Deployable Structures and Biological Morphology

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Folding and deployment of stored-energy composite structures

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Abstract
This lecture will present novel simulation techniques for lightweight deployable structures consisting of very thin sheets of fibre-reinforced composite materials. These structures are folded elastically and are able to self-deploy (dynamically) when they are released. Novel structural concepts of this kind were recently introduced on missions requiring parabolic reflector dishes for telecommunications and tubular deployable booms supporting sensors. A variety of related concepts are currently under consideration for future missions.

Designing these structures requires detailed predictions that capture both the overall, large displacement deformation of the structure, including the effects of contact and friction at the interfaces between parts of the structure that come into contact, and also the localised deformation of the most heavily deformed regions of the structure, in order to verify that no damage will occur in these regions.

Achieving this poses a number of novel challenges. First, the constitutive behaviour of thin composite materials differs fundamentally from that of standard composites and hence needs to be approached with suitable homogenization theories. Second, contact between heavily deformed (but still elastic) surfaces plays a key role in the tight packaging of these structures and the interaction between local and global instabilities that are encountered, even for the simplest configurations of current interest, is such that implicit solution schemes that attempt to capture the complexity of the physical situation become overwhelmed. Third, folding aids, such as jigs, straps, etc. are often used to facilitate the actual folding of a real structure without causing any damage and, although modelling this process in full detail is not necessary, any simulation needs to be steered through a maze of instabilities. The engineer must have confidence in the final predictions, because a fully detailed model of the folded configuration is required, to provide the initial conditions for any study of deployment.
Unfolding of potato flower as a deployable structure

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Abstract
A potato flower has triangle gussets between individual petals and five or six petals almost make a flat plane when they are fully unfolded. In this research, therefore, the folding and unfolding manners of a number of regular polygon paper models as the petals and gussets of a potato flower fully unfolded were investigated by vector analysis. It was found that the bud volume depends on the petal number, \( N \), and the bud with \( N = 5 \) has the largest volume. On the total kinetic energy during unfolding, however, the model with \( N = 5 \) or 6 became smaller than other models. Therefore, the energy about unfolding is more important for potato flower.

1. Introduction
Beautiful flowers often make us calmly and cure our injured heart. With increase of air temperature in spring, tiny bodies of leaves and flowers come out of their buds and start to grow. In their buds, therefore, various ideas to stow their bodies are often observed. For example, the leaves of common beech have a typical corrugated folding (Kobayashi et al. [2]), the fan-type bellows pattern is observed in maple leaves (Kobayashi et al. [3]), the petals of a morning glory flower are rolled spirally in a spindle-shaped bud (Kobayashi et al. [4]) and other deployable structures in plants are also shown by Delarue [1] and Kresling [5]. Thus, to examine the folding or unfolding manner in plants may be useful for us to design some artificial deployable structures, such as solar panels, antenna of satellites (Miura [6]), or deployable roofs.

A potato flower has triangle gussets between individual petals and petals and gussets make a almost flat plane when they are fully unfolded. Therefore, the study about unfolding of potato flower may be useful to consider the folding system or lapping system using a flat plane. In this research, the folding and unfolding manners of a number of regular polygon paper models with different petal number, the same area and valley/crest crease pattern following a potato flower were investigated by vector analysis. The volume of a bud (fully folded) and the kinetic energy during unfolding were calculated.

2. Observation of Potato Flower
A potato flower has usually five or six petals and foldable gussets are between individual petals, as shown in Figure 1(a). After unfolding fully, the petals and gussets make a plane like a pentagon or hexagon. Thus, the

Figure 1; Unfolded flower and bud of potato (a), hexagonal petal model (b) and unit right triangle (c).
plane made by the petals and gussets fully unfolded was modeled by a regular polygon with the number of side, \( N \), (corresponding to petal number) and area, \( A_0 \). As shown in Figure 1 (b), a regular polygon can be divided into \( 2N \) unit right triangles. A unit right triangle \( \triangle AOD (\angle ADO = 90') \) consists of a half petal (while area) and a half gusset (shadow area), as shown in Figure 1(c). Since the back of only petals can be seen in a bud (see Figure 1(a)), the gusset may be folded and inserted in the bud. In order to express this situation, we introduced two creases, a valley crease OB (dashed line) and a crest crease OC (solid line), into \( \triangle AOD \). These creases divide \( \triangle AOD \) into three triangles, therefore, the central angle, \( \alpha (=\angle AOD = \pi/N) \), is also divided into three partial angles, \( \beta_1 \sim \beta_3 \), as shown in Figure 1(c). From the symmetry of a regular polygon, we just consider the folding or unfolding of the unit right triangle after this.

### 3. Numerical Simulation of Flower Folding and Unfolding

#### 3.1 Coordinate System and Assumptions

For the numerical simulation, \( O-x'y'z' \) coordinates were adopted as shown in Figure 2. The plane I, \( x-z \) plane, and plane II, crossing to the plane I at \( z \)-axis with \( \alpha \), are the planes of symmetry. The origin, \( O \), is the centre of the polygon models including petals and gussets. Six polygons, regular triangle, square, pentagon, hexagon, octagon and dodecagon (i.e. \( N = 3, 4, 5, 6, 8 \) and 12), are considered here. When petals are fully unfolded, the unit triangle is on \( x-y \) plane, as shown by \( \triangle A'O'D' \) in Figure 2. By folding along the creases, its body changes to a zig-zag slender body with three partial triangles, \( \triangle AOB, \triangle BOC, \triangle COD \). The opening angle, \( \theta \), is defined as the angle between \( z \)-axis and the line OA. Before the petal starts unfolding, \( \theta = \theta_0 \). When the petal is fully unfolded, \( \theta = 90' \). The angles \( \phi \) and \( \psi \) are also defined as the angles between \( z \)-axis and the lines OB and OD, respectively. To calculate the location of points A ~ D during unfolding, we made a number of assumptions and limitations in the simulation:

1. Folding and unfolding are symmetrical about the plane I and II.
2. Even after the unit triangle is folded, its body must be in the space between plane I and plane II. This means that the lines OA and OD move on the plane I and II, respectively.
3. When the unit triangle is fully folded, the line OB is on the plane I.
4. \( \triangle AOB \) and \( \triangle COD \) are perpendicular to plane I and plane II, respectively, because of a regular polygon.
5. Three partial triangles, \( \triangle AOB, \triangle BOC, \triangle COD \) have the flat rigid bodies so that their deformation during unfolding can be neglected. The outstretched surface \( \triangle A'O'D' \) is, therefore, is identical to the folded one.

#### 3.2 Bud Volume and Dividing Angle

Figure 3 shows a regular pentagon with the area \( A_0 \) as an example of the polygonal models. When it is fully unfolded...
folded along the creases, it makes a pentagonal pyramid \(A-B_1B_2B_3B_4\) shown in Figure 3(b). We define the bud volume as the volume of the pentagonal pyramid here, although the tip triangles like \(\triangle AB_2B_3\) are left. We apply the same definition to other polygonal models. In order to determine three partial angles, \(\beta_i \sim \beta_n\) in the unit right triangle \(\triangle AOD\) (see Figure 1), the minimum bud volume condition is used. From the geometrical relations in the model fully folded, the bud volume \(V\) can be obtained by using the circumradius \(R\) of the polygon and the initial opening angle \(\theta_0\) as follows:

\[
V = \frac{N R^3 \tan \alpha}{3} \left( \frac{\sin^2 \theta_0 \cos \theta_0}{\tan^2 \alpha \sin \theta_0 + 1} \right)^3, \quad R = \frac{2 A_0}{N \sin 2\alpha} \tag{1}
\]

From equ.(1), it is found that \(V\) simply increases with the increase in \(\theta\), if \(\sin \theta < \frac{-\tan^2 \alpha + \sqrt{\tan^4 \alpha + 24}}{6}\). The most severe condition appears at \(N = 3\), which is \(\sin \theta < 0.46\). This means that it is enough to choose the smallest \(\theta_0\) to obtain the minimum \(V\). From the assumptions and the geometrical relations shown above, the following equations can be obtained:

\[
\begin{align*}
\tan \beta_1 &= \tan \alpha \sin \theta_0, & \tan \beta_3 &= \tan \alpha \sin \theta_0, & \beta_1 + \beta_2 + \beta_3 &= \alpha \\
\tan \theta_C &= \frac{\tan \theta_0}{\cos \beta_2}, & \cos \beta_2 &= \cos \theta_0 \cos \theta_C (\tan \theta_0 \tan \theta_C + 1) \tag{2}
\end{align*}
\]

where \(\phi_0\) is the initial value of \(\phi\) and \(\theta_C\) is \(\angle COE\). There are 5 equations and 6 unknowns, \(\beta_1, \beta_2, \beta_3, \theta_0, \phi_0\) and \(\theta_C\). Although the unknowns cannot be determined easily, a relation between \(\beta_3\) and \(\theta_0\) is derived as follows:

\[
\begin{align*}
A &= 1 + \tan \alpha \tan \beta_3, & B &= \frac{\tan \beta_3 (1 + 2 \tan^2 \alpha) - \tan^3 \alpha}{\tan \alpha}, & C &= \frac{\left(1 + \tan^2 \alpha\right)(\tan^2 \alpha - \tan^2 \beta)}{\tan \alpha} \tag{3}
\end{align*}
\]

Since the first equation in equ.(3) is a quadratic equation with respect to \(\tan \theta_0\), we can solve it easily by using the quadratic formula. Figure 4 shows the change of five angles with \(\beta_3\) in the case of \(N = 5\). As seen in Figure 2, \(\beta_i\) must be larger than \(\beta_1\), because \(\triangle BOC\) is inside \(\triangle AOB\). In the region up to the limit of \(\beta_3 = 10.53\degree\), the curve of \(\theta_0\) becomes convex upward. The minimum value of \(\theta_0\) appears on the limit line and \(\theta_0 = 18.12\degree\). From this, we can obtain \(\beta_1 = \beta_2 = 12.73\degree\). Figure 5 shows the change of \(\theta_0\) of the models with various \(N\) up to the individual limit lines. All curves are basically convex upward, however, the points (indicated by circles) at which the minimum \(\theta_0\) appears is divided into two groups. When \(N \leq 5\), the minimum \(\theta_0\) appears on the limit line, i.e. \(\beta_3 = 0\). The other group is on the line of \(\beta_3 = 0\). In this case, the unit triangle \(\triangle AOD\) is divided into two triangles. This means that the dividing manner of the unit triangle depends on petal number \(N\) under the

![Figure 4: Change of various angles with \(\beta_3\),](#)

![Figure 5: Change of \(\theta_0\) of different models.](#)
maximum bud volume condition. When $N \geq 6$, dual portioning is more suitable than division into three. For the vector analysis shown below, the numerical values of $\beta_1 \sim \beta_3$ determined here were used.

### 3.3 Energy during Unfolding

By using vector analysis, we can obtain the location of the partial triangles, $\triangle AOB$, $\triangle BOC$ and $\triangle COD$, see Figure 2. These triangles revolve about the axes on $xy$-plane during unfolding. Thus, if we use suitable moment of inertias of the partial triangles ($I_1$, $I_2$ and $I_3$, respectively), the kinetic energies of the triangles can be calculated. Since it is not enough space here, the details of the calculation are omitted. Figure 6 shows how the kinetic energies of $\triangle BOC$ and $\triangle COD$ change when $\triangle AOB$ is unfolded with a constant angular velocity $\omega$. The horizontal axis is the non-dimensional time $T^*$, i.e. $T^* = 0$ and 1 mean fully folded and unfolded, respectively. The petal model used here is a pentagonal model, i.e. $N = 5$. Since $\triangle BOC$ connects to $\triangle AOB$ through a crease $OB$, the energy of $\triangle BOC$ ($I_2 \omega^2$ in Figure 6) is similar to that of $\triangle AOB$, i.e. almost constant. However, $I_3 \omega^2$, which is the energy of $\triangle COD$, gradually increases because the effect of the movement of $\triangle AOB$ is reduced by the rotation of $\triangle BOC$ around OB. The area under the curves gives the kinetic energy used for fully unfolding. By adding the kinetic energy of individual triangles, the total energy of a model for fully unfolding can be calculated. In the calculation, the total unfolding time was kept to be constant. Figure 7 shows the total energy ratio $W^*$ to the energy of the model with $N = 5$. The bud volume ratio $V^*$ calculated by eqn.(1) is also shown. From this figure, the total energy of the model with $N = 5$ or 6 is smaller than that of other models. On the contrary, the bud volume of the model with $N = 5$ is largest. Potato flower has usually 5 or 6 petals, so it can be said that potato adopts the most effective number of petal for unfolding energy, although the bud is not compact.

### 4. Concluding Remark

In order to investigate the folding and unfolding manners of a potato flower with triangle gussets, a number of regular polygon petal models including valley and crest creases were considered and the bud volume and kinetic energy during unfolding was examined by using geometrical relations and vector analysis. It was found that the bud volume depends on the petal number, $N$, and the bud with $N = 5$ has the largest volume. On the total kinetic energy during unfolding, however, the model with $N = 5$ or 6 became smaller than other models. Therefore, potato flower seems to adopt the best petal number for the unfolding energy.

Figure 6; Kinetic energy of partial triangles. Figure 7; Total energy ratio and bud volume ratio.

### References

Bifurcation analysis for the multi-folding structures

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Abstract

We review the interesting bifurcation problem of a folding geometrical non-linear truss allowing for large deformation using both analytical and numerical approaches. An experimental approach by Holnicki et al.[1] showed an active shock-absorber based on the truss system. From these approaches the authors developed the concept of a pantograph truss to model the multi-folding and/or deployable structures. We have successfully obtained analytical numerical results, allowing for geometric nonlinearity, to trace control experiments of the basic pantograph system. In this paper, we investigate the mechanism of several folding patterns for a pantographic system with reference to the folding and/or deployable truss structures using bifurcation theory in the general classification of nonlinear mechanics[2] and numerical work to trace the folding patterns of the system.

1 Introduction

Recently, Holnicki et al.[1] designed an active shock-absorber system from which one of the authors suggested the concept of a pantograph truss as part of the Multi-Folding Method (MFM). We have successfully carried out control experiments on a fundamental framework. To develop this new structure, we must estimate the allowance vanishing energy on the multi-folding method or examine the defense of an important structure against impact. There are several different folding patterns on system as a means of controlling the stress of a member and using the folding characteristics, to actively regulate it. In this paper, based on the fundamental concept of MFM, we describe the correspondence of the numeric verification and theoretical method according to the difference with the gaps of asymmetric folds that are folded in fact. Our main objective is to understand the adequate behavior characteristics using the theory of elastic stability and to discover the elasticity of various folding patterns, establishing how to fold, and how to create theoretical interpretations through control experiments. To observe a large-scale deformation of the dynamic behavior, we analyze the equilibrium equation with only geometric nonlinearity for a basic truss model. Based on the concept of the multi-folding system under dynamic load, we have solved the numerical results and theoretical approach to compare with the folding process of the experiment described by Holnicki et al.. The numerical methods for this folding are taken by both of the static and dynamic nonlinear analysis. In particular, the dynamic analysis is used kinematic equation described by the ordinary differential equation. We have found that it is similar to the folding process for the multi-folding system and there is complex equilibrium curves during unstable state when it is called the post-buckling snapthrough behavior[2, 3, 4], then. This paper is presented some interesting mechanism and some results through the multi-folding simulation to develop a shock absorber structure.
2 Theoretical approach for folding truss

It is well-known that the snapthrough behavior for the two-bar truss model of von Mises truss shown in Fig. 1(a) is one of structural critical problems. This structural system shown in Fig. 1(b), has right-left symmetry. Hence in this paper the theoretical bifurcation analysis, is limited to considering a collapse with symmetric deformation. By allowing for symmetric models only, we can therefore consider the half model shown in Fig. 1(c) for the theoretical analysis.

Now, let us consider a theoretical estimation for the multi-folding truss model. We assume a periodic height for each layer of $h_i = \gamma_i L$ and where the width $L$ of the truss is fixed. Therefore, an initial length for each bar in the geometry of the figure is expressed as

$$\ell_i = \sqrt{L^2 + h_i^2} = L \sqrt{1 + \gamma_i^2}, \quad i = 1, \ldots, 3.$$  

The deformed length of each bar denoted as $\hat{\ell}_i$, is a function of the height and the nodal displacement variables

$$\hat{\ell}_i = L \sqrt{1 + (\gamma_i - Q_i + Q_{i+1})^2}, \quad i = 1, \ldots, 3$$

where $\gamma_i = h_i / L > 0$, $Q_i = v_i / L$, $(i = 1, \ldots, 3)$, $Q_{i+1} = Q_4 = 0$.

Using Green’s expression, we apply the strain in each elastic bar of the multi-folding truss as

$$\varepsilon_i = \frac{1}{2} \left\{ \left( \frac{\hat{\ell}_i}{\ell_i} \right)^2 - 1 \right\}, \quad \text{for } i = 1, \ldots, 3. \quad (1)$$

The total potential energy of a half of the system is given by

$$V = \sum_{i=1}^{3} \frac{EA_i \ell_i}{2} (\varepsilon_i)^2 - f^* Q_1 L \quad (2)$$

$$= \sum_{i=1}^{3} \frac{EA_i L \sqrt{1 + \gamma_i^2}}{2} \left\{ \frac{1 + (\gamma_i - Q_i + Q_{i+1})^2}{1 + \gamma_i^2} - 1 \right\}^2 - \frac{f}{2} Q_1 L \quad (3)$$

in where, $f^*$ is a half of load $f^* = f/2$, $Q_{i+1} = Q_4 = 0$. In the case of the same height for each layer, $\gamma = \gamma_i$ and the
same stiffness for each mamber, \( EA = EA_i \). Then, the total potential energy can be described as

\[
\mathcal{V} = \frac{\beta L}{8} \sum_{i=1}^{3} (Q_i - f_{i+1})^2 \left( (Q_i - Q_{i+1}) - 2\gamma \right)^2 - \frac{f}{2} Q_1 L
\]

(4)

\[
= \Lambda Q_1 + A_{11} Q_1^2 + A_{12} Q_2 Q_1 + A_{22} Q_2^2 + A_{23} Q_3 Q_2 + A_{33} Q_3^2
\]

\[
+ A_{111} Q_1^4 + A_{112} Q_1^2 Q_2 + A_{122} Q_2^4 + A_{222} Q_2^2 Q_3 + A_{233} Q_3 Q_2^2 + A_{333} Q_3^4
\]

\[
+ A_{1111} Q_1^4 + A_{1112} Q_1^2 Q_2 + A_{1222} Q_2^4 + A_{2222} Q_2^2 Q_3 + A_{2223} Q_2^2 Q_3^2 + A_{2333} Q_3^4
\]

\[
+ A_{3333} Q_3^4
\]

(5)

in where,

\[
\Lambda = \frac{f L}{2}, \quad A_{11} = \frac{\beta L}{2} \gamma^2, \quad A_{12} = A_{23} = -\beta L \gamma^2, \quad A_{22} = A_{33} = \beta L \gamma^2
\]

\[
A_{111} = -\frac{\beta L}{2} \gamma, \quad A_{112} = A_{223} = \frac{3 \beta L}{2} \gamma, \quad A_{122} = A_{233} = -\frac{3 \beta L}{2} \gamma, \quad A_{222} = A_{333} = 0
\]

\[
A_{1111} = \frac{\beta L}{8}, \quad A_{1112} = A_{222} = A_{2223} = A_{2333} = -\frac{\beta L}{2}, \quad A_{1122} = A_{2233} = \frac{3 \beta L}{4}, \quad A_{2222} = A_{3333} = \frac{\beta L}{4}
\]

where the stiffness parameter \( \beta(\gamma) = EA/(1 + \gamma^2)^{3/2} \), which is a function of \( \gamma \). From Eq. (5), we can obtain the equilibrium equations based on the principal of minimum energy in the following:

\[
F_i(\cdots, Q_i, \cdots) \equiv \frac{\partial \mathcal{V}}{\partial Q_i} = \frac{\partial \mathcal{V}}{\partial Q_i} \mathcal{L} = 0 .
\]

(6)

Hence, for the 1st, \( i \)-th and \( n \)-th equilibrium equations chained

\[
F_1 = -\frac{f}{2} \gamma^2 Q_1 - \gamma^2 Q_2 - \frac{3}{2} \gamma Q_2^2 + 3 \gamma Q_1 Q_2 - \frac{3}{2} \gamma Q_2^2 + \frac{Q_1^2}{2} - \frac{3 Q_1 Q_2^2}{2} - \frac{Q_3^2}{2} = 0
\]

(7)

\[
F_2 = -\gamma^2 Q_1 + 2 \gamma^2 Q_2 - \gamma^2 Q_3 + \frac{3}{2} \gamma Q_1 Q_2 + 3 \gamma Q_2 Q_3 - \frac{3}{2} \gamma Q_3^2
\]

\[
- \frac{Q_1^2}{2} + \frac{3}{2} Q_1^2 Q_2 - \frac{3}{2} Q_1 Q_2^2 + Q_1^2 - \frac{3}{2} Q_2^2 Q_3 + \frac{3}{2} Q_2 Q_3^2 - \frac{Q_3^2}{2} = 0
\]

(8)

\[
F_3 = -\gamma^2 Q_2 + 2 \gamma^2 Q_3 + \frac{3}{2} \gamma Q_2^2 - 3 \gamma Q_2 Q_3 - \frac{1}{2} Q_2^3 + \frac{3}{2} Q_2^2 Q_3 - \frac{3}{2} Q_2 Q_3^2 + Q_3^3 = 0
\]

(9)

### 3 Bifurcation analysis for simple model

Let’s consider solving equilibrium paths for the basic model shown in Fig. 1(c) based on the nonlinear bifurcation theory. The height of each layer is identical, \( \gamma = \gamma_i \). In order to solve for the variable \( Q_i \), we use the implicit function from Eq.(9), which shows the solutions in the following:

\[
Q_3 = \mathcal{F}_3(Q_2) \begin{cases} 
Q_2^2/2 & \text{for primary path,} \\
\frac{1}{4} \left( Q_2 \pm \sqrt{3Q_2^2 + 12\gamma Q_2 - 8\gamma^2} \right) & \text{for bif. path},
\end{cases}
\]

(10)

\[
Q_2 = \mathcal{F}_2(Q_1) \begin{cases} 
2Q_1/3 & \text{for primary path,} \\
-\gamma + Q_1 + \frac{\sqrt{3}}{2} \sqrt{-Q_1^2 + 6\gamma Q_1 - 5\gamma^2} & \text{for bif. paths}.
\end{cases}
\]

(11)

Since we have the expressions \( Q_3 = \mathcal{F}_3(Q_2) \) and \( Q_2 = \mathcal{F}_2(Q_1) \), we can now express the equilibrium equations for the primary and bifurcation paths: in terms of variable \( Q_1 \) in the following way:

\[
f_{\text{pri.}} = \beta \frac{Q_1}{3} \left( \frac{Q_1}{3} - \gamma \right) \left( \frac{Q_1}{3} - 2\gamma \right), \quad \text{for primary path},
\]

(12)

\[
f_{\text{bif.}} = \pm \frac{\beta}{3\sqrt{3}} \sqrt{- (Q_1 - \gamma)(Q_1 - 5\gamma)(Q_1 - 2\gamma)(Q_1 - 4\gamma)}, \quad \text{for bif. path}
\]

(13)

The primary equilibrium path and the bifurcation paths are obtained by the above equations, and these paths are shown in Fig. 2(a). Figure (b) shows the numerical equilibrium paths as the result based on incremental nonlinear computing method.
4 Conclusion

We found the folding mechanism of dynamic nonlinear numerical process for pantographic trusses system as well as the folding experiment based on the Multi-Folding-Method. It is successful to simulate the nonlinear equilibrium paths for smart passive structures and it is useful to get the capacity of the maximum impact energy and/or strain energy in the system. Although this model is very simple system, the analysis of this behavior near bifurcation points is more sensible to solve the equilibrium paths.

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References


Deployment Schemes for 2-D Space Apertures and Mapping for Bio-Inspired Design

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Abstract
This presentation first provides a review of deployment schemes and architectures for 2-D space apertures. Representative examples, flown or proposed, are shown for the various categories. Deployment schemes in biological systems are then discussed. Finally, opportunities and challenges in mapping between biological systems and the engineering of future deployment schemes are discussed.

1. Introduction
Demand for deployed aperture volume in space almost always exceeds spacecraft stowage volume. This is true whether for a large space telescope or a power array for a micro-satellite. At the extreme, solar sails for in-space propulsion may need to be 150 m or more in characteristic length, while active space-based radar has been studied at the 300 m length scale. Given the stringent launch volume and mass constraints, deployment and/or erection schemes must be used to transition from launch stowed volume to in-service size.

Deployment schemes for one-dimensional (1-D) space structures (whose length in one spatial dimension is much larger than in the other two dimensions) include inflation, stored strain energy, shape memory effect, and mechanical-deployable. Murphey [1] has provided a significant study of 1-D space truss architectures.

2. Two dimensional deployed space architectures
Deployment schemes for two-dimensional (2-D) space structures (whose lengths in two spatial dimensions are much larger than in the third) are more complicated, including not only the 1-D schemes but also structural configurations such as annular (perimeter), grid (branching) and cruciform (radial). Furthermore, methods for developing required deployed structural stiffness include intrinsic (material and geometry), stress, and coupled field schemes. As a first order taxonomy, deployed architectures for 2-D space structures (e.g., antennas, solar power arrays, and sunshields) could be organized in an array shown in Figure 1.

Shown in Figure 1 are examples of antennas such as the Rigid Rib Antenna (RRA) flown on the NASA Galileo mission, the Wrap Rib Antenna (WRA) flown on the Applications Technology Satellite (ATS), the Tension Truss (TT) antenna flown on the Highly Advanced Laboratory for Communications and Astronomy (HALCA), the Inflatable Antenna Experiment (IAE), the Inflatable Space Rigidized Reflector (ISRS), and the AstroMesh antenna (Tibert [2], Freeland et al. [3]). Solar power arrays are shown such as the Ultraflex (Piszczor [4]), the Teledesic and the Inflatable Torus Solar Array (ITSAT) (Grahne and Simburger [5]), as are solar sails such as the Heliogyro, and designs by ATK-ABLE and L’Garde (Garbe et al. [6]). (For completeness, the possible use of electrostatic or magnetic fields for developing deployed stiffness is also shown in the figure; however, no applications are known that fit any of the three deployed architectures.)
Deployment in biological structures

One important source of design inspiration is the biological world. Nature has elegantly crafted highly efficient solutions to many of the same or similar problems faced by engineers. Deployment schemes have evolved in several biological systems, including insect wings, insect proboscis, plant leaves and flower petals, and carnivorous plants such as the Venus Fly Trap (Vincent [7], Enos [8]).

Challenges in mapping for bio-inspired engineering

Bio-inspired engineering (BiE) uses biological structures and functions to either inspire or directly mimic existing biological designs in order to solve specific technical problems. Modern bio-inspired engineering has had a fruitful, if limited, history. BiE rests on the premise that evolutionary processes have produced biological structures and functions that are highly optimized solutions to diverse environmental pressures.

There are several impediments, however, to accessing biological solutions for engineering purposes, not the least of which is the enormous size of the biological data pool. Moreover, solutions exist on many scales, from molecular to organism to complex ecological systems. Most bio-inspired design today relies on an approach inverse to the normal engineering design process: a biological structure-function relation is understood and then a search is conducted for an engineering problem that may be solved with that knowledge. The usual forward approach (and the one most useful to technological advance) – an engineering problem in search of a biological solution paradigm – is today virtually unavailable (Ball [9]).

In the particular case of deployment, the mapping is further complicated since spacecraft engineers need solutions that work in zero-G, while many biological solutions exist in a gravity environment. In any case, the engineering problem must be well-posed for there to be any hope of finding design guidance from biology.

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References


Microstructure of foldable membrane for gossamer spacecrafts

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Abstract

The folding mechanics for two-dimensional deployable membrane are discussed to realize the spinning solar sail for gossamer spacecrafts. The structural and dynamic properties of the spinning solar sail are determined by the folding pattern and deployment mechanisms. Several fold patterns are introduced to examine the properties of the deployable and retraction characteristics in this paper under the view point of the microstructure of folds to examine the deployment and retraction properties. Finally, foldability and deployability are discussed.

1. Introduction

The folding mechanics for two-dimensional deployable membrane are discussed to realize the spinning solar sail for gossamer spacecrafts. The fold and deployment of large-scale membrane sails are significant to realize the solar sails. In NASA or Europe, extendible masts or deployable booms are adopted to deploy the membrane. In Japan, the spinning solar sail is adopted, which uses the centrifugal force to deploy the membrane. Since the membrane is large and it has many creases, we need to develop manufacture and retraction mechanisms. Also, we need to retract without large wrinkles to retract efficiently. The structural and dynamic properties of the spinning solar sail are determined by the folding pattern and deployment mechanisms.

2. Fold of membrane

The basic fold of membrane is fanfold as shown in Fig.1. However, as the fold of Fig.1-a cannot perfectly fold the membrane, the membrane requested to divided in several sections as shown in Fig.1-b.

The realize perfect fold of the membrane, spiral folds have been proposed. Fig.2-a is the folding apparatus proposed by Lanford. The basic feature is the wrapping fold of membrane around the circular centre hub with tensile forces around the cables. By extending the concept, Guest proposed another type of wrapping fold around the polygonal centre hub (Fig.2-b). Because the fold is consists of straight line folds, the retraction is pre-folded retraction. General spiral folds are discussed by Nojima as shown in Fig. 2-c. Because the spiral folds as shown in Fig.2 have z-fold properties, to manufacture the fold is relatively smooth, however, as the
retracted configuration is determined by the pitch of the spiral fold only, an arbitrary configuration is unable to construct.

Fig.2-a Spiral fold (Lanford, 1961)       Fig.2-b Spiral fold (Guest, 1992)

Fig.2-c Spiral fold (Nojima, 2003)

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The spiral fold based on the double corrugation fold is proposed Scheel (Fig.3). The fold is consists of the circular centre hub and double corrugation fold based on straight line. Extending the fold, the rotationally skew fold is proposed by applying the polygonal centre hub as shown in Fig.4. By considering the effects of physical thickness, the pseudo spiral fold is proposed by Natori. The double corrugation fold is possible to determine the retraction configuration more freely by the skew angle and fold pitch than the traditional spiral fold in Fig.2, although the fold is complicated.
Another way to design the retraction configuration is to introduce several types of fold patterns for folding the membrane. Figure 5 is the mixed spiral fold based on the traditional spiral fold indicated in Fig.2 and the simple z-fold. Because the mixed spiral fold is the single corrugation fold except of the centre area around the polygonal centre hub, to fold the membrane is simpler than the double corrugation folds. Also, the folding mechanisms are easily applicable to retract a very large membrane by extending the concept of the folding apparatus by Lanford. Fig.5 indicates the experimental photos for folding the membrane with the mixed fold.
Fig. 5 - Retraction experiments

References


Natural twist buckling in shells: from the hawkmoth’s bellows to the deployable Kresling-pattern and cylindrical Miura-ori

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Abstract
The analogy between the deployable folding pattern in a hawkmoth’s bellows (Wasserthal [10]) and the folding pattern of a cylindrical thin-walled shell, that is spontaneously buckling under torsional load, named “Kresling-pattern” (Poepppe in Stewart [9], Kresling [4,5], Hunt and Ario [2]), leads to a novel interpretation of buckling not as failure, but as a model strategy for designing and manufacturing “natural” deployable folding patterns with coupled mechanisms. Patterns of this type would operate nearly stress-free and could work reliably at both micro- and macro-level. The hawkmoth’s deployment pattern shows also some characteristics of the classic “Miura-ori” (Miura [7]), a pattern designed to deploy to 2D-arrays (maps, solar sails and other space applications…), found also in the leaves of deciduous trees (Kobayashi et al. [3]). Modified angle values of a series of fold lines would help deploy the structure in the 3-dimensional geometrical envelope of a cylinder.

Figure 1. Biological deployable patterns – generally microscopic – are based upon folded membranes with a specific geometry, that allows repeated deployment and refolding. [A/] The bellows intima of the abdominal air sac in the hawkmoth Acherontia atropos (Wasserthal [10], modified) are capable of contracting and expanding. The simultaneous two-directional expanding movement is combined with bending. The hawkmoth’s pattern is analogous to the geometry and coupled mechanism of two technical deployment patterns: [B] the spontaneous buckled “Kresling-pattern” and [C] the cylindrical “Miura-ori”, a geometrical and mechanical variant of the classic “Miura-ori” deployment pattern.
1. Introduction
During growth in insects, a series of fold patterns develops, which become highly functional deployment mechanisms at the last stage of development, during the metamorphosis form the larva to the adult. A folding pattern found in the air sac of the adult hawkmoth *Acherontia atropos* [10], a night-active insect, allows nearly stress-free pumping motion, by which the respiratory tracheal system is ventilated.

2. Experiments with natural twist buckling: “Kresling-pattern”
By experiments with thin-walled cylinders in paper or in polypropylene, it was discovered that a “natural” pattern – analogous to that of the hawkmoth – arises under torsional load. In the experiments a thin-walled sheet is wrapped around two coaxial mandrels, leaving a gap. When the mandrels are twisted, highly regular self-organized folding pattern appears, across the gap, formed by inclined and elongated parallelograms (mountain-folds), divided on their long diagonal by a valley-fold. For any given gap size there is a left-hand or right-hand twist-buckling pattern. An engineering study of the mechanism by Hunt and Ario [2] describes the twist-buckling pattern considering its accommodation to fold to a flat diaphragm. The obliqueness of the two series of fold lines changes while the collapsing cylindrical belt accommodates to its final folded position (Figure 2A). The triangulated “truss” then forms a 2-dimensional diaphragm perpendicular to the axis of the cylinder. Experiments done by the author with a great number of student groups showed, that at least two different buckling modes are responsible for this highly regular and reproducible pattern. The modes depend on respective (i.e. different!) “ideal” diameter-to-gap-size ratios, by which physical reactions to torque in the cylinder wall harmonise with the geometry of the final polygonal diaphragms (Figure 2B).

![Figure 2. Twist-buckling experiment: a thin-walled sheet of paper is wrapped around two coaxial mandrels forming a gap. When twisted, a highly regular self-organized folding pattern appears across the gap. [A] During buckling the angles $\alpha$ and $\beta$ adapt themselves to their final value in the folded diaphragm (Hunt and Ario [2], modified).[B] The mode depends on the buckle-wave that crosses either one dimple (“mode 1”) or two dimples (“mode 2”). The two basic parameters unchanged – paper cylinder $\phi$ = 36 mm; standard paper of 80 grs/m² – but a variable gap height (h), one obtains following results: at “mode 1” and $h =$ 6.5 mm, the cylindrical wall folds spontaneously to $n =$ 14 (fourteen-sided polygon); at “mode 2” and $h =$ 13 mm, the wall buckles and folds to $n =$ 12 (twelve-sided polygon).](image)
3. Natural twist buckling: "ideal" ratios, buckling modes and the geometry of resulting diaphragms

Twist buckling is a complex process with distinct stages, changing geometry and changing mechanical behaviour. 1) Even as the multiple buckled dimples begin to form across the gap, the cylinder wall already presents the definitive number of dimples. 2) Adaptation of folding angles, as described by Hunt and Ario [2], who observed mainly the positions of the outer mountain-folds. However, from a structural point of view, the inner oblique fold lines are, presumably, the most important elements of the system, as they are constantly under tensile force in one direction and give away to perpendicular compression loads (they also adapt their angle values). 3) Conical singularities (as described by Mahadevan [6] for developable surfaces) appear at the intersections of mountain- and valley-folds. 4) Final elongation of oblique folds while forming the diaphragm. (Eventual formation of “S”-curving of the outer folds at this stage is probably an artefact).

The question arises whether there is a precise ratio of diameter-to-gap size that would produce the most regular buckling pattern. Obviously, very small gaps result in a higher number of dimples than larger ones. But paradoxically, with the same parameters for the diameter of the mandrels and paper quality, but different gap sizes, one obtains a nearly identical number of dimples. By experiments performed by great number of student groups we tempted to elucidate this paradox and found ratio-specific modes of twist buckling. We observed the trajectory of a singularity on the ridge towards its “partner” singularity on the other ridge. In “mode 1” the trajectory crosses one dimple (buckled unit parallelogram) and crosses its valley-fold at approximately 90°. In “mode 2” the trajectory crosses not one but two dimples and is at approximately 90° to another line, in this case to the (virtual) longest diagonal of two neighbouring units (Figure 2 B). The observations allowed us to define two conditions for both buckling modes under which they produce very regular patterns:
- partner singularities of the two rippled ridges come to overlap,
- the resulting polygonal 2-D diaphragm is of the highest possible degree of symmetry.

4. Designing foldable cylinders with “Kresling-pattern” and the derived pattern of cylindrical “Miura-ori”

When reproduced artificially, cylinders with different “Kresling-patterns” can be designed. A twist-buckled belt may operate alone or be multiple, either in left-hand or right-hand version, or be inclined alternately to left and right. Twisted belts can be combined with smooth cylindrical parts and/or be “mode 1” (small folded belt) or “mode 2” (large belt) (Figure 3A). Finally folded parts can be used as “stringers” (stiffeners) that guide deployment or refolding. The most interesting property of these designs is their stress-reduced folding mechanism with a “natural” pattern of oblique compression and tension lines, crossing at right angles. Designs may profit from the possibility of a local “pop-up effect”, when controlled twist is applied to single belts (Figure 3). A derived pattern similar to a “Miura-ori” is obtained, when the triangles of at least two adjacent belts are truncated and the truncated parts joined together, by alternating left and right. The classic “Miura-ori” deploys on plane. In the cylindrical “Miura-ori” version where only the obliqueness of a series of valley-folds is modified, the whole folding pattern will move inside the geometry of a cylindrical envelope. (Figure 3 C/D).

5. Discussion and conclusion

Neither design, the “Kresling-pattern”, nor the cylindrical “Miura-ori” is new. Inflatable folded tubes by Guest and Pellegrino [1], and deployable structures for space applications by Sogame and Furuya [8] propose similar features – and beyond the designs in this paper, there are other interesting mechanisms and symmetries. But the results of “natural” buckling modes, presented here, could inspire some other designs, manufacturing processes and applications. In the fields of engineering, packaging, architecture at macro- or mega-scale, but also at micro-scale for simple and reliable mechanisms for surgical uses, micro-robotics and similar applications, finally for nano-scale materials and deployable devices with negative Poisson’s ratio and with “naturally” forming patterns – a guarantee for a strict minimum of energy expenditure when operating – could present innovating design strategies and open new perspectives for research topics.

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Figure 3. [A] Designed deployment mechanisms using “Kresling-patterns” and their modes. Depending on the orientation of the folds, axial movement is translated as “drilling”, “pumping”, “stiffening”, “squeezing” or “grasping”. [B] Cylindrical “Miura-ori”, on the geometrical basis of “Kresling-patterns” is foldable and extensible as a whole by simple axial motion. [C] The cylindrical “Miura-ori” may be combined with “Kresling-patterns” and their mechanisms. The combination allows the transition to smooth cylinders or stiffening integrated stringers.

References


