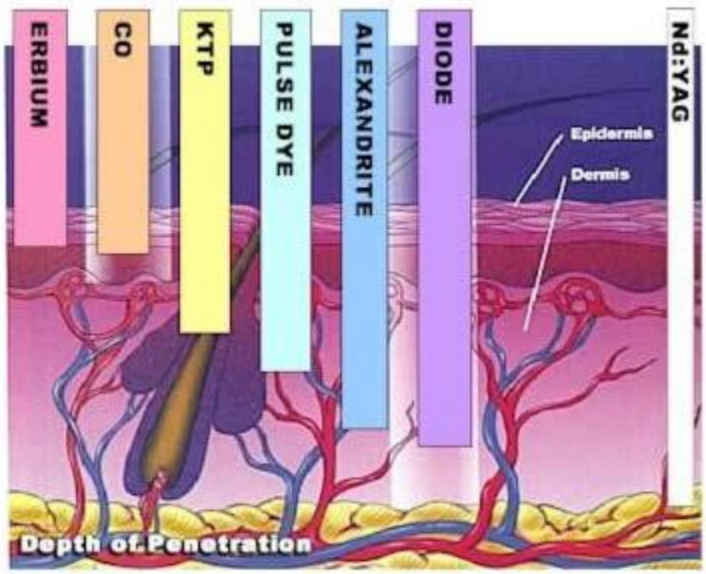
Laser Hair Removal:

Comparative Study of Light Wavelength and its Effect on Laser Hair Removal



http://www.body4real.co.uk/blog/wp-content/uploads/laser-hair-types.jpg

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Table of Contents

Executive Summary

For some people, hirsuteness (having excess body hair) can be an embarrassing problem. Many attempts have been made to find a solution to these problems, including electrolysis, tweezing, shaving, and waxing. However, most of these solutions are painful, are not useful in treating large areas of skin, or are not permanent. Laser hair removal stands out amongst these other methods as a permanent method of reducing hirsuteness that can cover large areas of the body, such as the chest or the legs. While laser hair removal is a widely used technology, few studies have explored the physical aspects of why it works so well. More specifically, there is a significant lack of computer models that show how temperature profiles look inside the hair and surrounding skin. Using the physical properties and dimensions of hair, we constructed a model of the hair that approximates how actual hair resides in the skin. Using COMSOL Multiphysics, we tested this model with five different lasers of varying wavelengths in order to determine the relationship between laser wavelength and temperature in the hair. Using a laser pulse duration of 0.01 seconds (10 milliseconds), we found a positive correlation between wavelength and temperature, with all wavelengths except the lowest (595 nm) achieving a temperature above the threshold temperature required for hair destruction. In addition, while all lasers caused a temperature rise in the surrounding skin, the extent of thermal damage was minimal. However, since we could not find physical properties of the hair follicle itself, we were forced to approximate those properties using the hair shaft properties, ultimately leading us to treat the follicle and shaft as one entity. This is a slight limitation with our design. Regardless, we have provided a greater understanding of the physiological temperatures involved in hair removal, and have reinforced the fact that laser hair removal can be a safe method and effective method for treating hirsuteness by showing that hair follicles can be heated to a temperature that kill them by using lasers, and that this heating does not severely or irreparably damage surrounding skin.

Introduction

Hirsutism is the presence of excessive body hair and can be extremely distressing to many patients. In most cases there is no obvious cause, but some patients have hormonal disturbances that can be attributed as the cause. Excessive hairiness of the body or face affects approximately 10% of women between the ages of 18 and 35. In addition, hirsutism is also a feature of menopause in a number of women. Hirsutism causes considerable psychological distress in many patients. This serious condition can prevent patients from confidently integrating into society (Jiang, 2002).

Over the past few years, laser hair removal has steadily developed and improved. Basically, laser hair removal is a medical procedure that utilizes laser light, which is an intense, pulsating beam of light to remove undesirable hair. Laser hair removal has been used for medical problems and cosmetic procedures. The efficacy of laser hair removal is generally accepted in dermatology and is widely practiced. To thermally destroy the hair follicle without harming the surrounding skin tissue, the laser has to target the melanin in the hair follicle with a specific wavelength in the 600-1100nm range. In recent years, selective photothermolysis based on laser light has been introduced into the aesthetic market for epilation or hair removal procedures. Epilation procedures are frequently used in cosmetic procedures to remove hair from unwanted places.

While selective absorption of light by hair chromospheres causes damage to hair follicles, it leaves the surrounding skin undamaged. This is based on the selective photothermolysis theory. It is theorized that thermal damage is most selective if the pulse duration is shorter than or equal to the relaxation time of the target. Although Hair Laser Removal (HLR) can be considered an inherently safe treatment, there may be approaches to optimize the benefit to risk ratio of this laser procedure. In the present project, we aimed to model the thermodynamics on the skin surface tissue and hair follicle during laser hair removal. Various types of lasers with different wavelengths and properties were modeled to determine which laser would be the most effective in destroying the hair follicle to prevent growth of hair after epilation treatments. The laser that both satisfies our criteria for hair destruction, while causing minimal damage to the surrounding skin would be the most effective one for use in epilation treatments.

Design Objectives

First, the model aims to define a temperature and duration at which the hair follicle will be completely removed. The temperature at which the hair follicle is completely removed is 65 °C (Gault, 1999). Second, we need to determine the guidelines for temperature control to avoid burning or damaging the surrounding skin. The temperature at which the skin tissue suffers thermal damage is 44 °C (Fiskerstrand, 2003). Finally, we need to determine a model for the time-varying application of heat from a laser to the skin and hair follicle.

Schematic

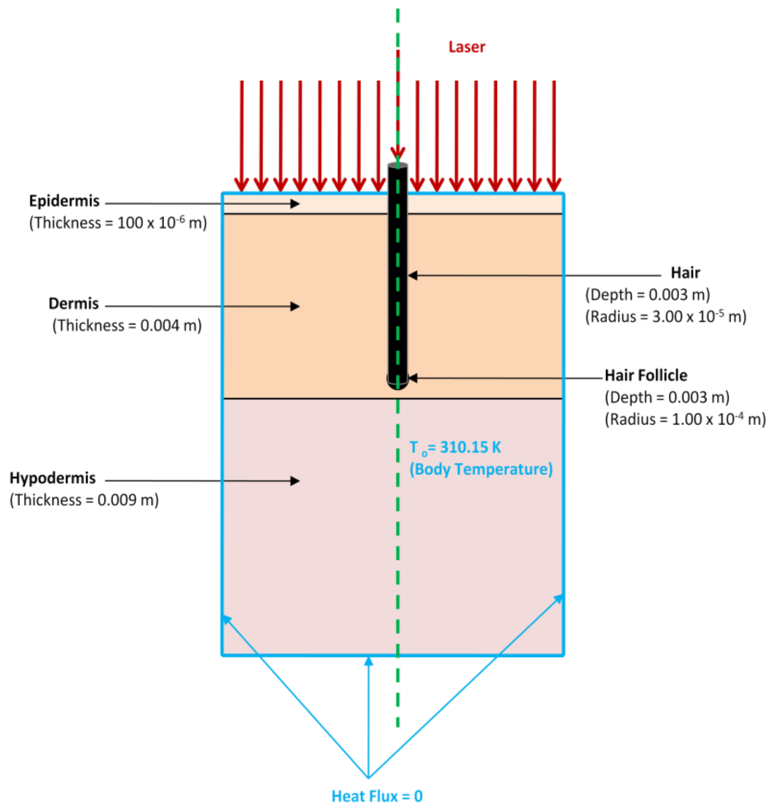

Figure 1: Schematic diagram depicting the layers of skin and hair follicle. Also includes boundary conditions and line of symmetry.

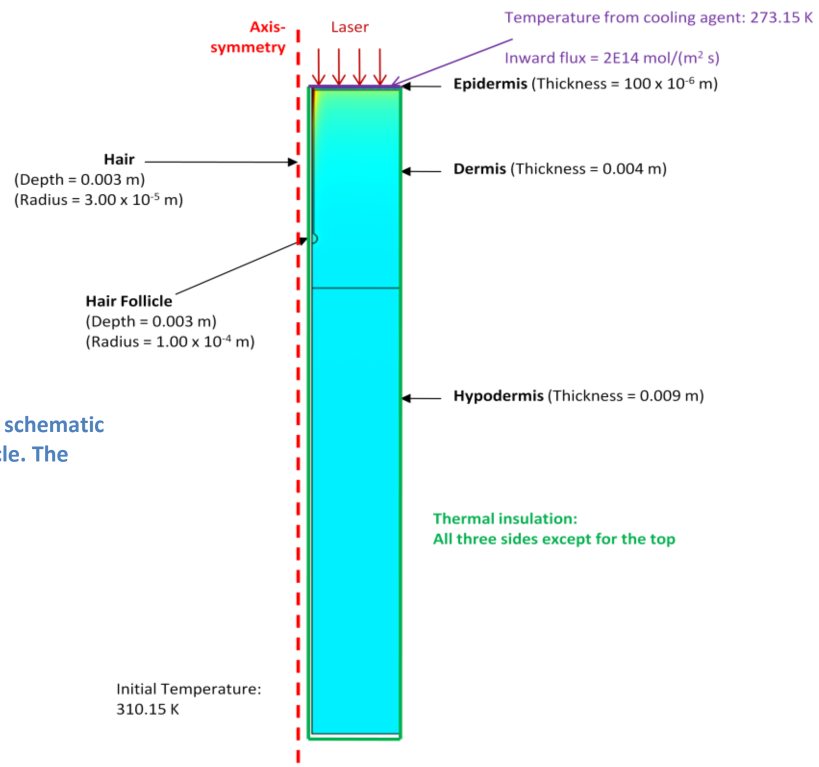

Figure 2 : Model as it appears in COMSOL. This schematic includes all the layers of skin and the hair follicle. The figure also indicates boundary conditions.

Governing Equations

$$governing\ equation\ (\ 2D\ axi\ symmetrical):\quad \frac{\partial T}{\partial t}=\frac{k}{\rho C_p}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\partial^2 T}{\partial z^2}\right]+\frac{Q}{\rho C_p}$$

$$light\ transport\ equations:\ \frac{\partial \emptyset}{\partial t}-D\nabla^2\emptyset+c_*\mu_a\emptyset=0,\ \ D=c_*\left[3\left(\mu_a+(1-g)\mu_s\right)\right]^{-1}$$

In COMSOL:

$$\delta_{ts}\frac{\partial c}{\partial t}+\nabla(-D\nabla c)=R\ ;\ \ c=concentration$$

$$\delta_{ts}=1,\ \frac{\partial c}{\partial t}=\frac{\partial \emptyset}{\partial t}R=-c_*\mu_a\emptyset$$

- D = optical diffusion coefficient [m^2/s] of skin
- ∅ = fluence rate [W/m^2] of the incident laser light at a location
- c_*= speed of light [m/s] in the particular tissue component (epidermis, dermis or blood vessel)
  - Assumed to be 2.19E-08 m/s
- μ_a = absorption coefficient [m^{-1}] of tissue
- μ_s = scattering coefficient [m^{-1}] of tissue
- $g=optical\ anisotropy\ factor$

$$boundary\ condition\ on\ the\ surface\ receiving\ laser\ radiation:\ (1-r)P(t)c_o=-D\frac{\partial \emptyset}{\partial n}$$

- P(t) = laser irradiance [W/m^2]
- r =ratio of reflected light to the laser power output
- c_o= speed of light [m/s] in vacuum
- n = normal direction

$$heat\ source\ term\ (\emptyset=fluence):Q=\mu_a\emptyset$$

Results and Discussion

While laser hair removal is a widely used technology, few studies have explored the physical aspects of why it works so well. More specifically, there is a significant lack of computer models that show how temperature profiles look inside the hair and surrounding skin. Using the physical properties and dimensions of hair, we constructed a model of the hair that approximates how actual hair resides in the skin. Using COMSOL Multiphysics, we tested this model with five different lasers of varying wavelengths in order to determine the relationship between laser wavelength and temperature in the hair.

As a representative display of temperature profiles in the human skin and in the human hair, contour plot (See Figure 11 in the Appendix) was taken at 0.1 s. The inward flux is constant for each wavelength. The laser irradiance enters the model on the surface where a cooling method is also being used (epidermis). The model show that there is very little temperature change observed in the epidermis, dermis, and hypodermis. This is result is supported by low values of absorption coefficient for the three components of skin in comparison to the human hair (See Table 3 in the Appendix) and past researches (see References). Figure 3 below is the main focus of the results. It shows the variation of temperature for YAG laser with λ = 1064nm used on the model as a function of time, from 0 to 0.01 s. Temperature will vary with many different factors such as scattering coefficient, thermal conductivity (See Table 3 in the Appendix).

Figure 3 illustrates the results obtained by a YAG Laser with a λ= 1064nm. The contour plot in Figure 3 shows that the skin remains relatively constantly at body temperature. However, for skin closer to the hair follicle, the temperature of the skin approaches 50°C. Since this value is greater than the 44°C, the skin may get damaged. In the line plot given in Figure 3, it shows the temperature is rising as time is extended at a range of points in the model. Point 6 is the center of the hair follicle. Points 11, 10, and 14 are varies points of the dermis that are heat above the 44°C at the end of 0.1 second. These temperatures higher than 44°C indicate that the dermis may be slightly damaged with the YAG laser. Furthermore, the graph shows that Point 6 is much greater than the other points in the model. Therefore, the hair follicle is being heated up and absorbing the heat source faster than the other points in the model.

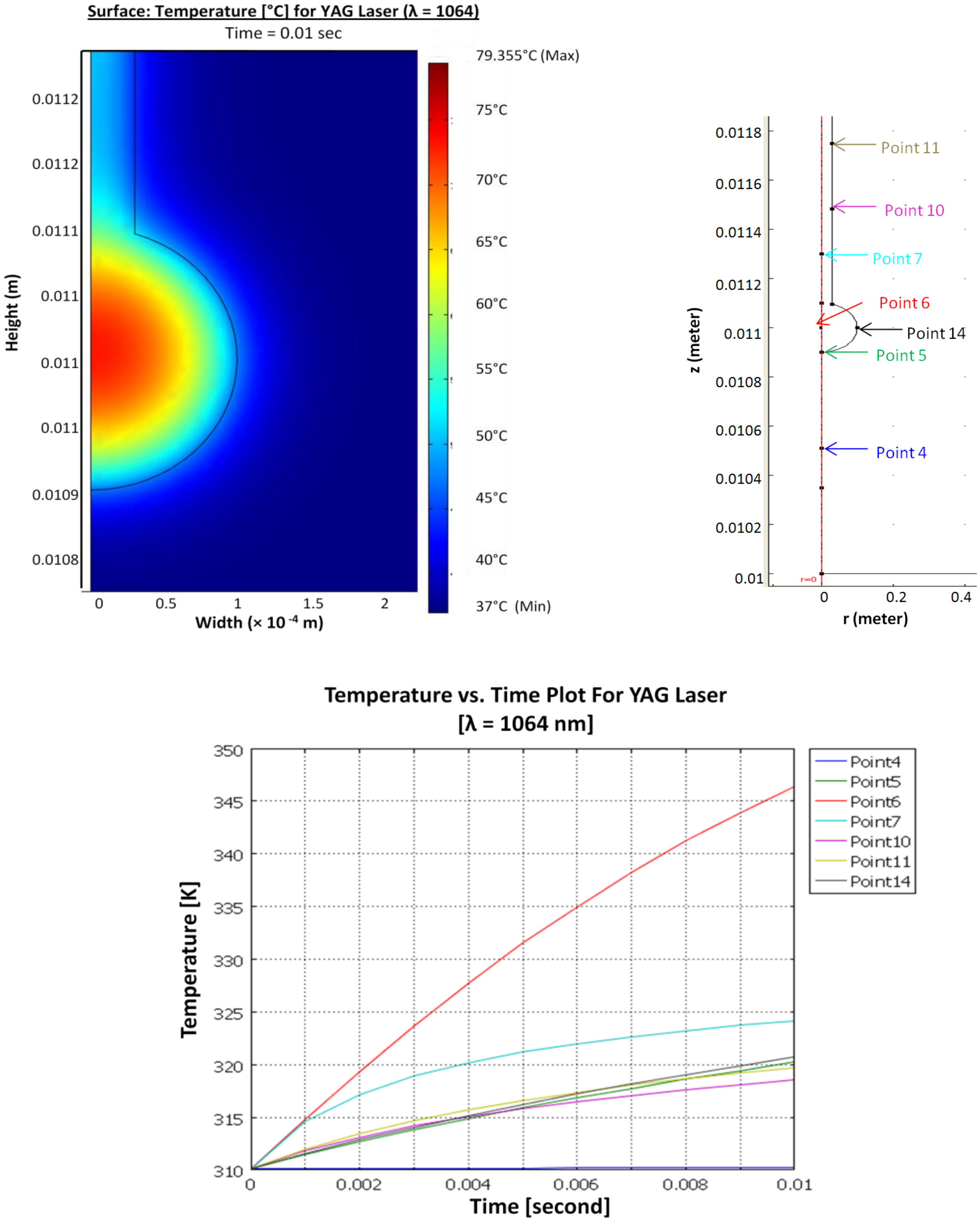

Figure 3: This figure shows the results obtained by a YAG Laser with a λ= 1064nm. In the contour plot on the top left corner, it shows that the skin remains constantly at body temperature. For skin closer to the hair follicle, the temperature of the skin approaches 50°C. In the line plot, it shows the temperature is rising as time is extended at a range of points in the model. Point 6 is the center of the hair follicle. In the graph, it shows that Point 6 is much greater than the other points in the model.

In comparsion to other lasers, YAG has the highest value for the wavelength. To analyze the differences and similaities of the five lasers, Figure 4 is created. In Figure 4, seven points in the model around the hair follicle and the dermis are analyzed. At point 6, it marks the center of the hair follicle, which also where the highest temperature is obtained for each laser. At 595nm, the highest temperature reading was roughly 40°C. For 694nm, the maximum temperature occurred in the hair follicle with a value of 50°C. At 755nm, the hair follicle reached a temperature of 55°C. As for Diode Laser with a wavelength of 800nm, the highest temperature of 60°C was recorded in the hair follicle. The YAG laser had its highest temperature at 79°C. Therefore, as wavelength is increased, the temperature is gradually increasing and killing the hair follicle. This is held true for all the points in the model as shown in the seven line plot in Figure 4.

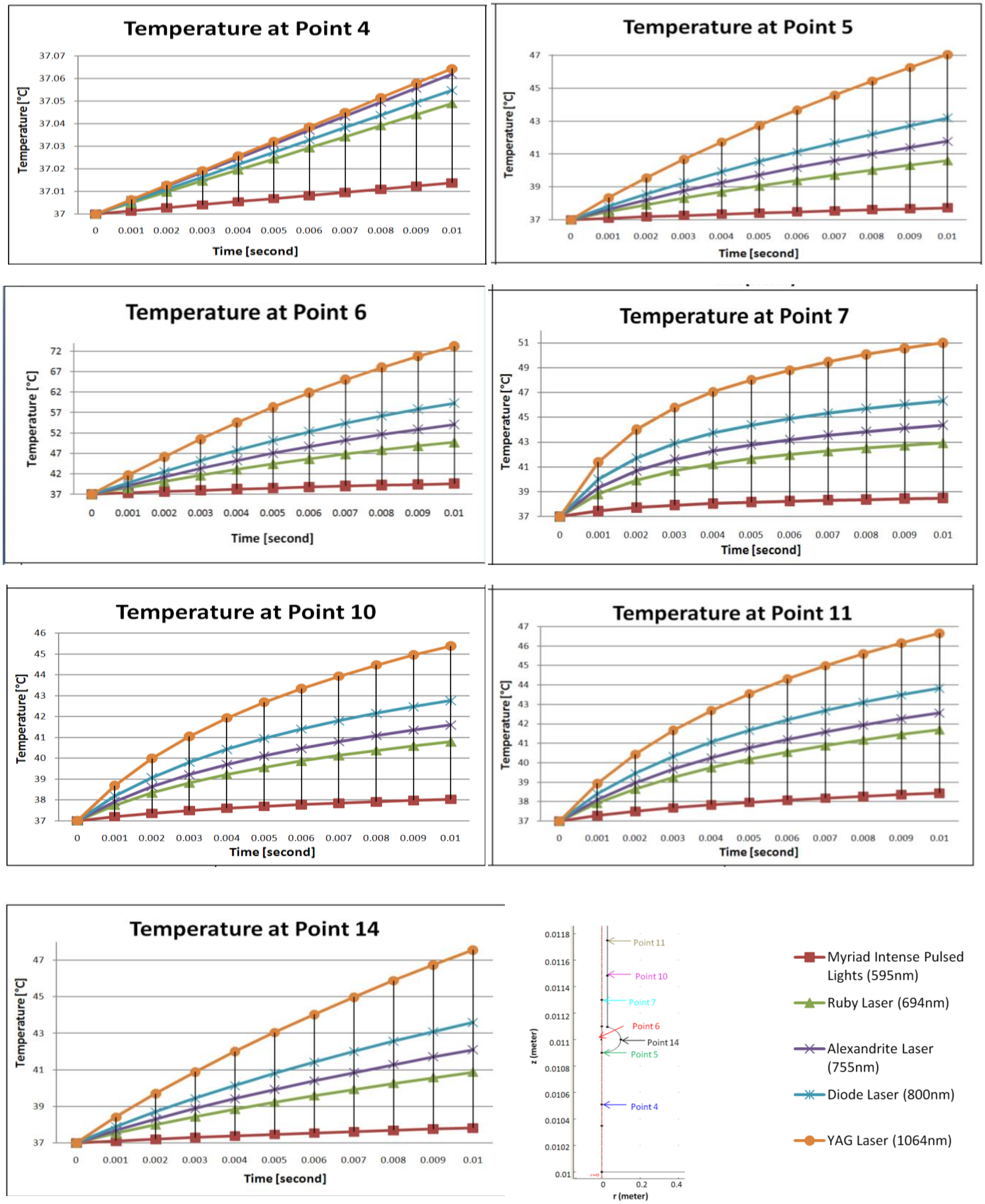

Figure 4: These line graphs show how temperature increase with as time continues. The lowest temperatures occur at Point 4, which is located within the dermis and far away from the hair follicle. Points 5, 7, 10, 11, and 14, have relatively the same range of temperatures. Point 6 is where more the heating occurs with the highest temperature above 72°C.

Figure 4 shows that the temperatures at varies point within the COMSOL model. As the temperature of the hair follicle rapidly rises, the other points in the model remain relatively constant with the body temperature of 37°C. Therefore, at a constant inward flux of 2×10^{14} W/m^2, the skin does not get severely damaged. Although, the inward flux is constant, the laser irradiance is not constant for each laser type. The inward flux is obtained from the many factors such as the speed of light, ratio of reflected light to the laser power output, and the power irradiance. While, the speed of light is constant, the ratio of reflected light varies at different wavelength (see Figure 10 in the Appendix). The laser irradiances for the five different lasers are recorded in Table 1. All values for the irradiance in Table 1 fall within the safety range for irradiance in skin.W/m

Table 1: For the model, the inward flux was a constant (2×10^{14} W/m^2), for each laser. This constant ensures that there is no variability in the flux. The ratio of reflected light to the laser power output (r) for each wavelength was obtained from a published graph by Shapshay (as shown in Figure 10 in the Appendix). With the known values for flux and ratio, the laser irradiance was calculated from the following equation: $(1-r)P(t)c_o = -D\frac{\partial \emptyset}{\partial n}$. The values were recorded in this table.

Wavelength	595 nm	694 nm	755 nm	800 nm	1064 nm
$P(t)\left(\frac{W}{m^2}\right)$	1.22×10^6	1.814×10^6	1.766×10^6	1.678×10^6	1.342×10^6
r	0.45	0.63	0.62	0.60	0.50

In a research done by Goldberg and Silpunt, they show that irradiance of 1.0×10^7 W/m^2 show no hyper pigmentation, hypo pigmentation or scarring at 3 months after treatment (Goldberg & Silapunt, 2001). It is optimal to have the lower value of irradiance to kill the hair. Therefore, YAG is the optimal laser under these specific conditions to kill the hair follicle. With a wavelength of 1064 nm, YAG has the second to lowest value for laser irradiance. In conclusion, the YAG laser under the conditions of this model has been shown to be the most effective and efficient to kill the hair follicle and maintain a low value for its irradiance.

Qualitative Description

With the exception of the IPL laser, all lasers tested satisfied our criteria for hair destruction and would have clearly destroyed the hair while causing minimal damage to the surrounding skin. What this means is that while the hair absorbs a high amount of the laser light, the various skin layers absorb very little. Depending on the wavelength of the light used (which depends on the laser), the highest temperature in the hair follicle could vary greatly. We found that the YAG laser attained the highest temperature in the hair follicle, with the diode laser being the next highest. However, while the YAG laser has been shown to be clearly effect, a higher temperature does not necessarily mean better results. The hair follicle is destroyed at a certain temperature, and attaining temperatures above this threshold value may not be more effective. In fact, it may cause more damage and result in undesirable effects, such as permanent scarring. A certain amount of thermal damage to the skin is expected, since the skin can feel mildly burned (like a sun burn) and can appear red after receiving laser hair treatment. For the lasers we examined, none caused extensive thermal damage.

Accuracy Check

Our model that we have created is predicated has several key assumptions that can be compared against previous research that has also been conducted on laser hair removal. In order to perform an accuracy check, our model for laser hair removal was compared to other thermal modeling and experimental validation of human hair and skin by broadband light.

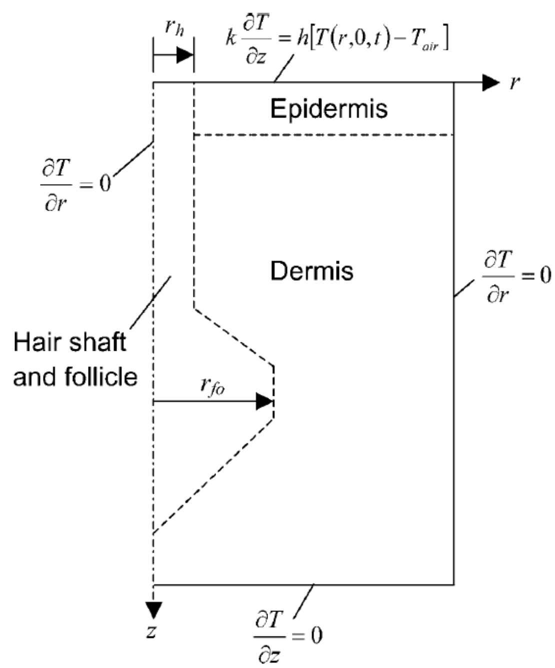

Figure 6: Thermal model by Sun et al.

Experimental validation of our thermal model of the heating of human hair and skin by laser light will be qualitatively compared to experimental results from a comparative analog (Sun, Chaney, Anderson, & Aguilar, 2009). According to Sun et al, the hair shaft and follicle are treated as one structure with the same properties. There is no differentiation between the hair shaft and follicle in terms of properties. As can be seen in Figures 9, the geometries of both our models by Sun et al and us have similar geometry and boundary conditions. Both models are axisymmetric and contain both the dermis and epidermis. However, our model considers the heating of the hypodermis by the laser. All the boundaries except for the one exposed to the laser are considered to be insulated. Both models used a laser wavelength of approximately of 400 to 1200 nm.

Even though the geometries of both models are similar, the depths of the models used in both projects vary. The depth of our model is 0.0191m, while the depth of the model by Sun et al is 0.0035m. However, the greater depth of our model can be attributed to our consideration of the hypodermis. The hair follicle only extends into the dermis and not the hypodermis, which is the reason why Sun et al did not include the hypodermis into their model. Excluding the hypodermis region, the depths of both models are more similar. Despite the differences in model depths, the similarities are qualitatively similar. In terms of the hair follicle radius used in both models, the radius for the model by Sun et al is five times greater than ours. They assumed that the hair follicle radius was much ticker. Despite these differences in model geometry, a qualitative comparison can be made analyzing the temperature ranges reached by both models. The fluence used by our model was for an IPL laser with a value of 30 J/cm^2. In the model created by Sun et al, the fluence was 5 J/cm^2 for three different pulses.

As can be seen from Figures 7 and 8, the ranges of temperatures observed in the contour plots of both models for the temperature within the tissue are similar. For the thermal model by Sun et al, within the region of interest, the hair follicle, the temperature ranges for both Sun et al and our model are approximately 50-80°C. Despite the differences in geometry, the temperature range reached within the follicle by both Sun et al and our model is the same. This indicates our model does achieve similar results, in terms of temperature, in the hair follicle. Also, as can be seen in the hair-dermis boundary, the temperature ranges experienced in that tissue for both models are remarkably similar. The temperature ranges of both Sun et al and our model are approximately 40-55°C. Even these are all qualitative comparisons, due to the different natures of both our geometries and experimental conditions, the similarities in the temperature ranges experienced by the tissue regions of interest, such as the hair follicle and the dermis, are encouraging. However, the current research into laser hair removal consists of primarily obtaining experimental results in lieu of modeling. In addition, there is much variability in the various geometries that are used by researchers in their attempts to model or experimentally determine the effects of laser hair removal. Therefore, it would be difficult to make a precise quantitative comparison.

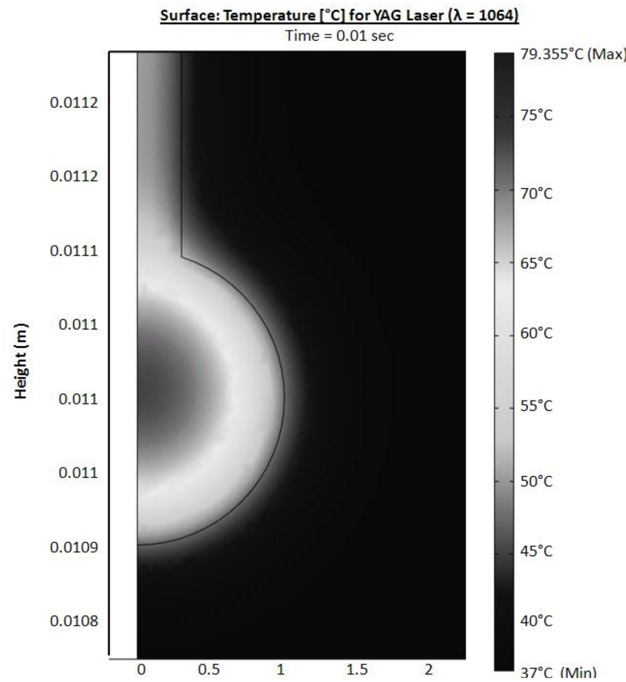

Figure 7: Temperature contour plot for thermal model for the YAG Laser

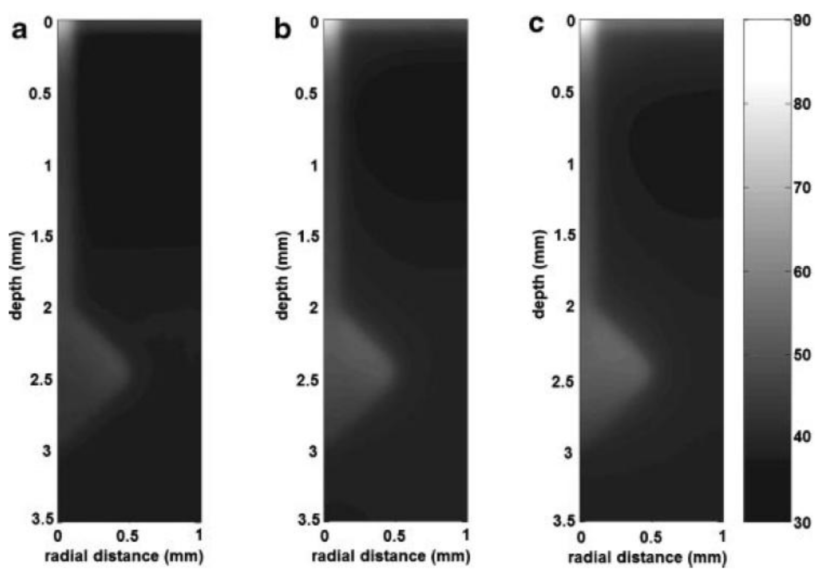

Figure 8: Temperature contour plot for thermal model by Sun et al.

Sensitivity Analysis

Sensitivity analysis was done on all the properties used in our model. Only one variable was changed at a time and the simulation was run using the 595 nm laser and the temperature over the full hair follicle was recorded. The ones with the greatest sensitivity are graphed below. The dermis anisotropy, the property of being directionally dependent, was particularly surprising since it affected the temperature in the hair the most by far. Also, most of the dermis properties changed the temperature of the hair more than the properties of the hair itself. Hypodermis properties were expected to not change the temperature of the hair by much and that is what we saw in the results of the sensitivity analysis. See appendix for full number tables.

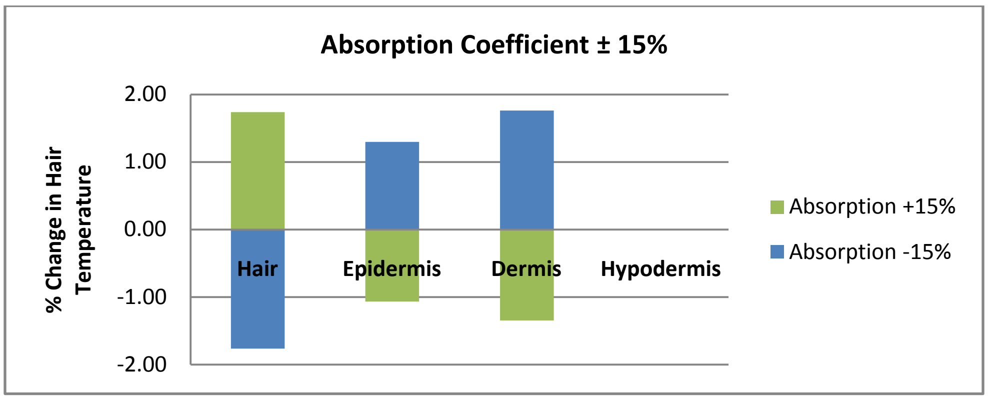

Figure 5.1—One of the graphs from the sensitivity analysis done. This bar graph ± 15% in absorption coefficient in the different parts of the model affect the hair temperature. This is down with a 0.01 second exposure to the 595 nm laser. With the exception of the hypodermis, the other components of the model show an effect on the hair temperature. When the absorption of the component is changed by 15%, the hair absorption has the greatest affect.

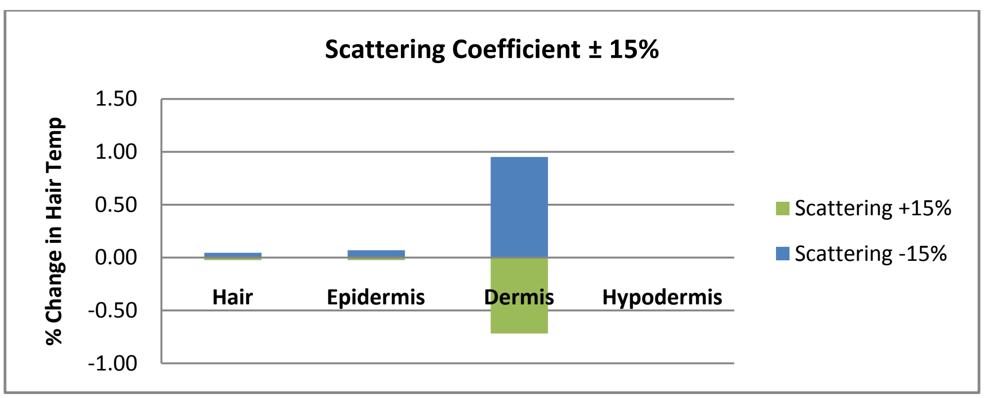

Figure 5.2-- One of the graphs from the sensitivity analysis done. This bar graph ± 15% in scattering coefficient in the different parts of the model affect the hair temperature. This is down with a 0.01 second exposure to the 595 nm laser. With the exception of the hypodermis, the other components of the model show an effect on the hair temperature. When the scattering coefficient of the component is changed by 15%, the scattering coefficient for the dermis has the greatest affect.

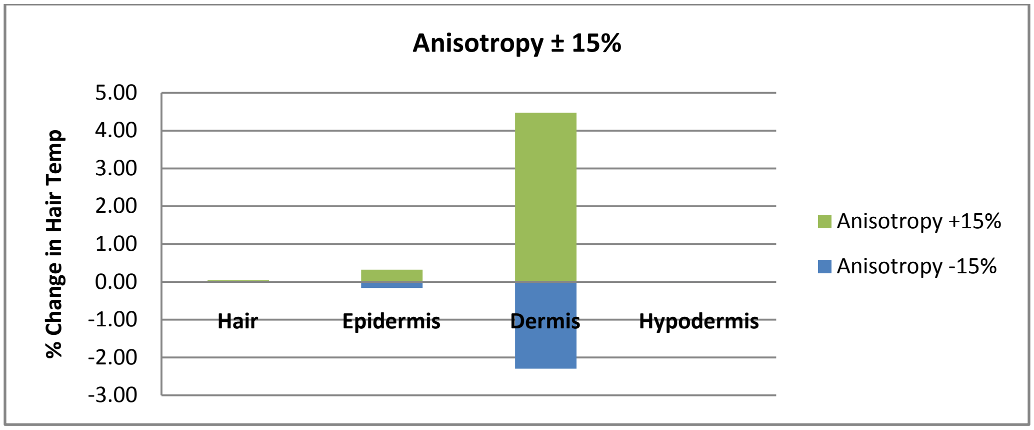

Figure 5.3-- One of the graphs from the sensitivity analysis done. This bar graph ± 15% in anisotropy in the different parts of the model affect the hair temperature. This is down with a 0.01 second exposure to the 595 nm laser. All of the components of the model show an effect on the hair temperature. When the anisotropy of the component is changed by 15%, the anisotropy value for the dermis has the greatest affect.

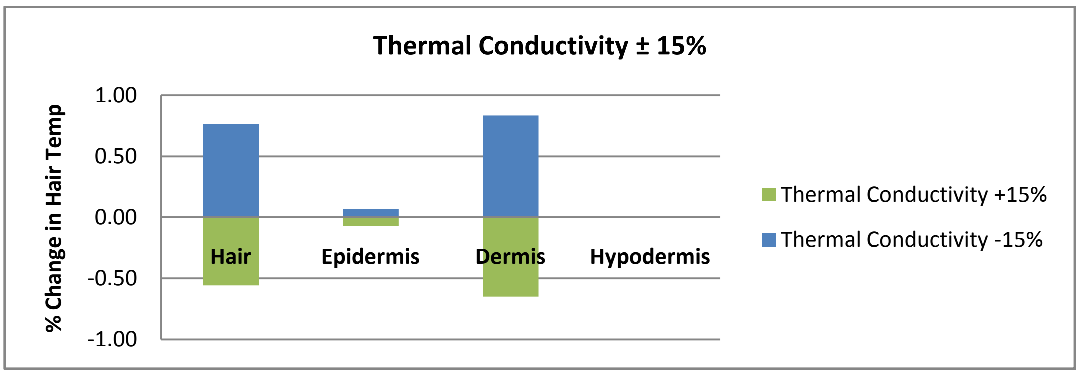

Figure 5.4-- One of the graphs from the sensitivity analysis done. This bar graph ± 15% in thermal conductivity in the different parts of the model affect the hair temperature. This is down with a 0.01 second exposure to the 595 nm laser. With the exception of the hypodermis, the other components of the model show an effect on the hair temperature. When the thermal conductivity of the component is changed by 15%, the thermal conductivity of the dermis has the greatest affect.

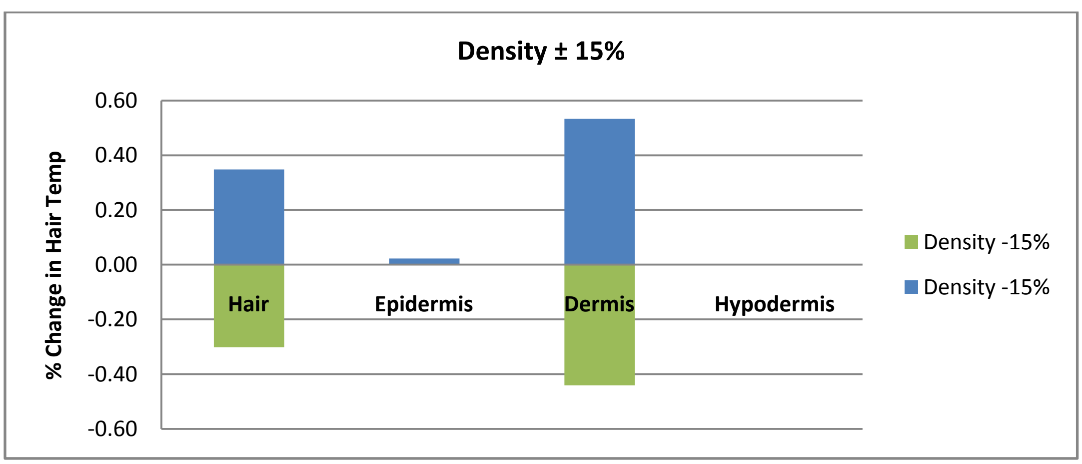

Figure 5.5-- One of the graphs from the sensitivity analysis done. This density ± 15% in absorption in the different parts of the model affect the hair temperature. This is down with a 0.01 second exposure to the 595 nm laser. With the exception of the hypodermis, the other components of the model show an effect on the hair temperature. When the density of the component is changed by 15%, the density of the dermis has the greatest affect.

Conclusion

In order for the laser to completely remove the hair follicle during the epilation procedure, a temperature of 65 °C will need to be reached or exceeded within the hair follicle. As can be seen in the results, the YAG laser was able to reach 79 °C in the hair follicle, which exceeds the threshold value of 65 °C (Gault, 1999). In order for the skin tissue to not suffer any thermal damage, the temperature of the skin tissue will need to be less than 44 °C (Fiskerstrand, 2003). For all the five laser types tested, the results clearly show the temperature of the skin that is in contact with the hair exceeded 44 °C. However, the temperatures in that region of skin barely exceed that value and can cause a small amount of thermal damage. An effective model was created to determine the time variation of heat generation from a laser to the skin and hair follicle. As can be seen from the results, the model is able to determine the guidelines for the temperatures that the hair follicle is removed and the skin tissue will suffer thermal damage.

One major surprise during the course of creating a comparative model of laser hair removal was the necessity of a cooling gel during the procedure. In the initial models of the procedure, there was no cooling gel considered. Without the cooling gel, the temperatures at the top surface of the epidermis and the hair shaft were extremely high and would have causes significant damage to the hair and the dermis tissue. With the addition of the cooling gel on the surface of the dermis covering the boundaries exposed to the laser light, the temperature significantly decreased from the previous models. The temperatures were feasible for a laser hair removal procedure to not cause serious burns to the patient.

Design Recommendations

Certain things must be taken into account when doing a procedure such laser hair removal. Laser hair removal should be easy enough to be done at home if the proper precautions are taken. One such precaution is the cooling of the epidermis. Some sort of cooling gel has to be on the surface of the skin or the patient could be severely burned. Cryogen spray is recommended for cooling of the skin. It is sprayed directly onto the skin and the laser is then applied. Another consideration is the type of laser. Our model tested various types of lasers with different operating wavelengths. The wavelength that performed the best on the model was the 1064nm (YAG Laser), because it killed the hair with the second to least amount of power. Another wise precaution would be a mechanism to set the exposure time for a very short period. Otherwise, a patient could easily get burned from over-exposure. Lastly, our model assumes darker hair and lighter skin types. Patients with lighter hair and darker skin may not find laser hair removal useful, or could possibly suffer skin damage as a result. One way to see how a person reacts is to do a small "test patch" before clearing large areas of hair. This way, someone who is not a good candidate for laser hair removal can find out before permanently damaging large areas of his or her skin. .

In stark contrast to many of the other projects, our COMSOL project was not meant to shed light on an experimental, new process. Instead, the main goal of our project was to map the temperature distribution and elucidate the underlying causes of why laser hair removal works. Laser hair removal is not very new or experimental, however. While once a controversial treatment for hirsuteness, it is now an accepted and proven technology used around the world for the purpose of hair removal. In terms of economic feasibility and manufacturability, hair removal has proven itself to be a worthy technology. There are many businesses which offer this service, and an individual can usually get the desired results for somewhere between (1000-2000) dollars on a specific area (chest, legs, etc.). However, laser hair removal does have its limitations. Due to the way laser light is absorbed by the skin and hair, the ideal candidate for laser hair removal has pale skin and dark hair. Laser removal does not work well for lighter-colored hair, and laser hair removal can have poor results on darker skin, sometimes even causing skin discoloration. So while laser hair removal is an economic solution to the problem of hirsuteness for some, it is clearly not the best solution for all.

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Appendices

In-Depth Sensitivity Analysis

Table 2: The Full Sensitivity Analysis. The three highlighted portions were used to on the graph in Figure 3.

	Absorption -15%	Avg Temp In Hair (C)	Actual Absorption (m^{-1})	Avg Temp In Hair (C)	Absorption +15%	Avg Temp In Hair (C)
Hair	5100.00	42.36	6000.00	43.12	6900.00	43.87
Epidermis	1452.23	43.68	1708.50	43.12	1964.78	42.66
Dermis	72.61	43.88	85.42	43.12	98.23	42.54
Hypodermis	458.90	43.12	539.88	43.12	620.86	43.12

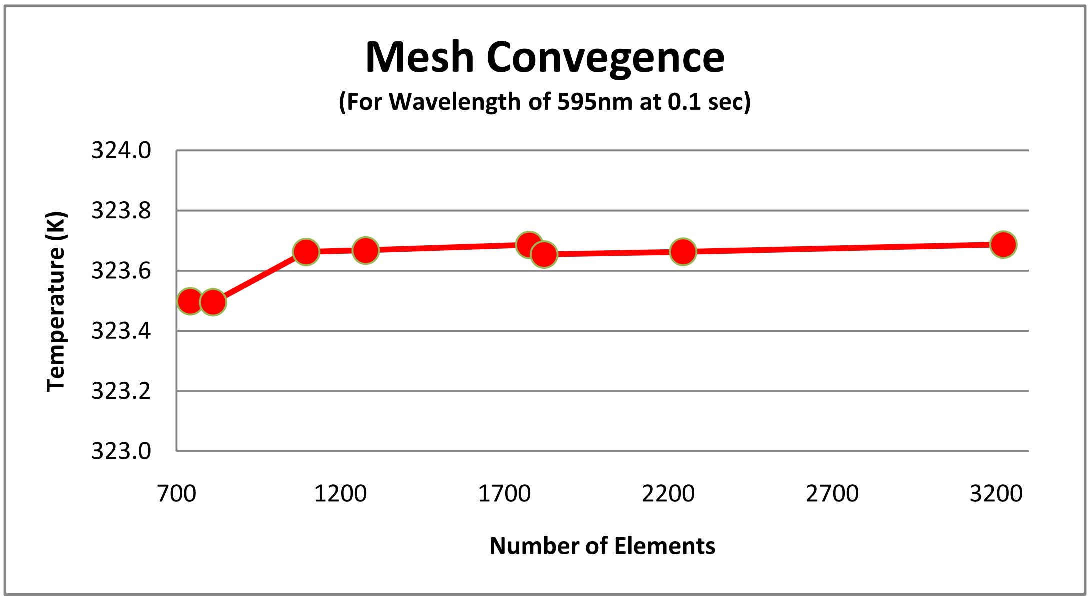
	Scattering -15%	Avg Temp In Hair (C)	Actual Scattering (m^{-1})	Avg Temp In Hair (C)	Scattering +15%	Avg Temp In Hair (C)
Hair	3268.13	43.14	3844.86	43.12	4421.59	43.13
Epidermis	20607.15	43.15	24243.70	43.12	27880.26	43.13
Dermis	20607.15	43.53	24243.70	43.12	27880.26	42.81
Hypodermis	8242.86	43.12	9697.48	43.12	11152.10	43.12

	Anisotropy -15%	Avg Temp In Hair (C)	Actual Anisotropy	Avg Temp In Hair (C)	Anisotropy +15%	Avg Temp In Hair (C)
Hair	0.77	43.11	0.90	43.12	1.04	43.14
Epidermis	0.77	43.05	0.90	43.12	1.04	43.26
Dermis	0.67	42.13	0.79	43.12	0.91	45.05
Hypodermis	0.77	43.13	0.90	43.12	1.04	43.11

	Thermal Conductivity -15%	Avg Temp In Hair (C)	Actual Thermal Conductivity (W/mK)	Avg Temp In Hair (C)	Thermal Conductivity +15%	Avg Temp In Hair (C)
Hair	0.31	43.45	0.37	43.12	0.43	42.88
Epidermis	0.18	43.15	0.21	43.12	0.24	43.09
Dermis	0.45	43.48	0.53	43.12	0.61	42.84
Hypodermis	0.47	43.12	0.55	43.12	0.63	43.12

_	Density -15%	Avg Temp In Hair (C)	Actual Density (kg/m^3)	Avg Temp In Hair (C)	Density +15%	Avg Temp In Hair (C)
Hair	1105.00	43.27	1300.00	43.12	1495.00	42.99
Epidermis	1020.00	43.13	1200.00	43.12	1380.00	43.12
Dermis	1020.00	43.35	1200.00	43.12	1380.00	42.93
Hypodermis	850.00	43.12	1000.00	43.12	1150.00	43.12

Mesh Convergence



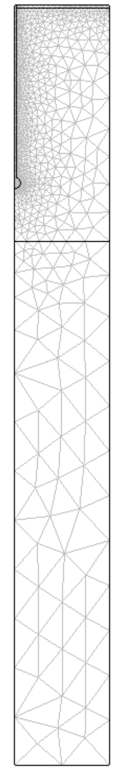


Figure 9: Complete mesh convergence. The final mesh can be seen on the right. We picked the number of elements between 1700 and 1800.

As the number of elements increase, the plot slowly descends to a common average concentration. This indicates a mesh convergence. So for our final mesh, we will most likely use a mesh that has the same number of elements as shown in the graph. We do not wish to increase the element number unnecessarily so we will use the elements where the average concentration begins to remain constant.

Properties

Table 3: Light properties of human hair and skin

Laser	Wavelength (nm)	ö (fluence) (J/cm2)	Layer	ì$_a$ (m^{-1})	ì$_s$ (m^{-1})	g (anisotropy)	Thermal conductivity (W/mK)	Heat capacity at constant pressure (J/kgK)	density (kg/m3)
myriad intense pulsed lights (IPL)	**595**	30[viii]	**Hair**	6000.00[i]	3844.86[ii]	0.90[iii]	0.37[iv]	1500[x]	1300[v]
			Epidermis	1708.49[vi]	24243.70[vi]	0.90[vii]	0.21[ix]	3600[ix]	1200[ix]
			Dermis	85.42[vi]	24243.70[vi]	0.79[vii]	0.53[ix]	3800[ix]	1200[ix]
			Hypodermis	539.88[vi]	9697.48[vi]	0.90[v]	0.55[ix]	3600[ix]	1000[ix]
Ruby	**694**	40[viii]	**Hair**	5300.00[i]	2291.69[ii]	0.90[iii]	0.37[iv]	1500[x]	1300[v]
			Epidermis	1000.00[vi]	20785.30[vi]	0.85[vi]	0.21[ix]	3600[ix]	1200[ix]
			Dermis	50.00[vi]	20785.30[vi]	0.85[vi]	0.53[ix]	3800[ix]	1200[ix]
			Hypodermis	316.00[vi]	8314.12[vi]	0.85[vi]	0.55[ix]	3600[ix]	1000[ix]
Alexandrite	**755**	50[viii]	**Hair**	4000.00[i]	1726.43[ii]	0.90[iii]	0.37[iv]	1500[x]	1300[v]
			Epidermis	745.89[vi]	19105.96[vi]	0.85[vi]	0.21[ix]	3600[ix]	1200[ix]
			Dermis	37.29[vi]	19105.96[vi]	0.85[vi]	0.53[ix]	3800[ix]	1200[ix]
			Hypodermis	396.00[vi]	7642.38[vi]	0.85[vi]	0.55[ix]	3600[ix]	1000[ix]
Diode	**800**	100[viii]	**Hair**	3800.00[i]	1421.08[ii]	0.90[iii]	0.37[iv]	1500[x]	1300[v]
			Epidermis	858.00[vi]	18031.25[vi]	0.85[vi]	0.21[ix]	3600[ix]	1200[ix]
			Dermis	24.00[vi]	18031.25[vi]	0.85[vi]	0.53[ix]	3800[ix]	1200[ix]
			Hypodermis	419.00[vi]	7212.50[vi]	0.85[vi]	0.55[ix]	3600[ix]	1000[ix]
YAG (neodymium: yttrium-aluminum-garnet)	**1064**	150[viii]	**Hair**	2500.00[i]	544.79[ii]	0.90[iii]	0.37[iv]	1500[x]	1300[v]
			Epidermis	226.03[vi]	13557.33[vi]	0.85[vi]	0.21[ix]	3600[ix]	1200[ix]
			Dermis	11.30[vi]	13557.33[vi]	0.85[vi]	0.53[ix]	3800[ix]	1200[ix]
			Hypodermis	304.00[vi]	5422.93[vi]	0.85[vi]	0.55[ix]	3600[ix]	1000[ix]

Table 4: Physical properties of human hair and skin

	Hair	Follicle	Epidermis	Dermis	Hypodermis
Thickness/Depth (m)	0.003[viii]	0.003[viii]	100 x 10^{-6} [ix]	0.004[ix]	0.009[ix]
Radius/Width (m)	3.00E-05[x]	1.00E-04[x]			

[i] (Kampen)
[ii] (Knappe, Frank, & Ewa, 2004)
[iii] (Xu, Lu, & Seffen, 2007)
[iv] (Fiskerstrand, Svaasand, & Nelson, 2003)
[v] (Steiner, Russ, Falkenstein, & Kienle, 2001)
[vi] (Svaasand & Nelson, 2004)
[vii] (Paithankar, Ross, Saleh, Blair, & Graham, 2002) &(Zhang, Verkkruysse, Aguilar, & Nelston, 2005)
[viii] (Lepselter & Elman, 2004)
[ix] (Babilas, et al., 2007)
[x] (Ross, Ladin, Kreindel, & Dierickx, 1999)

Other Properties: Speed of light = 2.19 x 10^8 m/s

Ratio of reflected light to the laser power output (r)

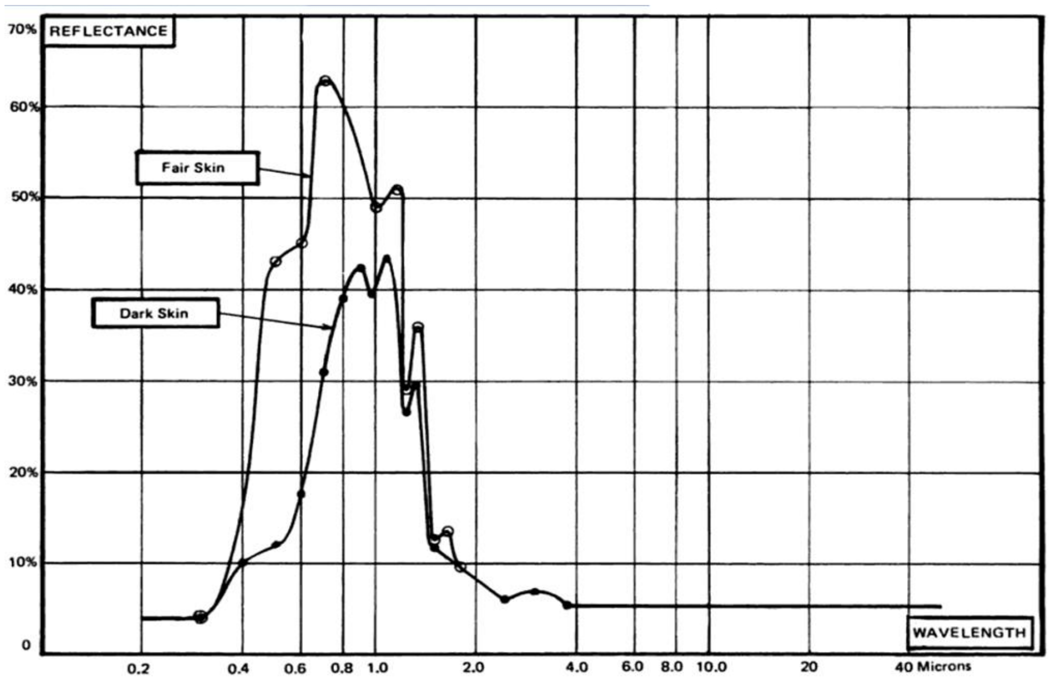


Figure 10: This graph show the spectral variation of epidermal reflectance for fair-complexioned and dark-complexioned individuals, as plotted by the author from the data of Jacquez and Kuppenheim for the visible and near-infrared, and from the data of hardy et al. for the ultraviolet and far-infrared regions of the spectrum. (Shapshay 1987).

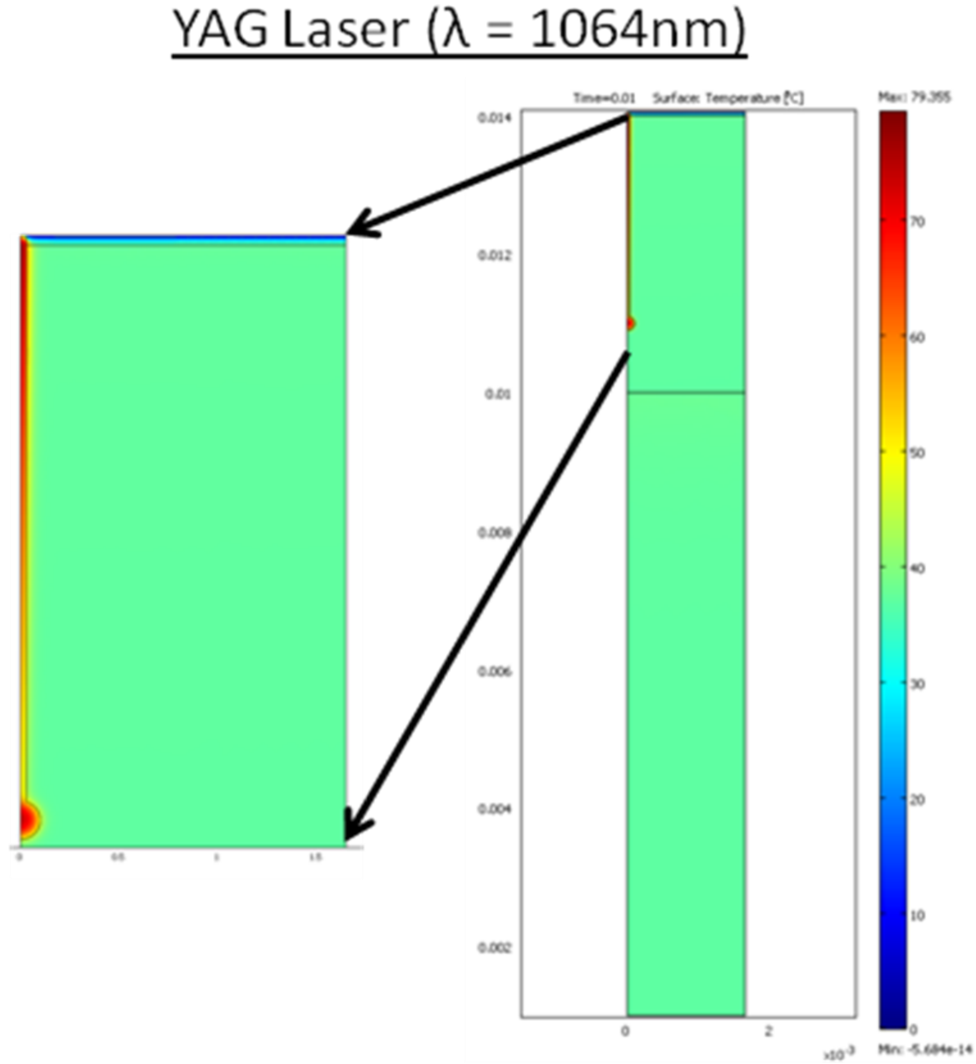


Figure 11: Overall view of the laser hair removal model at 0.01 seconds for YAG Laser with laser irradiance of 1.342×10^{6} W/m^{2}. Under this wavelength, the hair follicle reaches 79°C as the maximum temperature.