

## **Analysis and design of materials and structures for attenuating vibration and acoustic response**

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### **Abstract**

The ability to design to design and manufacture materials with complex structure and novel material response offers the opportunity to consider the control of the dynamic response of structures. This problem is well suited for a multi-scale approach in which the material characteristics and material layout are considered as part of the structural analysis and design problem.

Three approaches towards the design of band-gap materials and structures are presented and representative materials and structures are shown. In addition to material design employing Bragg scattering, the use of compliant mechanisms in sandwich structures is demonstrated. Lastly, inertial amplification is shown to provide a third method of inducing band-gap phenomena into materials and structures using embedded amplification mechanisms.

### **1. Introduction**

The importance of designing structures with desired dynamic response characteristics is increasing, particularly in the ground vehicle industry. This is driven by customer demands for quiet vehicles and for well-tuned ride characteristics. In addition, safety requirements also are impacted by the dynamic response of materials, especially for military vehicles. Competing with these requirements is the continuing need to reduce the weight of vehicles to improve fuel economy. To this end, significant research has been directed towards the design of new composite materials.

Of particular focus in the present work is to construct systematically materials that enable structures to have designed vibration or acoustic response (spectral gaps) or to have significantly reduced response in desired frequency bands. Using a multi-scale approach, the desired structural response can be obtained by careful design of the material response.

Spectral gaps in the band structure of periodic media have been an ongoing research endeavor since the 1950s. In the last decade, there has been growing interest in computing and designing the phononic band structure of 2D and 3D periodic systems comprising various materials. Of particular focus has been obtaining complete phononic band gaps, which forbid the propagation of elastic or acoustic waves regardless of mode or wave vector. Practical applications of these systems include mechanical filters, sound and vibration isolators, and acoustic waveguides.

The two different widely published means of generating phononic band gaps in periodic media are Bragg scattering and local resonances. In Bragg scattering, a gap appears due to destructive interference of the wave reflections from the periodic inclusions within the media. Band gaps can also be generated via local resonators, which impede wave propagation around their resonance frequencies. A third approach towards phononic band gap generation is possible in which the effective inertia of the wave propagation medium is amplified via embedded amplification mechanisms. We classify this alternate approach as inertial amplification.

These three approaches towards the design of band-gap materials and structures are presented and representative materials and structures are shown. In addition to material design employing Bragg scattering, the use of compliant mechanisms in sandwich structures is demonstrated as a novel application of local resonances. Lastly, inertial amplification is shown to provide a third method of inducing band-gap phenomena into materials and structures using embedded amplification mechanisms.

## 2. Bragg scattering

Wave propagation in heterogeneous media is dispersive, i.e., the media causes an incident wave to decompose into multiple waves with different frequencies. A medium with periodic heterogeneity has distinct frequency ranges in which waves are either effectively attenuated or allowed to propagate. These frequency ranges are referred to as *band-gaps* (or stop bands) and *bands* (or pass bands), respectively, and are attributed to mechanisms of wave interference within the scattered elastic field, known as Bragg scattering. From a practical perspective, it has been shown that under certain conditions bounded structures formed from periodic materials can exhibit similar frequency-banded wave motion characteristics. By controlling the layout of constituent material phases and the ratio of their properties within a unit cell, a periodic composite can be designed to have a desired frequency band structure (the size and location of stop bands and pass bands). Figure 1 depicts various designs of a bi-material unit cell that exhibit different frequency band-gap responses. In Hussein *et al.* [1], an optimization problem was constructed to identify unit cell topologies that could maximize the bandwidth of the stop-band behavior across a broad frequency response domain.

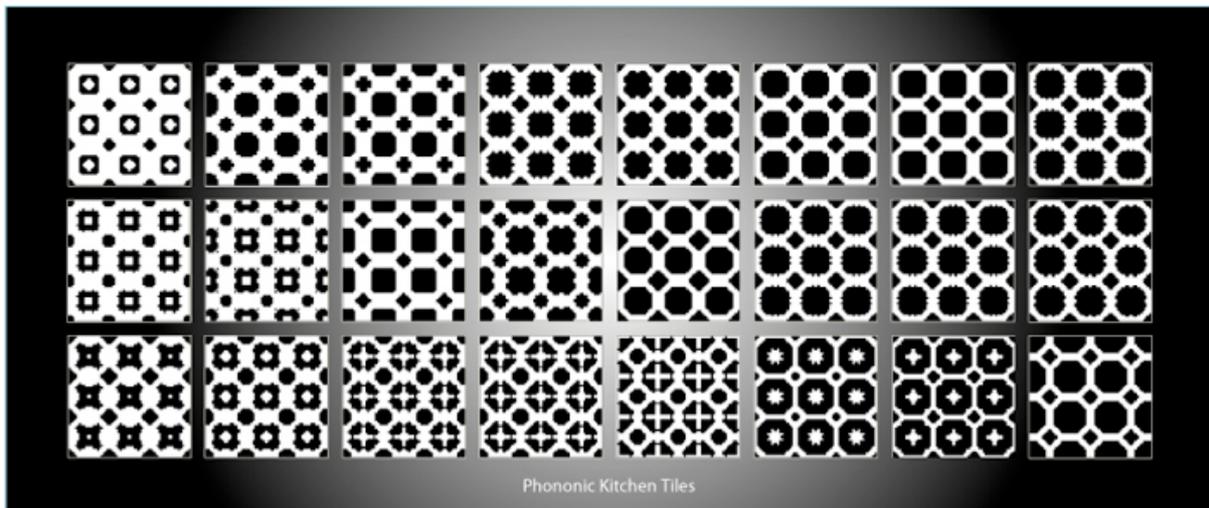


Figure 1: Examples of bi-material layouts to optimize phononic band-gaps using Bragg scattering (courtesy of Prof. Mahmoud I. Hussein)

## 3. Integral compliant mechanisms

The design of structures to mitigate structural vibration and acoustic response in mid-frequency spectrums (1-10 kHz) often has relied upon periodic lattices and structures, in particular, sandwich structures. The high stiffness-to-weight ratio and mid-frequency isolation attributes of sandwich structures both are attractive for the design of practical engineering structures. We have explored the novel use of compliant mechanisms as the core topology to attenuate mid-frequency structural response of sandwich structures; see, e.g., Dede and Hulbert [2]. While compliant mechanisms have been an active research field for the past 20 years, their application as a building block for vibration attenuation of sandwich structures presents a new application field. Figure 2 depicts an optimized compliant cell unit topology and a sandwich structure constructed from an assembly of the unit cells. The ability to optimize the compliant mechanism topology is described in Dede and Hulbert [2].

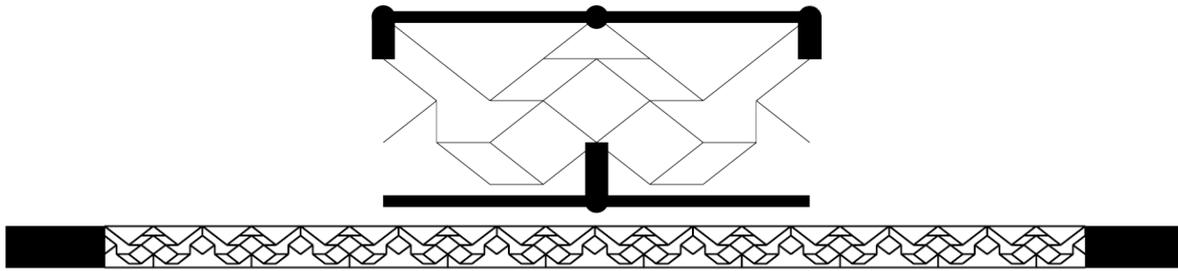


Figure 2: A reduced unit cell compliant mechanism topology (top); compliant sandwich structure comprising ten parallel unit cells (bottom).

#### 4. Inertial amplification

A significant and practical challenge is to design systems that possess wide, low-frequency band-gaps. The lowest frequency gap due to Bragg scattering is of the order of the wave speed (longitudinal or transverse) of the medium divided by the lattice constant. Thus, a low-frequency Bragg gap requires low wave speeds (i.e., heavy inclusions in a soft medium) or a large lattice constant. On the other hand, by choosing low resonator frequencies, one can place local resonator induced band-gaps at much lower frequencies than that can be obtained by Bragg scattering. Low-frequency local resonances can be realized by embedding rubber-coated dense metal spheres or cylinders in an epoxy matrix. However, to obtain wide band-gaps at low frequencies, large volume filling fractions are required. Since the average density of the coated inclusions, e.g., rubber and dense metal, is more than an epoxy matrix, large volume fractions imply even larger mass fractions. Consequently, to obtain wide band-gaps at low frequencies, heavy resonators are needed that form a large fraction of the overall mass of the medium. Alternatively, by amplifying the effective inertia of the wave propagation medium using embedded amplification mechanisms, it is possible to circumvent the disadvantages described.

One of the first designs that made use of amplified effective inertia employs a single stage vibration isolator consisting of a levered mass in parallel with a spring. Such systems are used to isolate massive objects from vibrations. The lever in the system generates large inertial forces by amplifying the motion of a small mass, which in turn effectively increases the inertia of the overall system by lowering its resonance frequency. Furthermore, the isolator also introduces an anti-resonance frequency when the inertial force generated by the levered mass cancels the spring force. In Yilmaz *et al.* [3], this inertial amplification concept is utilized to generate band-gaps in infinite periodic systems. Their simple yet effective geometry allows them to be easily embedded into two or three-dimensional lattices, as illustrated in Figure 3. It is shown that the widest low-frequency band-gaps are obtained when most of the mass within the lattice is concentrated on very stiff amplifiers that can generate large amplifications. However, with smaller mass fractions on amplifiers, wide low-frequency band-gaps can still be obtained, provided that amplifiers are moderately stiff and can generate reasonably large amplifications. This is in contrast to obtaining wide low-frequency gaps via local resonators, which require heavy resonators that form a large fraction of the overall mass of their unit-cell. Moreover, unlike Bragg scattering, wave speeds and the lattice constant do not limit the lower frequency limit of a band-gap. Hence, this alternative method of generating band-gaps is particularly attractive for low-frequency applications.

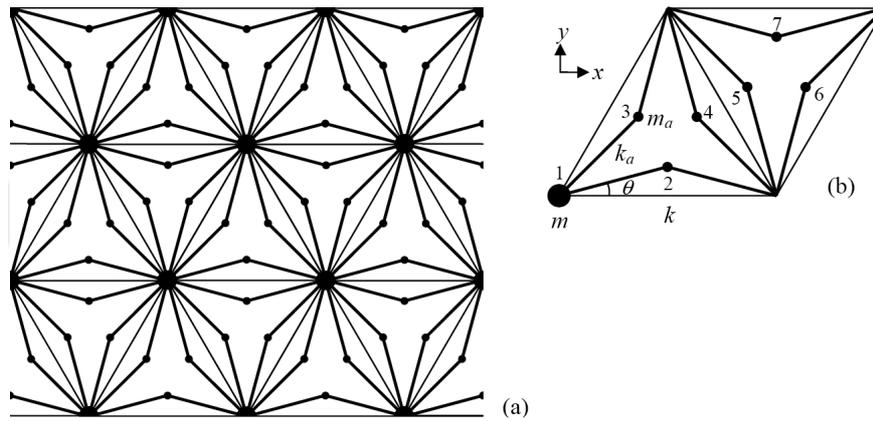


Figure 3: The infinite periodic lattice with inertial amplification (a); its irreducible unit cell (b).

## References

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