Analysis and Design of Materials and Structures for Attenuating Vibration and Acoustic Response

Gregory M. Hulbert

Department of Mechanical Engineering
University of Michigan

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Outline

- Motivation and background
- Multiscale approach
- Inertial coupling and amplification
- Compliant mechanisms
- Conclusions

Collaborators

Mahmoud Hussein, U Colorado-Boulder

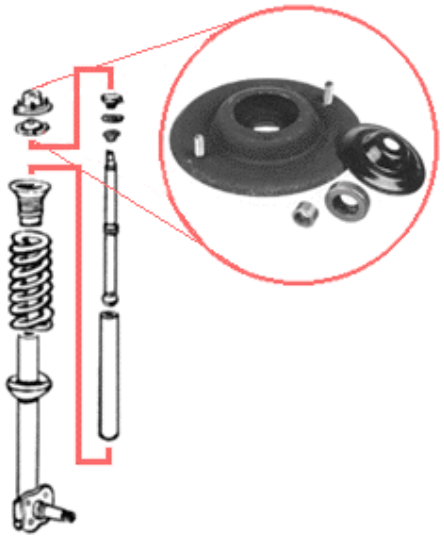
Cetin Yilmaz, Bogazici University

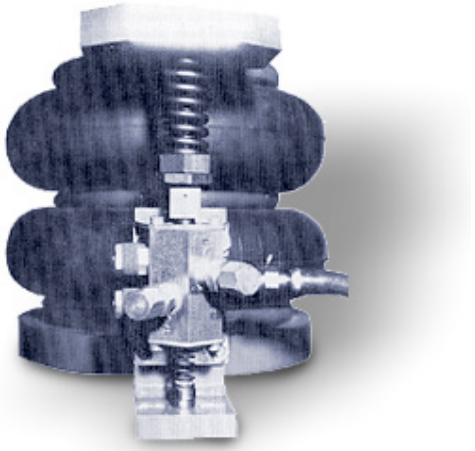
Ercan Dede, UM

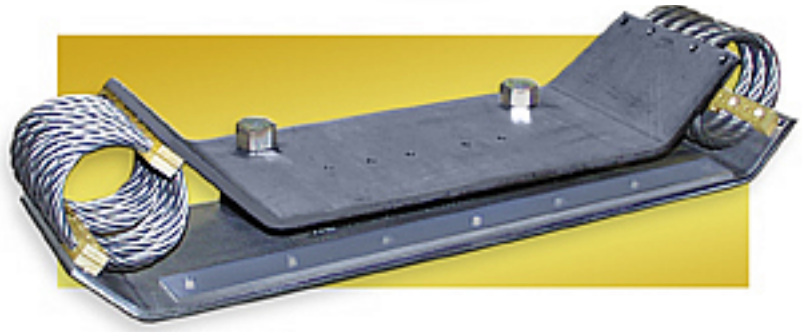
Noboru Kikuchi, Kazu Saitou, Karim Hamza

Motivation

- Low Frequency Structural-Borne Vibration and Noise
  - Automotive body and engine mounts
  - Machinery vibration isolators







Motivation

- Mid-Frequency Structural-Borne Noise

"NASA has long been aware that certain elements of the station…are much noisier than they were supposed to be, thanks to a host of valves, pumps, and fans."

"…noise levels average more than 70 decibels."

– Science Magazine

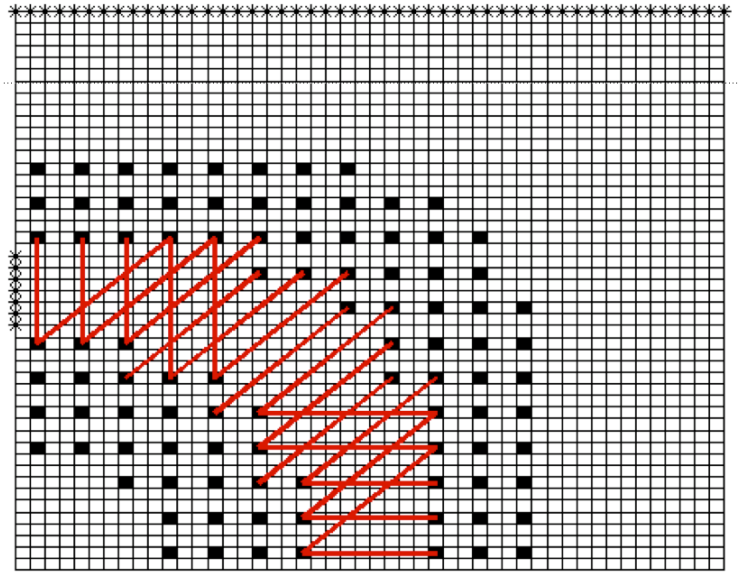
"Long-term space flight has produced 30-50 dB…of NIHL in the 4 kHz - 6 kHz range."

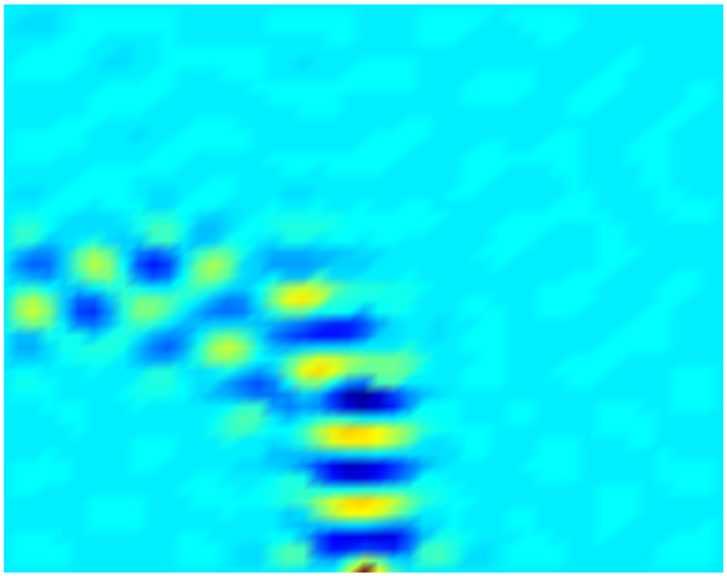
"There is concern that the Astronauts…are not able to obtain "auditory rest." "

– Military Audiology Association,
NOPHER 2000 International Symposium

Motivation

- High Frequency Structural-Borne Vibration / Shock
  - Equipment protection and isolation: electrodynamic waveguides





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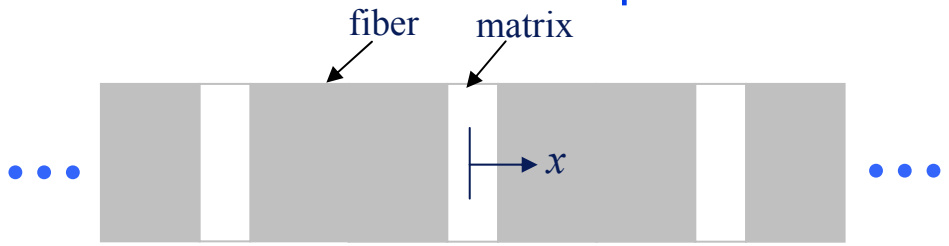
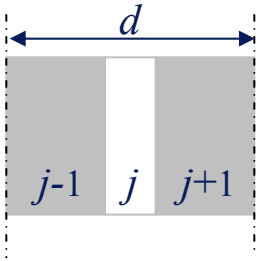
Multiscale Approach

- High-frequency application: Bragg scattering
  - One-dimensional example



Infinite Layered Medium



Repeated Unit Cell

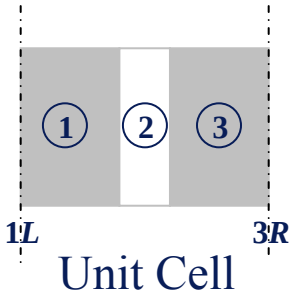
Wave Equation: $$c^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$$

Time Harmonic Solution: $$u(x,t) = \left[A_+^{(j)} e^{ik^{(j)}x} + A_-^{(j)} e^{-ik^{(j)}x} \right] e^{-i\omega t}$$

Floquet Theorem: $$u(x+d,t) = e^{ikd} u(x,t)$$

Multiscale Approach: Unit Cell Analysis

- Eigenvalue Problem
  Esquivel-Sirvent et al.,1994

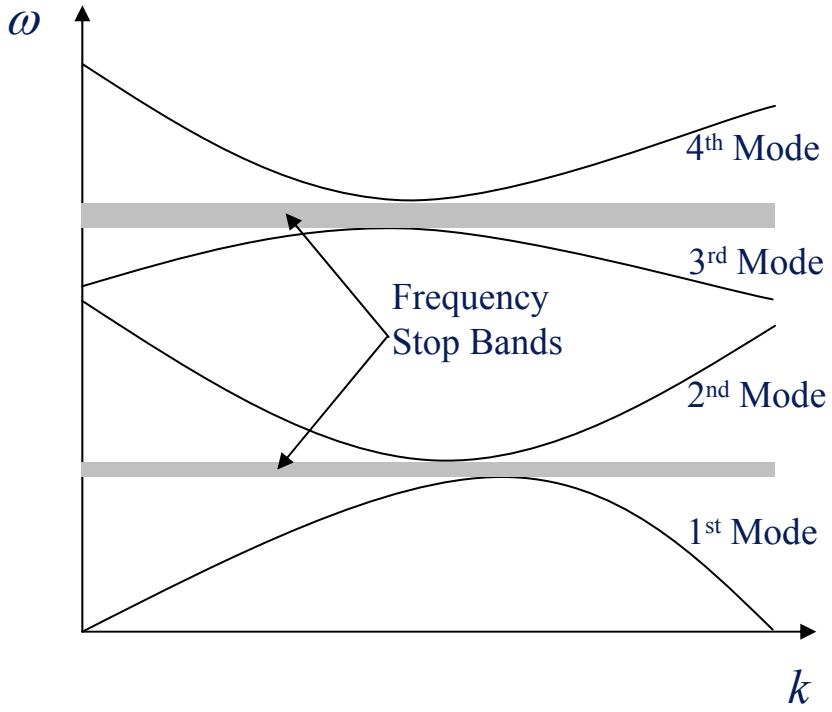


Transfer Matrix: $$\begin{bmatrix} u \\ \sigma \end{bmatrix}_{nR} = \mathbf{T}(\omega) \begin{bmatrix} u \\ \sigma \end{bmatrix}_{1L}$$

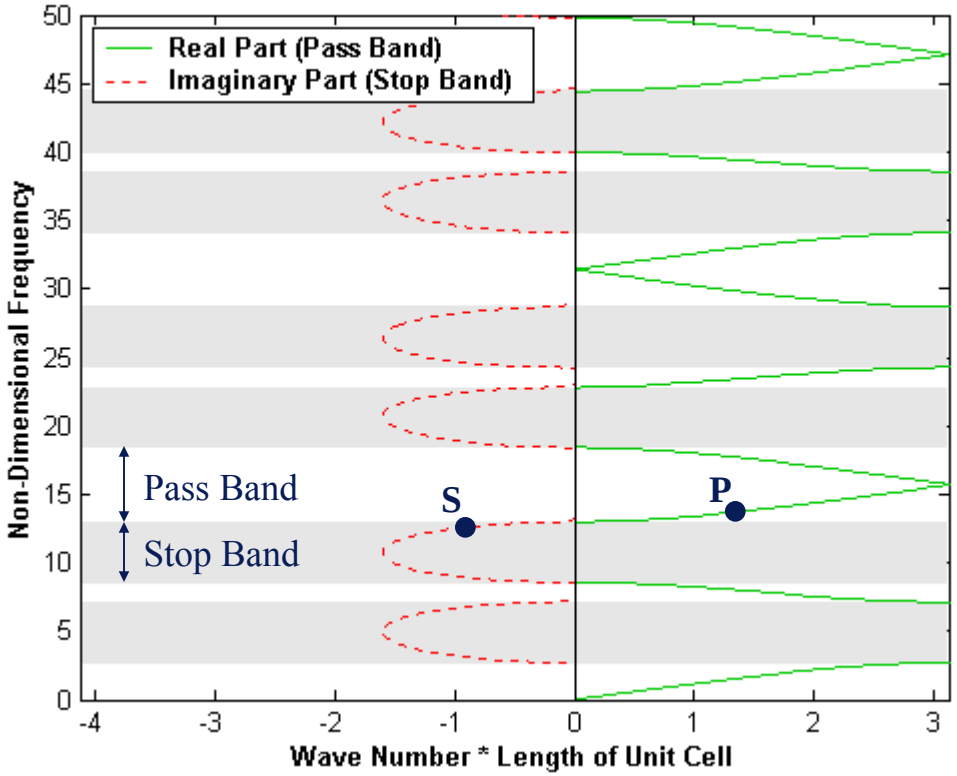
Floquet Theorem: $$\begin{bmatrix} u \\ \sigma \end{bmatrix}_{nR} = \mathbf{I}e^{ikd} \begin{bmatrix} u \\ \sigma \end{bmatrix}_{1L}$$

Eigenvalue Problem: $$[\mathbf{T}(\omega) - \mathbf{I}e^{ikd}] \begin{bmatrix} u \\ \sigma \end{bmatrix}_{1L} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

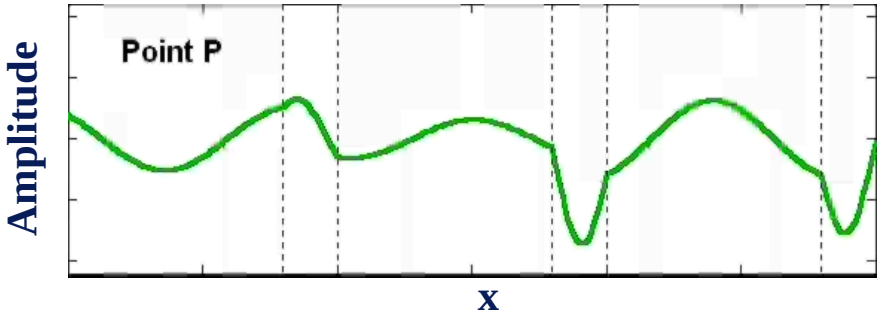
Frequency Spectrum of Periodic Medium



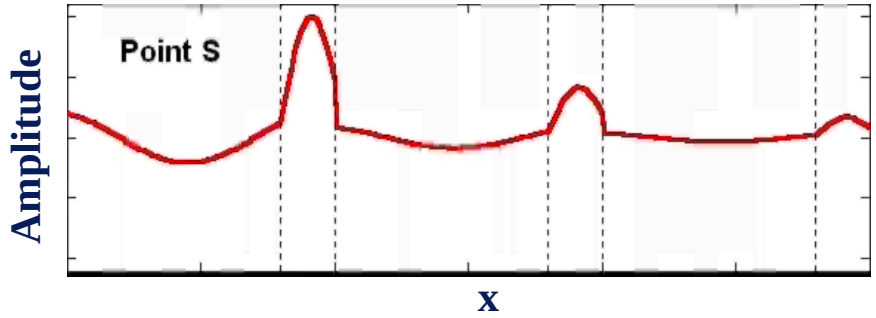
Frequency Spectrum of Periodic Medium



Displacement Mode Shapes



Pass Band



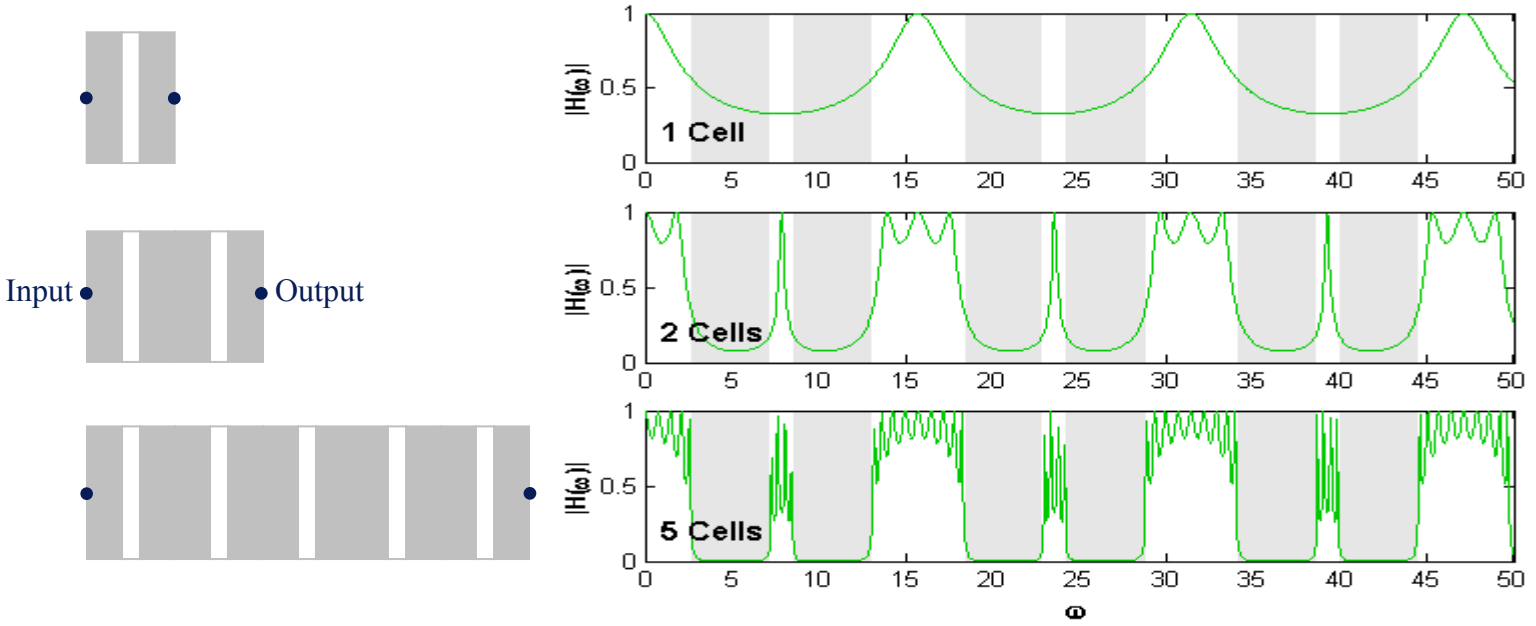
Stop Band

Transmission Function for Finite Periodic Structure

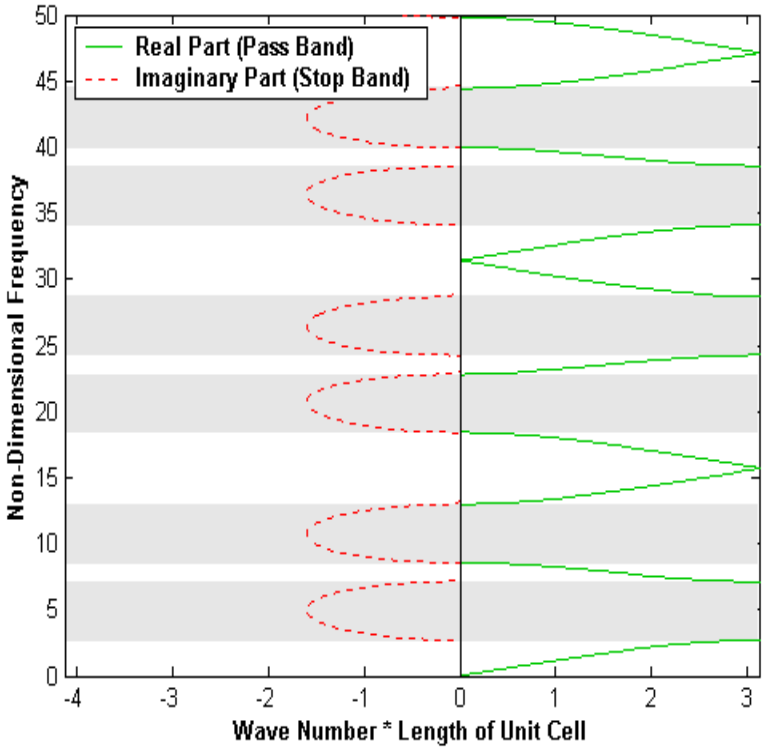
- Finite Periodic Structure

Transmission Function: $H(\omega) = \frac{1}{\overline{T}_{11}} e^{-ik\,Nd}$

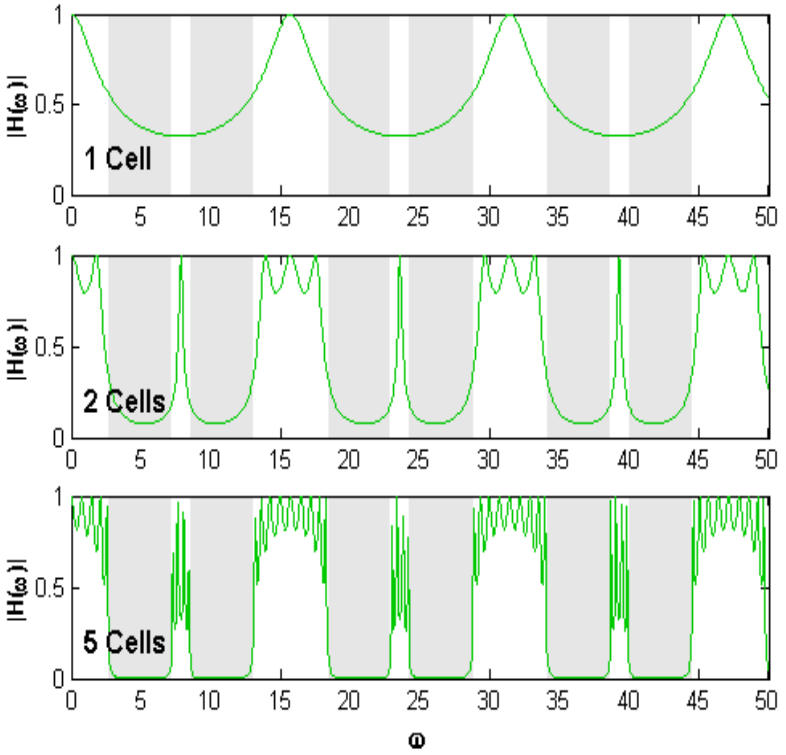
Day et al., 1993



Band Formation: Infinite vs. Finite Structures

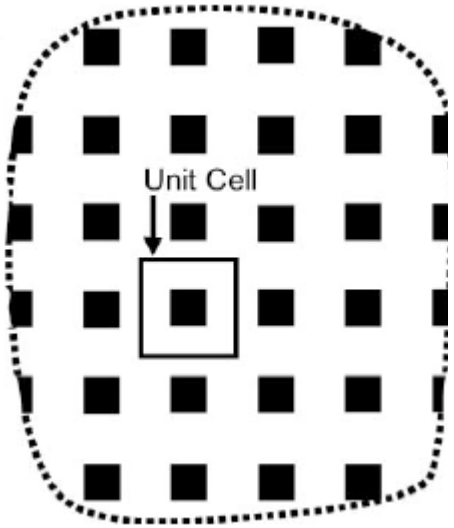


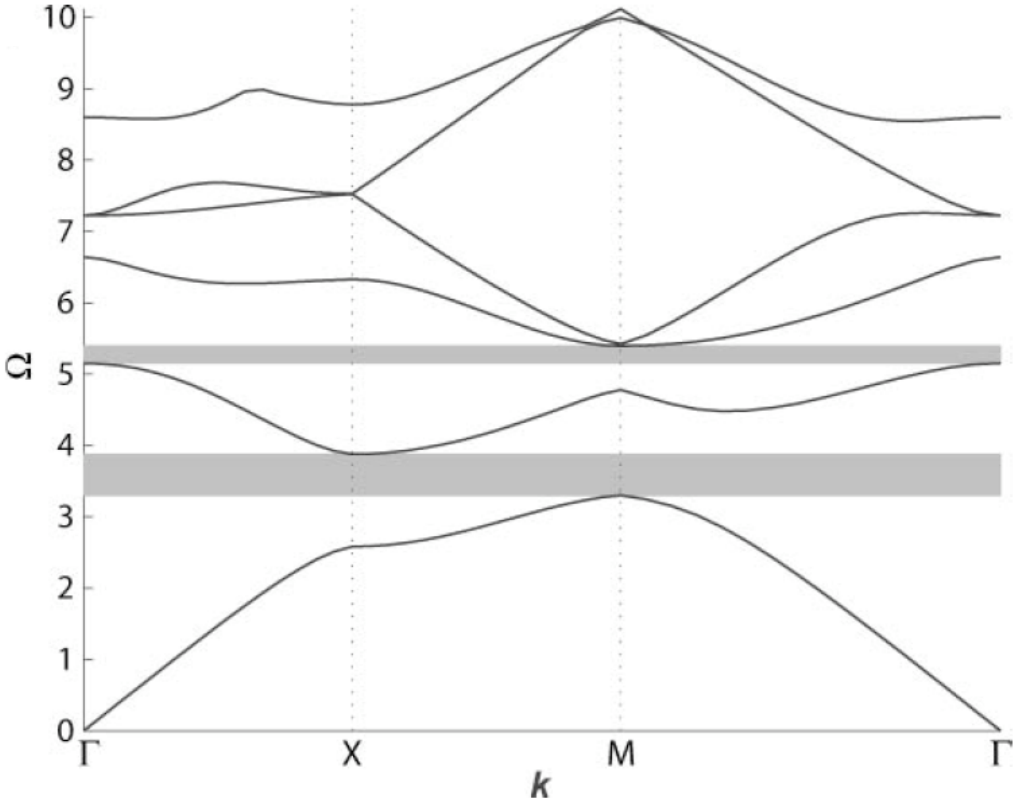
Infinite Periodic Structure



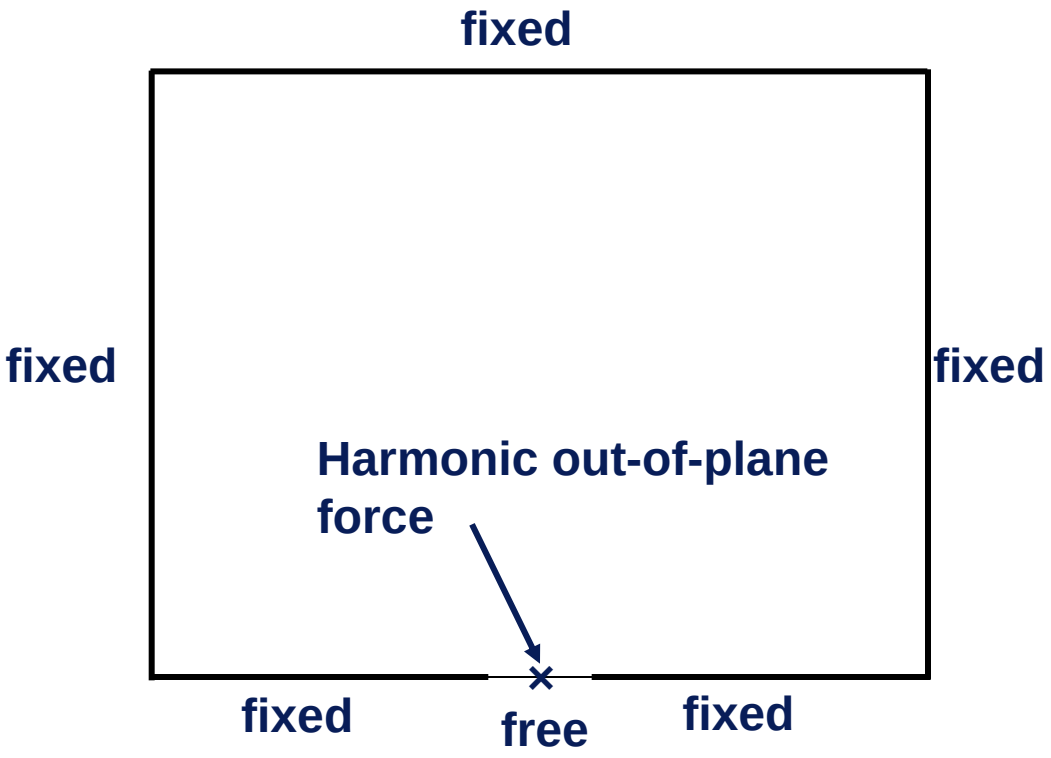
Finite Periodic Structure

2D Phononic Materials

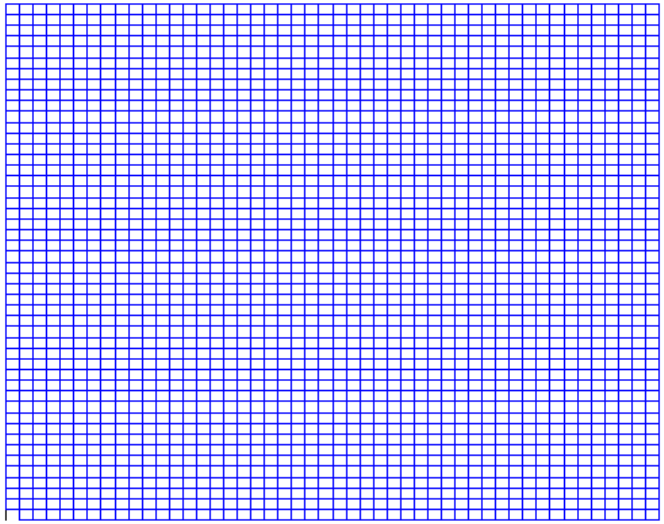


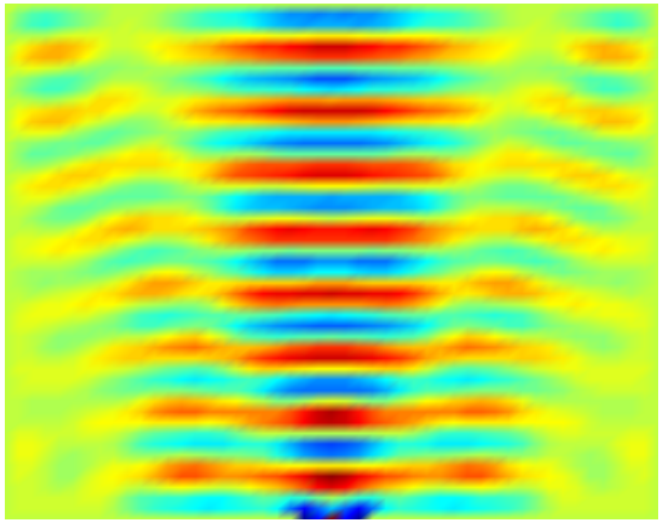


Waveguide Example 1

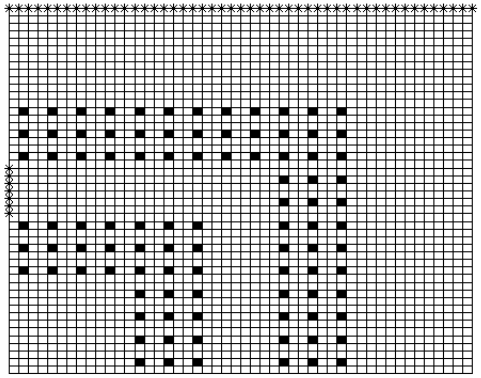


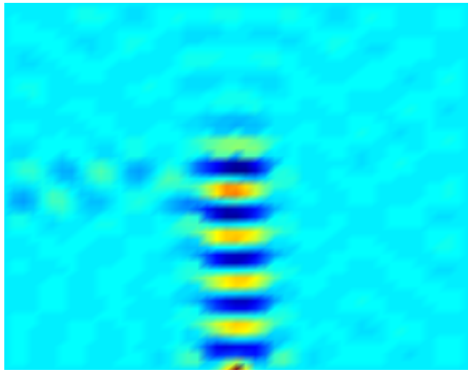
Waveguide Example: Homogeneous Material

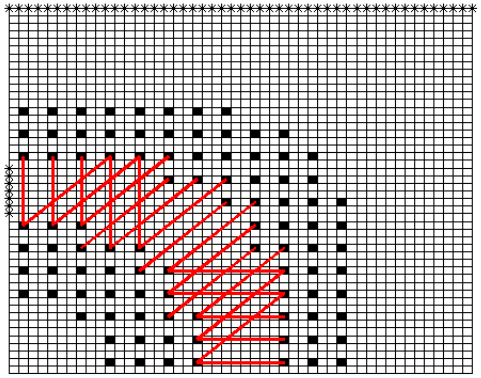


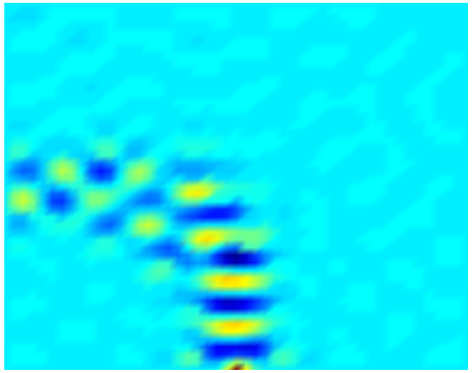


Waveguide Example: Material Design and Placement









Optimal Synthesis of Phononic Crystals

Design Variable:
Material type of pixel

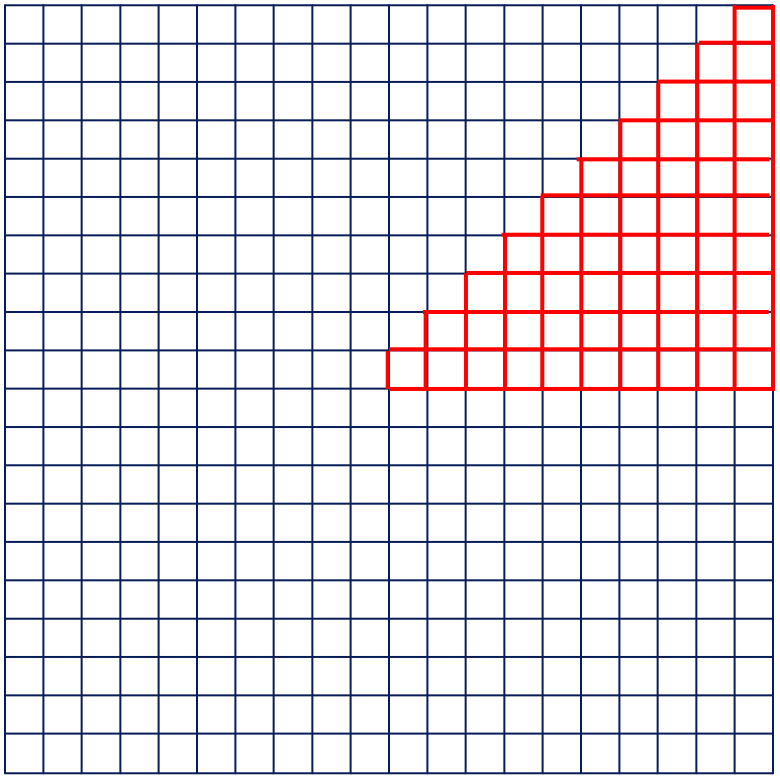
Vector of binary variables:

$$\boldsymbol{u} = \{0,1\}^n$$

n: number of pixels

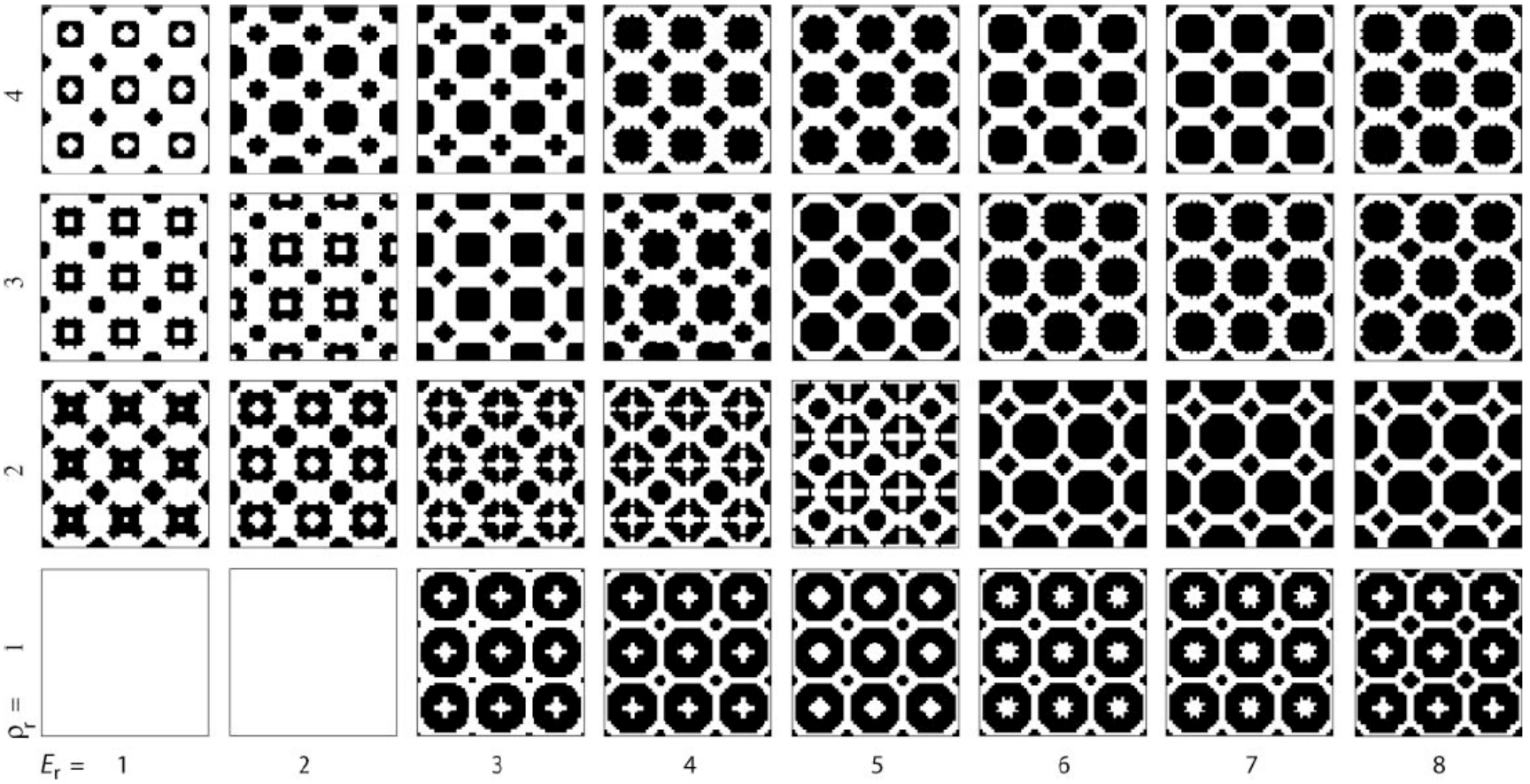
Binary Code:
0: Material A
1: Material B

Choice of n:
n = 55

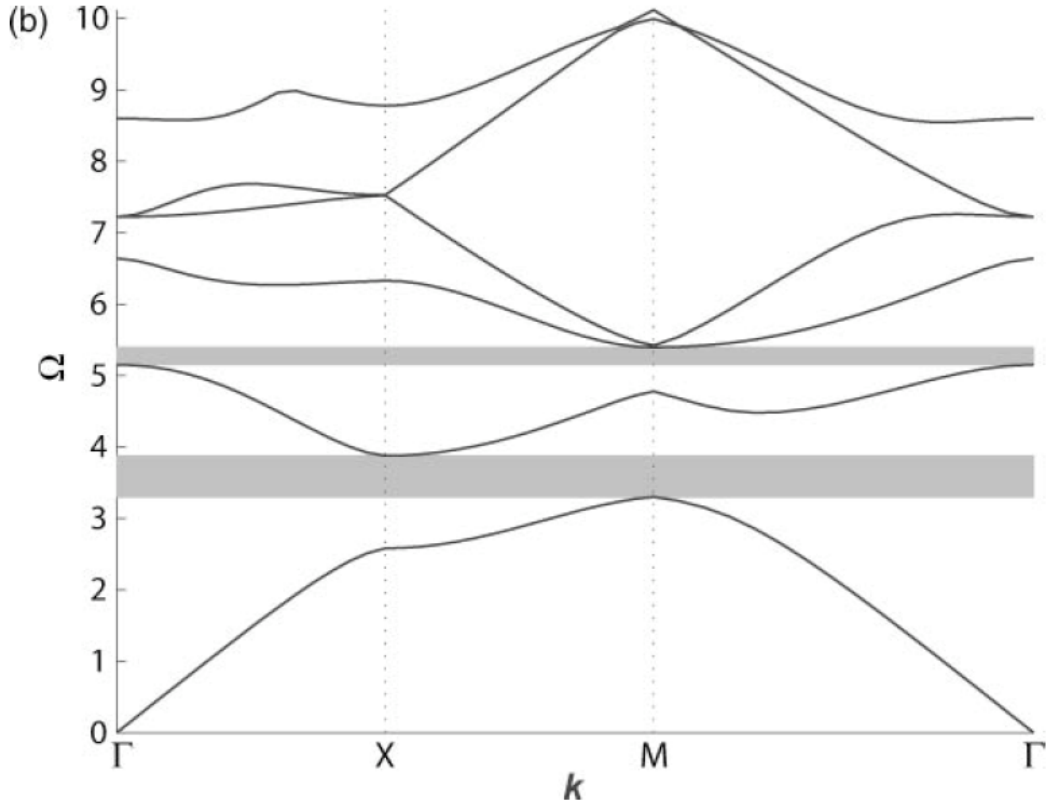


Unit Cell

Optimal Synthesis of Phononic Crystals

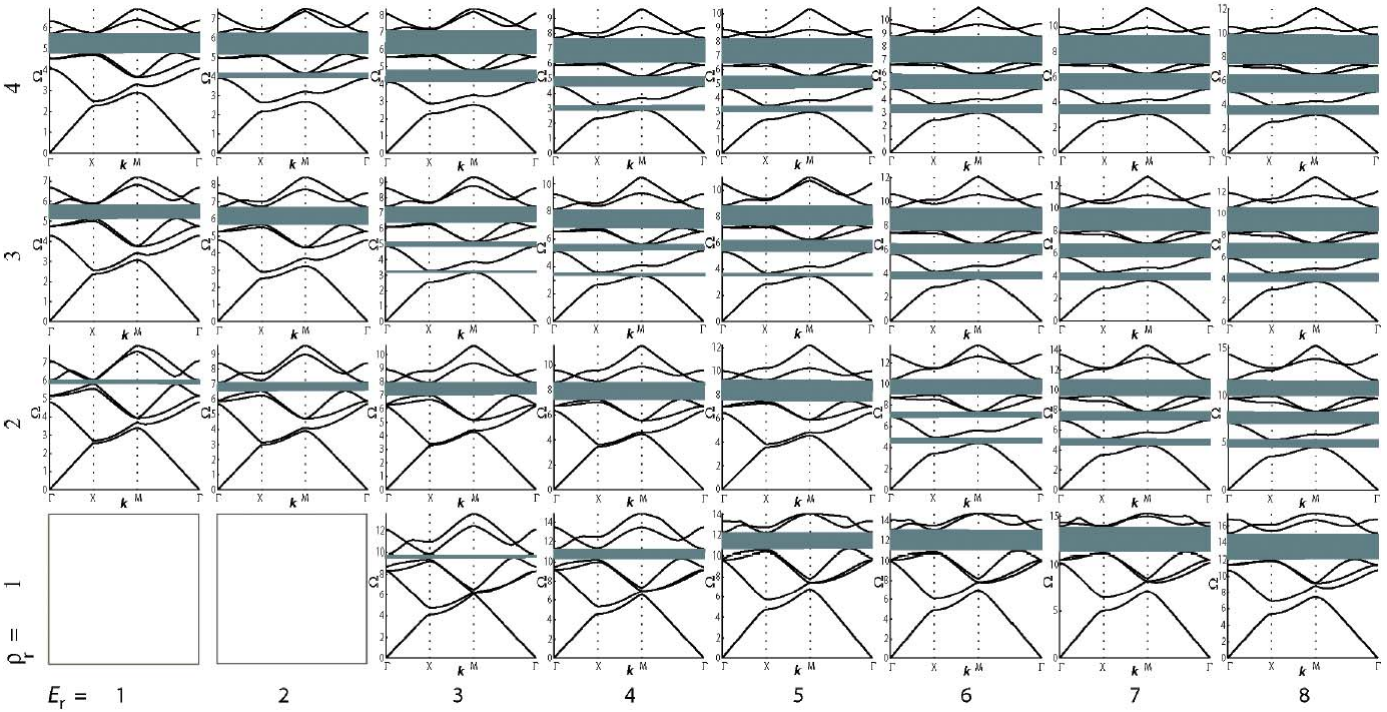


Optimal Synthesis of Phononic Crystals

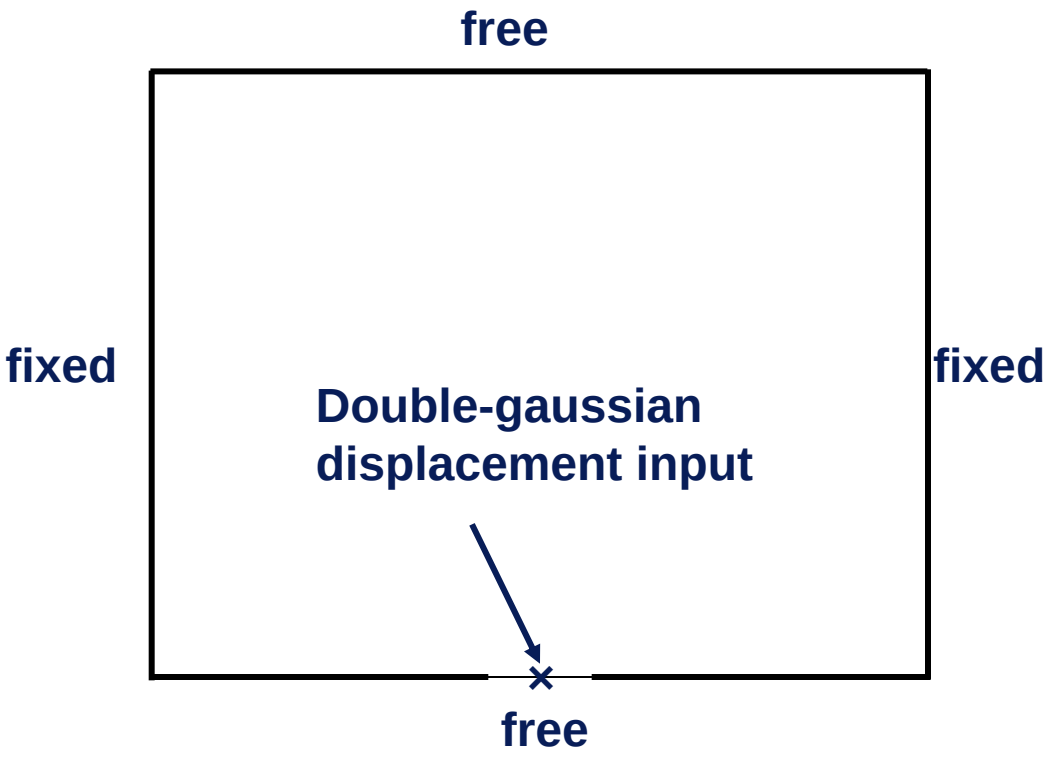


Maximize sum of the widths of first 5 bandgaps normalized by gap center frequency

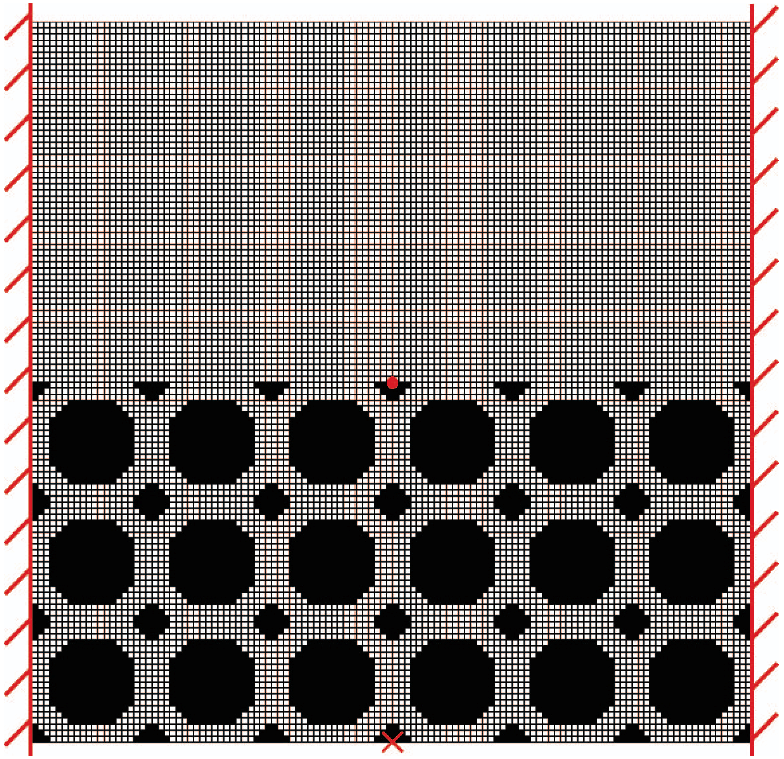
Optimal Synthesis of Phononic Crystals



Waveguide Example 2

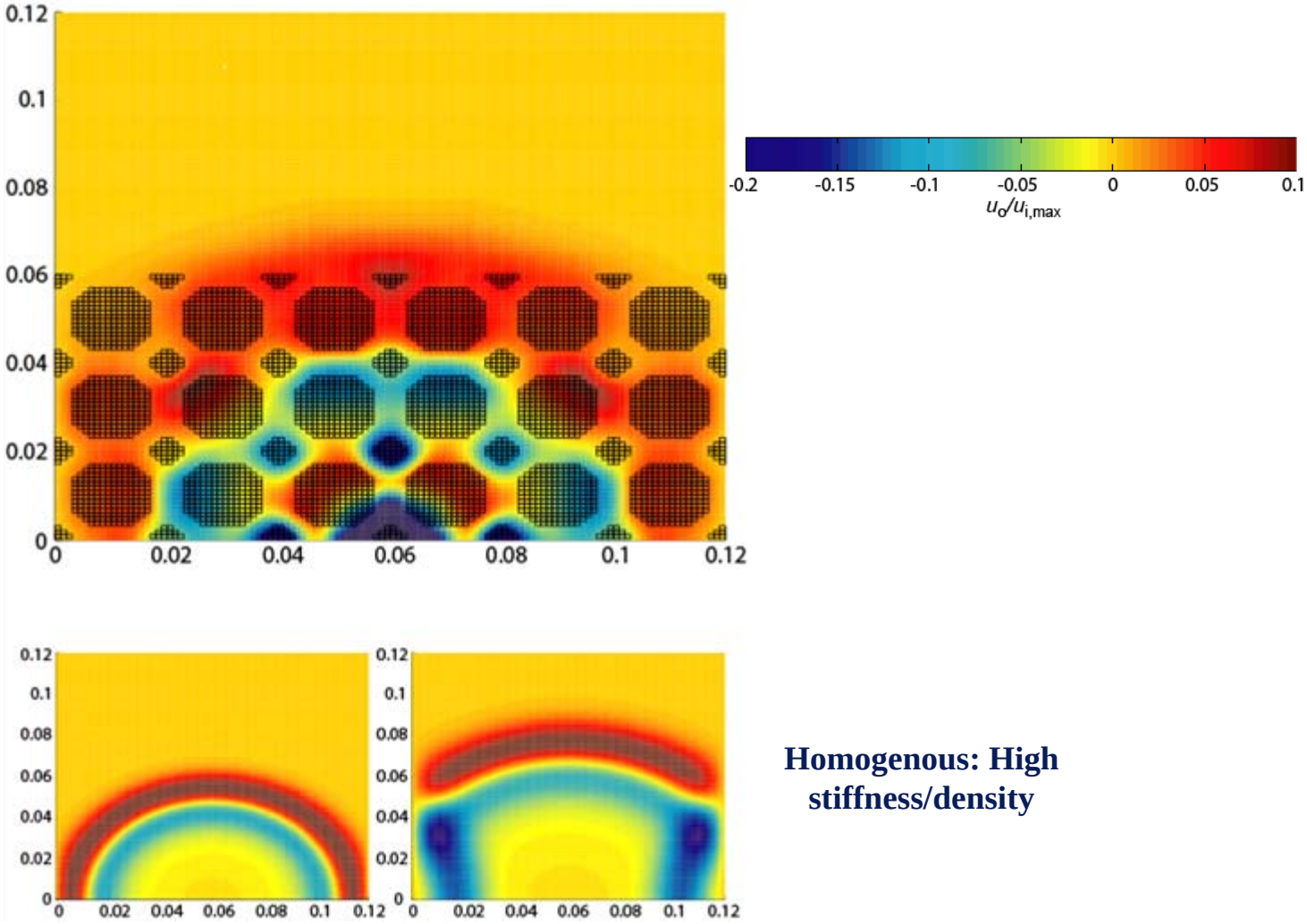


Waveguide Example 2



$$E_r = 8$$
$$\rho_r = 4$$

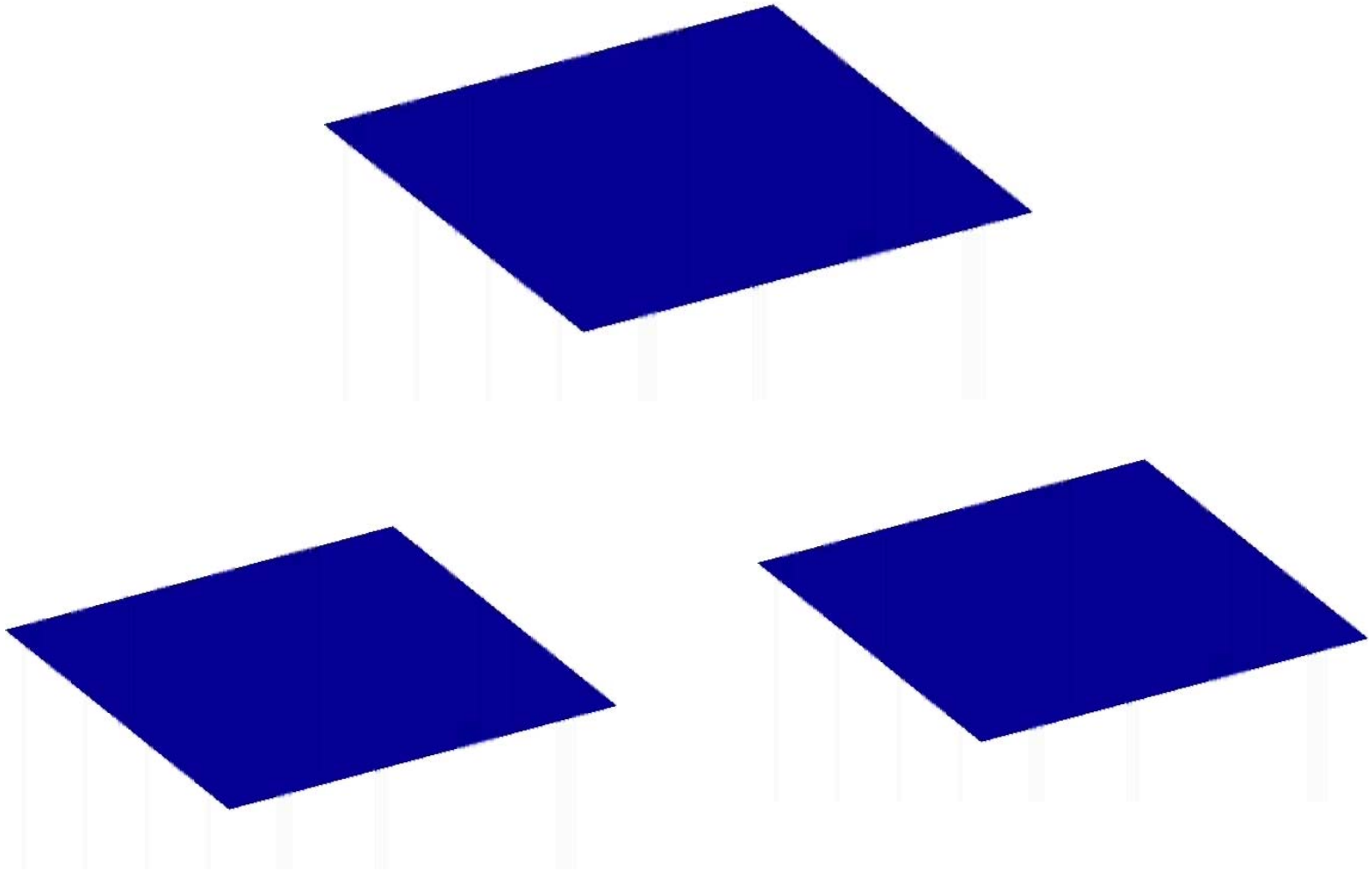
Waveguide Example 2



Homogenous: Low stiffness/density

Homogenous: High stiffness/density

Waveguide Example 2

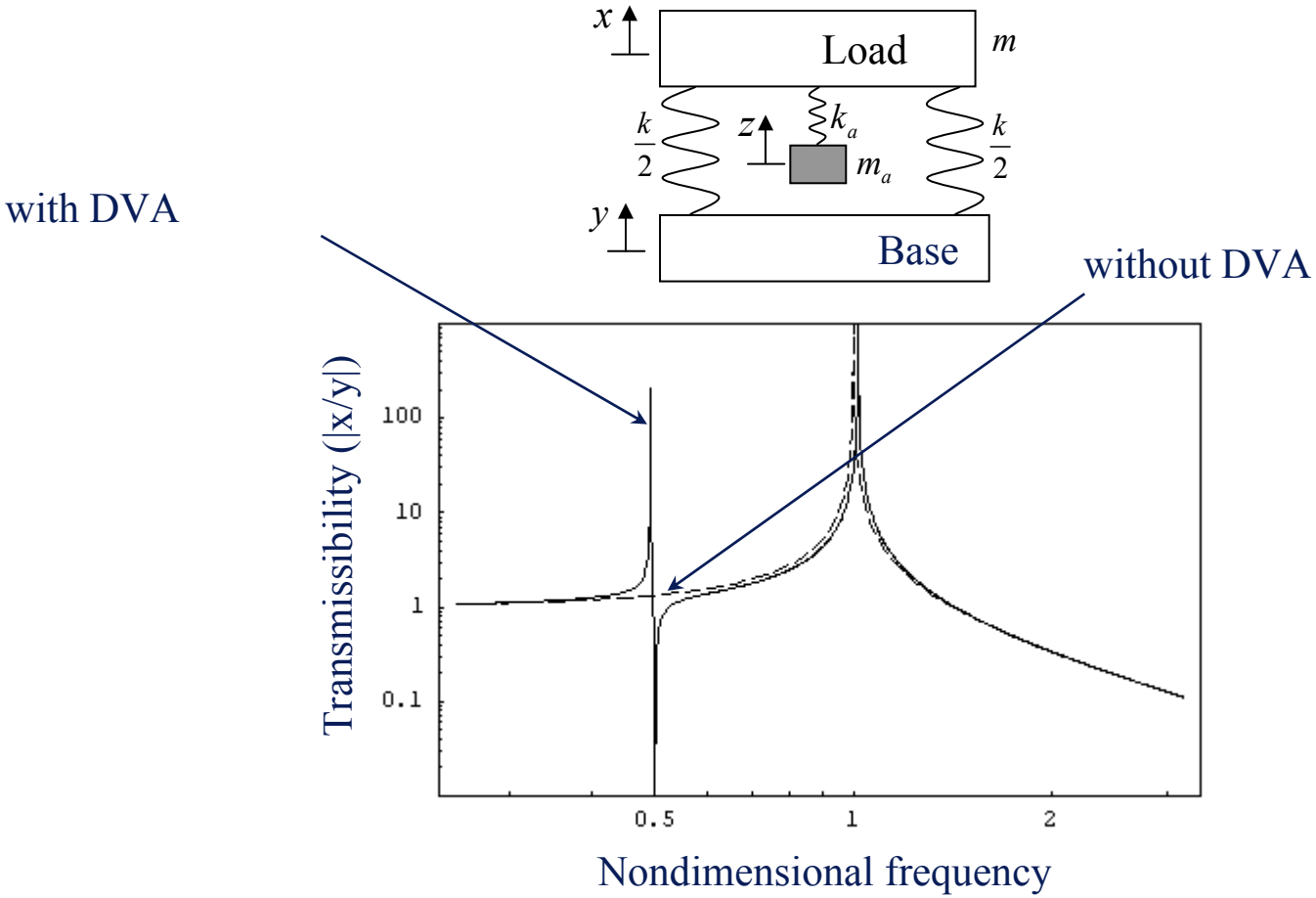


Overview

- Motivation and Background
- Multiscale approach
- Inertial coupling and amplification
  - Low frequency application
- Compliant mechanisms
- Summary

Classical Vibration Control Techniques

- Dynamic Vibration Absorber (DVA) *Frahm 1911*



Classical Vibration Control Techniques

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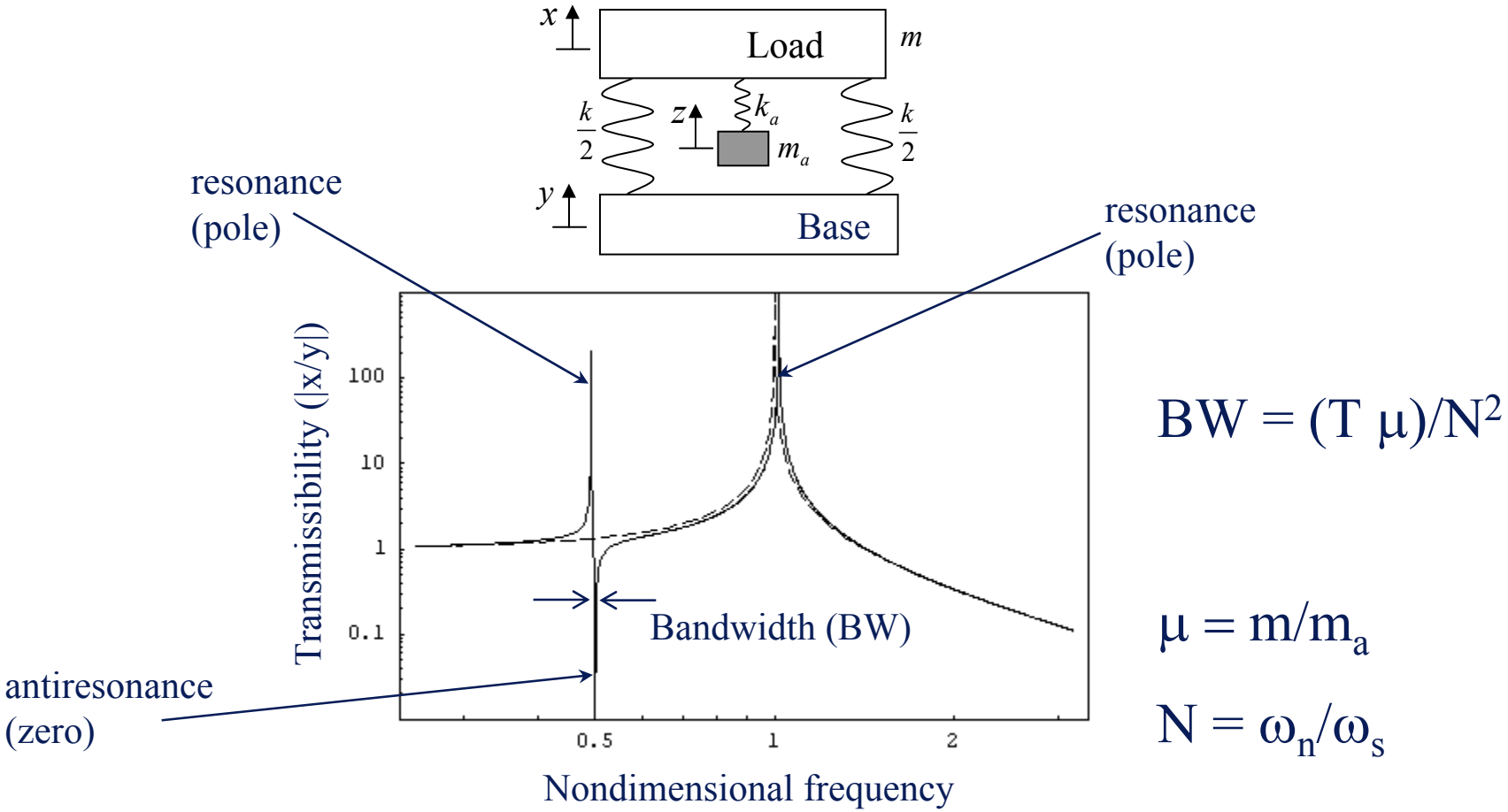


$BW = (T\ \mu)/N^2$

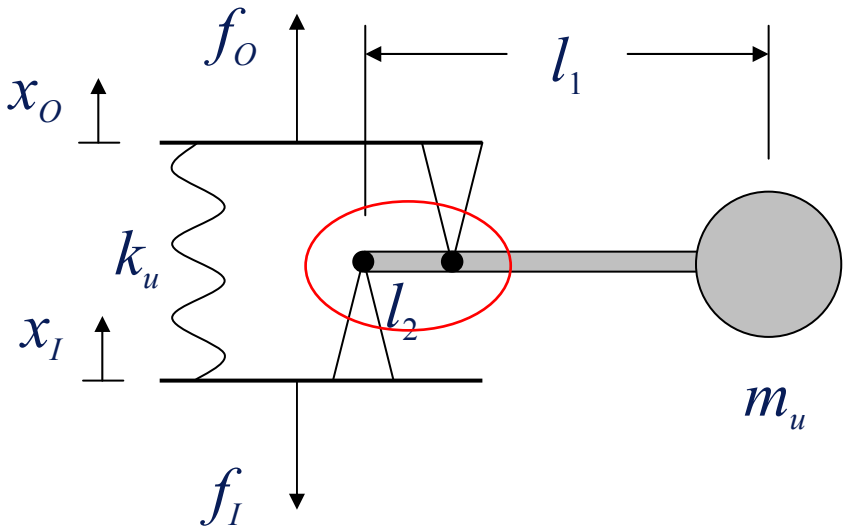
$\mu = m/m_a$

$N = \omega_n/\omega_s$

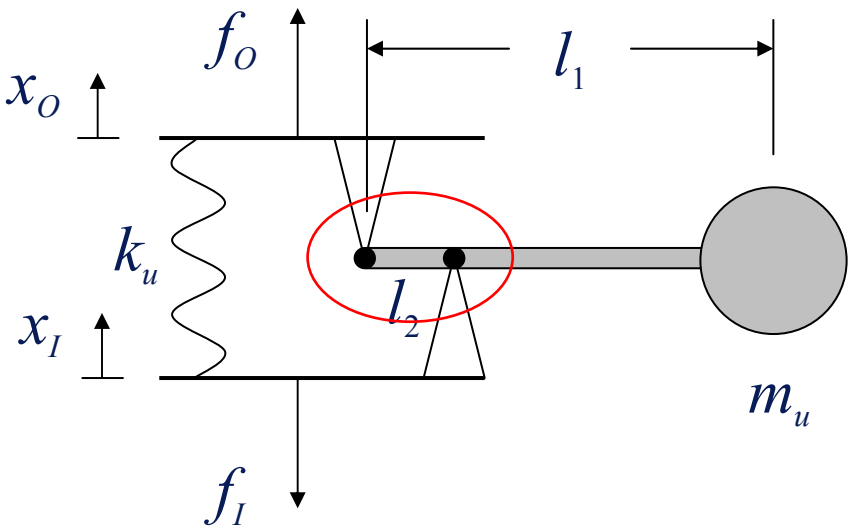
Inertial Amplification / Coupling

- Dynamic Anti-Resonant Vibration Isolator *Flannelly 1967*

Type I Isolator

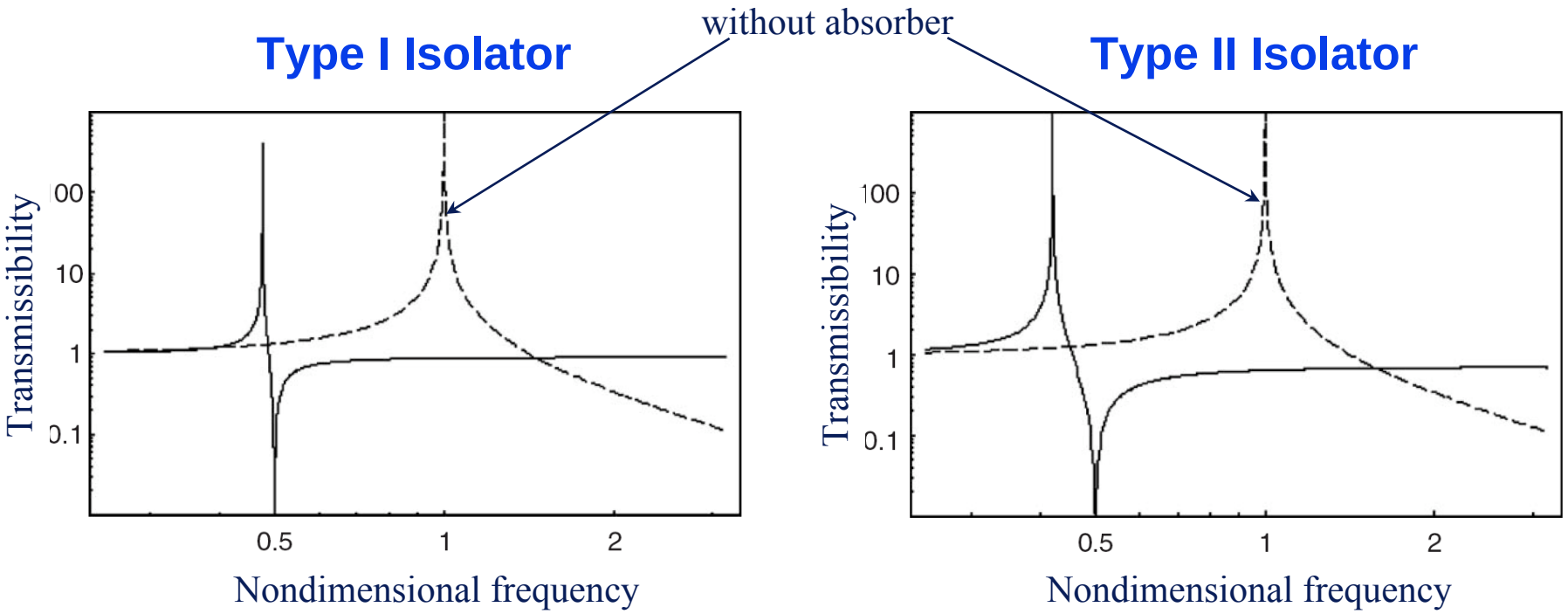


Type II Isolator



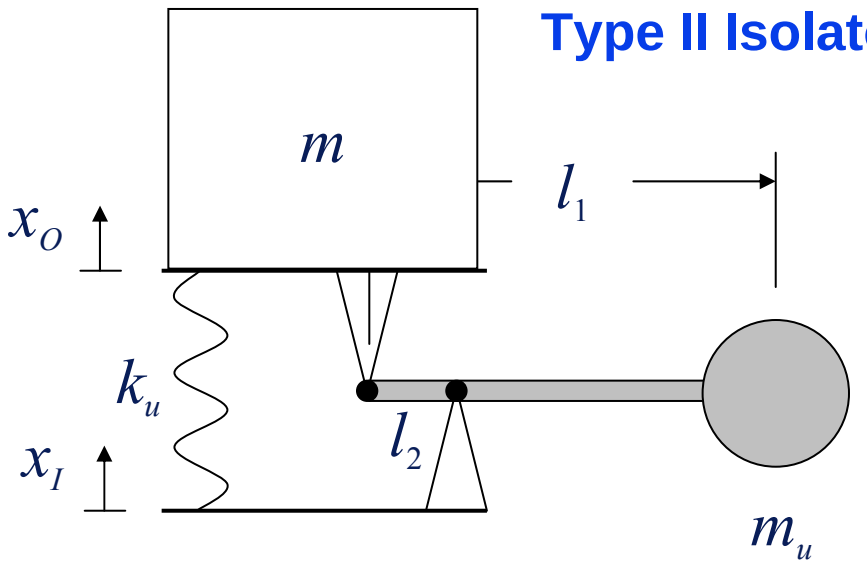
$$\alpha = l_1 / l_2$$

Inertial Amplification / Coupling



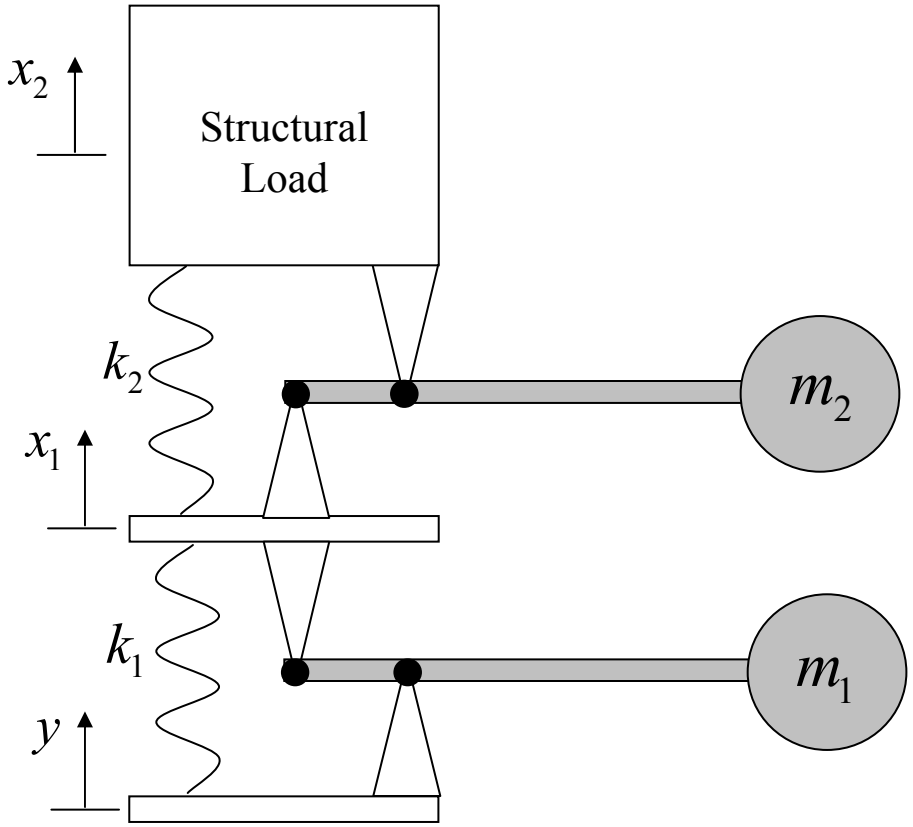
- Inertial coupling optimizes isolator performance for a given total mass and stiffness
- Type II slightly outperforms Type I

Inertial Amplification / Coupling



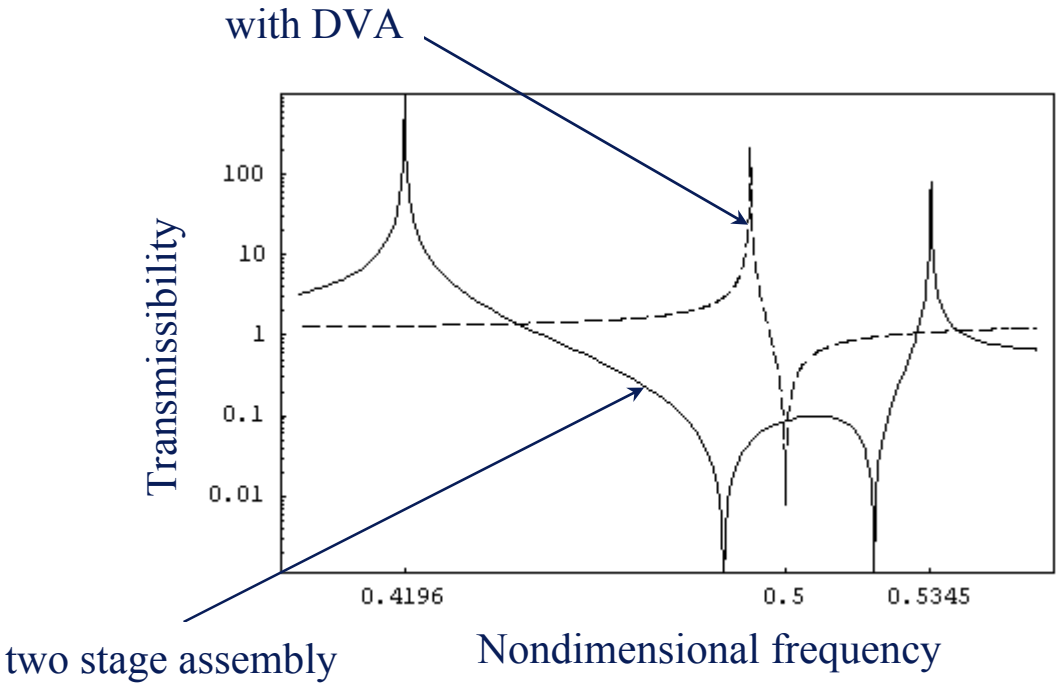
- DVA: $BW = (T\ \mu)/N^2$
- Type I: $BW = T(1/N^2 - 1/\alpha)$
- Type II: $BW = T(1/N^2 - 1/(\alpha - 1))$

Inertial Amplification / Coupling



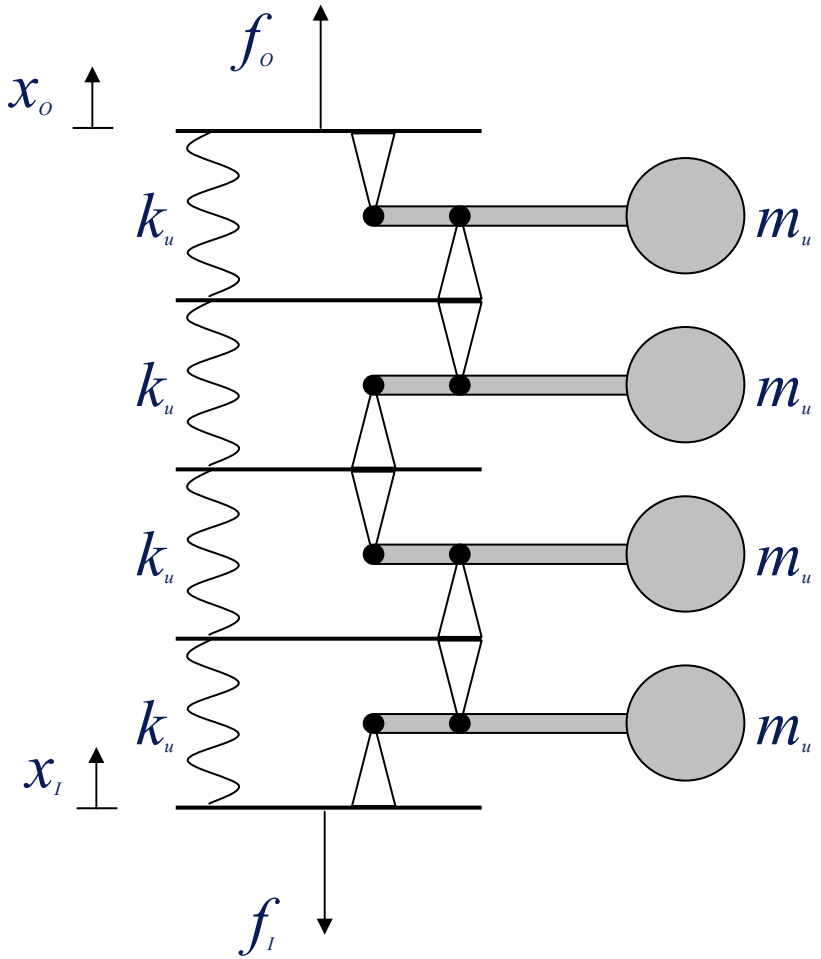
- Two-stage (cell) assembly

Inertial Amplification / Coupling



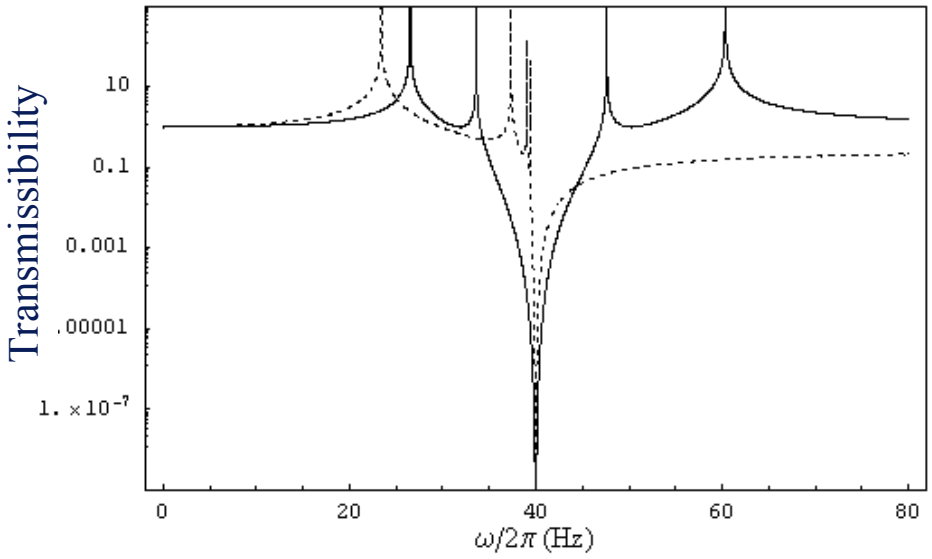
- Extra stages provide more flexibility

Inertial Amplification / Coupling

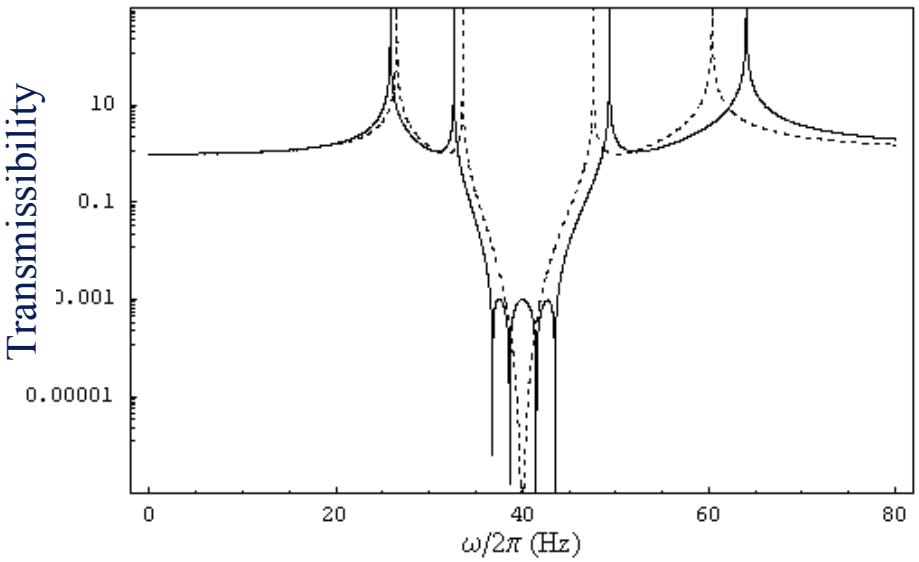


- Four-stage (2 cell) assembly

Inertial Amplification / Coupling



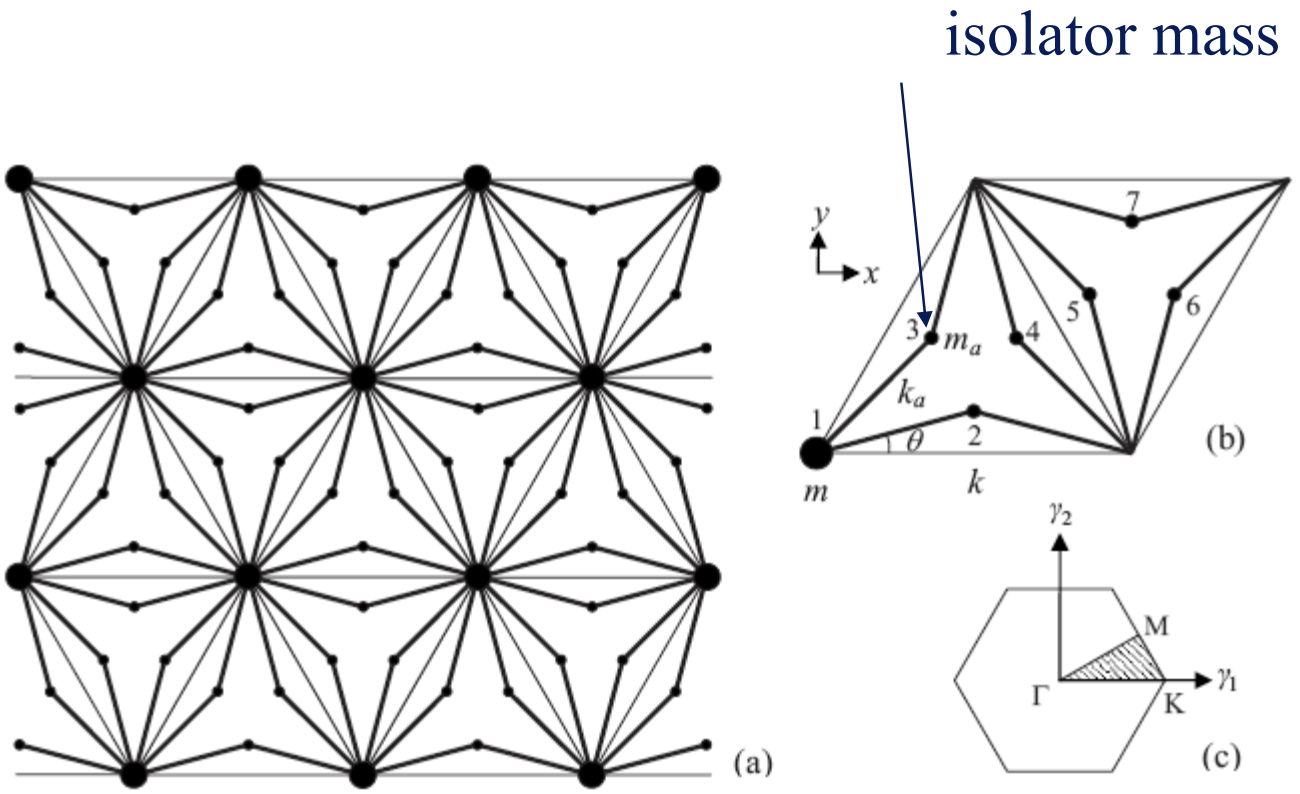
Periodic stages



Aperiodic stages

- Four-stage (2 cell) assembly
- Aperiodicity enables spread of zero placement

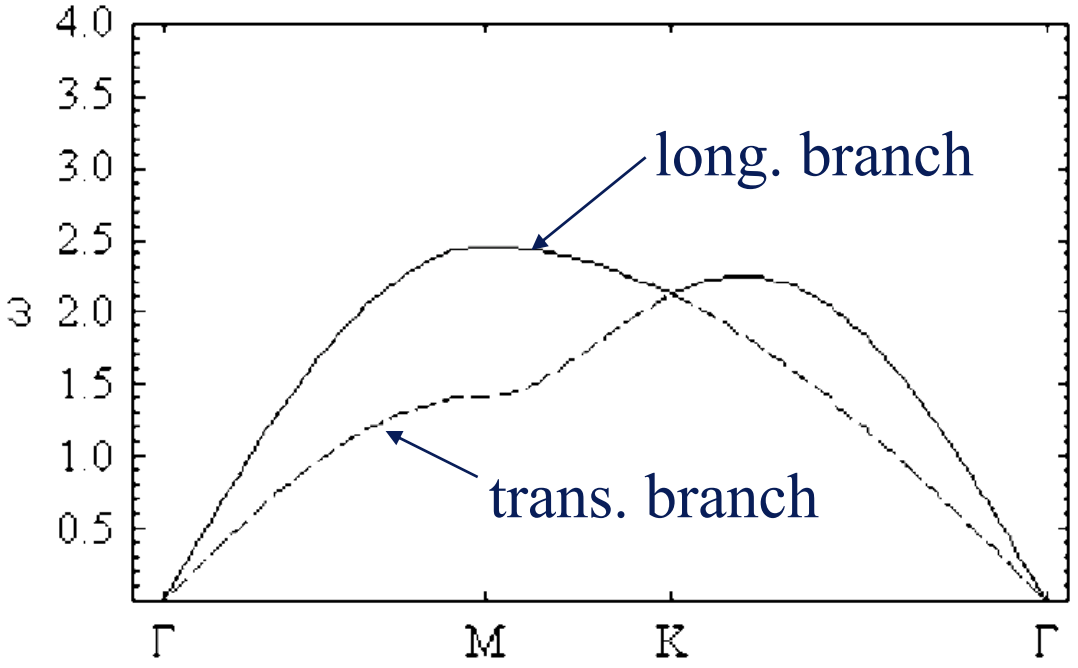
Phononic Bandgaps: Inertial Amplification



- Response controlled by isolator mass, m_a,and stiffness, k_a

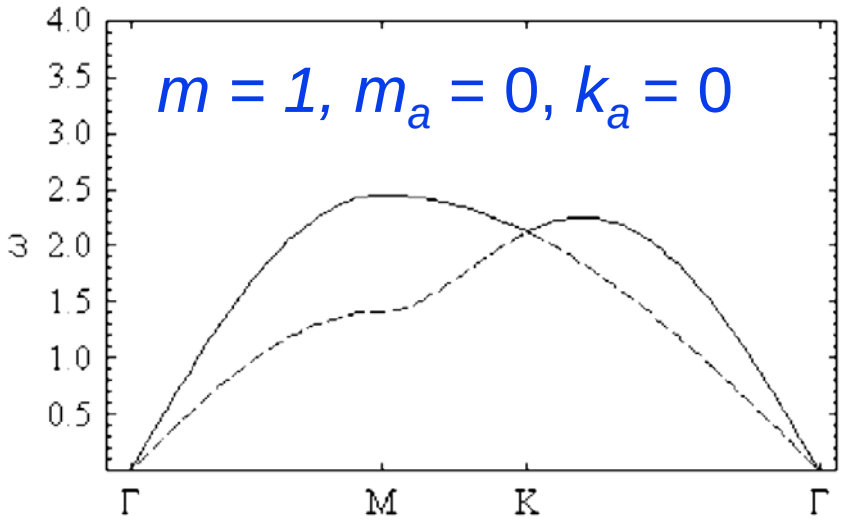
Phononic Bandgaps: Inertial Amplification

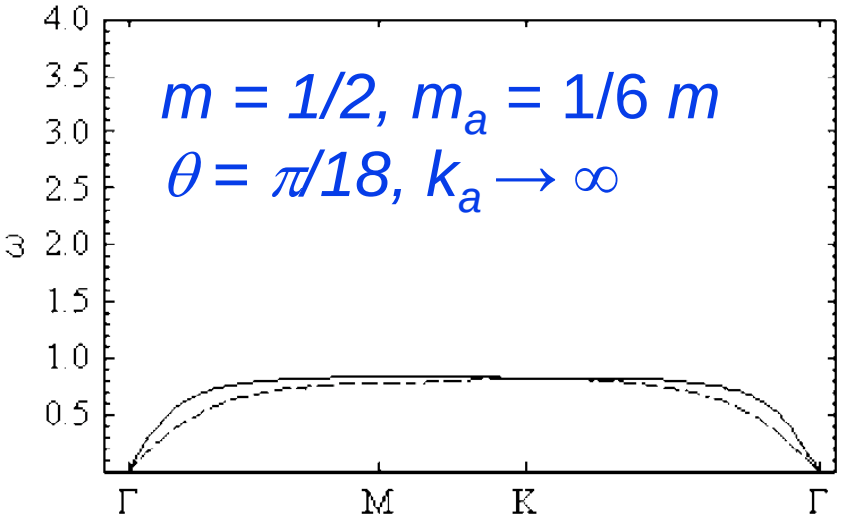
- Lattice response with no amplification
  - $m_a = 0$, $k_a = 0$ ($m = 1$, $k = 1$)
- Low-pass system



Phononic Bandgaps: Inertial Amplification

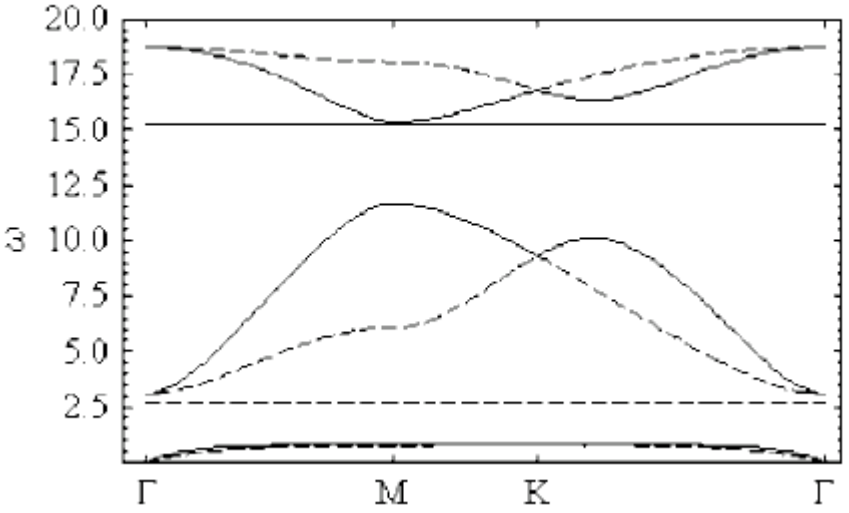
- Lattice response without and with amplification
- Low-pass response

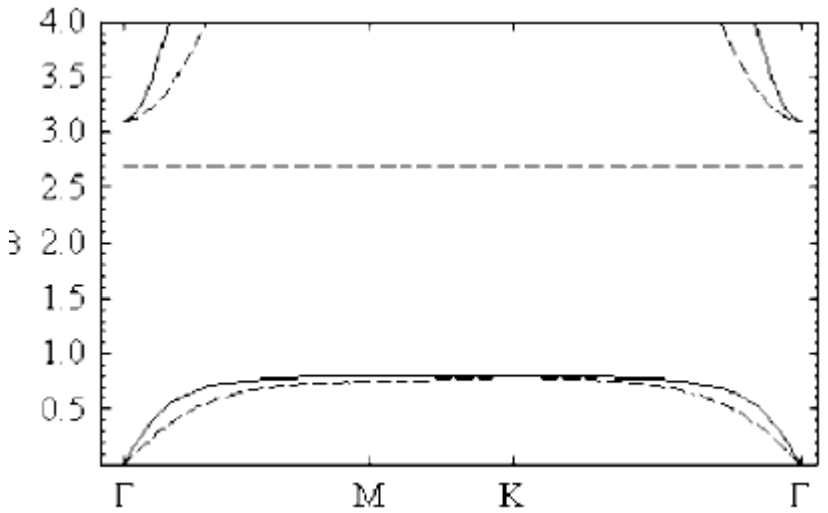




Phononic Bandgaps: Inertial Amplification

- Lattice response with amplification (compliance)
  - $m_a = 1/6\ m$, $k_a = 10\ k$, $\theta = \pi/18$ ($m = 1/2$, $k = 1$)
- Bandgaps created with much lower mass fractions than local resonators
- Bandgaps controlled by m_a , k_a , and θ





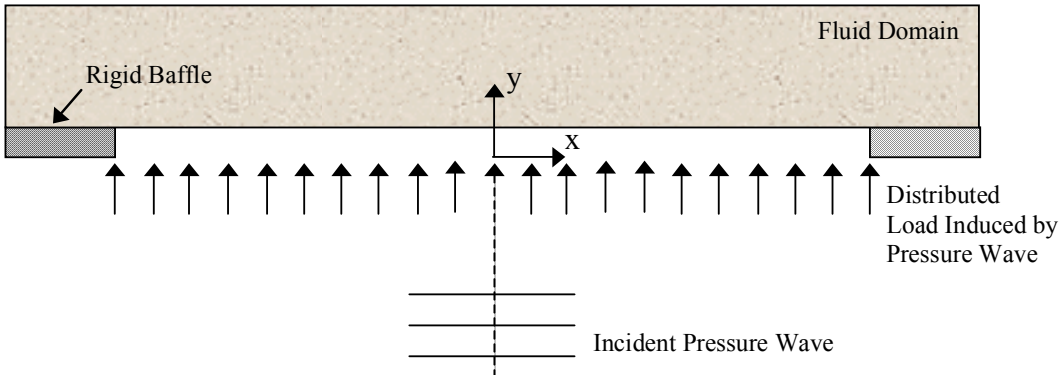
Overview

- Motivation and Background
- Multiscale approach
- Inertial coupling and amplification
- Compliant mechanisms
  - Mid-frequency application
- Summary

Vibro-Acoustic Response of Truss Structures

- Mechanical filter characteristics
  - *Signorelli & Flotow 1988*
  - *Bondaryk 1997*

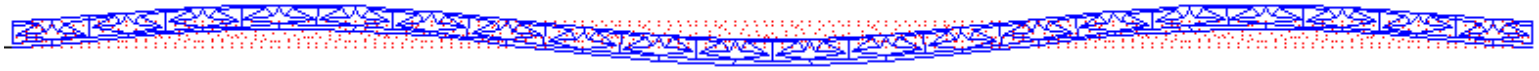
- Vibro-acoustic response
  - *El-Raheb & Wagner 1997*
  - *Ruzzene 2004*



Vibro-Acoustic Response of Truss Structures

- Global to local transition with increasing frequency

40 Hz

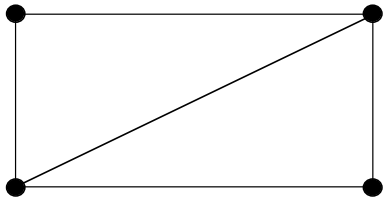


150 Hz

3000 Hz

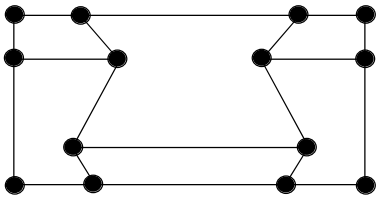
Vibro-Acoustic Response of Unit Cell & Truss

- Vibration and sound radiation of honeycomb sandwich panels
  - Two analyses:
    - Unit cell
      - Square core



Vibro-Acoustic Response of Unit Cell & Truss

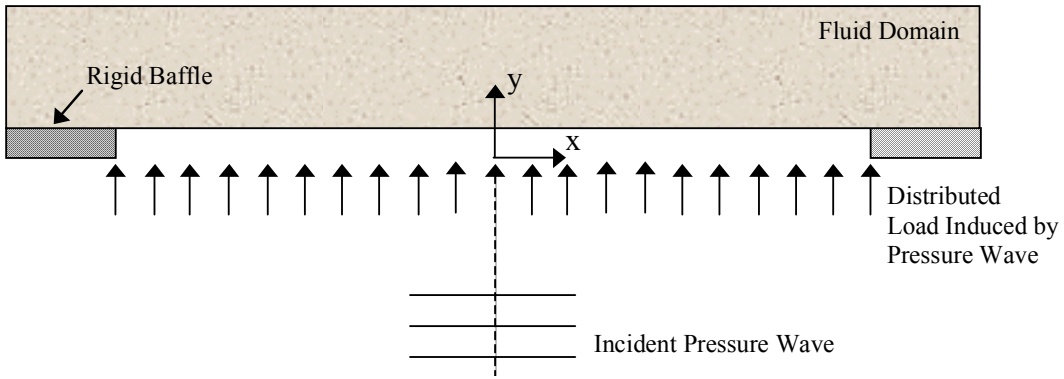
- Vibration and sound radiation of honeycomb sandwich panels
  - Two analyses:
    - Unit cell
      - Square core
      - Re-entrant core



Ruzzene 2004

Vibro-Acoustic Response of Unit Cell & Truss

- Vibration and sound radiation of honeycomb sandwich panels
  - Two analyses:
    - Unit cell
      - Square core
      - Re-entrant core
    - Full truss
      - Baffled rectangular radiator



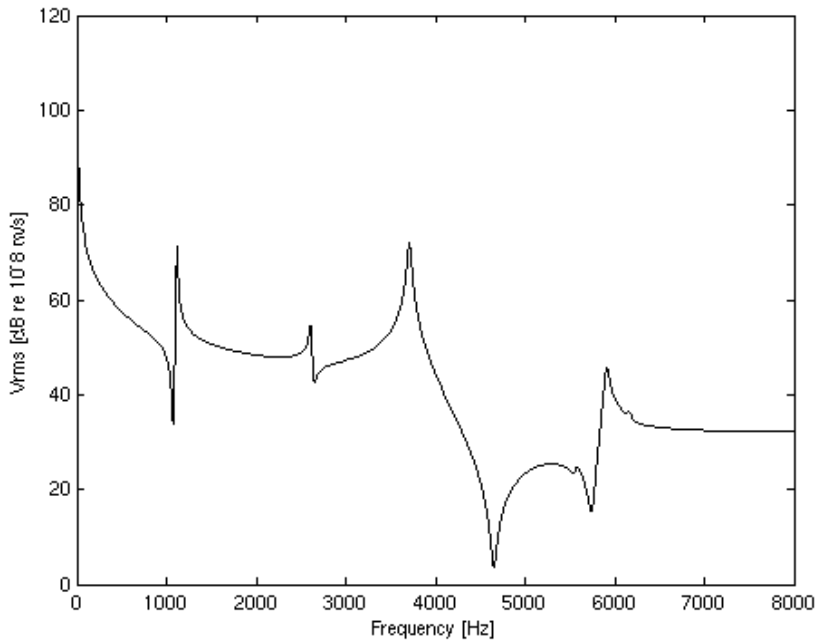
Vibro-Acoustic Response of Unit Cell & Truss

- Vibration and sound radiation of honeycomb sandwich panels
  - Two analyses:
    - Unit cell
      - Square core
      - Re-entrant core
    - Full truss
      - Baffled rectangular radiator
- Figure of merit: Output side nodal RMS velocity
  El-Raheb 1997

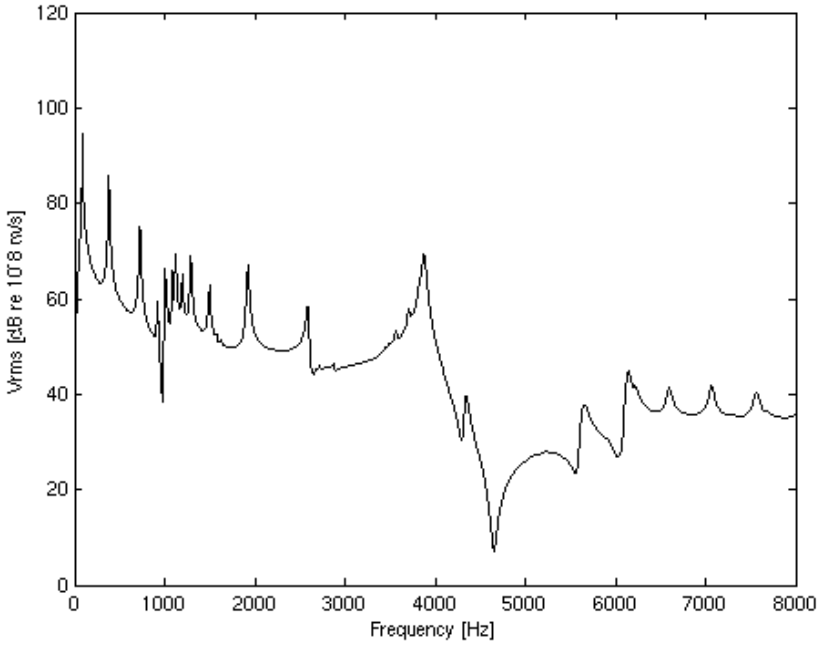
$$v_{rms} = 20\log 10\left[\frac{\omega}{v_{ref}}\left\{\frac{1}{N}\sum_{j=1}^{N}\left|\hat{v}_t\right|_j^2\right\}^{1/2}\right]$$

Vibro-Acoustic Response of Unit Cell & Truss

Square Core



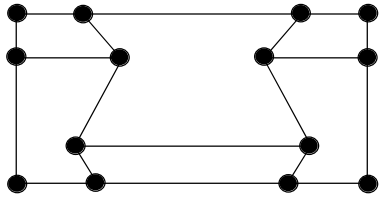
Unit Cell



Truss

Compliant Mechanism Unit Cell & Truss

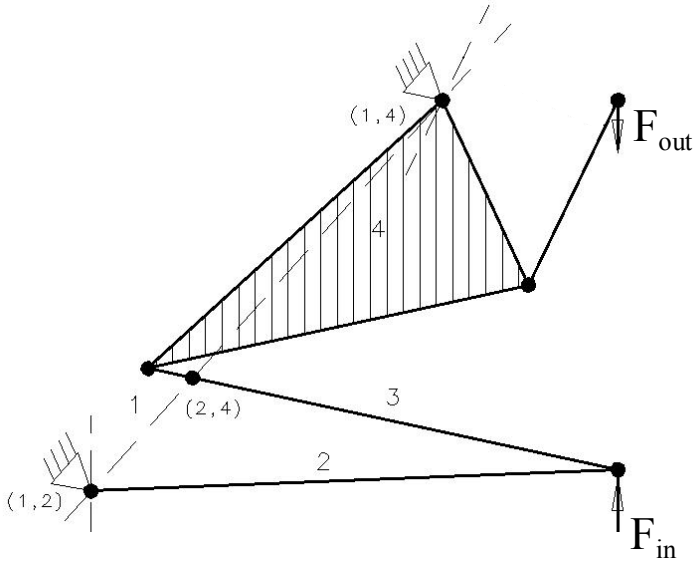
- Re-entrant core shape suggests compliant mechanism application



- Compliant Mechanism (CM) Design
  - Static analysis, design, and optimization - *Nishiwaki 1998, Kota et al. CSDL1998-2008, Howell 2001, Lobontiu 2003*
  - Dynamic analysis and design - *Li & Kota 2002, Tantanawat et al. 2004*

Compliant Mechanism Unit Cell

- Rigid link kinematic diagram of stroke reducer mechanism
  - *Hetrick & Kota 2000*
  - Concept of Mechanical Advantage (M.A.)

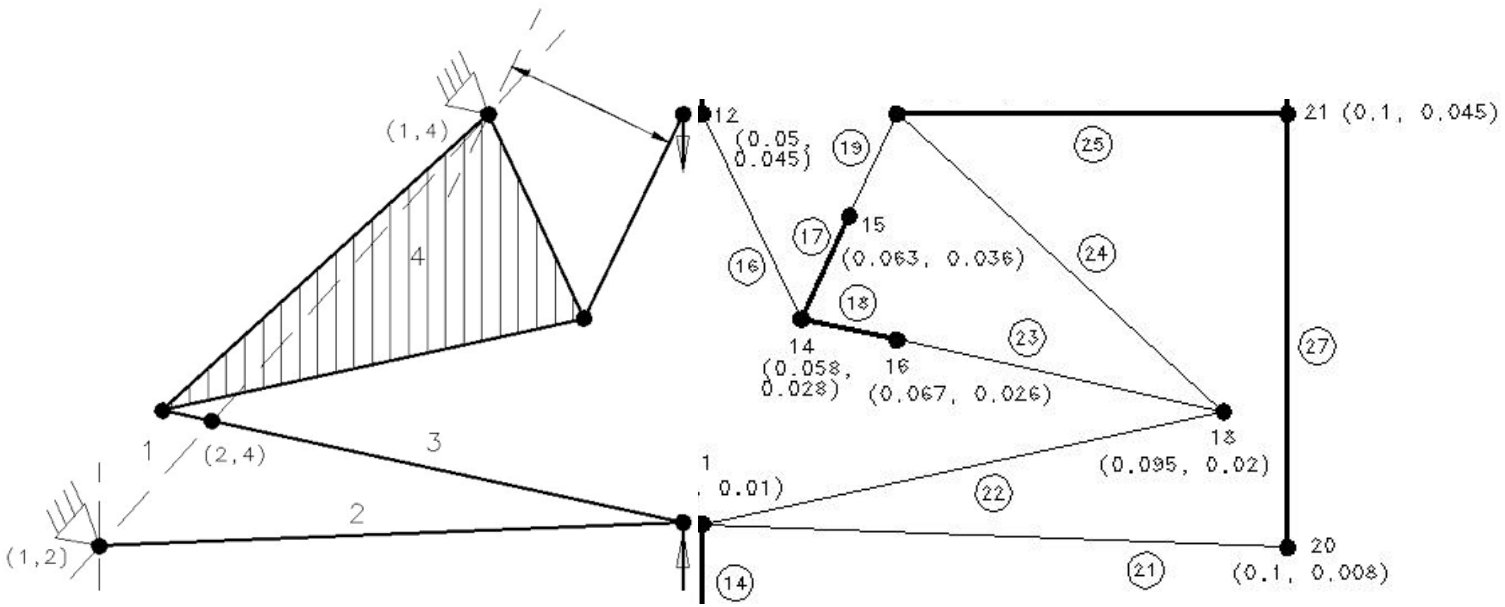


$$M.A. = \frac{F_{out}}{F_{in}} = \frac{v_{in}}{v_{out}}$$

$$M.A. = 8.3$$

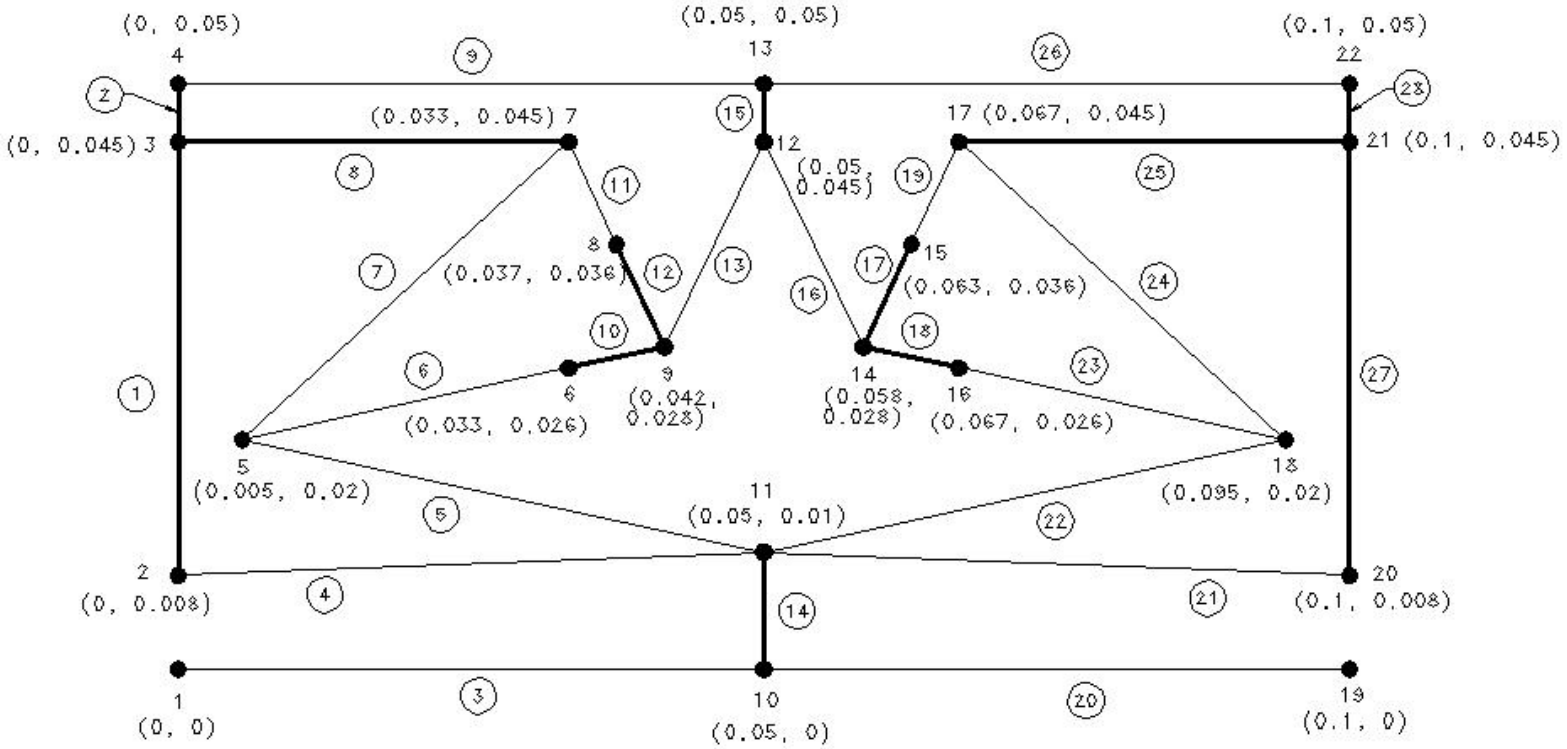
Compliant Mechanism Unit Cell

- Compliant mechanism unit cell embodiment



Compliant Mechanism Unit Cell

- Compliant mechanism unit cell embodiment

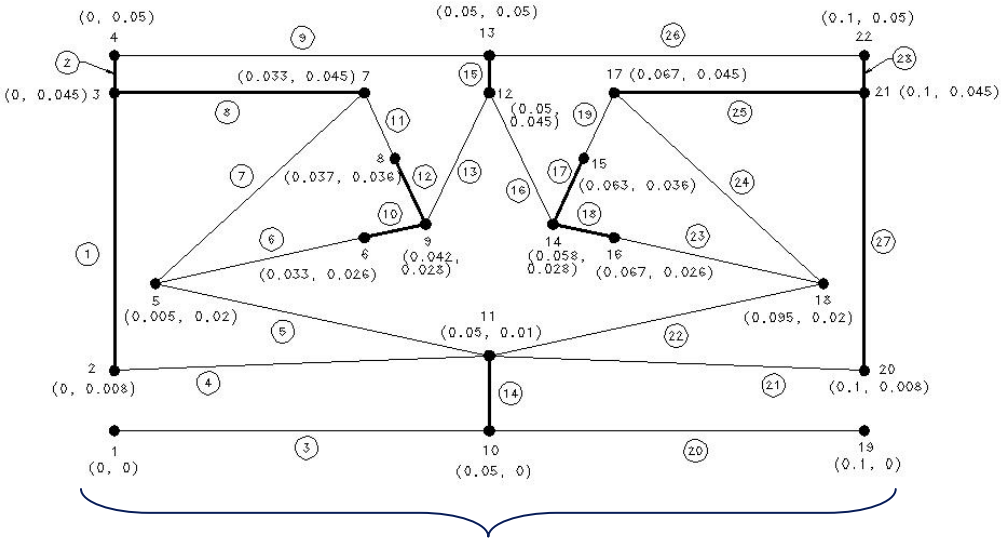


Compliant Mechanism Unit Cell & Truss

- Predicting the attenuation zone
  - Wavenumber frequency analysis

$$\omega = \left(\frac{2\pi}{\lambda}\right)^2 \sqrt{\left(\frac{EI}{\rho A}\right)}$$

$$f_{AZ} = \frac{\omega_{AZ}}{2\pi} = 2325 Hz$$



$$k = \sqrt{\omega}\left[\frac{\rho A}{EI}\right]^{1/4}$$

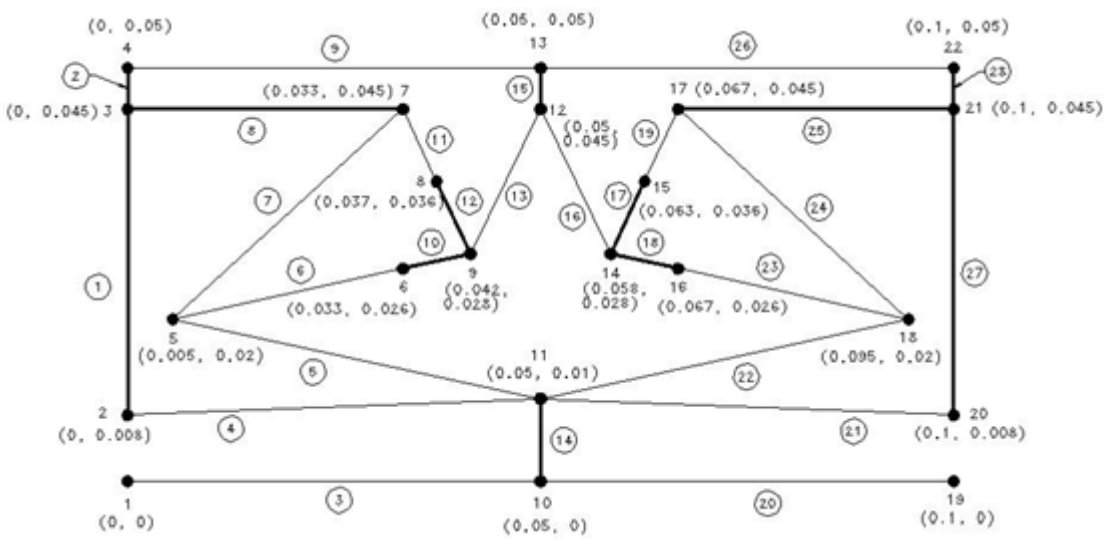
$$\lambda = \frac{2\pi}{k}$$

Compliant Mechanism Unit Cell & Truss

- Predicting the attenuation zone
  - Wavenumber frequency analysis
  - Mechanism frequency response function (FRF)
    - Output / input displacement ratio
    - Output / input phase relationship

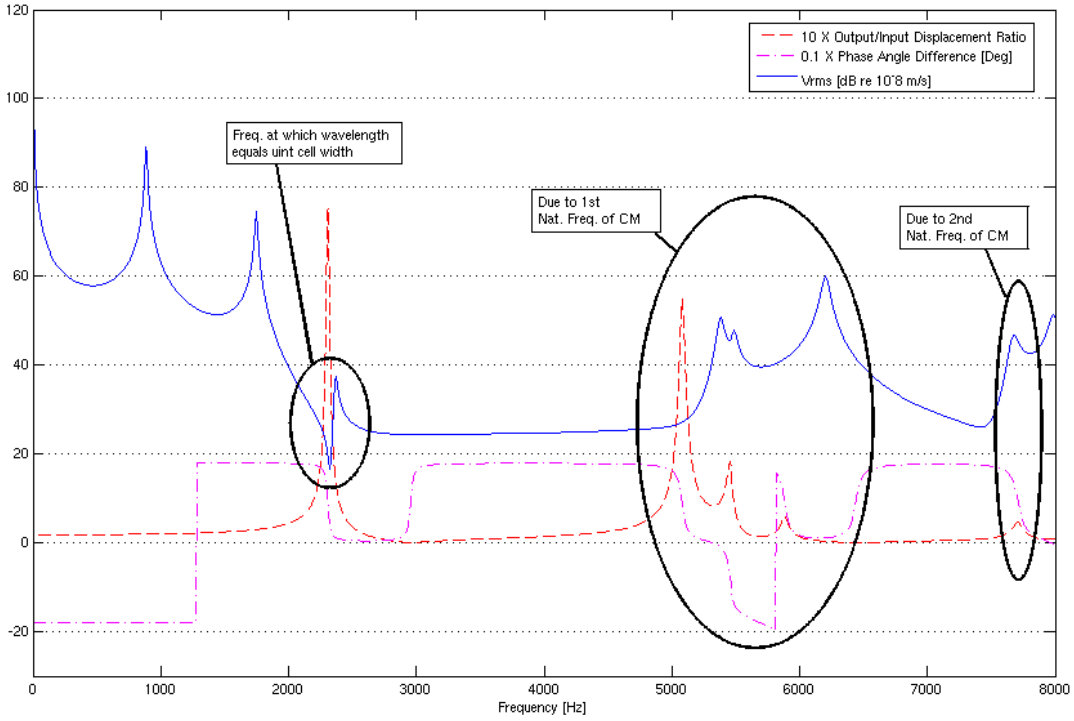
$$Amplitude = \left| \frac{\hat{v}_{13} - \hat{v}_4}{\hat{v}_{10} - \hat{v}_2} \right|$$

$$Phase = \frac{180}{\pi} \left[\angle(\hat{v}_{13} - \hat{v}_4) - \angle(\hat{v}_{10} - \hat{v}_2) \right]$$



Compliant Mechanism Unit Cell & Truss

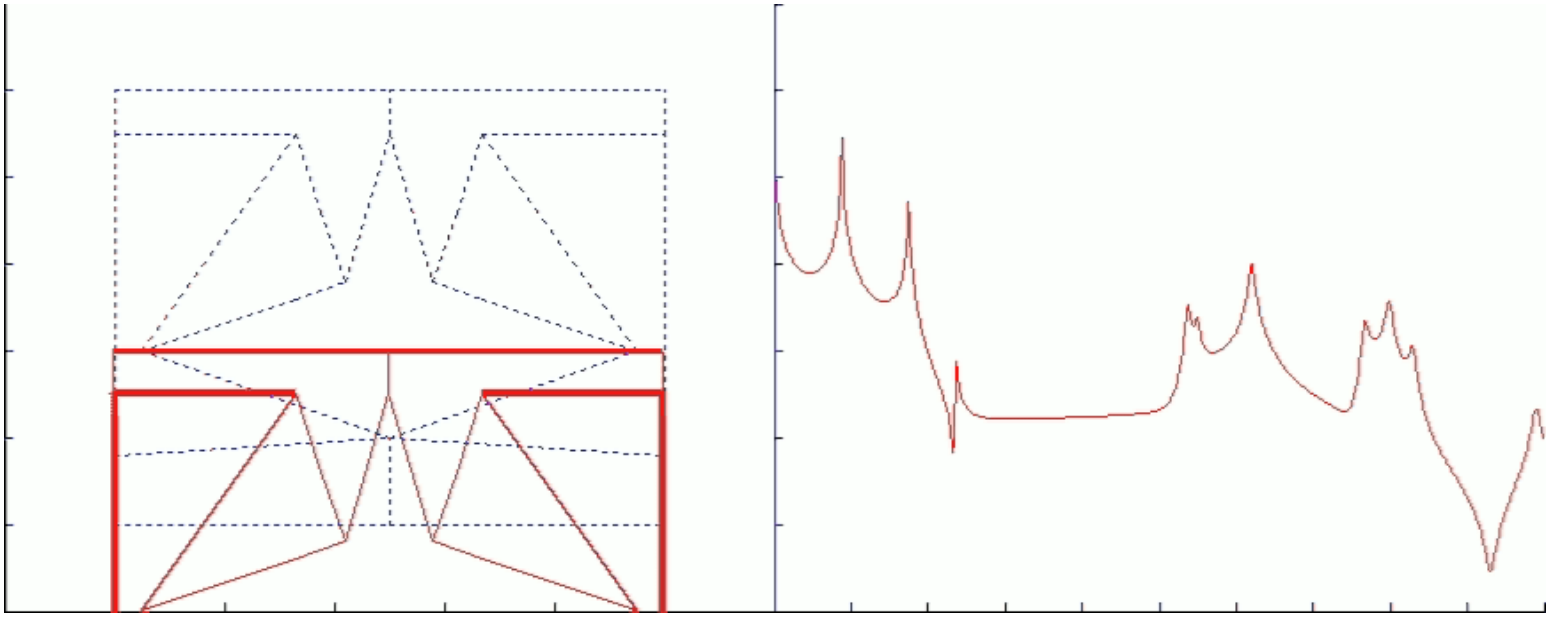
- Predicting the attenuation zone
  - Wavenumber frequency analysis
  - Mechanism frequency response function (FRF)



Broadband attenuation zone from 2500 Hz - 5000 Hz, 45 dB reduction at 3700 Hz

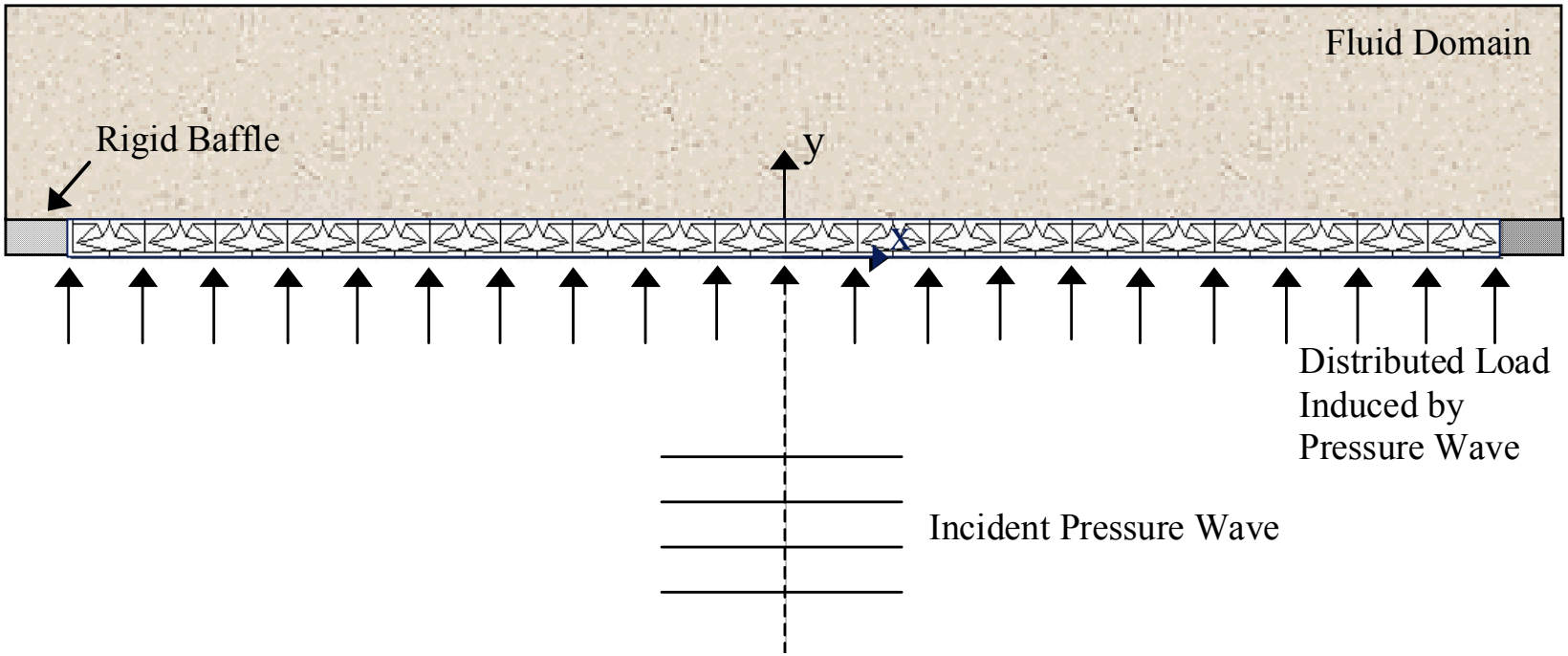
Compliant Mechanism Unit Cell & Truss

- Predicting the attenuation zone
  - Wavenumber frequency analysis
  - Mechanism frequency response function (FRF)
    - Output / input displacement ratio
    - Output / input phase relationship
  - Mode shape / strain energy frequency plots

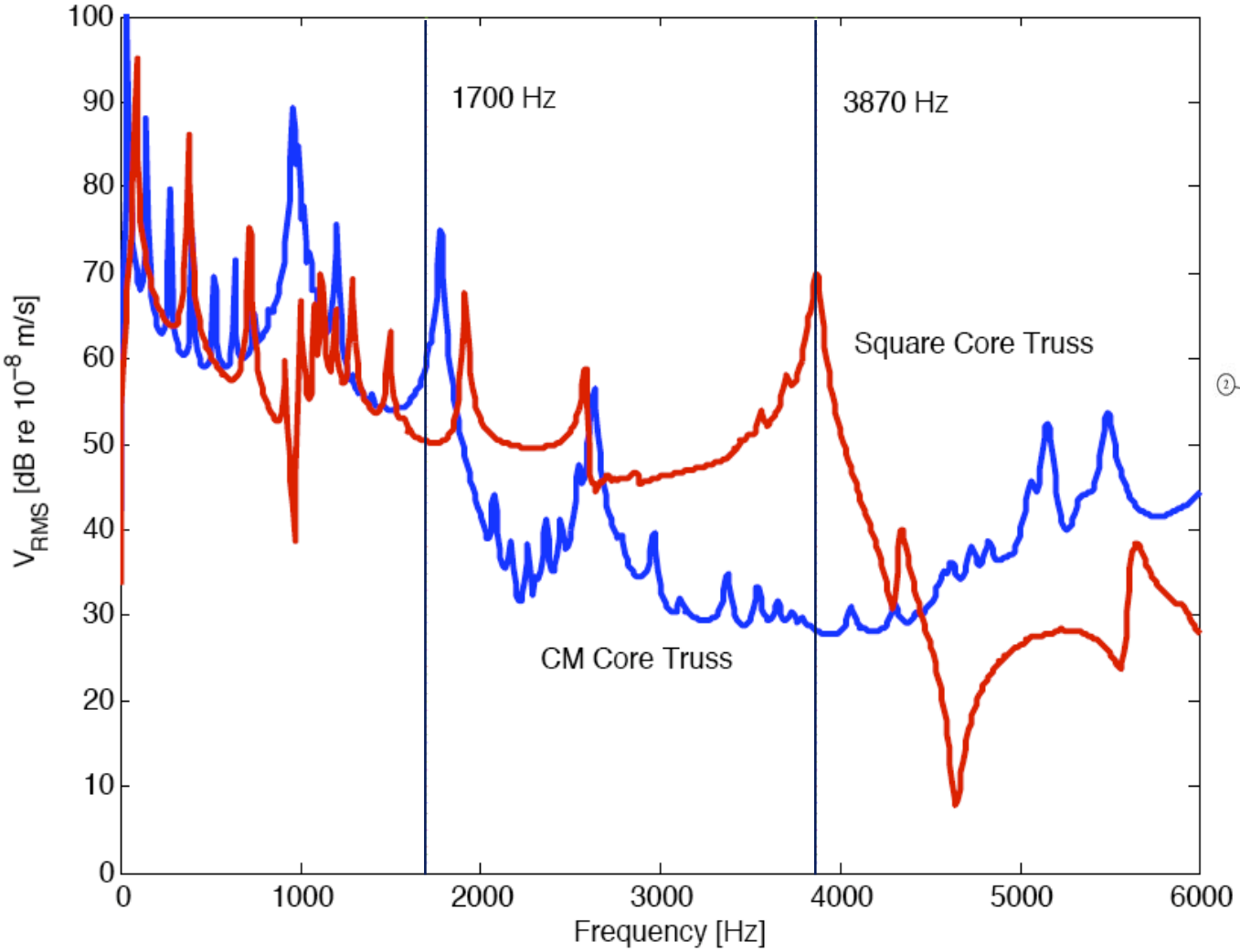


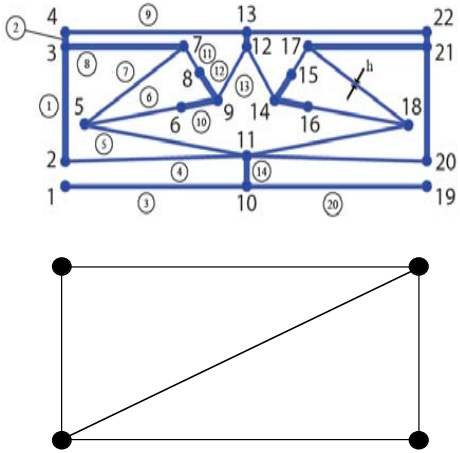
Compliant Mechanism Unit Cell & Truss

- CM truss structure

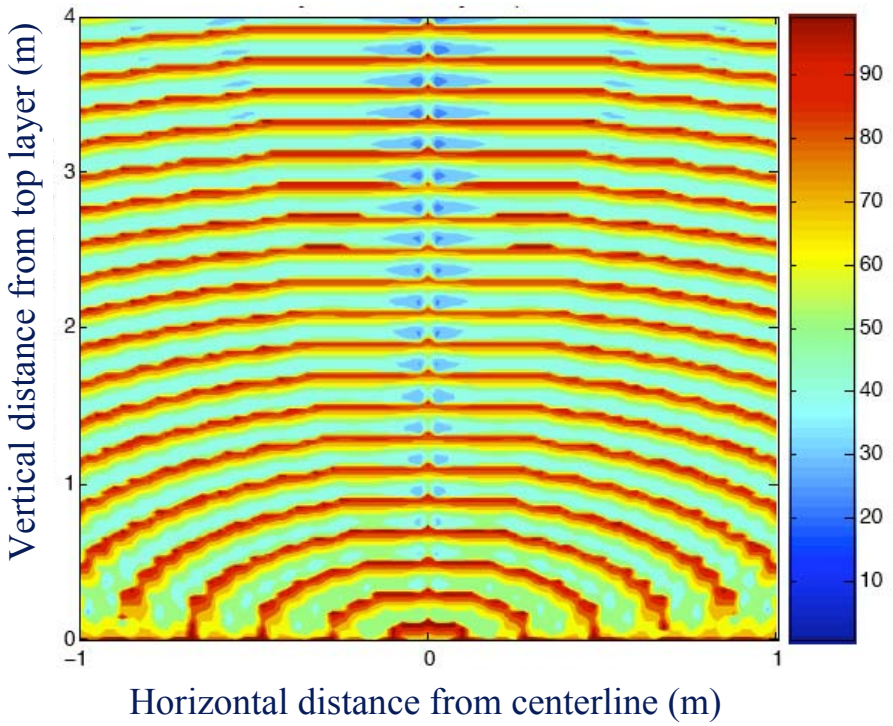


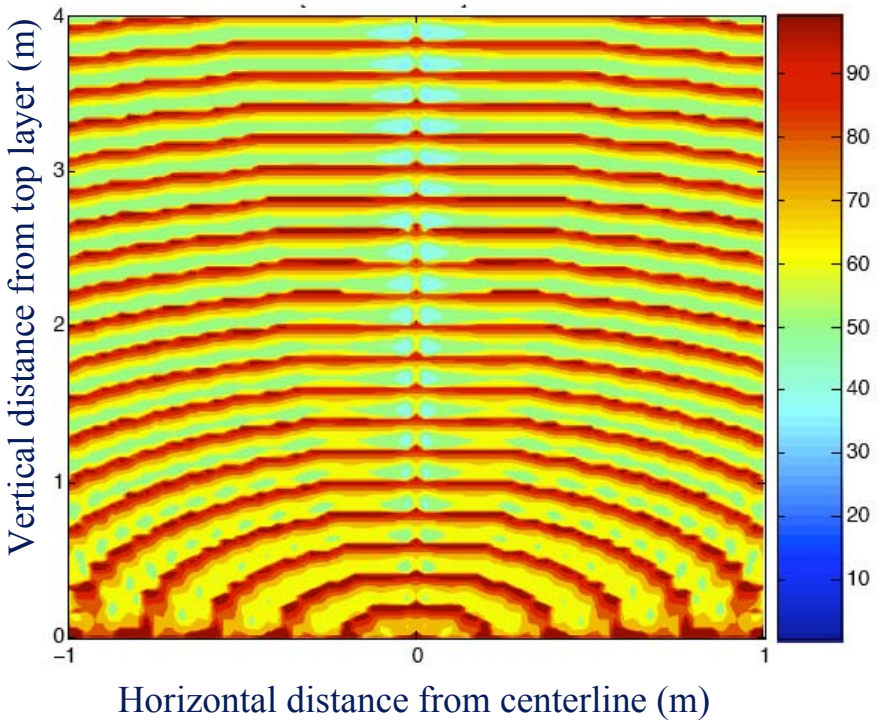
Comparison of Truss Core Responses





Comparison of Truss Core Responses



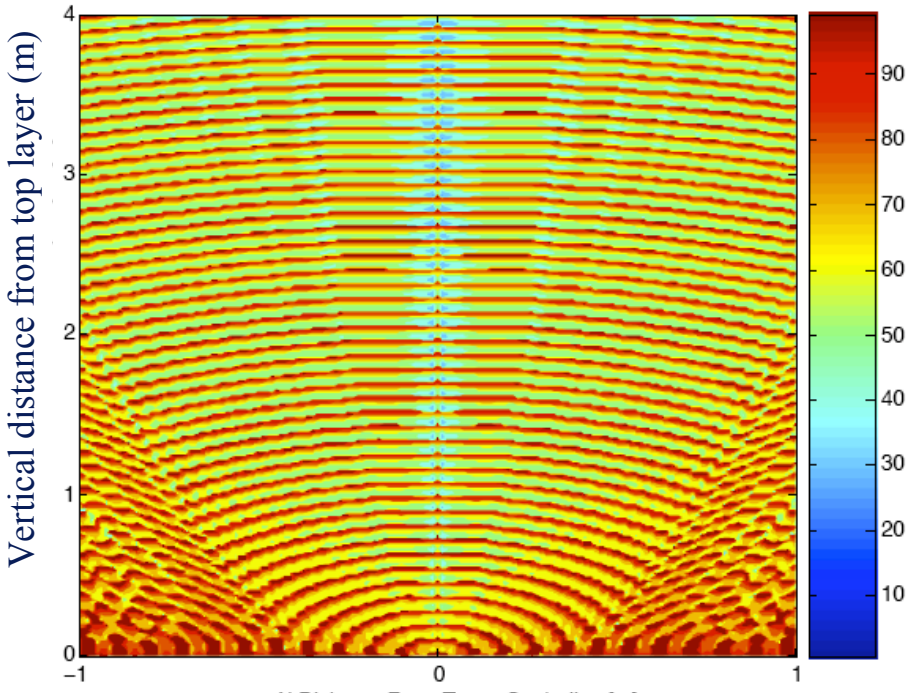


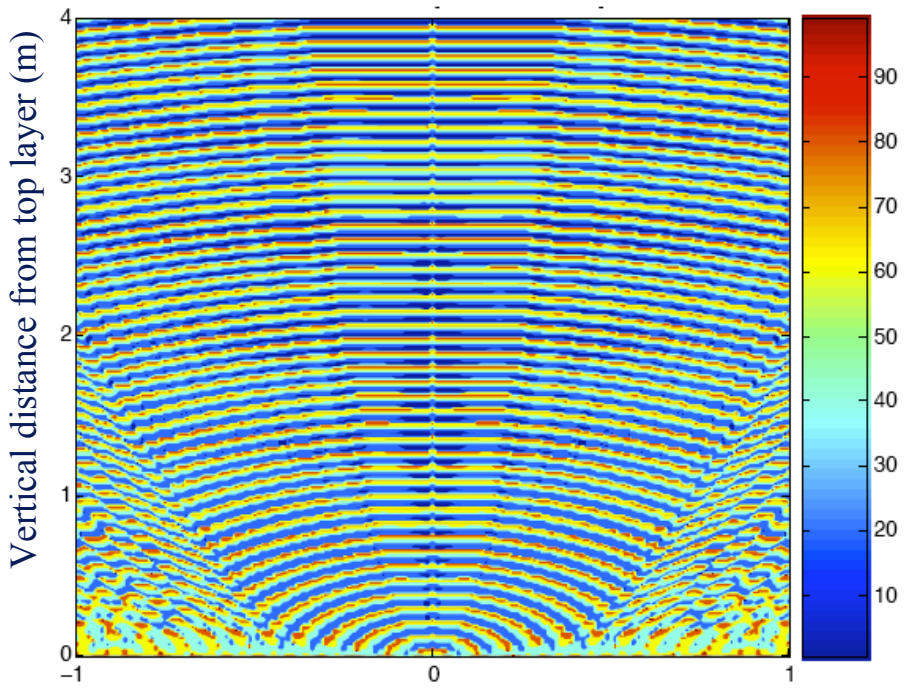
Square core section

CM core section

1700 Hz

Comparison of Truss Core Responses





Square core section

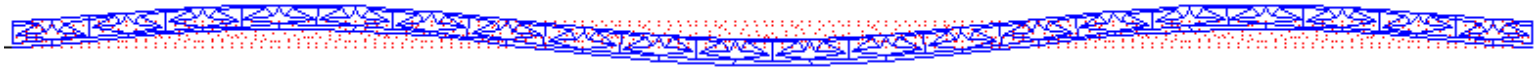
CM core section

3870 Hz

Compliant Mechanism Unit Cell & Truss

- Transition from global to local deformation

40 Hz

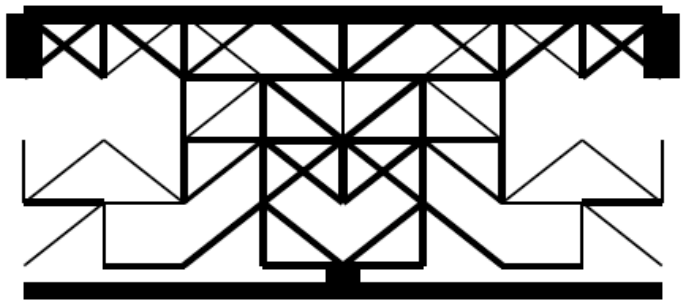


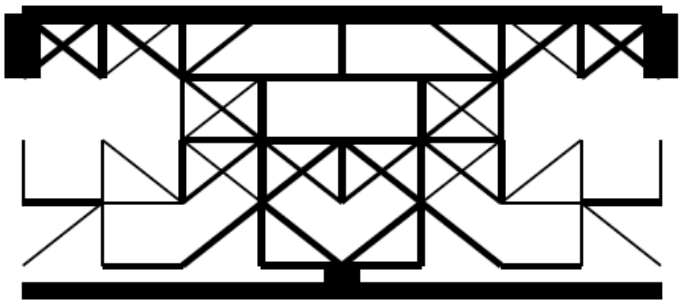
150 Hz

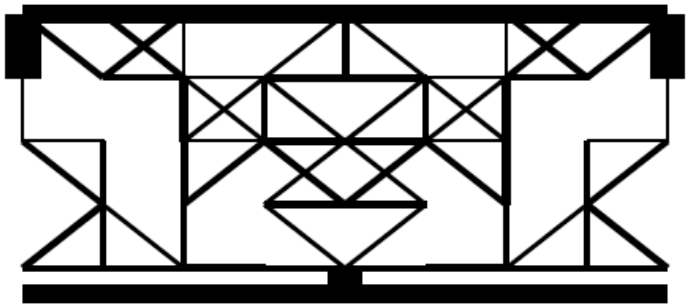
3000 Hz

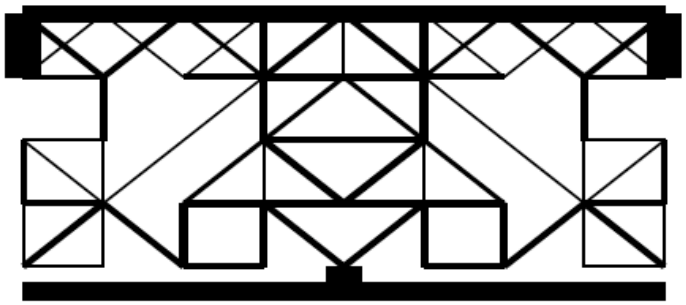
Topology Optimization of Compliant Mechanism Unit Cell

- Genetic algorithm-based optimization with sandwich core of networked frame elements
- Minimize v_{rms} at 2320-2330 Hz





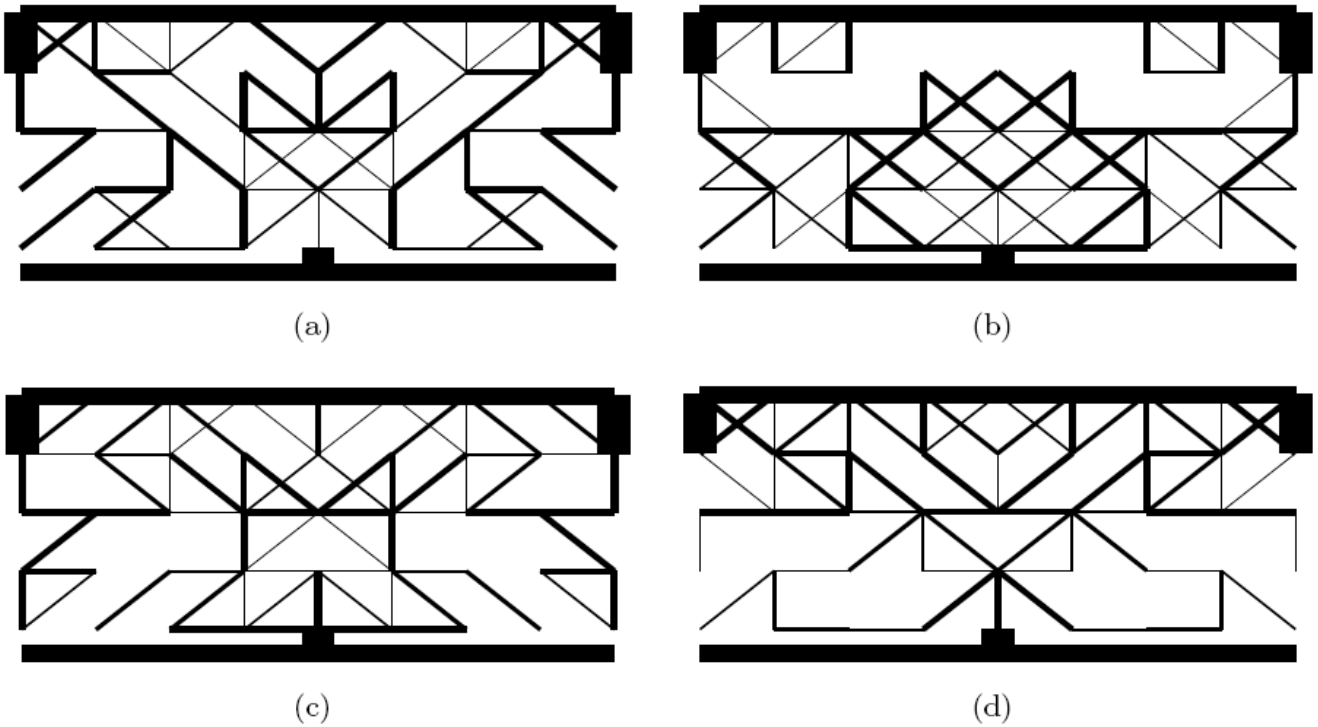




No mass constraint

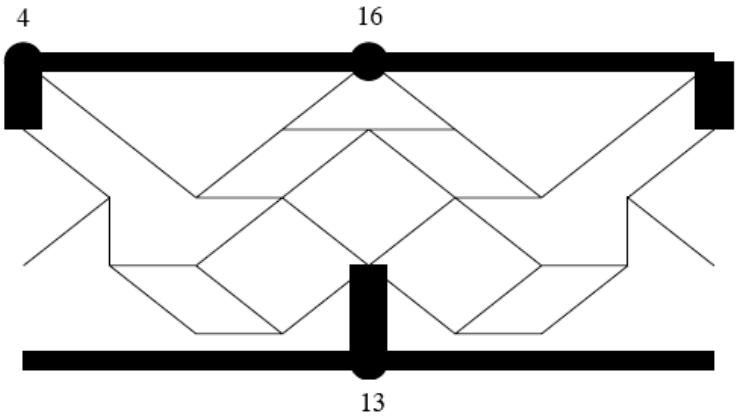
Topology Optimization of Compliant Mechanism Unit Cell

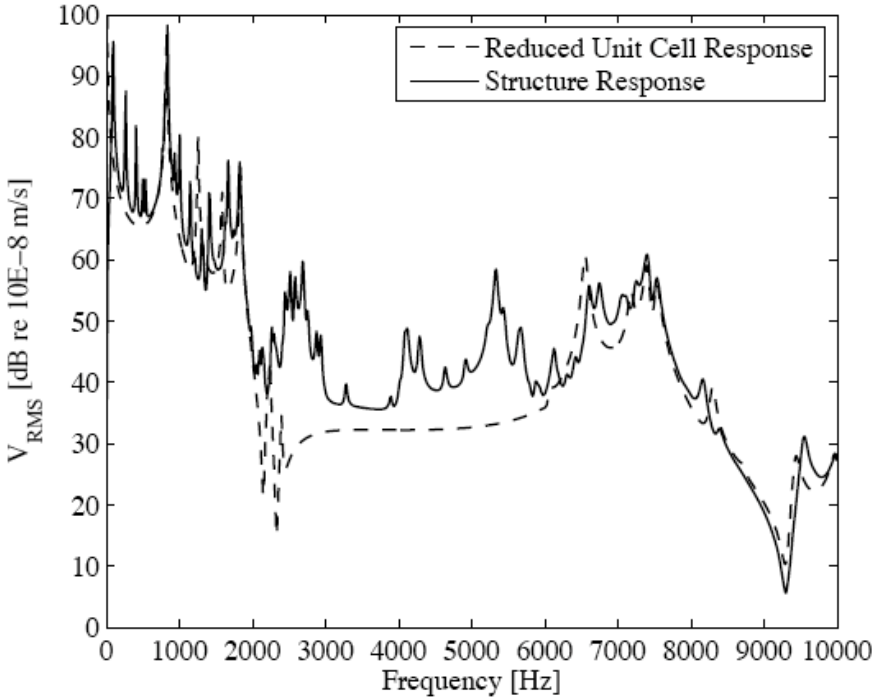
- Minimize v_{rms} at 2320-2330 Hz
- Mass <= 0.71 maximum mass



Compliant Mechanism Optimization

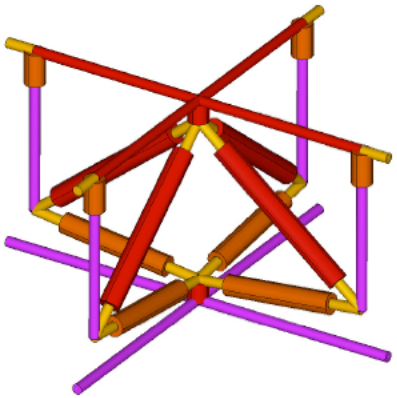
- Simplified design for manufacturing



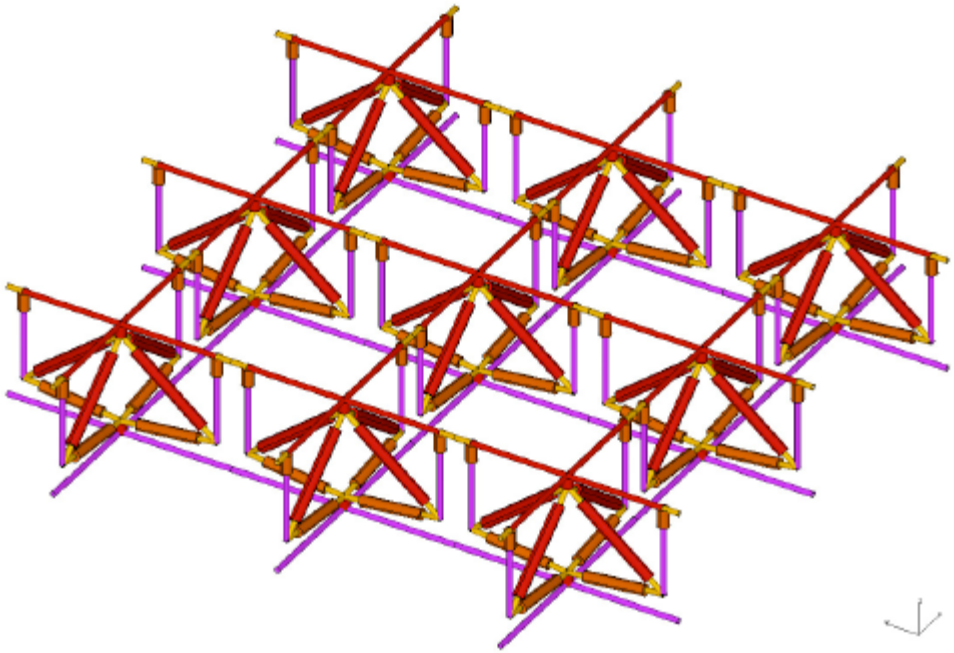


Future Concepts

- Extension to surfaces (3D mechanisms)



Unit cell



Panel arrangement

Conclusions

- New approaches towards structural vibration control
- Modern perspective on inertial amplification and coupling
- Novel application of compliant mechanisms
- Combine inertial amplification and compliant mechanism topology generation and optimization