q)-Strings

Only specific configurations of D-branes of various dimensions are BPS, and the non-BPS configurations can usually be argued to be unstable to decay to some BPS configuration of lower energy. Specifically, a D1-brane can end and break on a D3-brane; this can be understood at the S-dual of an F-string breaking on a D3-brane [50]. Further, applying the general $SL(2, \mathbb{Z})$ duality of Type IIB string theory, the string that breaks on the brane can be of type (p, q). Just as F-strings provide an electric source for the $U(1)$ gauge field on the brane, (p, q)-strings source the $U(1)$ as (p, q) dyons on the brane worldvolume. Again, the $U(1)$ must be present as a local field for the (p, q)-string to be allowed to break. Then, in the absence of other effects,

> *(p, q)-strings are stable when they are localised to a throat in which there are no D3-branes, or D7-branes which have a locally free $U(1)$ field.*

4.2.2 String Breaking on "Baryons"

The models of KKLMMT [28] have moduli stabilised by various fluxes, and D-branes couple to those fluxes. Specifically at the end of a KS throat, there are

M units of 3-form flux on the $\mathbb{S}^3$, which supports it against collapse. A D3-brane wrapped on the $\mathbb{S}^3$, appearing as a 4D particle of mass $\sim \tau_3(g_s M\alpha')^{3/2}$ couples electrically to the 4-form potential $C^{(4)}$, and the corresponding 5-form field strength is sourced by the Chern-Simons coupling $B^{(2)} \wedge F^{(3)}$. On the worldvolume of the wrapped D3-brane, this coupling becomes

$$\oint_{\mathbb{S}^3} F^{(5)} = \oint_{\mathbb{S}^3} dC^{(4)} + MB^{(2)}.$$

Therefore since F-strings are electric sources for $B^{(2)}$ on a brane worldvolume, such a wrapped D3-brane can only exist if it is attached to M F-strings. Stated another way, if M F-strings are coincident and macroscopically extended, they will be unstable to breakage by generating pairs of wrapped D3-branes, and will decay.

For $M = 1$, this is an instability of a single F-string, however in the flux stabilised models of inflation, $M > 1/g_s$; $g_s < 1$ for weak coupling, and $g_s M \gg 1$ in order for the supergravity approximation to hold. Consequently $M \gtrsim 15$. Now although there are no bound states of many F-strings which could be anything more than accidentally coincident, a $(M+p, q)$ string is a bound state when $M+p$ and q are relatively prime, and can decay in this way to a (p, q)-string. This instability leads to a cut-off on $|p|$ of $M/2$ for (p, q)-strings. It is uncertain however, how important this result is for cosmological purposes, since the Kibble production of anything other than F- and D-strings will be highly suppressed; while it is causally necessary that singly-wound defects form at a rate of one per Hubble volume at the end of inflation, it is causally improbably for the junction of neighbouring Hubble volumes to generate multiply wound defects - the exact suppression of more highly wound strings would be interesting to calculate more precisely. At any rate, a model with only primordial F- and D- strings would likely dynamically produce low p and q (p,q)-strings.

4.2.3 Axionic Strings and Domain Wall Boundaries

A charged string must couple to a 2-form field, and in 4 dimensions a 2-form field is dual to an axion. The string charge under the 2-form field translates to a topological source for the axion:

$$
\begin{aligned}
dC^{(2)} &= *_4 d\phi. \\
Q^{(2)} &\equiv \frac{1}{2\pi}\oint *_4 dC^{(2)} = \frac{1}{2\pi}\oint d\phi. \qquad (4.6)
\end{aligned}
$$

Now, any light scalar such as the axion must gain a mass at some high scale; in particular, the axion gains a mass due to an instanton amplitude and the axion potential must be periodic in ϕ. The axion winding (4.6) when there is a periodic axion potential means there must be an axion domain wall at the point where the axion field jumps between successive minima of its potential [109]. The consequent tension of the domain wall causes any such axionic string to rapidly collapse. This line of argument was one which ruled out superstrings as cosmic strings before the advent of D-branes [109].

4.2.4 Competing Instabilities: Domain Walls versus Breakage

As was studied in [42], complications arise when a string has two sources of instability: axion domain walls, and breakage. Naïvely, if the string bounds an axion domain wall, then it cannot break since a boundary has no boundary; conversely if a string can break, it cannot bound an axion domain wall. The Chern-Simons coupling of the bulk 2-form field to a D3-brane $U(1)$, A is

$$\mu_3 \int C^{(2)} \wedge 2\pi\alpha' dA, \tag{4.7}$$

which leads to an effective 4D equation of motion for $C^{(2)}$

$$d *_4 dC^{(2)} = \frac{1}{(2\pi)^2 \alpha' M_{\rm Pl}^2} dA,$$

which is satisfied if the axion is defined by $[(2\pi)^2\alpha' M_{\rm Pl}^2] dC^{(2)} = *_4(d\phi + A)$. Therefore, CMP [42] argue, the brane gauge field, A is Higgsed removing the 4D axion in the process. This implies that $C^{(2)}$ is massive and any source of $C^{(2)}$ charge will be screened. Then, since there is no conserved charge to protect against D-string breaking, it will do so.

This interpretation has been challenged in [101], who find a string-like solution to the equations of motion for A and $C^{(2)}$ on a D3-brane worldvolume (in some compactified 6-manifold). The physical understanding of these strings is that they are D1-branes coincident with D3-branes. It is well known that such systems are unstable - there is a tachyonic 1-3 string. However, the D1-brane will induce a flux of dA on the D3-brane because of the Chern-Simons coupling (4.7), and [101] argue that in this background, the tachyon mass could be lifted. This issue is not definitively resolved. CMP argue that the strings of [101] are certain axionic strings which will not be produced post-inflation because the breaking of the relevant D3-brane $U(1)$ occurs before D$\bar{\text{D}}$ annihilation, before inflation.

This seemingly innocuous point is so important because if D-strings are long-lived and can be produced coincident with D3-branes, it greatly increases the ubiquitousness of cosmic strings in string theory models of inflation. In such a case, conditions on the KKLMMT-type models which lead to cosmic strings are greatly relaxed, and they should almost certainly exist in most such models.

4.2.5 Strings Non-BPS with Respect to the Orientifold

The flux stabilisation methods of GKP [13] avoided no-go theorems regarding warped compactifications of string theory by inclusion of local negative tension objects, orientifold planes (the F-theory constructions are not considered in this section). Orientifold planes are defined as local objects which reverse the orientation of strings passing through them [106]. This orientation reversal projects out spacetime fields which are odd under the orientifold. Orientifold compactifications can be usefully viewed in the covering space, in which there is an orientation reversed mirror image of the original compactification glued to the location of the orientifold plane.

The GKP compactifications require O3-planes, and under the projection of the O3-plane the bulk 2-form fields are odd [13]. Hence $B^{(2)}$ and $C^{(2)}$ have no zero modes, and therefore are not massless 4D fields. This does not mean they are completely projected out of the theory however, since the O3-plane is localised away from the strings which source the 2-forms. The best way to understand this is by observing the covering space picture of the orientifold compactification, in which a F- or D- string (or even (p,q)-) at the end of the warped inflationary throat is mirrored by an anti F- or D-string in the mirror throat. Then, integrating $B^{(2)}$ or $C^{(2)}$ over the covering space gives zero contribution (and they are not present as massless 4D fields), however locally in the compact manifold, at scales less than the distance between the end of the throat and the O3-plane, the 2-form fields exist as 10D excitations. The calculations of Chapter 5 shall rely crucially on these musings, because they ignore the orientifolding; this is valid since those calculations are local interaction calculations which occur at the string scale, below the relatively large compactification scale at which the 2-form fields must be ignored.

This argument then clearly shows that no string type is BPS [42], since any string in such a geometry can be understood as the bound state of a string and an anti-string, which have the possibility of annihilating when the tachyonic strings linking them attain expectation values. Fortunately though, since the strings are localised at the end of a warped throat (and the anti-strings at the end of a mirror throat), there is a large potential barrier for the tachyonic strings to overcome in order to attain the true vacuum. Although the metric distance between the string and anti-string is only a few string lengths, this tunneling is heavily suppressed because the string tension is suppressed by the warp factor and tunneling is not efficient at lowering the energy of the system (and the relative barrier height is much greater). As explained in [42], the tunneling probability is consequently suppressed by an exponential in the warp factor,

$$\mathcal{P}_{\text{decay}} \sim e^{-10^4}.$$

F-, D- and (p, q)-strings can therefore be considered effectively stable in KKLMMT models to this channel of decay.

CHAPTER 5

PROPERTIES OF COSMIC STRINGS FROM STRING THEORY

Many of the important properties of cosmic string physics follow from the behaviour of cosmic string networks. The evolution of string networks is governed by the details of string interactions, and much of the resulting late-time physics will depend on these interactions. In this section, the stringy cosmic string interactions are reviewed, and the cosmological consequences are discussed.

Naïvely, averaging over a string network will yield an energy density which scales like a^{-2} (where a is the cosmological scale factor) [110]. This can be easily seen because the energy-density of non-interacting objects of worldvolume n will scale like in spacetime of dimension D like a^{-D+n}; in 4D radiation scales like a^{-4}, dust like a^{-3} and dark energy like a^{0}. Without interactions a^{-2} scaling would be disastrous, since the string network would quickly come to dominate the universe. Adding interactions, however, generally leads to "scaling solutions," where if sub-dominant, the string network will match onto the scaling behaviour of the dominant energy species.

A very simple (but not entirely accurate [83]) method to estimate the evolutionary behaviour of a string network with interactions is the "single-scale model" introduced by [111]. Start by assuming that there is a single scale which characterises the string network, $L(t)$; this is for instance the characteristic string length. A single string aligned along the x-axis will have an energy-momentum tensor $T^{\mu}_{\nu} = \rho \text{ diag}(1, 1, 0, 0)$, and averaging over all string orientations, a fluid of strings will therefore have $T^{\mu}_{\nu} = \rho \text{ diag}(1, \frac{1}{3}, \frac{1}{3}, \frac{1}{3})$, *i.e.* pressure $p = -\frac{1}{3}\rho$ [110]. ρ can therefore be written in terms of the tension μ as $\rho \sim \mu/L^2$. Assume also there is a definite probability of string self-interactions, $\mathcal{P}$, and the equation of motion for the string fluid becomes

$$\nabla_\nu T^\nu_0 = \{\text{interactions}\}, \qquad \longrightarrow \qquad \dot\rho = -2H\rho - \mathcal{P}\frac{\rho}{L}.$$

This can be solved for $L(t) = t\xi(t)$, which in a 4D universe dominated by objects of worldvolume $n < 2$ has a fixed point $\xi(t) = \mathcal{P}\frac{(4-n)}{2(2-n)}$. The consequences of this are that firstly, with these interactions, $\rho \sim (\mathcal{P}t)^{-2} \sim (\mathcal{P}^2 a^{4-n})^{-1}$ which is the scaling of the dominant energy species. However, simulations reveal that although this picture is qualitatively correct, the model is a little simplistic, and the true scaling of the network is like $\rho \sim (\mathcal{P}t^2)^{-1}$ [83, 112]. Physically, the interaction is allowing string loops to break off from long excited strings; the loops then decay into gravitational radiation, and the network energy density is damped, leading to the scaling behaviour. Also, since the density scales like the inverse of the interaction probability squared, a lower interaction probability leads to enhanced string density, hence the effects shall be augmented.

Therefore, one way in which a cosmic string network which arises in string theory can differ strongly from that in a field theory model is in its interaction probabilities, $\mathcal{P}$. The field theory solitons have essentially $\mathcal{P} \simeq 1$ [113], so to calculate any deviation from this would distinguish string theory cosmic strings. Further, in string theory there is the possibility of producing (p, q) strings which can have 3-string junctions [42] (see § 5.1); networks of such string types could also lead to deviations from scaling, so their interactions are important. In this chapter, the detailed calculations of string theory F-, D- and (p, q)- string interactions are presented with a view to seeking these distinguishing features.

5.1 (p, q)-strings

The strings of superstring theory come in a number of flavours, and this fact is responsible for many of the distinctions between cosmic string models from string theory and those from field theories. One of the insights of the recent work [42] was that (p, q)-strings could play a rôle in cosmic string networks born of string theoretic inflation. These string types differ from the $U(1)$ vortices mainly considered in the cosmic string literature in that because they contain two types of charges, they can form networks with 3-point string junctions when two strings intersect, rather than just admitting a reconnection interaction. Although this feature is not distinctively stringy and for instance could be generated in a field theory model of strings which involves vortices of a non-Abelian gauge theory, these strings are common in many brane inflation models, in particular in the flux stabilised models where the standard model branes are outside the inflationary throats.

The p and q of (p, q)-strings label the two types of fundamental charges a Type IIB string can carry; charge under the NS-NS 2-form, B, and the RR 2-form, $C^{(2)}$, respectively. There are a number of ways in which one can deduce the properties of these strings and understand their physics:

- Type IIB supergravity admits BPS string-like brane solutions which can be charged under the two 2-form fields [114], and the $SL(2, \mathbb{R})$ duality of this supergravity can take such a brane with any charges (p, q) into one with any other (p, q). The full Type IIB string theory is conjectured to have an $SL(2, \mathbb{Z})$ duality group, which accounts for the quantisation of (p, q); this group of dualities takes any integral (p, q) into any other pair of integers. Since the duality can take F- to D- strings and vice versa, this is a strong-weak coupling duality.

- A IIB F-string has unit charge under B, and a IIB D1-brane has unit charge under $C^{(2)}$. Parallel, stationary F- and D- strings experience attraction, and at $g_s < 1$, the lighter F-strings will move toward the D-string and dissolve into it. The dissolution leads to a new BPS state, the tension of which satisfies a BPS bound of the spacetime supersymmetry algebra [50].

- Since the F-string carries B charge which must be conserved when the F-string dissolves, the D-string worldvolume theory must account for the B charge. This can be done by considering the DBI and Chern-Simons actions for the q D-strings (with only $U(1)$ fields excited),

$$\begin{aligned} S_{\text{D1}} &= -\tau_D \text{Tr} \int d^2x \sqrt{-\det P[g+B] + 2\pi\alpha' F} \\ &= \int B \wedge \frac{q\tau_D 2\pi\alpha' *F}{\sqrt{1+(2\pi\alpha' F)^2}} + \ldots, \\ S_{\text{CS}} &= \tau_F \text{Tr} \int \left[C^{(2)} + C^{(0)}\right] \wedge e^{2\pi\alpha' F + B} = \int B \wedge q\tau_F C^{(0)} + \ldots. \end{aligned}$$

Ignoring the complications of the RR scalar, $C^{(0)}$, B charge appears when $(B + 2\pi\alpha' F)$ is non zero, so the dissolved F-string endpoints are electrically charged under the D-string $U(1)$ gauge field. $p\tau_F$ is conventionally this charge under B, and labeling it so gives a relation between p and the flux $f \equiv 2\pi\alpha' F_{01}$:

$$p \equiv q \left(\frac{f}{g_s\sqrt{1-f^2}} + C^{(0)} \right) \tag{5.1}$$

The tension of such objects can be calculated from any of these three viewpoints. For instance, the worldvolume method sees that the D-strings with flux have tension

$$\tau_{(p,q)} = \frac{q\tau_D}{\sqrt{1-f^2}} = \tau_D\sqrt{q^2 + \left(g_s p - qC^{(0)}\right)^2} \tag{5.2}$$

when re-expressed in terms of p via (5.1).

Understanding the classical behaviour of (p, q)-string networks is as simple as writing down the conditions to balance forces at any three-string junction [114, 115, 116, 117, 118, 119]. Three string junctions are allowable given that the total F- and D- charges entering the junction is zero,

$$p_1 + p_2 + p_3 = q_1 + q_2 + q_3 = 0,$$

and the angle between any two strings is given by

$$\cos\theta_{i,j} = \frac{e_i \cdot e_j}{|e_i||e_j|}, \qquad e_i \equiv ([p_i - C^{(0)}q_i]g_s, q_i),$$

since the vertices are essentially massless.

5.2 Cosmic String Interactions: Basic Probabilities

In [90] it was recognised that the important string interaction probabilities for string theoretic cosmic strings could be suppressed by the spread of the strings in the extra dimensions. The argument applied to orientifold torus models. In the flux stabilised model [28], this reasoning must be modified because the strings are localised by a potential and see only some effective volume; also different string types can be localised in different throats and the alteration of interaction probabilities is not so clear [42].

The explicit string theory interaction probabilities for macroscopic strings are calculated in [120]. The procedure employed is to first calculate flat-space interaction probabilities and then to embed these into the various brane inflationary

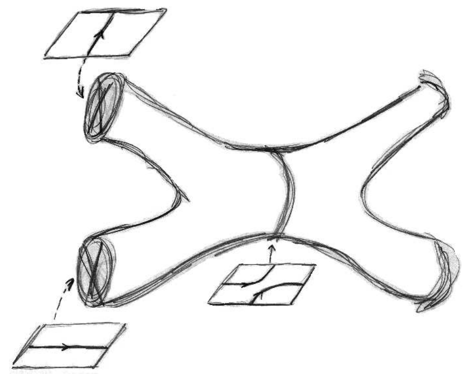

Figure 5.1: The string diagram corresponding to recombination of macroscopic F-strings.

models by considering the effective volumes of localisation. These calculations are reviewed below.

5.2.1 F−F Interaction

The basic calculation of the string reconnection process, if the strings involved are F-strings, is a simple string scattering calculation, using the tricks applied to the calculation for the Bosonic string in [121]. In fact the result shall be the same, although the strings are superstrings; this is because the macroscopic limit erases the low-level oscillator information. Incidentally, the result for D-strings in the bosonic and superstring theories shall be very different, because for instance in the supergravity approximation when the strings are separated in the extra dimensions appreciably, the D-strings of the Bosonic theory are uncharged whereas those of the superstring theories have 2-form charge just like the F-strings.

The calculation is set-up as shown in Figure 5.1. Compactifying two of the 4 spacetime dimensions on a torus of radii l, with sides at an angle θ, the incoming macroscopic strings can be represented by closed strings wound on the torus; the final state string is a bent doubly wound closed string. The transverse dimensions are compactified on a torus of volume $V_\perp$. The straight wound strings are easily represented by winding vertex operators; the bend string would have an infinite number of oscillator excitations in the macroscopic limit, so the optical theorem is used to calculate the sum of amplitudes to all final states, and the final state vertex operator need not be used. The imaginary part of the forward scattering diagram shown in Figure 5.1 is the required sum of squared amplitudes to final states. Finally, to further simplify the calculation, the string states are included without any oscillator excitations; this amounts to switching the GSO projection by inserting a factor of $(-1)^{\boldsymbol{F}}$ ($\boldsymbol{F}$ being spacetime fermion number) in the trace over the directions of the torus. Adding a finite number of excitations in the macroscopic, $l \to \infty$, limit cannot change the result.

The quantisation conditions for the singly wound string states are

$$p_L^2 = p_R^2 = \frac{2}{\alpha'}, \qquad\qquad p_{L/R} = p \pm \frac{L}{2\pi\alpha'},$$

and the vertex operators in the $(-1,-1)$ and $(0,0)$ pictures are

$$\begin{aligned} \mathcal{V}^{(-1,-1)} &= \frac{\kappa}{2\pi\sqrt{V}} :e^{-\phi-\tilde{\phi}+ip_L\cdot X+ip_R\cdot\tilde{X}}: , \\ \mathcal{V}^{(0,0)} &= \frac{\kappa}{2\pi\sqrt{V}}\frac{\alpha'}{2}(\psi\cdot p_L)(\tilde{\psi}\cdot p_R) :e^{ip_L\cdot X+ip_R\cdot\tilde{X}}: . \end{aligned}$$

Here $V = V_\perp l^2 \sin\theta$ is the eight-dimensional compactification volume, and the factor of $V^{-1/2}$ is from the zero modes.

The diagram Figure 5.1 represents the correlator

$$\mathcal{A} = \left\langle \mathcal{V}_1^{(0,0)}\mathcal{V}_2^{(0,0)}\mathcal{V}_3^{(-1,-1)}\mathcal{V}_4^{(-1,-1)} \right\rangle , \tag{5.3}$$

since the sphere requires two (holomorphic and anti-holomorphic) vertex operators in the -1 picture, and three c-ghosts fix the first three operator positions to $z = 0$, $z = 1$ and $z \to \infty$. Also, using the optical theorem, $p^{3,4}$ shall be set to $-p^{1,2}$. The normalisation of the path integral is crucial, and is $N_{\mathbb{S}^2} = 32\pi^3 V/\kappa^2\alpha'$ [50]; the additional volume of compactification, V comes from the zero-mode integrals. To simplify the amplitude which results from (5.3), note that there is no momentum in the wound directions, so $p_{Li}\cdot p_{Lj} = p_{Ri}\cdot p_{Rj}$. The invariant amplitude is [120]

$$\mathcal{M} = -\frac{4\kappa^2}{V\alpha'}\frac{\Gamma\big(-\frac{\alpha'}{4}s\big)\,\Gamma\big(-\frac{\alpha'}{4}t\big)\,\Gamma\big(-\frac{\alpha'}{4}u\big)}{\Gamma\big(1+\frac{\alpha'}{4}s\big)\,\Gamma\big(1+\frac{\alpha'}{4}t\big)\,\Gamma\big(1+\frac{\alpha'}{4}u\big)},$$

where s, t and u are the Mandelstam variables, constructed from either of p_{Li} or p_{Ri}.

To construct the angled F-string pair wrapped on the torus, with one string stationary and the other traveling toward it at velocity v, the momenta are

$$p_1 = \left[\left(\frac{l}{2\pi\alpha'}\right)^2 - \frac{2}{\alpha'}\right]^{\frac{1}{2}}(1,0,0,0,\mathbf{0}), \qquad L_1 = l(0,1,0,0,\mathbf{0}),$$
$$p_2 = \left[\left(\frac{l}{2\pi\alpha'}\right)^2 - \frac{2}{\alpha'}\right]^{\frac{1}{2}}[1-v^2]^{-\frac{1}{2}}(1,0,0,v,\mathbf{0}), \qquad L_2 = l(0,\cos\theta,\sin\theta,0,\mathbf{0}).$$

For $l \gg \sqrt{\alpha'}$ with fixed small t (corresponding to momentum transfer in the transverse directions) the amplitude is in the Regge region, the $s\to\infty$ limit averages the poles of the amplitude into a cut leaving

$$\mathcal{M} \xrightarrow[t\to 0]{s\to\infty} -\frac{\kappa^2}{V}\frac{s^2}{t}(\alpha' s/4)^{\alpha' t/2}e^{-i\pi\alpha' t/4}.$$

The normalisation of the $t=0$ pole agrees with graviton exchange calculated in an effective field theory, and the full form is determined by this normalisation plus the Regge behavior. By an extension of this observation the general reconnection probability of F-strings with other strings can be obtained from field theory.

Inserting the relevant kinematic factors the probability is

$$\begin{aligned} P &= \frac{1}{4E_1E_2v}2\,\mathrm{Im}\,\mathcal{M}|_{t=0} \\ &= \frac{\kappa^2}{\alpha'\pi V_\perp}f(\theta,v), \qquad f(\theta,v) = \frac{(1-\cos\theta\sqrt{1-v^2})^2}{8\sin\theta\, v\sqrt{1-v^2}}. \end{aligned} \tag{5.4}$$

The factors of l have canceled out to give a finite $l \to \infty$ limit. The result is the same as for the bosonic string [121].

As discussed in [120], this result suffers higher order corrections, which cannot be so quickly calculated by the unitarity trick. The imaginary part of the one-loop diagram will contain contributions for excited but not-recombined strings which have passed through one another. The reconnection is however a local interaction which is independent of l as is manifest in the tree-level calculation above, whereas excitation of the wound strings would diverge in the $l \to \infty$ limit; this allows for the possibility of extracting the appropriate piece of the imaginary part of the one-loop amplitude describing the recombination.

5.2.2 F-(p, q) Interaction

When one of the interacting strings has D-string charge and the other remains a pure F-string, a tree-level open string diagram can describe the lowest order term in the process. The F-string splits on the D-string to form an open string attached to a D-brane, and the calculation [120] is a generalisation (and supersymmetrisation) of the calculation [122] describing the decay of macroscopic open strings. The D-string can be turned into a (p, q)-string by adding electric flux on the disc boundary [50].

The amplitude of Figure 5.2 is

$$\mathcal{A} = \left\langle \mathcal{V}_2^{(0,0)} \mathcal{V}_4^{(-1,-1)} \right\rangle .$$

The vertex operators are those of the previous subsection, and the (p, q)-string is expressed by first adding a q-valued Chan-Paton factor to the string boundary, and then by activating constant electric flux on the boundary; the relationship between

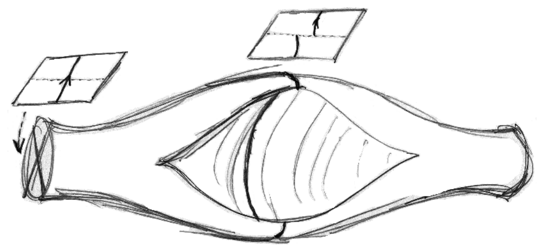

Figure 5.2: The open string tree-level diagram describing the splitting of an F-string in the presence of a (p, q)-string, represented by a D-string boundary condition with quantised electric flux.

the flux and p is given in (5.1). This condition follows from matching the energy of a D-string with f-units of electric flux (calculated from the appropriate DBI for instance) with the energy of a (p, q)-string from the BPS tension formula (5.2). The flux is appropriately quantised for integer p. In terms of the worldsheet, this corresponds to adding an operator to the boundary

$$i \oint_{\partial\Sigma} d\tau F_{\mu\nu} \left\{ \dot{X}^\mu X^\nu + \frac{\alpha'}{2} \psi^\mu \psi^\nu \right\},$$

precisely as described in § 3.1.2, which has the effect of inducing the OPE's

$$\begin{aligned} \left\langle X^\mu(z_1) X^\nu(z_2) \right\rangle &= -\frac{\alpha'}{2} \eta^{\mu\nu} \ln(z_1 - z_2), & \left\langle \psi^\mu(z_1) \psi^\nu(z_2) \right\rangle &= \frac{\eta^{\mu\nu}}{z_1 - z_2}, \\ \left\langle X^\mu(z_1) \tilde{X}^\nu(\bar{z}_2) \right\rangle &= -\frac{\alpha'}{2} G^{\mu\nu} \ln(z_1 - z_2^*), & \left\langle \psi^\mu(z_1) \tilde{\psi}^\nu(\bar{z}_2) \right\rangle &= \frac{G^{\mu\nu}}{z_1 - z_2^*}. \end{aligned} \qquad (5.5)$$

The boundary conditions define the open string metric [69]

$$G^{\mu\nu} = \begin{pmatrix} -\frac{1+f^2}{1-f^2} & -\frac{2f}{1-f^2} & 0 \\ \frac{2f}{1-f^2} & \frac{1+f^2}{1-f^2} & 0 \\ 0 & 0 & -\mathbb{1} \end{pmatrix},$$

where the $-\mathbb{1}$ arises from the Dirichlet boundary conditions in the directions transverse to the (p, q)-string.

The diagram Figure 5.2 can be evaluated with these rules; the forward scattering, macroscopic string $l \to \infty$ limit lead to the interaction probability [120]

$$\begin{aligned} P &= \frac{1}{2E_2 v} 2\mathrm{Im}\,\mathcal{M}|_{t=0} \\ &= g_s^2 \frac{V_{\min}}{V_\perp} h_{p,q}(\theta, v), \quad h_{p,q}(\theta, v) = \frac{q^2v^2 + \left[g_s p - \cos\theta\sqrt{(1-v^2)(g_s^2p^2+q^2)}\right]^2}{8\sin\theta\, v g_s \sqrt{(1-v^2)(g_s^2p^2+q^2)}}. \end{aligned} \tag{5.6}$$

Again, the normalisation of the disc path integral is crucial and is

$$N_{\mathbb{D}^2} = \frac{2\pi^2 l_1 q\sqrt{1-f^2}}{2\pi\alpha' g_s}.$$

This can be obtained from the standard disc partition function normalisation, $2\pi^2 V_9 \tau_9$, by T-duality and taking into account the Chan-Paton factors and background fields. Note that at zero velocity the numerator vanishes when the angle between the strings is $\tan\theta = q/g_s p$, which is the angle at which the strings are mutually BPS and there is no long-range force between them [114].

The reconnection probability above reduces exactly to (5.4) when $p \to 0$, $q \to 1$. This may be surprising since the two interaction probabilities are obtained from very different string amplitudes, however in the forward scattering limit $t \to 0$, in the t-channel both processes are seen to pick up the supergravity pole; the imaginary part of the full amplitude is then obtained from an average over the s-channel poles in the macroscopic $s \to \infty$ limit. Thus both cases lock onto a result which could be obtained by a supergravity calculation and consequently (5.6) should reduce to (5.4).

5.2.3 (p, q)-(p', q') Reconnection

The calculation of the interaction when both strings have D-string charge is less straightforward. The perturbative method of [120] is to first calculate the probability of string pair production between the moving angled D-strings, then to estimate the number of string pairs that must be produced to "stick" the D-strings together long enough such that they will recombine, and finally to calculate the probability of that number being produced.

The first part of the calculation was first performed for parallel Dp-branes by Bachas [123] and has been reapplied many times. The procedure is to calculate the one-loop open string vacuum amplitude, the annulus diagram, where either boundary has the boundary conditions of one of the (p, q)-strings. To understand this general case for angled (p_1, q_1) and (p_2, q_2)-strings traveling toward each other, the spectrum must first be understood. As discussed, the $q_{1,2}$ are Chan-Paton factors added to the two boundaries of the annulus, and the $p_{1,2}$ are related to electric flux on those boundaries by (5.1). The boundary conditions can be written as [69]

$$A^\mu{}_\nu X'^\nu(0) = B^\mu_{(1)\nu}\dot{X}^\nu(0), \qquad [A \cdot R]^\mu{}_\nu X^{\nu\prime}(\pi) = [B_{(2)} \cdot R]^\mu{}_\nu \dot{X}^\nu(\pi),$$

in which B depends on the fluxes, $f_{1,2}$, which are related to the F-string charges $p_{1,2}$,

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \qquad B_{(i)} = \begin{bmatrix} 0 & f_i & 0 & 0 \\ f_i & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

and the rotation matrix, R, is expressed in terms of the angle between the strings, θ, and the velocity of the moving brane, $v \equiv \tanh(\pi\epsilon)$

$$R = \begin{bmatrix} \cosh \pi\epsilon & 0 & 0 & \sinh \pi\epsilon \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ \sinh \pi\epsilon & 0 & 0 & \cosh \pi\epsilon \end{bmatrix}.$$

Here are some papers by some friends of mine [124,125,126,127]. With X a general linear combination of $e^{i\omega(\tau+\sigma)}$ and $e^{i\omega(\tau-\sigma)}$, $e^{2\pi i\omega}$ must be an eigenvalue of

$$R^{-1}(A - B_{(2)})^{-1}(A + B_{(2)})R(A + B_{(1)})^{-1}(A - B_{(1)}). \qquad (5.7)$$

This matrix is easily seen to be $\in SO(3,1)$ and there are two real and two imaginary eigenvalues

$$\omega = \pm \left\{ \frac{\tilde{\theta}}{\pi}, i\tilde{\epsilon} \right\}.$$

These can be understood as an effective rotation angle, and an effective rapidity, which respectively reduce to the actual angle between branes, θ and the actual rapidity $\tanh^{-1}(v)/\pi$ when the fluxes vanish. The precise eigenvalues, obtained in [120] are given by the eigenvalues of (5.7),

$$\cos(\tilde{\theta} + i\pi\tilde{\epsilon}) = \left(\frac{1+f_1^2}{1-f_1^2}\right)\left(\frac{1+f_2^2}{1-f_2^2}\right)\cos(\theta + i\pi\epsilon) - \left(\frac{2f_1}{1-f_1^2}\right)\left(\frac{2f_2}{1-f_2^2}\right)$$

With the spectrum given by the moding above and an impact parameter y, it is straightforward to extend the results of [123] to this case [30, 31, 50, 99],

$$\mathcal{M}(y) = -\frac{i}{2}(q_1 q_2)\int_0^\infty \frac{dt}{t} e^{-ty^2/2\pi\alpha'}\left[\eta^6(it)\ \Theta_1\big(i\frac{\tilde{\theta}t}{\pi}\big|it\big)\Theta_1\big(\tilde{\epsilon}t\big|it\big)\right]^{-1}$$
$$\times\left\{\sum_{k=2}^{4}(-1)^{k-1}\Theta_k\big(0\big|it\big)^2\Theta_k\big(i\frac{\tilde{\theta}t}{\pi}\big|it\big)\Theta_k\big(\tilde{\epsilon}t\big|it\big)\right\}.$$

The imaginary part of this amplitude can be understood as the pair production rate (or directly the probability in this case where the strings are confined to a point) of open strings between the D-strings. Similar calculations in [123, 128] can be understood as the production rate of charged particles in an electric field, which is the string-theory version of the field theory calculation of this phenomenon due to Schwinger [129]. As emphasised in [120], the pair production rate will not be the reconnection probability, but it can be deduced from it. The probability of producing *at least one* pair of strings is given by the sum of disconnected annuli diagrams,

$$\mathcal{P}_{\rm pp}(y) = 1 - |e^{i\mathcal{M}(y)}|^2 = 1 - e^{-2{\rm Im}\,\mathcal{M}(y)}.$$

The imaginary part arises from the poles of $\Theta_1\left(\tilde\epsilon t\middle|it\right)^{-1}$ on the real t-axis at $t = n/\tilde\epsilon$, which must all be traversed on the same side [123].

$$\begin{aligned}
\operatorname{Im}\mathcal{M}(y) &= \frac{q_1q_2}{2}\sum_{n=1}^{\infty}\frac{1}{n}\Big[(-1)^{n+1}Z_B(\tfrac{n}{\tilde\epsilon}) + Z_F(\tfrac{n}{\tilde\epsilon})\Big],\\
Z_B(t) &\equiv \sum_{\text{bosons}\,i} e^{-2\pi\alpha' t m_i^2} = e^{-ty^2/2\pi\alpha'}\frac{\Theta_3\left(0\middle|it\right)^3\Theta_3\left(i\frac{\tilde\theta t}{\pi}\middle|it\right) - \Theta_4\left(0\middle|it\right)^3\Theta_4\left(i\frac{\tilde\theta t}{\pi}\middle|it\right)}{2\eta^9(it)\, i\Theta_1\left(i\frac{\tilde\theta t}{\pi}\middle|it\right)},\\
Z_F(t) &\equiv \sum_{\text{fermions}\,j} e^{-2\pi\alpha' t m_j^2} = e^{-ty^2/2\pi\alpha'}\frac{\Theta_2\left(0\middle|it\right)^3\Theta_2\left(i\frac{\tilde\theta t}{\pi}\middle|it\right)}{2\eta^9(it)\, i\Theta_1\left(i\frac{\tilde\theta t}{\pi}\middle|it\right)}.
\end{aligned}$$

The residues can be easily summed to give

$$\mathcal{P}_{\text{pp}}(y) = 1 - \left[\frac{\prod_{\text{fermions}\,j}(1-x_j)}{\prod_{\text{bosons}\,i}(1+x_i)}\right]^{q_1q_2}, \qquad x = e^{-2\pi\alpha' m^2/\tilde\epsilon}, \tag{5.8}$$

where m is the mass of the given stretched string state at minimum separation.

The small velocity limit of this expression is just given by the expansion in the open string channel in which the lightest modes dominate - the tachyon and 4 massless fermions (times q_1q_2 to account for the different Chan-Paton factors); higher modes are suppressed by $\exp(-2\pi\alpha' m^2/\tilde\epsilon)$. The lightest state is the boson with $m^2 = y^2/(2\pi\alpha')^2 - \tilde\theta/2\pi\alpha'$ [30]. As $\tilde\epsilon \to 0$, for impact parameters smaller than the critical separation $y^2 < 2\pi\alpha'\tilde\theta$ when there is a tachyon in the spectrum, the tachyon completely dominates (5.8) and $\mathcal{P}_{\text{pp}} \to 1$. The rolling of the tachyon when the stationary branes are at a non-zero angle describes the reconnection of the branes [34, 35]. In the strict small velocity limit, integrating (5.8) over a 6-volume gives a classical black-sphere cross section

$$\sigma = \int d^6y\, \mathcal{P}_{\text{pp}}(y) = (2\pi^2\alpha'\tilde\theta)^3. \tag{5.9}$$

Of course, at $g_s = 1$ F-strings and D-strings are identical under duality and so the reconnection probabilities should become equal. This is not evident in the small-velocity limit, where the D-D interaction approaches a constant while the

F-F result (5.4) diverges as $1/v$. Higher order effects must cut the latter off. The relativistic velocity limit of (5.8) can be obtained from a modular transformation of the theta-function expressions [120], however for most cosmic applications, that limit is inapplicable; in cosmic string networks, the velocities are moderately relativistic, so that a typical string collision will have $v \sim 0.7$ or $\epsilon \sim 0.3$ [83, 130].[1] This is not so different from the small velocity limit, in that only the lightest open strings are produced.

As previously stated, this probability of pair generation is not the probability of (p, q)-string recombination; basically because at weak string coupling F-strings are much lighter than D- or (p, q)-strings, so a single F-string pair will not strongly perturb the motion of the (p, q)-strings. Suppose only a small number (compared to g_s^{-1}) of F-string pairs are created in the collision; then the (p, q)-strings will pass through each other and continue on their trajectories, stretching the F-strings as they do so. The F-strings pairs are localised about the point of intersection, and they will quickly find each other and annihilate, leaving excited (p, q)-strings, with the excitation energy provided by a net slowing of the strings. If, however, a number of F-strings sufficient to glue the (p, q)-strings together is produced, the tachyonic mode will rapidly roll and the strings will reconnect.

This reasoning appears in [120], where a precise condition is formulated to determine the reconnection probability. Firstly, for N F-string pairs, balance of forces implies that the angle between the F-strings and D-strings in the rest frame of the junction is $\frac{1}{2}\pi + \phi$, where $N = \sin\phi\sqrt{p^2 + q^2/g_s^2}$. The effect of the collision can only travel with the speed of light, so on each string there are two kinks traveling away from the point of the collision. If the centre-of-mass frame speed,

[1]Of course, in an evolving network there will be a distribution of collision parameters.

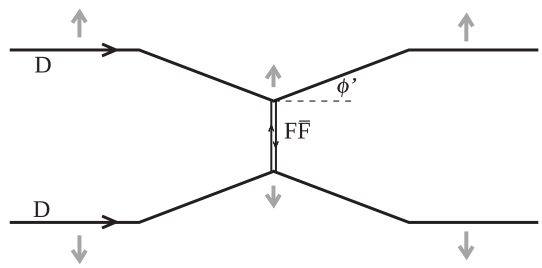

Figure 5.3: F-string pair creation between colliding (p,q)-strings. ϕ' is the junction angle, boosted into the frame in which the 3-string vertex moves. The kink disturbance on the (p,q)-strings cannot travel faster than the speed of light.

u, satisfies $u > \tan\phi$ then the system contains sufficient energy to stretch the F-string pairs and separate the (p,q)-string junctions. For lower speeds, the junctions do not separate and the (p,q)-strings remain in contact, and the tachyon will be given time to roll down its potential, thus allowing the strings to combine. Thus the condition for recombination is

$$N > \frac{1}{g_s}\sqrt{(g_s^2 p^2 + q^2)}_{\max}\ \sinh\left(\frac{\pi}{2}\epsilon\right), \qquad \epsilon \equiv \frac{2}{\pi}\tanh^{-1}(u). \tag{5.10}$$

Here, if the two strings (p,q)-strings have different charges, it is the maximum value of (p,q) which is relevant, since both (p,q)-strings are required to stay fixed at the interaction point for recombination, and this only occurs when there are sufficient F-strings between them that neither F-(p,q) vertex can move.

The annulus calculation of the Schwinger pair production result only dealt with the production of at least one pair. For the case for which the string impact parameter vanishes, in the low velocity limit, it is simple to calculate the probability of producing N F-string pairs. Being localised to the intersection region, the tachyon pairs are produced in a squeezed state. For a squeezed state of a single oscillator, if the probability of producing at least one pair is p, then the probability of producing at least k pairs is just p^k. Here, $p \simeq 1 - e^{-\tilde{\theta}/\tilde{\epsilon}}$ and so $p^k \simeq \exp(-ke^{-\tilde{\theta}/\tilde{\epsilon}})$.

Subtracting the four massless fermionic pairs which will necessarily be produced, the probability that the reconnection condition (5.10) is satisfied is

$$\mathcal{P} = \exp\left(\left[4q_1q_2 - \frac{1}{g_s}\sqrt{(g_s^2p^2+q^2)}_{\max}\ \sinh\left(\frac{\pi}{2}\epsilon\right)\right]e^{-\tilde{\theta}/\tilde{\epsilon}}\right). \qquad (5.11)$$

$\mathcal{P}$ decreases as $g_s \to 0$, because the reconnection condition (5.10) becomes more stringent while the probability of producing a given number of pairs is constant. In fact, it falls as $e^{-O(1/g_s)}$ and so is non-perturbative, even though it was deduced from a perturbative calculation.

This asymptotic suppression of $\mathcal{P}$ does not set in until below the GUT value $g_s \sim 0.05$. For two D-strings for instance, with $g_s \sim 0.05$ and $\epsilon \sim 0.3$, $\mathcal{P} \sim \exp(-6e^{-\theta/0.3})$. At $\theta \gtrsim 1$, $\mathcal{P}$ is at least 0.8; for $\theta \sim 0.6$ it falls to around 0.5 and then begins to rise again due to the higher states in the expansion (5.8). Thus for this choice of parameters there is a range of small angles where D-strings will sometimes pass through one another, but this will have likely have a small effect on the network behavior. This discussion extends directly to the general case. The general conclusion is that reconnection almost always occurs unless g_s is very small.

5.2.4 Vertex interactions

As the string network evolves, pairs of trilinear vertices will collide, and for detailed simulations of (p, q)-string networks, the subsequent evolution needs to be determined.

In the Figure 5.4, the strings are aligned at the supersymmetric configuration in which string i is in the direction $(p_i g_{\rm s}, q_i)$, and define a general configuration by rotating strings 3 and 4 by an angle ψ around the string segment. The simplest

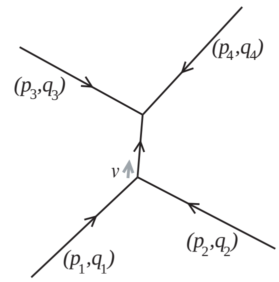

Figure 5.4: The interaction of two (p, q)-string vertices, which can decouple into separated (p, q)-strings.

case is two F-strings ending on a (p, q) string, so that

$$(p_i, q_i) = (p, q),\ (-1, 0),\ (1, 0),\ (-p, -q)\ , \quad i = 1, 2, 3, 4\ .$$

The two F-strings have the same orientation, so the endpoints can annihilate and the F-string disconnect from the (p, q) string.

The probability for this interaction can be obtained by the same general strategy as for the previous F-string processes. To set up the macroscopic open string states, two spectator strings are introduced on which the other ends of the F-strings are fixed. Note that to lowest order the (p, q) strings do not bend when a single F-string attaches. These are at separation R in the 2-3 plane, and R is taken to ∞ at the end of the calculation to remove the spectator strings and make the F-strings macroscopic. As before the opposite of the usual GSO projection is taken so as to get the simpler scalar ground state; this is equivalent to taking the spectators to be $(p, -q)$ strings. The optical theorem is again used to obtain the contribution from all final states.

The disconnection process is characterised by the relative velocity of the endpoints and the relative angle ψ of the two strings (figure 6). The open string vertex

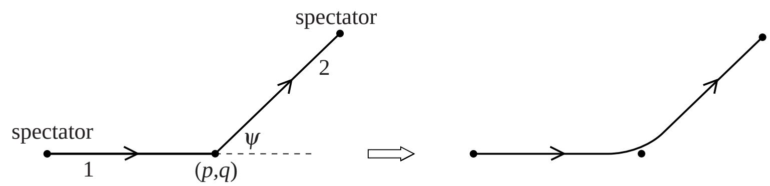

Figure 5.5: The (p, q) decoupling interaction depends on an additional angle, in another plane, so it is not merely the inverse of the F-(p, q) combination process. The calculation is setup by representing the F-strings as open strings stretched from the (p, q)-brane to spectator branes which shall be removed as the F-strings are made macroscopic.

operators for the strings stretched between the branes are

$$\mathcal{V}^{(-1)} = \lambda g_o \, {:}e^{-\phi} e^{ip_L \cdot X + ip_R \cdot \tilde{X}}{:},$$

$$\mathcal{V}^{(0)} = \lambda g_o \sqrt{2\alpha'} (\psi \cdot p){:}e^{ip_L \cdot X + ip_R \cdot \tilde{X}}{:}.$$

Adjoint $U(q)$ Chan-Paton factors λ have been added to the F-string vertex operators. Note also that there are no factors of $V_\perp$ since the open string wavefunctions are localised in the extra dimensions by the D-strings.

With the (p, q)-string aligned along the 1-direction, the first F-string is placed at rest aligned along the 2-direction, and the second with velocity along the (p, q)-string and aligned at an angle ψ in the 2-3 plane. The momenta are therefore

$$p_{1L,R} = \left[\left(\frac{R}{2\pi\alpha'} \right)^2 - \frac{1}{2\alpha'} \right]^{\frac{1}{2}} [1 - f^2]^{\frac{1}{2}} (1, 0, 0, 0, \mathbf{0}) \pm \frac{R}{2\pi\alpha'} (0, 0, 1, 0, \mathbf{0}),$$
$$p_{2L,R} = \left[\left(\frac{R}{2\pi\alpha'} \right)^2 - \frac{1}{2\alpha'} \right]^{\frac{1}{2}} \left[\frac{1 - f^2}{1 - v^2} \right]^{\frac{1}{2}} (1, v, 0, 0, \mathbf{0}) \pm \frac{R}{2\pi\alpha'} (0, 0, \cos\psi, \sin\psi, \mathbf{0}).$$

The initial and final F-strings are now 1,2 and 3,4 respectively. The vertex operator for the stationary string can be obtained, for example, by T-duality along the 1-direction, and then the other is obtained by a boost. Using the contractions (5.5), each pair of vertex operators leads to a factor of $e^{2\alpha' p_i * p_j}$, where

$$p_i * p_j = \frac{1}{4} (p_{iL} \cdot p_{jL} + p_{iR} \cdot p_{jR} + p_{iL} \cdot G \cdot p_{jR} + p_{jL} \cdot G \cdot p_{iR}) \ .$$

One can check the mass shell condition $p_i * p_i = 2/\alpha'$.

The amplitude is then

$$\mathcal{M} = -\tilde{N}_{\mathbb{D}^2} g_o^4 \mathrm{Tr}\,(\lambda^1\lambda^2\lambda^{2\dagger}\lambda^{1\dagger}) \frac{\Gamma(-\alpha' s)\,\Gamma(-\alpha' t)}{\Gamma(1+\alpha' u)},$$

where the Mandelstam variables are defined by $s = -(p_1+p_2)*(p_1+p_2)$ and so on. The Chan-Paton trace is simply 1 (in all other channels it vanishes), and the path integral is normalised as in § 5.2.2, $\tilde{N}_{\mathbb{D}^2} = \pi\sqrt{1-f^2}/\alpha' g_s$ (the 1-direction is treated with continuum normalisation, and the explicit Chan-Paton trace is separated out), while $\tilde{N}_{\mathbb{D}^2} g_o^2 = 1/\alpha'$ holds in general by unitarity. Using these normalisations and taking the imaginary part as in earlier calculations,

$$\mathrm{Im}\,\mathcal{M} = \frac{g_s s}{\sqrt{1-f^2}}.$$

With the usual kinematic factors, the disconnection probability is

$$P = \frac{2\mathrm{Im}\,\mathcal{M}}{2E_1 2E_2 v} = \frac{g_{\mathrm{s}}}{v}\sqrt{(1-f^2)}[1-\sqrt{1-v^2}\cos\psi].$$

When three or four of the strings carry D-string charge, there is no simple CFT description of the system. However, the rule of thumb that in this situation the open string tachyons will almost always take the strings to their lowest energy state. Which of strings 3 and 4 string 1 will join onto must still be determined. In fact, the supersymmetric configuration shown is neutrally stable in both directions, but any nonzero ψ increases θ_{13} and decreases θ_{14} so that the latter reconnection is favored. In particular, when $(p_1, q_1) = -(p_4, q_4)$, the strings disconnect.

5.3 Cosmic String Interactions: Bound State Effects

The string reconnection probabilities (5.4, 5.6, 5.11) all depend on the compactification volume as $V_{\mathrm{min}}/V_\perp$, reflecting the fact that the strings have to come roughly within a string radius in order to interact [85, 90, 112]. It is therefore essential to determine the effective value of $V_\perp$.

Naïvely it would seem that one could obtain very small values of $\mathcal{P}$ in models with large compact dimensions. However, from the point of view of the worldsheet field theory, the position of the string in the compact dimensions is a scalar field, which is not protected by any symmetry. One therefore expects that at some scale this modulus will be fixed, like the compactification moduli. That is, there is an effective potential which localises the string. Moreover, the behavior of scalar fields in 1+1 dimensions implies that the effective volume over which the string wavefunction spreads depends only logarithmically on the mass scale of the moduli, as the cube of the logarithm, to be precise [42]. As a result, $\mathcal{P}$ can be suppressed somewhat, but not by many orders of magnitude.

5.3.1 Worldsheet Scalar Fluctuations

The effective action for a (p, q)-string moving in a general warped metric,

$$ds^2 = H^{-1/2}(Y)\eta_{\mu\nu}dX^\mu dX^\nu + H^{1/2}(Y)g_{ij}(Y)dY^i dY^j,$$

where X^μ, Y^i are the non-compact and compact directions respectively, can be obtained by inserting the metric into the relevant worldsheet action for a (p, q)-string,

$$\begin{aligned} S &= -\frac{1}{2\pi\alpha'}\int d^2\sigma\, \nu(-\det h_{ab})^{1/2}, \\ h_{ab} &= H^{-1/2}(Y)\eta_{\mu\nu}\partial_a X^\mu \partial_b X^\nu + H^{1/2}(Y)g_{ij}(Y)\partial_a Y^i \partial_b Y^j, \end{aligned}$$

where the string tension is proportional to $\nu = (p^2 + q^2 e^{-2\Phi(Y)})^{1/2}$. General dilaton dependence on the compact coordinates $\Phi(Y)$ gives a position-dependent string tension. Choosing the static worldsheet gauge $X^0 = \sigma^0$, $X^1 = \sigma^1$, $Y^i =$ constant, the action then reduces to a potential

$$V(Y) = -\mathcal{L} = \frac{\nu(Y)}{2\pi\alpha' H^{1/2}(Y)}. \tag{5.12}$$

Compactifications with string tensions below the Planck scale generally have branes which produce a nontrivial warp factor and/or dilaton, so that the potential (5.12) depends non-trivially on the compact dimensions. The strings will then sit near the minimum of the potential. For strings that are supersymmetric with respect to all the branes the classical potential can cancel [90], and it will then be necessary to go to higher order or even to non-perturbative physics to find the leading effect.

Notice that if the dilaton is nontrivial then the position of the minimum will depend on p and q. For the strongly warped geometries [28], the variation of the dilaton is negligible. However, for F-theory compactifications (see for example [13]) it should be noted that this effect has the possibility in principle to localise the different (p, q) strings far enough apart that they will evolve as essentially independent networks. Roughly speaking they must be separated by more than a string length for this to happen.

To understand the fluctuations around the minima, expanding the action to second order in the Y^i and choosing coordinates such that the minimum is at $Y = 0$ and $G_{ij}(0) = \delta_{ij}$, the worldsheet theory becomes that of 6 massive scalars

$$S \approx -\int d^2\sigma \left\{ V(0) + \frac{1}{2}\partial_i\partial_j V(0) Y^i Y^j + \frac{\nu(0)}{4\pi\alpha'}\partial_a Y^i \partial^a Y^i \right\},$$

and the average spread of the Y^i will give an effective volume within which the strings are confined. For a single massive scalar field in two dimensions,

$$S = -\frac{Z}{2}\int d^2\sigma \left(\partial_a\phi\partial^a\phi + m^2\phi^2 \right)$$

the worldsheet fluctuations are logarithmic in the mass and UV cutoff scale

$$\begin{aligned}\langle\phi^2(0)\rangle &= \frac{1}{Z}\int^\Lambda \frac{d^2k}{(2\pi)^2}\frac{1}{k^2+m^2}\\ &= \frac{1}{4\pi Z}\ln\frac{\Lambda^2+m^2}{m^2}.\end{aligned}$$

For the present purposed, the UV cutoff is the string scale: $\Lambda^2 \sim 1/\alpha'$ as seen by a ten-dimensional observer, but in four-dimensional units this is red-shifted to $\Lambda^2 \sim 1/\alpha' H^{1/2}(0)$. Rotating coordinates to make $\partial_i\partial_j V(0)$ diagonal gives

$$\begin{aligned}\langle Y^i Y^i\rangle &= \frac{\alpha'}{2\nu(0)}\omega_i, \qquad \text{(no sum on } i\text{)},\\ \omega_i &= \ln\left[1+\frac{\nu(0)}{2\pi\alpha'^2 H^{1/2}(0)V_{,ii}(0)}\right] \end{aligned} \tag{5.13}$$

The fluctuations of the string in the transverse dimensions scale only logarithmically as the scale of the potential that localises the string is lowered; the linear scale of the fluctuations goes as the square root of the logarithm, and the volume goes as the cube of the logarithm. The fluctuation (5.13) is proportional to

$$\frac{1}{\nu(0)} = \frac{g_{\rm s}}{\sqrt{p^2 g_s^2 + q^2}},$$

so for strings with D-brane charge it vanishes to leading order in perturbation theory.

This calculation is meaningful only when Λ^2/m^2 is large, meaning that $V_{,ii}$ is small in string units. This corresponds to the geometry varying slowly on the string scale[2]. In this case it is expected to be able to combine the flat spacetime calculation with an effective wavefunction for the string calculated as above. When $V_{,ii}$ is of order one in string units, so that there is no separation of scales, there is no way to use the flat spacetime calculation. It is then necessary to do a full

[2]Also in this limit the effective quartic coupling is small, so the only-loop calculation that has been done is valid.

perturbative string calculation in curved space. It is not clear whether there is any physical situation in which $V_{,ii}$ becomes much greater than one in string units, but if there is it will require some complementary method of calculation.

5.3.2 Effect on reconnection

The effective value of $1/V_\perp$ is the density Ω of the wavefunction where the strings coincide. For example, the effective density for an F-D collision is obtained from the field theory fluctuations for an F-string relative to a fixed center (since the fluctuations of the D-string are much smaller):

$$\begin{aligned}\Omega_{\rm FD} &= \left\langle \delta^6(Y) \right\rangle \\ &= \int \frac{d^6k}{(2\pi)^6} \left\langle e^{ik\cdot Y} \right\rangle \\ &= \int \frac{d^6k}{(2\pi)^6}\, e^{-k_i k_j \left\langle Y^i Y^j \right\rangle/2} \\ &= \frac{1}{(\pi\alpha')^3 \prod_i \omega_i^{1/2}}.\end{aligned}$$

This has the expected logarithmic behavior, with the coefficient of the logarithm now determined. This assumes that the F- and D-strings are localised at the same point. If the F-string potential is minimised at $Y = 0$ and the D-string potential at $Y = Y_{\rm D}$ then

$$\begin{aligned}\Omega_{\rm FD} &= \left\langle \delta^6(Y - Y_{\rm D}) \right\rangle \\ &= \frac{1}{(\pi\alpha')^3 \prod_i \omega_i^{1/2}} \exp\left[-\sum_i \frac{Y_{\rm D}^i Y_{\rm D}^i}{\alpha'\omega_i} \right].\end{aligned} \tag{5.14}$$

This illustrates the expected large suppression when the separation of the minima is larger than the string scale. For an F-F collision one has separate fields Y and

Y', and so

$$\begin{aligned}\Omega_{\rm FF} &= \left\langle \delta^6(Y-Y') \right\rangle \\ &= \frac{1}{(2\pi\alpha')^3 \prod_i \omega_i^{1/2}}.\end{aligned} \tag{5.15}$$

Since the F-strings are of the same type their minima are coincident.

For the general $(p,q)_1$-$(p,q)_2$ collision the effect of a nonzero impact parameter y is to replace

$$x \to x e^{-y^2/2\pi\alpha'\tilde{\epsilon}}$$

in the general interaction probability (5.8). Applying this in the case that only the bosonic tachyon and the massless fermions are important, as holds over most of the parameter space,

$$1 - \mathcal{P}_{\rm pp}(0) \simeq \left[\frac{(1 - e^{-y^2/2\pi\alpha'\tilde{\epsilon}})^4}{1 + e^{\tilde{\theta}/\tilde{\epsilon}} e^{-y^2/2\pi\alpha'\tilde{\epsilon}}} \right]^{q_1 q_2}. \tag{5.16}$$

Again for strings in different minima, the interaction probability falls rapidly for separations large compared to the string scale. For strings in the same minima, the quantum fluctuations (5.13) imply that the result (5.16) must be averaged over a Gaussian wavefunction of this width.

A typical value of the correction factor for (p,q)-strings is

$$e^{-y^2/2\pi\alpha'\tilde{\epsilon}} \sim \exp\left[-\frac{\sum_i \omega_i}{2\pi\nu(0)\tilde{\epsilon}} \right], \tag{5.17}$$

assuming strings of the same type, thus summing fluctuations as in (5.15) which contributes a factor of 2 to the exponent. For D-strings, $\nu^{-1}(0) = g_s$, so this is formally higher order in perturbation theory. However, the fluctuations in the different directions add to give an effective factor of 6, and so for the typical $\epsilon \sim 0.3$ the exponent can be of order one if the logarithm ω_i is large. This would lift the suppression due to the fermion zero modes. The probability to produce tachyon string modes remains large until the suppression factor (5.17) approaches $e^{-\theta/\epsilon}$, but if the scale of $V_{,ii}$ is low this can be the case, at least for some range of angles. Thus there is the possibility that D-strings can pass through one another without reconnecting.

When the typical value of the exponent in (5.17) becomes large, the dominant contribution to the reconnection probability comes from those collisions that happen to occur at small impact parameter, near the center of the Gaussian distribution. In this case the reconnection probability is given by the ten-dimensional cross-section, which is given to reasonable approximation by the low-velocity limit (5.9), times the peak density

$$\Omega_{\rm DD} = \Omega_{\rm FF}|_{\omega_i \to g_s \omega_i};$$

the factor of g_s reflecting the smaller fluctuations of the heavier D-string.

5.3.3 Model parameters

The KKLMMT model

In the KKLMMT model [28], inflation takes place in a highly warped throat whose local geometry is given by the Klebanov-Strassler solution [108]. The warp factor also produces a potential well for the transverse coordinates of the string. The geometry near the base of this solution is locally $\mathbb{R}^3 \times \mathbb{S}^3$,

$$g_{ij}(Y)dY^i dY^j = dr^2 + r^2 d\Omega_2^2 + R_3^2 d\Omega_3^2 \ ,$$

where $R_3^2 = bg_s M\alpha'$; M is an integer characterising the number of flux units. The warp factor near the origin depends on the radial coordinate of $\mathbb{R}^3$,

$$H(Y) = H(0)\left(1 - \frac{b' r^2}{g_s M\alpha'}\right).$$

The energy scale of inflation in this model is of order 10^{-4} in Planck units, so $H^{-1/4}(0) \sim 10^{-4}$. The constants $b \approx b' \approx 0.93$ [131] will be treated as 1 here. The dilaton in this solution is constant. There are also three- and five-form fluxes in the compact directions, but these do not enter into the string action. The product $g_s M$ must be somewhat greater than one in order for the supergravity approximation to be valid.

This solution has the special property that the warp factor has its minimum not at a point but on the entire three-sphere at $r = 0$. Considering first this geometry as it stands (and then considering corrections), the effective $V_\perp$ is given by combining the volume of the $\mathbb{S}^3$ with the quantum fluctuations on the $\mathbb{R}^3$,

$$\Omega_{\rm FF} \approx \frac{1}{4^{5/2}\pi^{7/2}\alpha'^3 (g_s M)^{3/2}} \frac{1}{\ln^{3/2}(1 + g_s M)}. \qquad (5.18)$$

Also, $\Omega_{\rm FD} = 2^{3/2}\Omega_{\rm FF}$.

The fact that the potential is constant on the $\mathbb{S}^3$ reflects an $SU(2) \times SU(2)$ symmetry of the Klebanov-Strassler solution. This local geometry is part of a larger Calabi-Yau solution, which can have no isometries. The breaking of the symmetry will generate an effective potential along the S^3, which will localise the strings. The warp factor $H^{-1/4}(0)$ is a measure of the size of the tip of the Klebanov-Strassler throat in terms of the underlying Calabi-Yau geometry [13]. It therefore governs the extent to which the throat feels the curvature of the geometry, and so size of the $SU(2) \times SU(2)$ breaking and the size of the potential. Assuming that the curvature of the Calabi-Yau manifold will have an effect on the throat geometry of order the warp factor squared, $H^{-1/2}(0) \sim 10^{-8}$, relative to the other scales in the throat, so that $\omega_i \sim \ln H^{1/2}(0)$ in the $\mathbb{S}^3$ directions[3]. Then

$$\Omega_{\rm FF} = \frac{1}{(2\pi\alpha')^3 \ln^{3/2}(H^{1/2}(0)) \ln^{3/2}(1+g_s M)}. \tag{5.19}$$

In this case $\Omega_{\rm FD}$ contains an additional factor of 8, but it may easily be the case that the F and D strings are localised at different points of the $\mathbb{S}^3$, leading to the additional suppression (5.14).

Obviously, the spread of the string wavefunctions in the extra dimension can only grow as much as the size of the $\mathbb{S}^3$, so whichever density of (5.18, 5.19) is larger shall be the appropriate one to use. In the fluctuation calculation, the log of the ratio of scales times the world-sheet coupling $1/g_s M$ is becoming large, and the renormalisation group must be used to improve the calculation.

For D-D collisions, the relevant quantity is

$$\frac{y^2}{2\pi\alpha'\epsilon} \sim \text{Min}\left[\frac{g_s M}{2\pi\epsilon}, \frac{3g_s \ln H(0)}{8\pi\epsilon}\right], \tag{5.20}$$

depending on whether the quantum fluctuations fill out the $\mathbb{S}^3$. Again, the fermion zero modes are lifted to the extent that this is nonzero, and the tachyonic modes are not excited for collisions at angles less than $\epsilon \times$ (5.20).

General Brane Inflation Models

For models in which n dimensions have periodicity $2\pi R$ and $6-n$ have the minimum periodicity $2\pi\sqrt{\alpha'}$ [90]. The four- and ten-dimensional gravitational couplings are

$$\kappa_4^2 = \kappa^2 (2\pi R)^{-n} (2\pi\sqrt{\alpha'})^{n-6}.$$

[3] A more complete analysis might give a different power of $H(0)$ inside the logarithm. This will affect some of the numerical estimates, but not the overall logic.

It is convenient to rewrite this as

$$G\mu_{\rm D} = \frac{g_s}{16\pi}\left(\frac{\alpha'}{R^2}\right)^{n/2} .$$

Assuming that the effects that fix the moduli and break supersymmetry produce modulations of the warp factor and/or the dilaton by a factor of order $\delta \lesssim 1$, as opposed to the large warping of the KKLMMT model. Then for F-strings, $V_{,ii} \sim \delta/2\pi\alpha' R^2$ in the large directions, and so

$$\begin{aligned} \omega_{\rm large} &\sim \ln\left(\frac{R^2}{\alpha'\delta}\right) \\ &\sim \frac{2}{n}\ln\left(\frac{g_s}{16\pi G\mu_{\rm D}\delta}\right), \\ \Omega_{\rm FF} &= \frac{1}{V_\perp}\left(\frac{2\pi}{\omega_{\rm large}}\right)^{n/2} . \end{aligned} \tag{5.21}$$

In $\Omega_{\rm FD}$ there is an additional $2^{n/2}$ but the likelihood of a suppression factor from separated minima. For D-D collisions, the relevant quantity is

$$\frac{y^2}{2\pi\alpha'\epsilon} \sim \frac{g_s}{\pi\epsilon}\ln\left(\frac{g_s}{16\pi G\mu_{\rm D}\delta}\right) . \tag{5.22}$$

5.4 Final Interaction Probabilities

Having assembled all of the relevant calculations, it is interesting now to insert some typical parameter values and obtain estimates for $\mathcal{P}$. Estimations of the self-interaction probabilities, as already explained, will yield useful estimates of the enhancement of the string network density relative to standard cosmic string models, given that $\mathcal{P}$ describes the efficiency of the lossy interaction responsible for string loops breaking off and radiation energy away from the network. The probabilities for the interactions of strings of different types will be necessary for detailed simulations of (p, q)-string networks, but shall not be applicable to the simple estimates of this section. The range of interaction probabilities indicates the extent to which it might be able to probe stringy physics by measuring the various intercommutation probabilities.

Consider first F-F reconnection. The reconnection probability is the earlier result (5.4) with one of (5.19, 5.18, 5.21) in place of $1/V_{\perp}$. The function $f(v,\theta)$ is roughly 0.5 when averaged over angles and velocities. The results are summarised in Table 5.4, which also has the final numerical probabilities for appropriate values of the model parameters. In Table 5.4 for the general brane inflation models (which are defined here as those models in which there is no highly warped region), to obtain the estimates the typical values of $\delta = 1$ and $n = 2 \to 6$ have been used, and the fact that the scale $G\mu_{\rm D}$ is likely to lie between 10^{-6} and 10^{-12} [27,90].

For the D-D reconnection probability, the results (5.20, 5.22) are combined with appropriate values of the model parameters in Table 5.4. The multiplicative contribution of each would-be fermion zero mode to (5.16) is $(1-e^{-y^2/2\pi\alpha'\epsilon})$, so the fermion zero modes are largely lifted in all cases, and for larger values of g_s (but still less than 1) the production of fermionic open strings is negligible. The suppression of producing tachyonic strings is angle dependent, and the effective value of the impact parameter determines the range of angles for which recombination is suppressed.

For (p,q)-(p',q') collisions the result depends primarily on the values of q and q'. Making one or both of these larger than 1 enhances reconnection in two ways: the production of strings is enhanced by the Chan-Paton degeneracy, and the fluctuations are decreased due to the greater tension of the (p,q) string; however, the great tension also requires that more F-string pairs are produced between the (p,q)-strings in order to allow the tachyon to roll.

Table 5.1: Final interaction probabilities for the different string types in the various brane inflation models. In the KKLMMT models, $g_s M \gg 1$ to ensure that the supergravity approximations made are valid, and that the curvature at the tip of the inflationary throat is small compared to the string scale. Individually, g_s can be between its G.U.T. value, 1/20 and 1 (in some F-theory compactifications); M must be at least 12, perhaps somewhat larger [14]. The KKLMMT probabilities are generally somewhat less than 1, but shown is the range from best to worst case of the model parameters.

	KKLMMT		
Quantity	$\mathbb{S}^3$ Localised	$\mathbb{S}^3$ Filled	General Brane Inflation
$\mathcal{P}_{\mathrm{FF}}$	$\frac{1.5 g_s^2}{\ln^{3/2}(1+g_s M)}$	$\frac{100 g_s^{1/2}}{M^{3/2}\ln^{3/2}(1+g_s M)}$	$0.5 g_s^2 \left[\frac{\pi n}{\ln(g_s/16\pi G\mu_{\mathrm{D}}\delta)}\right]^{n/2}$
	$10^{-2} \leftrightarrow 1$		$10^{-3} \leftrightarrow 1$
$\frac{y^2}{2\pi\alpha'\epsilon}$	$15 g_s$	$0.5 g_s M$	$g_s \left\{\mathcal{O}(10 \to 25) + \ln(g_s/\delta)\right\}$
$\mathcal{P}_{\mathrm{DD}}$	$10^{-1} \leftrightarrow 1$		$10^{-2} \leftrightarrow 1$

For F-(p,q) reconnection, $\mathcal{P}$ contains an extra factor of $2^{k/2}/g_s$, where $k = 6, 3, n$ for KKLMMT with fluctuations localised on the $\mathbb{S}^3$, fluctuations filling the $\mathbb{S}^3$, and the general models respectively. Also the kinematic function f is replaced with $h_{p,q}$ from (5.6), which varies in roughly the same range as f except for an extra factor of q. Thus the F-(p,q) reconnection probability is somewhat larger than the F-F reconnection probability, *if* the strings are coincident in the transverse directions (when the perturbative calculation gives $\mathcal{P} > 1$ it is assumed that the probability is approaching saturation, $\mathcal{P} \to 1$). If the strings sit at separated minima in the transverse directions, the F-(p,q) reconnection can be suppressed by an arbitrary amount, and can easily be negligible.

5.5 From String Theory to Astrophysics

In the preceding chapters, somewhat formal string theory arguments were outlined to indicate that cosmic strings are a general consequence of $D\bar{D}$ inflation. Particularly in the present chapter, string theory calculations of the basic properties of these cosmic strings were presented. At this juncture the interface of these basic properties with the large-scale astrophysical characteristics of the string networks needs to be understood. While some rudimentary estimates of the connection have been made, understanding here is incomplete mainly because detailed simulations of string network behaviour and evolution are required with these new "stringy" properties.

5.5.1 Current Bounds and Detection at $\mathcal{P} = 1$

Since most of the work on cosmic strings to date considers "standard" 4D cosmic strings, the bounds and detection sensitivities are formulated for $\mathcal{P} = 1$. As outlined in *e.g.* [83, 130], cosmic string networks have a variety of astrophysical signatures:

Pulsar Timing

The variation in pulse times depends on the stocastic gravitational wave background, which in turn depends on the cosmic string network. Since the stocastic gravitational wave background is sensitive to the string network density, it will vary with $\mathcal{P}$. The $\mathcal{P} = 1$ bound on string tension - $G\mu \lesssim 10^{-6}$ - is weak compared to more recent bounds from CMB measurements.

Cosmic Microwave Background

Cosmic strings were originally considered an alternative to inflation because they seed density perturbations and structure formation. This fact and the WMAP data [19] supporting inflation place a constraint on the contribution of a string network to the temperature fluctuations, and so on the string tension. The current bound at $\mathcal{P} = 1$ is $G\mu < 7 \times 10^{-7}$ [112], which is well satisfied by most string models: strings in KKLMMT have $10^{-10} \lesssim G\mu \lesssim 10^{-9}$ [28] and more general brane inflation models have $10^{-12} \lesssim G\mu \lesssim 10^{-6}$ [90].

Also a cosmic string network leads to significant B-mode polarisations in the CMB above that expected from gravitational waves in slow-roll inflationary models [112]. For the most optimistic tensions, these could be detectable by future ground and space-based polarisation measurements.

Gravitational Wave Bursts

As described by Vilenkin and Damour [132], dynamical excitations - cusps and kinks - on cosmic strings, which are caused by their interactions, give rise to gravitational wave bursts. Each individual gravitational wave burst depends on the string tension, but there will be $\mathcal{P}$ dependence on the net signal, since $\mathcal{P}$ affects the total string network energy density. The $\mathcal{P} = 1$ estimates for the sensitivity to such gravitational wave bursts are:

- LIGO II, in its frequency band, can detect $G\mu$ as low as 10^{-11} [132].
- LISA would be able to detect bursts in its frequency band from cusps on strings of tension $G\mu > 10^{-13}$ [132].

Gravitational Lensing

It is well-known that an isolated line-like source of stress-energy, as a realistic 4D cosmic string will approximate on some scale, generates a conical spacetime. The deficit angle of the cone is proportional to $G\mu$. Such a spacetime provides a means of lensing a distant object, which importantly shall lead to an even number of images. Given that point-like or spherical lensing objects lead only to odd numbers of images, the string lensing signature is therefore distinctive. String tension could be measured from the image separation.

While there are recent suggestions of such lensing events being detected ([133, 134] for instance), there is of course no definitive evidence yet. A particularly promising proposal for the detection of lensing events is the Square Kilometre Array radio-telescope, which in principle would have milli arc-second resolution and such a wide field of view that an extensive search for such lensing events would be possible[4]. Importantly, this resolution equates to an ability to detect string of tensions down to $G\mu \gtrsim 10^{-10}$, and so all of the strings of KKLMMT models, and most of the more general high-scale supersymmetric models.

5.5.2 Avenues to Detecting $\mathcal{P} < 1$

In order to ascertain whether astrophysical cosmic strings have "stringy" characteristics, the above section needs to be revised for $\mathcal{P} < 1$. What follows is largely speculative, since most of the important supporting calculations are yet to be done.

F-string reconnection probabilities were found to be in the range $10^{-3} \lesssim \mathcal{P} \lesssim 1$, and those for D-strings $0.1 \lesssim \mathcal{P} \lesssim 1$. As described, the network energy density

[4] Ira Wasserman brought this possibility to the attention of the author.

scales with string tension and loop-formation probability like

$$\rho \sim \frac{\mu}{\mathcal{P}t^2}.$$

This behaviour for $\mathcal{P} < 1$ can infect many of the conclusions of the previous section. The pertinent question is, given that 4D gauge theory strings have interconnection probability $\mathcal{P} = 1$ exactly, whether there are any experimental measures which can determine $\mathcal{P}$ and μ independently. Fortunately of the various proposed observable effects of cosmic strings on astrophysical phenomena, each has a different sensitivity to $\mathcal{P}$ and μ; not all effects are sensitive only to the energy density of the network. It is still important to reiterate that any detection of $\mathcal{P} < 1$, while not a definitive test of D$\bar{\text{D}}$ inflation and string theory, would be extremely convincing evidence.

Although the stocastic gravitational wave background depends on the overall string network density, pulsar timing measurements are not a promising avenue for detecting cosmic strings; there are numerous other more standard sources of variability in the pulses. The string network influences the CMB by the combination $\mu^2/\mathcal{P}$ [112]. Individual gravitational wave bursts are sensitive to μ only, whereas the burst frequency (per month, say) depends on the network energy density and hence, $\mathcal{P}$, although the precise dependence on $\mathcal{P}$ and μ is not yet known. For these astrophysical effects, because μ and $\mathcal{P}$ cannot be individually determined from any one measurement, at least two different experiments would be needed to disentangle μ and $\mathcal{P}$ dependence, exposing any "stringy" properties.

Finally and perhaps most promising are the lensing experiments (particularly the proposed Square Kilometre Array). These can detect the tension of individual strings. The number of such lensing events would give a measure of the string network density and therefore of $\mathcal{P}$. Further a survey of double images in the sky would be able to map a spectrum of string tensions, and ascertain if heavier strings were present, whether they were (p, q) or Kaluza-Klein excitations, giving an clear window into the microscopic origins of the universe.

5.6 Conclusions

As proposed in [27, 84, 90], cosmic strings can be a new and exciting window into the microscopic physics of string theory. In this work, it was described how this prediction from various brane anti-brane inflationary models is quite generic to most such models in string theory, although the details of their construction vary considerably. This prediction of cosmic strings, made possible by the enlightening advances in string field theories and other non-perturbative techniques in string theory, is a conceivable way of testing string theory. The lessened string interaction probability is one clear signal of - at least the presence of extra-dimensions, and more optimistically - string theory itself. The possibility of a spectrum of string tensions from either Kaluza-Klein like excitations of a basic string, or from (p, q)-strings is a signature that could give a more detailed illustration of the physics of hidden dimensions and the string scale.

These results render such phenomena extremely worthy of attention, particularly in that so much of both the underlying string construction and the astrophysical properties of these modified strings are yet to be understood. With progress in these directions, the fascinating possibility of seeing strings in the sky may not be too distant.

APPENDIX A

LIST OF ABBREVIATIONS AND ACRONYMS

BPS - Bogolmo'nyi, Prasad, Sommerfeld; usually referring to BPS solitons in supersymmetric theories, which exhibit no force between them.

BSFT - Boundary String Field Theory.

CFT - Conformal Field Theory.

CMB - Comsic Microwave Background.

DBI - Dirac, Born, Infeld, specifically the DBI action of nonlinear electrodynamics.

D$\bar{\text{D}}$ - Brane anti-brane.

F- & D- - Fundamental and Dirichlet strings (D1-branes).

GKP - Giddings, Kachru, Polchinski [13].

GSO - Gliozzi, Scherk, Olive [135], usually the GSO projection which projects worldsheet supersymmetric theories into spacetime supersymmetric and modular invariant theories.

$G\mu$ - The dimensionless ratio of cosmic string tension to Planck mass squared which is relevant in 4D cosmic string physics.

KKLMMT - Kachru, Kallosh, Linde, Maldacena, McAllister, Trivedi [28].

KKLT - Kachru, Kallosh, Linde, Trivedi [14].

KS - Klebanov, Strassler [108].

LIGO - Laser Interferometer Gravitational Wave Observatory.

LISA - Laser Interferometer Space Antenna.

NS-NS - Closed string modes arising from holomorphic and antiholomorphic Neveua Schwarz worldsheet fermions.

RR - Closed string modes arising from holomorphic and antiholomorphic Ramond worldsheet fermions.

RS - Randall, Sundrum [11, 12].

WMAP - Wilkinson Microwave Anisotropy Probe.

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