Essays in Firm Heterogeneity and Productivity

A Dissertation Presented to the Faculty of the Graduate School of Cornell University in

Accordance with the Requirements of the Degree of Doctor of Philosophy

by

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May 2022

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Cornell University 2022

Abstract

"Essays on Firm Heterogeneity and Productivity" explores the connections between firm heterogeneity, financial markets, firms' outcomes, and economic aggregates. The thesis aims to further understand the linkages between investment decisions of heterogeneous firms, the financial intermediaries' lending behavior, and the economy's aggregate output. The thesis consists of three essays organized in chapters. Chapter I, "Firm heterogeneity and misallocation" investigates the aggregate consequences of asymmetric information about firms' expected returns in financial markets. The main result is that if lenders cannot distinguish between firms with high and low potential returns, they offer them more similar funding conditions than otherwise, generating inefficient firm sizes and output losses. Chapter II,

“Liability protection and firm growth,” investigates the role of limited liability as insurance for entrepreneurs. The main result of the essay is that limited liability fosters entrepreneurial activity because it incentivizes risk-taking in two dimensions, firm size and the adoption of riskier and more profitable technologies. Quantitatively, this accounts for sizable output gains. Chapter III, “Aggregate effects of wealth redistribution on technology adoption and productivity,” shows how carefully targeted wealth redistribution can generate sizable output gains. The key is to redistribute towards entrepreneurs that might adopt more profitable technologies if they were more affluent. The primary mechanism is that increasing the wealth of entrepreneurs makes them more likely to run risks, adopt more profitable technologies and set up larger firms.

Biographical Sketch

Raul Alejandro Morales Lema is originally from Santiago, Chile. He obtained a B.A. and M.A. in Economics from the University of Chile. Before enrolling at Cornell, he worked as an economic and financial analyst at the Central Bank of Chile (2014-2016) and as an economic advisor to the Chilean Ministry of Finance (2013-2014). During his time at Cornell University, he has specialized in macroeconomics and firm dynamics, conducting research on the different causes and consequences of firms' performance. After graduation, he will join the World Bank in Washington, D.C.

Acknowledgements

I would like to thank my academic advisor, Julieta Caunedo, for her dedication, encouragement and support during the six years of my Ph.D. She always supported my research agenda (often showing more enthusiasm than myself), and constantly showed her disposition to help me when I struggled. In addition, Julieta introduced me to firm dynamics, a topic of which I knew nothing about, and quickly became my passion. I also want to thank my committee members Kristoffer Nimark and Mathieu Taschereau-Dumouchel, for their support, comments, and suggestions. Without their insights and personal support, my research would not have been the same. I want to thank Penny and Seth Sanders. This Ph.D. would not conclude in such a happy way without all of their work and help during the Job Market Process. I will be forever grateful to the faculty, students, and staff in the Department of Economics at Cornell University, who have been invaluable contributors to my research.

Working in my dissertation, far from my home country and family, would have been very dull without my friends - Nikolai Boboshko, Vitor Costa and Zhou Fan. I will always cherish those road trips for concerts with Nikolai, the politics and soccer debates with Zhou, and the contemplative talks with Vitor. I am happy to take the memories and experiences that we have together with me. Furthermore, I will be forever grateful to my fiancee, Valentina Ardiles, whose love, companionship, and unconditional support made this process possible. You have helped me pursue my dreams like nobody else and kept the faith up for both of us when I could not. Finally, I want to thank my family. My siblings have been a constant source of comfort and support, and my parents always encouraged me to "break the mold" and dream big. Without them, I would have never started this Ph.D.

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Chapter 1: **Firm Heterogeneity and Misallocation**

1 Introduction

Recent literature on firm dynamics shows that firms' performance can be explained to a large extent by permanent characteristics of firms that are already fixed when they start operating (e.g. entrepreneurial ability). This ex-ante and permanent heterogeneity implies that firms are created with different expected profits, size, and growth potential (Sterk et al, 2021). In this paper, I study the consequences of this result for the financing contracts offered to entrepreneurs. However necessary, these potential returns are challenging to be observed by financial intermediaries. At the same time, entrepreneurs could have incentives to misrepresent them to obtain better credit conditions than otherwise. This interaction complicates allocating credit and can generate inefficiencies with sizable aggregate consequences. In addition, once firms are already in the market, they suffer different series of shocks (e.g. demand shocks) that put them in different trajectories (Eslava and Haltiwanger, 2020), adding a layer of complexity to the lenders' problem.

This paper aims to analyze and evaluate quantitatively the effect of asymmetric information on firms' ex-ante heterogeneity on their financing and size in a general equilibrium setting. The main contribution is studying the relevance of permanent heterogeneity for financial contracts, which the canonical studies on the matter have not considered. To do this, I propose an economy where firms' productivity is heterogeneous along these two dimensions: they have different permanent productivity and suffer different productivity (transitory) shocks in every period. Crucially, investors (or lenders) cannot observe either: both are private information to the entrepreneur. The intuition for both mechanisms is clear. First, unobservable heterogeneous permanent productivity speaks to the problem for financial intermediaries of distinguishing between good and bad projects. Anecdotal evidence, such as the cases of WeWork and LeSports (both firms had billion dollars valuations and failed spectacularly), shows that this problem is inherent to the relationship of startups and financial institutions. From a more academic perspective, it has been present in the finance literature at least since the first generation of credit rationing models (Jaffee and Russell, 1976; Stiglitz and Weiss, 1981), and recent papers on firms dynamics show its actual relevance for firms' outcomes (Sterk et al, 2021). Second, transitory heterogeneity, which has been already studied in the context of asymmetric information and financial frictions, points to the fact that firms might divert resources from paying back to their investors or not exert the optimal amount of effort to make the project succeed (Cole, Greenwood and Sanchez, 2016. Verani, 2018).

In the first part of the paper I introduce the model. I take the canonical model of Clementi and Hopenhayn (2006), where firms are subject to transitory revenue shocks that are private information, but where the expected revenue (conditional on inputs) is homogeneous for all firms. I modify it to include two types of firm: high and low productivity (expected revenue), which is permanent and private information to the entrepreneur. This way, the lenders are uncertain about the ex-ante expected revenue and the period by period realizations. Consequently, they do not know a priori how much funding allocate to each firm, so they must rely on a truth-telling mechanism. Then I solve for the optimal contract in this economy and show that this mechanism benefits less productive firms and harms the most productive ones: if the lenders cannot ex ante distinguish between them, they provides both of them a more similar amount of funding than they would otherwise. This generates inefficient firm sizes: high productivity firms are born smaller and take longer to achieve their inefficient size, while the opposite is true for low productivity firms. Interestingly, allowing lenders to eventually learn the type of each entrepreneur does not change this results.

Then I use Chilean data to show that the main mechanism proposed in the paper is plausible: I show that in economic sectors where there is larger uncertainty about the permanent component of productivity, firms with higher productivity face tougher financial conditions, which is the main prediction of the model. For this I use a nationally representative survey of firms that contains rich information about firms characteristics and financial statements.

After that I calibrate this model for the Chilean Economy. To compute the permanent component of firms' revenue, I use a simple factor model to extract firms' idiosyncratic component from their TFP. I quantitatively evaluate the model and find that the implied output losses add up to 24.6% of GDP and wages are 15.9% lower compared to a frictionless benchmark. As comparison, the economy with only transitory differences, generates an output loss of 15.9% and a reduction in wages of 10.2%. Thus, asymmetric information on the permanent component of productivity generate losses of 8.7%. In terms of the mechanism, low productivity firms more than double their size compared to the unconstrained level: they are benefited from the depressed wages and the asymmetric information. On the other side, more productive firms see their sizes reduced in 24%. Finally, I evaluate the effects of a financial that increases the reliability of financial information, allowing lenders to ex-post observe firms' revenue but not to distinguish between entrepreneurs with different permanent productivity. I find that it can undo most of the losses from this friction, achieving the frictionless level of output. This suggest ample opportunities for developing economies of closing the gap with the developed ones engaging in these sort of reforms.

2 Literature Review

This paper builds on the literature about financial frictions. In general, this points to imperfections in the credit markets as one of the factors contributing to income and productivity differences between develop and developing economies. Papers such as Bento and Restuccia (2020) show that in a cross country comparison, indicators of more severe financial frictions are associated with lower output and smaller firms. From a purely empirical perspective, Schmalz, Sraer and Thesmar (2017) and Ek and Wu (2018) show that shocks that relieve the financial constraints of entrepreneurs increase firm growth. However the aggregate consequences of these financial constraints are still part of an ongoing debate. Different papers have heterogeneous findings in terms of to which extent financial frictions matter, and which of them are relevant in aggregate terms. Midrigan and Xu (2014) show that the existence of collateral constraints does not generate sizable output losses in steady state as entrepreneurs are able to accumulate savings and self-finance their firms. In contrast, Moll (2014) shows in a similar setting that in productivity shocks are persistent enough, output losses generated in the transition to steady state can be considerable. Buera, Kaboski and Shin (2011) show how limited commitment of financial contracts endogenously generate collateral constraints that prevent the reallocation from unproductive sectors with low financing needs, to high productivity ones but with higher financing needs. They compute that their model can explain the majority of the output differences between Mexico and the US.

The two closest papers to this one are Cole, Greenwood and Sanchez (2016) and Verani (2018). Cole, Greenwood and Sanchez (2016), consider an economy where firms face a productivity ladder, with a certain probability of experiencing an upgrade. In their setting, lenders do not know firms' productivity, and these have an incentive to misrepresent it in order to divert funds from paying back to the investors. Calibrating this economy for

Mexico and India, they find output losses in line with the differences between those countries and the US. The crucial difference between their paper and this one, is that ex-ante, investors are able to observe the type of entrepreneurs, so they know how much funding to allocate to them. Verani (2018) evaluates two sources of financial frictions, limited commitment, because firms can repudiate their financial contracts, and asymmetric information, because firms suffer revenue shocks that are non-observable to the entrepreneur. This allows them to divert resources from paying back to the investor. He finds that limited liability has a sizable impact on output, but asymmetric information does not. The difference with this paper is also the source of asymmetric information, in Verani (2018) firms are ex-ante homogeneous, so the lender does not need to distinguish between them.

The main contribution of this paper comes from including non-observed permanent heterogeneity as a source of financial frictions. This is consistent with recent findings in the literature. Sterk et al (2021), exploiting the autocovariance structure of the employment series at the firm level in the US, find that ex-ante heterogeneity explains a large fraction of the cross sectional dispersion among firms. In the first year after entry, it accounts for 90% of the dispersion, and even after 20 years it explains about 40%. This is related to papers that show that firm characteristics at the moment they are born play a relevant and persistent role in explaining their outcomes. In a different context, Belenzon, Chatterji and Daley (2017) show that firms that bear the name of their owner have superior performance. They explain this as high productivity entrepreneurs trying to get a good impression for their firms. Similarly, McDevitt (2014) explains eponymy as a tool to signal quality. Choi et al (2021) analyzes a different set of characteristics. They show that the quality of human capital in the founding team of startups is an important determinant of their performance. Even more, they show that it is fundamental to the firm, to the point that if one of these founding members quits, the likelihood of failure increases significantly. From a completely different perspective, Guzman and Stern (2020) present estimates of entrepreneurial quality by establishing a relationship of firm characteristics at the moment of business registration with their posterior performance. They confirm that entrepreneurship is heterogeneous and dependent of factors intrinsic to the entrepreneurs and more general, economic variables. In a different setting, Foster, Haltiwanger and Syverson (2016), explain the slow growth of new plants as consequence of a demand accumulation process. However, ex-ante determined characteristics, such as being part of a larger conglomerate, are also relevant.

3 The Environment

The economy is a modified version of the one in Clementi and Hopenhayn (2006). The main feature is the interaction that takes place in financial markets. Entrepreneurs own blueprint technologies and need funds to set up and operate firms. Financial intermediaries have imperfect information regarding entrepreneur's technology. The source of asymmetric information lies on revenue shocks: in every period t, there is a shock that makes the production in t worthless with some firm specific probability, p_i. Thus, conditional on inputs, firm have permanently different expected revenue. In addition, in every period they can experience a different transitory revenue shock. Financial intermediaries do not observe p_i and the realizations of the shock. This means that they are not able to distinguish among firms with different expected returns before lending them resources, and once that production has taken place, they do not know how successful the firm was. The second type of imperfect information has been widely studied in the financial frictions context (Clementi and Hopenhayn, 2006; Cole et al., 2016), my innovation is to consider the first one.

3.1 Workers

There is a mass λ of workers. They chose consumption c_t, savings a_{t+1}, and labor l_t to maximize their lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t [c_t - A_w l^{\frac{1+\eta}{\eta}}]$$

where c_t is the unique consumption good. Their budget constraint is: $c_t + a_{t+1} \geq a_t(1+r) + w_t l_t$, with the natural borrowing limit ($a_{t+1} \geq \underline{\text{a}}$). r is the risk free rate and w_t is the wage rate.

3.2 Entrepreneurs and firms

There is a unit mass of entrepreneurs. They maximize their lifetime expected utility, $\sum_t \frac{1-\gamma}{1+r} c_t$, where γ is the probability of dying exogenously at the end of each period. Upon death, they are replaced by new entrepreneurs at the start of next period. Entrepreneurs have no wealth and they cannot work as employees. They have access to a blue print technology $f(x) = Ax^\theta$, with $\theta < 1$. x is a compound input with cost q as in Cole, Greenwood and Sanchez (2016):

$$x = k^{\alpha} l^{1-\alpha}$$

$$q = min_{\{k,l\}} \{r_r k + wl : k^{\alpha} l^{1-\alpha} = 1\}$$

Where r_r is the rental rate of capital. In order to produce, entrepreneurs need to pay an initial investment I_0 and, later on, pay for inputs in advance using working capital R:

$$qx \leq R$$

Every period t, entrepreneur i faces a revenue shock (v_{it}): given x, with probability p_i, her production is worth $f(x)$ ($v_{it} = 1$) and with probability $1 - p_i$ it is worth 0 (v_{it}=0). For simplicity I assume that there is only two types of entrepreneurs $i = \{H, L\}$, with $p_H > p_L$. They draw a probability p_i from a distribution **G** such that:

$$Pr(p_i = p) = \begin{cases} g & if\ p = p_H \\ 1 - g & if\ p = p_L \\ 0 & otherwise \end{cases} \tag{1}$$

The v_{it}s are *iid* across t and i. Note that with this structure we will observe permanent and transitory differences among firms. The permanent variation comes from differences in the expected value of production: conditional on x, the expected revenue, $p_i f(\bar{x})$, is itself a random variable. Transitory variation is caused by different v_{it} realizations for a given firm over time.

3.3 Financial intermediaries

Financial intermediaries will take deposits from households and lend them to firms. For this, they will design a set $\sigma = \{\sigma_i\}_{i \in \{H,L\}}$ of lending contracts, with each one (σ_i) intended for an specific type (p_i) of firm. A lending contract is a schedule of history dependent working capital advancements (R) and payments from the entrepreneur to the intermediary (τ). Ideally, the lender would make these contingent on the type (”i”) of the firm and the revenue shocks history ($s_{it} = \{v_{i1}, v_{i2}...v_{it}\}$). However, both of them are private information to the entrepreneur, so the contract will be dependent on the information provided by them:

$$\sigma_{i^R} = \{R_{i^R t}(s^R_{it-1}), \tau_{i^R t}(s^R_{it})\}$$

Where i^R is the firm type reported by the entrepreneur and s^R_{it} is the reported history of shocks until period t. The revelation principle applies, so we know that the optimal contract must induce truthful revelation of p_i and v_{it}.

The **timing** in every period is as follows:

1. Mass γ of entrepreneurs is born. They draw a probability p_i.
2. Lenders post the set of available contracts $\{\sigma_i\}_i$
3. Entrepreneurs decide what contract to sign.
4. Agreed working capital is delivered to the entrepreneurs. They buy inputs.
5. Production takes place and the productivity shocks (v_{it}) are realized.
6. Entrepreneurs decide what to report and makes the corresponding payment.
7. With probability γ entrepreneurs die.

Lenders will be profit maximizing institutions, but there is free entry, so they will have zero profits in equilibrium.

This economy has two different sources of inefficiencies that will operate through the financial contracting problem. First, there is an adverse selection problem because entrepreneurs might have incentives to lie about their type and sign a contract not designed for them. This is my innovation in this framework, but it has the same underlying idea as the canonical credit rationing models (Stiglitz and Weiss, 1981, among others) or papers of venture capitalism such as Cole, Greenwood and Sanchez (2016). Second, there is a dynamic moral hazard problem, which arises from the fact that firms can misreport their revenue. This was already incorporated in the model of Clementi and Hopenhayn (2006).

3.4 The Contracting problem

For a firm type i, the contract designed for type j (σ_j) and the reported history of shocks s_{it}^R will imply an expected present value of surplus ($W_i(\sigma_j, s_{it}^R)$) and its distribution between the entrepreneur ($V_i(\sigma_j, s_{it}^R)$) and lenders ($B_i(\sigma_j, s_{it}^R)$):$W_i(\sigma_j, s_{it}^R) = V_i(\sigma_j, s_{it}^R) + B_i(\sigma_j, s_{it}^R)$.We refer to them as equity and debt, respectively. $V_i(\sigma_j)$ is the initial equity, before any revenue shock has been realized. Note that if $i \neq j$, $V_i(\sigma_j)$ is the value the entrepreneur gets by misreporting her type, which is a priori unknown because she might misreport about the revenue realizations too, but we need to pin it down in order to solve the contracting problem. The lender's optimization problem is:

$$\text{(P0)}: Max_{\{x(s_{it}),\tau(s_{it})\}} \sum_i g\Big(\sum_t \beta^t \sum_{s_{it}} P(s_{it})\big(p_i\tau(s_{it}) - qx(s_{it})\big)\Big)$$

$$\text{s.t.: } V_i(\sigma_i) \geq V_i(\sigma_j), \ \ \forall i, j \tag{1}$$

$$V_i(\sigma_i, s_{it}^R) \leq V_i(\sigma_i, s_{it}), \ \ \forall i, \ s_{it} \tag{2}$$

$$\tau_i(s_{it}) \leq f(x(s_{it-1})), \ \ \forall i, \ s_{it} \tag{3}$$

Where (1) and (2) are the type and revenue truthful revelation constraints, this is, they ensure that revealing p_i and v_{it} is in the entrepreneur's best interest. (3) implies there is limited liability.

3.5 Two benchmark economies

In this section I present two alternative versions of the economy, and show why the financial contracts they generate would fail in my framework. First, I consider a frictionless benchmark, where types p_i and the revenue shocks v_{it} are public information. Then, I consider that the revenue shocks v_{it} are directly observable, but the types p_i are private information to the entrepreneur.

3.5.1 The contract under perfect information

Consider a situation where there is no asymmetric information: the lender can observe directly the entrepreneurs' types and realized revenue. Here, since there is perfect competition, the lender will still maximize the aggregate expected surplus but there is no need to make contracts incentive compatible. This implies that the problem can be considered static, with no history dependence. The contracting problem can be reduced to:

$$Max_{x_i} p_i f(x_i) - qx_i$$

The optimal level of inputs x_i^* will be given by: $f'(x_i^*) = 1/p_i$. Notice that in this case, firms are always producing at their socially optimal level, and they never grow or shrink. The total surplus generated by firm i will be:

$$W_i^u = \left(\frac{p_i f(x_i^*) - x_i^*}{r}\right)$$

Note that this contract does not induce type or revenue truth-telling and it will fail in both aspects. First, if firms are guaranteed to receive the same level of inputs x_i^* in every period, they will always report being unsuccessful. In this case, they will make no payments to the lender and keep all of the profit for themselves. In addition, since higher types are given more working capital ($x_H^* > x_L^*$), firms would always report being the highest possible type, and then keep all the revenue, avoiding paying back to the lender.

3.5.2 Non-observable types

Now consider the same economy, but with unobservable types (p_i). The contracting problem would be the same as in (P0), except for restriction (2), which is the revenue truthful revelation constraint. The problem can also be reduced to a static version:

$$Max_{\{\tau_i, x_i\}} g\left(f(x_H) - \tau_i\right) + (1-g)\left(f(x_L) - \tau_L\right)$$

$$s.t.: \; p_L\left(f(x_L) - \tau_L\right) \geq p_L\left(f(x_H) - \tau_H\right) \tag{*}$$

$$g\left(qx_H - p_H x_H\right) + (1-g)\left(qx_L - p_L x_L\right) \geq 0 \tag{**}$$

In this case, it will also be true that the optimal input advances yield $f'(x_i^*) = 1/p_i$. The optimal payments τ_i will be determined by the non-profit condition and the type truthful revelation constraint. Note that from the type revelation constraint (equation *), we know that in case of being succesful, both types of entrepreneurs will be left with the same amount of revenue. Thus, the contract would achieve social optimality simply by distributing revenue in a incentive compatible way, but not distorting the production level.

Again, in my framework this contract would fail because it does not induce revenue truthful revelation. Entrepreneurs would never reveal they revenue shock and make the corresponding payment τ_i if they are guaranteed the same level of capital advances in the future. Plus, it does not induce type truthful revelation either: since H is granted more resources, L would always sign its contract.

3.6 Optimal Contracts

In this economy there are two different information problems: adverse selection and repeated moral hazard. It turns out, the problem of the lender can be analyzed in two stages that deal with them separately. The first problem, related to adverse selection, will be how much initial equity $V_i(\sigma_i)$ promise to each type such that they reveal their true type and maximizes lenders' expected profit. The second one, related to repeated moral hazard, to choose $R(s_{it})$ and $\tau(s_{it})$ such that it induces truthful revelation of the revenue shocks and maximizes firm value. To solve this, I assume that given a promised initial equity, $V_i(\sigma_i)$, all entrepreneurs are able to forecast what R and τ will be in the future, in other words, they know the problem the lender will face.

3.6.1 Second stage: repeated moral hazard

The repeated moral hazard problem is the same as in Clementi and Hopenhayn (2006). The contracting problem is recursive, with the continuation equity summarizing the complete history of shocks up to that point. At the beginning of each period, given promised V_i, the the lender chooses working capital R, payment τ and continuation equities for the next period in case of high and low revenue shocks, V_i^H and V_i^L, such that such that they deliver V_i and induce revenue truth telling. To derive the complete series of R and τ, we only need to feed our problem with an initial V_i that would be determined in the first stage.

$$\text{(P2)} : W_i(V_i) = Max_{\{x, V_i^H, V_i^L\}} p_i f(x) - qx + \beta\Big(p_i W_i(V_i^H) + (1-p_i) W_i(V_i^L)\Big)$$

$$s.t. : V_i = p_i(f(x) - \tau) + \beta\Big(p_i V_i^H + (1-p_i) V_i^L\Big) \tag{4}$$

$$f(x) - \tau + \beta V_i^H \geq f(x) + \beta V_i^L \tag{5}$$

$$f(x) \geq \tau \tag{6}$$

Constraint (4) ensures that the promised equity is delivered; (5) and (6) are the same as (2) and (3), revenue truth telling and limited liability, respectively.

3.6.2 First Stage: adverse selection

In the first stage, the lenders decide how much equity to promise to each entrepreneur so they sign the contract designed for them. We assume that the entrepreneurs know the problem of the lender in the second stage. Thus for a given initial equity (for them or others), they can derive the complete series of R and τ and anticipate their optimal strategies under any contract.

$$
\begin{aligned}
&(\text{P1}) : Max_{\{V_H, V_L\}} \Big(W_H(V_H) - V_H \Big) + \Big(W_L(V_L) - V_L \Big) \\
&s.t. : V_H(\sigma_L) \geq V_H(\sigma_L) \qquad (7) \\
&\qquad V_L(\sigma_L) \geq V_L(\sigma_H) \qquad (8)
\end{aligned}
$$

Where V_H and V_L are the equities promised to entrepreneurs type H and L respectively. Equations (7) and (8) are the type truthful revelation constraints for H and L respectively.

3.7 Characteristics of the optimal contract

The first result of the paper states that by solving problems (P1) and (P2) separately, we will reach the same solution as the first problem. This simplifies things greatly. First, because (P2) has already been solved, so we already know some of the characteristics of the optimal contract. Second, because since (P2) can be stated recursively, so can (P1). This way, the dimensionality of the problem is greatly reduced, requiring significantly less computing power.

Proposition 1. *The set of contracts implied by the solutions of ($P1$) and ($P2$) solve the full contracting problem ($P0$).*

From the solution of Clementi and Hopenhayn (2006) to ($P2$) we know the following features of the optimal contract. For each type of firm there is some equity level V_i^* such that if the promised level V_i is higher than V_i^*, then the firm is granted the socially optimal working capital advancement x_i^* perpetually, so there is no surplus loss. The payments τ and continuation equities are not determined. Consider now $V_i(s_{it}) < V_i^*$. In this area, the capital advancements will be lower than x_i^* with $x_i'(V_i) > 0$. After a positive (zero) cash-flow shock, the lender will increase (decrease) the continuation equity V_i^H (V_i^L). The equity of the firm is expected to grow at a constant rate since $E(V') = (1/\beta)V$. Finally, the optimal repayment policy will be $\tau = f(x)$ because this way the can grow out of the constrained area faster. What is left is to determine is the relationship between the equity granted to different types of entrepreneurs. For this, we need to determine the value of misreporting their types for entrepreneurs.

Proposition 2. *For an entrepreneur type L under a contract designed for H, (σ_H), the optimal strategy after a high revenue shock ($v_{it} = 1$) is to misreport it as low. This implies that the value of signing that contract is $V_L(\sigma_H) = \frac{p_L}{P_H} V_H(\sigma_H)$. It also implies that the upward looking type truth-telling constraint will be binding.*

The intuition behind this results is the following: to induce truthful revelation of a high revenue shock, the lender must compensate the entrepreneur for making her payment τ, by promising her a higher equity in the future (V_i^H). It does it in such a way that restriction (5) binds. However, under the same contract (σ_H), a less productive entrepreneur assigns a lower value to this compensation because she is less likely to experience high revenue shocks in the future. Thus, if L signed σ_H, she would always misreport high revenue shocks. This out of equilibrium strategy, implies that $V_L(\sigma_H)$ is a linear function of $L's$ productivity and $H's$ equity. Note that there is a one to one relationship between $V_H(\sigma_H)$ and $V_L(\sigma_L)$, which means that we can pin down the equity offered to each firm from the zero profit condition of the lender:

$$V_H(\sigma_H) = sup_{V_H}\{V_H : g\left(W_H(V_H) - V_H\right) + (1-g)\left(W_L(\frac{p_L}{p_H}V_H) - \frac{p_L}{p_H}V_H\right) = I_0(1+r)\}$$

Given that the profit function is strictly concave, we know that there exists a unique equilibrium level of V_H that satisfies this condition. If lenders were able to distinguish among types of firms, each contract should yield zero profits on its own. Under private information, this induces lower types would pretend to be high ones. In order to make them report their type truthfully, the lender will offer more (less) initial equity to less (more) productive types than he would otherwise. This is the next result.

Proposition 3. *Lenders offer strictly positive equity V_i to all types. The initial equity offered to the H (L) type is strictly lower (higher) than if the types p_i were public information.*

This result is the main theoretical result of this paper, and it will be quantitatively important. Since the lenders cannot determine how productive is an specific firm they end up promising sub optimal levels of equity to entrepreneurs. The intuition is as follows: if the lender were to offer both types the same equities that if their types were public information, the firm type L would sign the contract designed for H. To avoid this, the lender must offer L more equity than he would otherwise, actually losing money in this contract. At the same time, she offers H less equity, making enough profit on the contract to break even in total. This is a type of correlated distortions, in the sense that it affects more negatively entrepreneurs with higher productivity or revenue. Eslava and Haltiwanger (2020) show that these are particularly important in explaining newborn firms' outcomes, which is also a feature of my model.

A corollary of proposition 3 is that the working capital advancements will be (weakly) higher for the H, given any history s_t. However, the difference will be (weakly) smaller. This speaks directly to inefficient firm sizes. Asymmetric information about firm types, lowers the difference between inputs (and output) of more and less productive firm to an inefficient level. For the low productivity types, their capital advancements and expected production is weakly higher than under observable productivity, so for this type, the presence of adverse selection could diminish the output losses generated by the moral hazard problem. The opposite is true for high productivity types.

Finally, I introduce the notion of equilibrium in this economy, which is the same as Verani (2018). **Definition 1:** A Recursive Stationary Equilibrium consists of prices w and r, sequences of consumption and labor supply $\{c_t, l_t\}_t$ for entrepreneurs and workers, set of contracts $\{\sigma(p)\}_p$, and stationary distribution ψ of entrepreneurs such that:

1. Consumption, debt and labor sequences solve the optimization problems of workers and entrepreneurs.
2. Financial contracts maximize the aggregate value of firms.
3. Intermediaries make ex-ante zero profits. This is, they aggregate expected profits are zero.
4. Labor an capital markets clear.

Two additional results are necessary to close the model: the existence of an ergodic distribution and the existence of a unique recursive stationary equilibrium. Both follow closely Verani (2018), so they are omitted in this paper but can be found in the appendix.

3.8 Financial Reform

In this section I consider the effects of a financial reform on the baseline economy. The reform allows the lender to observe the realized revenue at the end of the period, and to recover a fraction ϕ of it if she has been lied to. This is similar to the financial reforms studied in papers like Verani (2018) and Cole, Greenwood and Sanchez (2016). The reformed economy can be thought as a developed benchmark, a country with perfect or near perfect financial information and a judiciary system that allows high recovery rates for the investors (Neira, 2017), like the US and other OECD countries. Notice that this addresses the moral hazard problem directly, because now lenders can monitor revenue. However, it does not adresses adverse selection: lenders do not observe entrepreneurs' types.

Assume the lender can costlessly monitor the revenue realization at the end of the period. She cannot recover anything if she has been lied to, but she can punish the entrepreneur providing a lower equity in the future. In this case, the lender optimally terminates the contract if the entrepreneur misrepresented the period's equity. If not, a low revenue shock provides continuation equity $V_i^L = V$ and a high one $V^H = \frac{1-\beta p_i}{\beta(1-p_i)} V$ and the working capital advancement is given by $X_i = min\{x_i^*, V^H/\beta\}$. With this, the new second stage problem (P2') will be:

$$\text{(P2')} : W_i(V_i) = Max_{\{x,\tau,V_i^H,V_i^L\}} p_i f(x) - qx + \beta\Big(p_i W_i(V_i^H) + (1-p_i) W_i(V_i^L)\Big)$$

$$s.t. : V_i = p_i(f(x) - \tau) + \beta\Big(p_i V_i^H + (1-p_i) V_i^L\Big) \tag{4'}$$

$$f(x) - \tau + \beta V_i^H \geq f(x) \tag{5'}$$

$$f(x) \geq \tau \tag{6'}$$

We can see that the only difference is in the right hand side of the constraints that ensure truthful revelation of the revenue shocks (equations 5 and 5'). In the reformed economy, the continuation equity after misreporting a positive revenue shock is zero.

Proposition 4. *In the reformed economy, if L signed the contract σ_H, she would misreport any positive revenue shock. The value of lying is now:* $V_L(\sigma_H) < V_H(\sigma_H) \frac{p_L}{p_H} \frac{1-\beta(1-p_H)}{1-\beta(1-p_L)}$.

The first part of proposition 4 bears the same intuition as before: after a positive revenue shock, the lender needs to compensate H for reporting it truthfully, and paying back τ, with higher equity V_H^H. However, under the same contract (σ_H), this equity is worth less to L, to the point that she would rather keep the entire revenue $f(x)$ and avoid paying τ. This implies that after that period, the lender would terminate the contract. The second part is the key point of this reform, reducing asymmetric information in this fashion, has a dual effect. On the one hand, the lender is able to achieve higher surplus level since she does not need to induce revenue truthful revealing. On the other hand, it makes the incentives to lie even higher in proportional terms. The ration between the initial equities for both types is even lower than before. However, as in the (second) benchmark case, now the planner can resolve this by distorting the production less than before and redistributing equity.

4 Testable Predictions

The main testable prediction from the model is that uncertainty about permanent productivity harms (benefits) the funding opportunities of firms with higher (lower) permanent productivity. In this section I use Chilean data to document that this is empirically plausible.

4.1 Data

I use the microdata of the Chilean survey "ELE" (Spanish acronym for Longitudinal Survey of Firms), which is nationally representative of all firms in industry, construction, and services, with sales larger than $USD\$2300$, approximately. It includes information about firm characteristics and their finances, backed by financial statements, among other items. It includes firms from all sectors in the economy, which allows me to analyze differences between sectors that are usually not included by the literature working with manufacturing exclusively. Crucially, this survey it is designed as a rotating panel: a fraction of firms is followed from one round of the survey to the next one. I use the data from the third to the fifth round, because it allows me to observe the financial information from 2012 to 2017 continuously.

To distinguish between the permanent and transitory component of productivity I need to characterize the productivity process. I do this using two alternative measures of productivity: TFP and value added per worker. To compute the TFP, I assume that for each four digit sector "j", firm i's revenue is price (P_j) times a Cobb-Douglas production function with decreasing returns to scale:

$$Y_{it} = P_j e^{z_i} (k_{it}^{\alpha_j} l_{it}^{\beta_j} m_{it}^{1-\alpha_j-\beta_j})^{\theta_j}, \;\; \theta_j \in (0,1), \; \alpha_j + \beta_j < 1$$

Note that the inputs elasticities are sector specific. I obtain the production function parameters from their corresponding shares in the total revenue of each sector and compute the TFP as the Solow residual. Note that I compute revenue TFP, so I am unable to distinguish between different sources of revenue shocks, such as true productivity or demand shocks. I use fixed assets as capital, payroll as employment and the expenditure in intermediate goods as materials. As Hsieh and Klenow (2009) suggest, using the payroll rather than number of workers/hours, can correct the discrepancies in the quality of human capital. I extend this logic to materials. After that, following Bailey, Jones, and Pratap (2020) I assume that the productivity can be decomposed between a set of sector j specific year fixed effect ($\triangle_j$), firm specific permanent effects (μ_i) and a purely transitory component(ε_{it}):

$$z_{it} = \mu_i + \triangle_j + \varepsilon_{it}$$

I estimate this equation (for both measures)directly using firm fixed effects. With this I recover the variation in the permanent component of TFP (μ_i) for each sector. As measure of uncertainty about the permanent component of productivity in each sector I use its standard deviation normalized by the average (total) productivity. Note that this a sector level measure, and that it is constant through time.

I use two classical measures of firms' access to external funds **XXX**: the ratio of assets to total liabilities and the ratio of financial expenditure to liabilities. The first one captures the ability of the firm to finance its investment with external funds. The second one is a proxy measure of the interest rate, which shows the cost of obtaining external funds, but also includes other expenses like transaction costs and other fees charged by financial institutions.

Then I divide firms in two groups: high productivity firms, whose permanent TFP is in the top half of the distribution, and low productivity ones, which are in the lower half. The idea of doing this, is to compare the financial conditions of high and low productivity firms, in sectors with different degrees of uncertainty about permanent productivity. Recall that the model predicts that higher uncertainty will shrink the difference in financial conditions of high and low productivity firms. So the variable of interest is the difference in these conditions rather than their raw value. For each measure I compute this difference by subtracting the average value of low productivity firms in each sector-year from the level of high productivity firms.

4.2 Results

The regression of interest is:

$$Dif(measure_{it}) = \alpha + \beta * uncertainty + \gamma * Controls + \varepsilon_{it}$$

Where $Dif(measure_{it})$ is the difference between the measure of financial conditions of firm i at time t, and the average of low productivity firms in its sector. *Uncertainty* is the sectorial measure of uncertainty defined before and *Controls* is a group of control variables, such as firm's payroll, age, productivity and year fixed effects. I run this regression only for high productivity firms.

Table 1: **Financial Conditions Regressions**
(Standarized coefficients)

Productivity measure:	**V. Added/Worker**		**TFP**	
Financial measure:	**Interest rate**	**Liabilities/Assets**	**Interest rate**	**Liabilities/Assets**
Uncertainty	0.044***	-0.123***	0.122***	-0.041***
Productivity (perm.)	0.031	-0.015	0.019	0.010
Size (payroll)	0.010	0.039***	0.013	0.060***
Age	0.013	-0.123***	0.061***	-0.044***
Year FE	YES	YES	YES	YES
Cons	YES	YES	YES	YES
N	$2,089$	$4,623$	2,267	5,061
R^2	0.026	0.034	0.027	0.01

(*, ** and *** = significant at 1, 5 and 10% level)

In table 1 we can observe that the main prediction of the model appears in the data. The uncertainty measure is negatively correlated with the debt ratio for high productivity firms, and positively correlated with the cost of external funds they face. Thus everything else equal, high productivity firms operating in sectors with more uncertainty, face worse credit conditions.

5 Quantitative Evaluation

In this section I evaluate the model for Chile with the objective of computing output losses and dispersion. I consider a one sector economy, which will represent the average sector. I evaluate four alternative versions of this economy: first, the economy introduced in this paper: the permanent component of productivity (p_i) and the transitory shocks (v_{it}) are private information to the entrepreneur. Second, I evaluate the Hopenhayn and Clementi (2006) economy, where the permanent component (p_i) is public information, but the transitory shocks (v_{it}) are private to the entrepreneur. The third economy is the benchmark against which output losses will be computed: the "developed" economy where the p_is are private information, but lenders can observe the realized revenue (v_{it}) at the end of each period. This economy corresponds to one under the financial reform presented in the theoretical part of this paper. Finally, I also consider a version of the baseline economy proposed in this paper, where financial intermediaries can learn the true type of firms at the end of each period with a probability λ. As in the benchmark, I assume that the lender cannot recover any resources, but she can cancel the contract for the future. Crucially, all of these economies are computed separately so I allow for wages to adjust to clear the labor market. Compared to the benchmark, my model generates output losses of 24.6%, exclusively in the intensive margin. In turn, the Hopenhayn and Clementi (2006) economy generates an output loss of approximately 15.9%, which is in line with the findings of Verani (2018) calibrating the same type of economy for Colombia. This implies that the mechanism proposed in this paper can account for an output loss of 8.7%.

5.1 Parametrization

Table 2 shows the values of the parameters' values for this economy. My strategy is to characterize the productivity process using the Chilean survey data from ELE for those cases where this is posible, and take the rest from external sources.

To characterize the productivity process, I need to obtain values for the sector TFP shifter A, the shares of high TFP firms g, and their respective success probabilities p_H and p_L. I choose them to match four moments of the empirical TFP distribution: first, the average TFP, its variance, the average (across sectors) variation in the permanent component of productivity, and the kurtosis of the TFP distribution. With these four moments, I am capturing the differences in TFP between firms, and the relative weight of them in the distribution. At the same time, by including the total variance in TFP I directly take into account the contribution of within and between differences to TFP variation. Finally, including the average TFP normalizes these variations. I set θ and α to match the average profits and labor shares in the database. γ is calibrated as the exit rate implied by the age distribution of firms in the data.

I also need to obtain values for two parameters than cannot be directly computed from the ELE database: the initial investment "I_0" and the exit rate γ. I choose I_0 to match the average cost of starting a business according to the Doing Business ranking compiled by the World Bank during the period of the survey (2012-2017). This report states that on average it takes 6.2 full days of work to start a business in Chile, so I choose I_0 to match the corresponding cost in terms of yearly salary, which is 2.3%. Note that this will adjust if the wage changes in the economy. Since Chile is a small open economy, the interest rate would be determined exogenously. I set the risk free interest rate (r) to match the average real annual interest rate of the bonds issued by the Chilean Central Bank. The rental rate of capital is set to be $r_f = r + \delta$, where δ is the average depreciation rate, taken from the Penn World Table 10.0. I set the learning probability, λ to 0.5. Finally the parameter A_w, which governs the desutility of labor, is chosen such that in the baseline specification, the number of hours worked in the economy matches the average observed during the period, and η is set to 1.0, a typical value.

Table 2: **Model Parameters**

Parameter		**Value**
A. **Productivity Process**		
TFP Shifter	A	4.37
P high	P_H	0.738
P low	P_N	0.254
Share of H firms	g_H	0.57
B. **Calibrated with survey data**		
Extent scope	θ	0.918
Labor share	α	0.519
Exit prob.	γ	0.091
B. **External Sources**		
Interest rate	r	0.040
Depreciation	δ	0.042
Initial investment (% wage)	I_0	0.023
Elasticity of labor	η	1.000
Utility scaling	A_w	27.723
Learning Probabilities	λ	0.5

5.2 Output loss and firm size

Table 3 shows the results of the quantitative evaluation on my model. As in Verani (2018), private information regarding revenue or TFP shocks can generate sizable output losses, in my economy they add up to 15.9%

of GDP compared to the benchmark. Accordingly, it also depresses wages they are 10.2% lower than in the benchmark. My economy, with private information about productivity and revenue shocks generates an output loss of 24.6% compared to the benchmark economy and depresses wages by 15.9%. This implies that private information about the permanent component of productivity can account for an output loss of 8.7%. Regarding firm sizes, the same pattern emerges: while the Hopenhayn and Clementi economy generates an average decrease of 13.6%, in my economy, firms are a 23.1% smaller than in the benchmark. Finally, note that in the economy where lenders can randomly learn the true type of the firms, output, wages and firm size are similar to the baseline economy. This occurs because low productivity firms derive the expected value of lying about their types mostly by missreporting a positive revenue shock in the first period, so that they avoid making the corresponding payment and keep the full revenue. At the same time, lenders can only learn at the end of each period, when the value of (keep) lying is substantially lower. Note that I assumed that the lender cannot recover anything after she finds out the entrepreneur misreported her revenue. This is another source of differences between developed and developing economies (Neira, 2017), which could potentially generate larger gains from learning.

Table 3: **Steady state allocation**
(% of benchmark)

	Baseline	**W/ learning**	**HyC**
Output	75.4	77.4	84.1
Wages	84.1	85.5	89.8
Average firm size (inputs)	76.9	78.8	86.4

We can see the main mechanisms in place in Table 5. First, private information exclusively about the revenue shocks (HyC, Hopenhayn and Clementi economy), decreases firm size for both types, but it does so significantly more for the low type. In terms of distortions, this would imply that within an economic sector, the wedges are negatively correlated with TFP, which is contrary to previous findings in the literature. In my economy, high TFP firms are harmed and the low type are benefited, relative to the benchmark. Recall that if the lender cannot distinguish between them, it ends up giving both more similar resources than otherwise (compared to the HyC economy). This means, increasing resources for L and decreasing them for H. As consequence, H firms produce 73% of their unconstrained level, while L firms 274%, on average. This ends up generating sizable, additional, output losses. Similar to before, the economy with learning is not substantially different from the benchmark in any variable. This highlights the importance of financial information for economic growth, specially for the growth of high productivity firms.

Table 4: **Distribution**
(% of benchmark)

	Baseline	**W/ Learning**	**HyC**
Average firm size (inputs), Type H	$76,9$	$78,8$	86
Average firm size (inputs), Type L	$274,3$	$249,4$	6
Average output, type H	$73,6$	$77,4$	$84,1$
Average output, type L	$252,1$	$231,4$	4

6 Conclusion

Recent literature o shows that permanent and transitory heterogeneity play important roles in explaining differences in firms' outcomes. In this paper, I show the implications of this for financial frictions in a world where investors do not know ex-ante the potential of each firm. As I discussed, the intuition behind this has been present in the finance literature since the earlier papers of credit rationing (Stiglitz and Weiss, 1981; Jaffee and Ruslell, 1976). However, the macro development literature has not considered that firms permanent characteristics, which make them ex-ante different, can play a significant role in the allocation of financial resources. Papers such as Cole, Greenwood and Sanchez (2015), typically consider firms that are ex-ante equal, so lenders know how much resources to allocate to them.

I proposed a modified version of the Clementi and Hopenhayn (2006) economy, where firms show heterogeneity in the permanent and transitory components of their productivity. Conditional on inputs, they have different expected revenue right from the moment they are born, and they suffer different revenue/productivity shocks in every period. By solving the optimal contract, I show that if investors cannot distinguish between firms with different potential, they end up providing more (less) resources to low (high) productivity firms that they would otherwise. The mechanism is simple: this is the only way to prevent firms from overstating their expected productivity. I consider a benchmark economy, where there is perfect financial information: entrepreneurs to directly observe the revenue of each entrepreneur. This economy should be thought of as a developed economy, where financial information is available and reliable. Using data from Chile, I evaluate the model in three different versions: the baseline economy proposed in this paper, the Hopenhayn and Clementi (2006) economy where only revenue shocks are private information, but lenders can observe the permanent component of productivity; and a version of the baseline economy of this paper, where lenders are allowed to learn randomly the true productivity of firms. I find that this model generates substantial output losses: total output is 24.6% lower than in the benchmark economy. Wages fall 15.9% compared to the same economy. The role of unobservable permanent heterogeneity explains a sizable fraction of these losses: in the Clementi and Hopanhayn (2006) economy, where revenue shocks are private information, but lenders know the type of each firm, output and wages suffer only losses of 15.9 and 1.02%, respectively, which is in line with the results of Verani (2018) for Colombia. When I evaluate the economy with learning, I find that output losses are similar to those of the baseline. It is note worthy, that these losses are found exclusively at the intensive margin.

This paper emphasizes that financial information plays a crucial role for economic growth. It allows firms with high productivity to differentiate from low productivity ones, and obtain the funding necessary for their growth. This is specially relevant as it speaks directly to recent findings in the literature that show that an important fraction of the output and productivity growth can be accounted by ”gazelles” or high productivity-high growth firms. However, several developing economies have adopted the OECD standards in financial information (Chile and Colombia, for example), which has not been reflected in the output gains predicted by the literature. Future research should look more intensively into the causes of this asymmetrical information, specially on those that are beyond the direct decision of regulators.

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A Proofs

First I show the following intermediate remarks.

Remark 1. *The limited liability and revenue truth-telling constraints will be binding.*

Proof. Recall that I only allow the L type to lie about its type. Thus I only need to prove this for entrepreneurs type H.

First consider limited liability. If $f(x(s_{t-1}) > \tau(s_t)$, the lender could just increase τ by δ and decrease it by $\frac{\delta}{P_1\beta}$ the following period if the entrepreneur reports high revenue in t. This does not change the value of the objective function and the contract would still satisfy the revenue-truth telling and limited liability in $t+1$. Even more, this changes the value of lying for type L ($V_L(\sigma_H)$) in $-\delta + \beta P_L \frac{\delta}{P_H\beta} < 0$. Thus the type-truth telling constraint would also be satisfied.

Now consider the revenue truth-telling constraint:

$$f(x(s_{t-1}) - \tau_i(s_t) + \beta(V_i^H(s_t) \geq f(x(s_{t-1}) - \beta V_i^L(s_t))$$

And suppose it does not bind at some node for the entrepreneur type H. Then, the lender could increase $f(x_H(s_t-1))$ and $\tau_H(s_t)$ by δ, increasing her expected profits in $\beta^t P_H \delta$. If this increases the value of lying for an entrepreneur type L, it will do so by $\beta^t P_L \delta$ maximum. So the lender could just increase $V_L(\sigma_L)$ by this amount and still obtain higher profits by an order of $\beta^t(P_H - P_L)\delta$.Note that this does not change the equity of H at any node, so it does not affect the truth-telling constraints in any period.

Now suppose it does not bind for an entrepreneur type L. Then, the lender can just increase $f(x_H(s_t-1))$ and $\tau_H(s_t)$ by δ, increasing profits. Since the H type cannot lie, it does not conflict with any type truth-telling constraint. Thus, in either case, the contract could have not been optimal in the first place. □

Remark 2. *For the optimal contracts, σ_L^* and σ_H^*, the continuation equities after any history s_t is increasing in p_i, this is, $V_H(\sigma_j, s_t)/P_H > V_L(\sigma_j, s_t)/P_L$.*

Proof. Consider any reporting strategy S^R that L could follow under contract σ_H starting from history s_u. **The idea is to prove that under the same strategy, H would obtain more equity**. Consider the following mixed strategy: in every period that H experiences a positive revenue shock, she does what L would do with probability $\frac{P_L}{P_H}$, and with probability $1 - \frac{P_L}{P_H}$ reports it as low (lies). Define $x_H^*(s_{u+t}|S^R)$ and $\tau_H^*(s_{u+t}|s^R)$ as the working capital advancements and payments after history s_t and reporting strategy S^R. Under this strategy, the probabilities of reaching any future node/history of reports in the game are exactly the same as for L, and for each one the expected period's payoff is $p_H(f(x_H(s_{u+t}|S^R)) - ind(s_{u+t}|s^R) \times \tau_H(s_{u+t}|S^R)) \geq p_L(f(x_H^*(s_{u+t}|S^R) - ind(s_{u+t}|S^R) \times \tau_H(s_{u+t}|S^R)) >$, where $ind(s_{u+t}|S^R)$ takes value 1 if L reports truthfully the period's revenue, and 0 if not. In addition, with probability $p_H - p_L/p_H$, H experiences a positive revenue shock and denies it, obtaining a payoff of $f(x_j^*)$, thus, if both followed the same strategy s_T^R their equities would be:

$$V_L(\sigma_H, s_u)/P_L = \sum_{s_{u+t}} \beta^t P_L(s_{u+t}|L)\Big(f(x_H(s_{u+t}|S^R)) - \tau_H(s_{u+t}|S^R) * ind(s_{u+t}|S^R)\Big)$$

$$V_H(\sigma_H, s_u)/P_H = V_L(\sigma_H, s_u)/P_L + \sum_{s_{u+t}} \beta^t P_H(s_{u+t}|H)(1 - P_L/P_H) f(x_j(s_{u+t}|S^R))$$

Where s_{u+t} only counts histories conditional on s_u. In addition, we know that truthful revealing the revenue shocks in every period is weakly dominating for H, so it must provide at least as much equity as strategy S^R. Thus, from here it is clear that $V_H(\sigma_j, s_u)/P_H \geq V_L(\sigma_j, s_u)/P_L$. $\square$

Now I proceed to prove proposition 2.

Proof of proposition 2.

First define $V_i^H(\sigma_H, s_t)$ and $V_i^L(\sigma_H, s_t)$ as the continuation equities after a positive and null revenue shock in t, implied by the solution to $P0$ for entrepreneur i under contract σ_H after history s_{t-1}. Define a history $s_t^L = \{v_0 = 0, v_1 = 0.....v_{t-1} = 0\}$, only null revenue shocks up to t. Consider a period t_0 such that $V_H^L(\sigma_H, s_{t_0}^L) = 0$. From remark 1 we know that the revenue truth-telling constraint satisfies $f(x(s_{t_0})) = \beta V_H^H(\sigma_H, s_{t_0})$. Thus, using remark 2 we obtain $f(x(s_{t_0}^L)) \geq \beta V_L^H(\sigma_H, s_{t_0}^L)$. If L were under contract σ_H, had a history of shocks $s_{t_0}^L$ and experienced high revenue in t_0, she would lie about it and report it as unsuccessful. Also note that:

$$V_H^L(\sigma_H, s_{t_0-1}^L) = \beta p_H V_H^H(\sigma_H, s_{t_0}^L) = \beta p_H f(x(s_{t_0-1}^L))$$

and:

$$V_L^L(\sigma_H, s_{t_0-1}^L) = \beta p_L f(x(s_{t_0-1}^L))$$

Consider now the immediately previous node $s_{t-1}^L = \{v_0' = 0, v_1' = 0.....v_{t-2}' = 0\}$. Here the revenue truth-telling constraint for H is:

$$f(x(s_{t-1}^L)) = \beta(V_H^H(\sigma_H, s_{t-1}^L) - V_H^L(\sigma_H, s_{t-1}^L))$$
$$f(x(s_{t-1}^L)) = \beta(V_H^H(\sigma_H, s_{t-1}^L) - p_H f(x(s_{t-1}^L)))$$

Thus, using remark 2 we have:

$$f(x(s_{t-2})) < \beta(V_L^H(\sigma_H, s_{t-1}') - p_L f(x(s_{t-1}')))$$

Which implies she would also lie about a positive revenue shock in this node. Iterating this procedure backwards, we obtain that in the first period ($t = 1$), if L were under σ_H and experienced $v_1 = 1$, she would also lie about it:

$$f(x)) < \beta(V_L^H(\sigma_H, s_1') - V_L^L(\sigma_H, s_1'))$$

This means that the initial equity of lying for L is: $V_L(\sigma_H) = \sum_t \beta^t p_L f(x(s'_{t-1}))$ where s'_t is the history of exclusively null revenue shocks until period t.

Consider now the initial equity for H: $V_H(\sigma_H)$. From remark 1 we obtain that:

$$V_H = p_H f(x) + \beta V_H^L(\sigma_H)$$

$$V_H = p_H f(x) + \beta(p_H f(x(s'_1) + \beta V_H^L(\sigma_H, s'_1))$$

And thus iterating forward, we obtain that $V_H(\sigma_H) = \sum_t \beta^t p_H f(x(s'_{t-1}))$. consequently, $V_L(\sigma_H) = \frac{p_L}{p_H} V_H(\sigma_L)$.

Now I proceed to prove proposition 1.

Proof of proposition 1.

Define $V_i(\sigma_i)$ as the initial equity for type i implied by the solution to the sequential problem $P0$. We know that under this contract $V_H(\sigma_H) = \sum_t \beta^t P_H f(x_H(s_t^L))$ and $V_L(\sigma_H) = \sum_t \beta^t P_L f(x_H(s_t^L))$ where $s_t^L = \{v_0 = 0, v_1 = 0.....v_{t-1} = 0\}$. Now consider problem (P2), the distribution of continuation equities, working capital and payments such that they deliver some promised level of equity and suppose these initial equities are $V_H(\sigma_H)$ and $V_L(\sigma_L)$, respectively. From Clementi and Hopenhayn (2006) we know that in this case the revenue truthful revealing and limited liability constraints will also bind, so just like (P0) we have that $V_H(\sigma_H) = \sum_t \beta^t P_H f(x_H(s_t^L))$ and $V_L(\sigma_H) = \sum_t \beta^t P_L f(x_H(s_t^L))$. Thus, the contract implied by (P2) given the initial equities $V_H(\sigma_H)$ and $V_L(\sigma_L)$ also satisfy the type truthful revelation constraint.

Now consider (P1), choosing the optimal initial equities for both types. Called them V_H^I and V_L^I. It is clear that the profits generated by this problem cannot exceed the value of (P0) because in that case the sequence of x and τ implied by (P1) and (P2) would yield higher profits for the entrepreneur and satisfy all constraints. Then (P0) would not be maximized. It also cannot be lower because in that case $V_H(\sigma_H)$ and $V_L(\sigma_L)$ would yield higher profits, and still satisfy the type truthful revealing constraints, so (P1) would not be maximized otherwise.

Proof of proposition 3.

First I prove that $V_H\frac{p_L}{p_H}=V_L$. We know from proposition 2 and the type incentive compatibility constraint that $V_L\geq V_H\frac{p_L}{p_H}$. Now suppose that $V_L>V_H\frac{p_H}{p_L}$. Then from remark 2 we know that if H signed the contract for L he would get higher equity:

$$V_H(\sigma_H)/p_H<V_L(\sigma_L)/p_L\leq V_H(\sigma_L)/p_H$$

$$V_H(\sigma_H)<V_H(\sigma_L)$$

Thus, $\{\sigma_H,\sigma_L\}$ cannot be incentive compatible. Hence, it must be that $V_H\frac{p_L}{p_H}=V_L$. This means that the no profits condition of the lender is:

$$g\Big(W_H(V_H)-V_H\Big)+(1-g)\Big(W_L\Big(\frac{p_L}{p_H}V_H\Big)-\frac{p_L}{p_H}V_H\Big)=(1+r)I_0$$

Next, note that if the P_is were public information, competition would force both contracts to make zero profits on their own. Define V_i^{PI} as the equity offered to entrepreneur i in this benchmark. We have that

$$W_H(V_H^{PI})-V_H^{PI}=(1+r)I_0$$
$$W_L(V_L^{PI})-V_L^{PI}=(1+r)I_0$$

Note that $\frac{p_L}{p_H}W_H(V)\geq W_L(V)$ for all V. This implies that the contract under public types is not incentive compatible. To see this, consider any V_L^{PI} such that the lender breaks even:

$$W_L(V_L^{PI})-V_L^{PI}=(1+r)I_0$$

And consider a incentive compatible $V_H^{PI}=\frac{p_H}{p_L}V_L^{PI}$. We have that:

$$W_L(V_L^{PI})\leq\frac{p_L}{p_H}W_H(V_L^{PI})\leq\frac{p_L}{p_H}W_H\left(\frac{p_H}{p_L}V_L^{PI}\right)$$

Since $p_H/p_L>1$ and W_H is an increasing function. This implies that $V_H^{PI}=\frac{p_H}{p_L}V_L^{PI}$ cannot make zero profits:

$$W_H\left(\frac{p_L}{p_H}V_L^{PI}\right)-V_L^{PI}>W_L(V_L^{PI})-V_L^{PI}=I_0(1+r)$$

In a similar way we can prove that if $V_L(\sigma_L) < V_L^{PI}$, the lender makes profits with both contracts, σ_H and σ_L. Thus, it must be that $V_L(\sigma_L) > V_L^{PI}$: the L type gets more equity than under observable p_is. This implies that:

$$W_L(V_L(\sigma_L)) - V_L(\sigma_L) < (1+r)I_0$$

And since the lender must break even, it must be that the lender makes some profit on σ_H:

$$W_H(V_H(\sigma_H)) - V_H(\sigma_H) > (1+r)I_0$$

Since W_H is a concave function, this condition implies that $V_H(\sigma_H) < V_H^{PI}$.

Proof of proposition 4.

This follows exactly the steps in proposition 3.

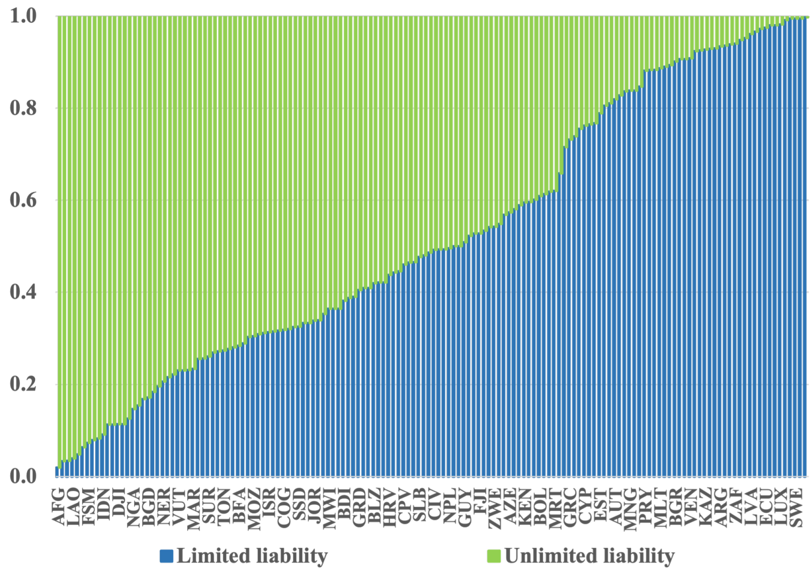
Chapter 2: **Liability Protection and Firm Growth**

1 Introduction

Entrepreneurship is risky. Entrepreneurs devote their time and a considerable fraction of their assets to projects that might not succeed, potentially losing sizable amounts. Thus, firm owners look for different instruments to protect themselves against failure, and one of the most common ones is limited liability. Entrepreneurs who set up firms with limited liability are not personally liable for firms' losses, so they only need to surrender the firm's assets in the case of default. On the other side, entrepreneurs operating under unlimited liability also need to pledge their assets to pay off firms' debts. Figure 1 shows the cross-country distribution of firms in the World Bank's Enterprise Survey between those who have limited and unlimited liability. Most countries count with a legal status that provides limited liability, which is used by 52% of all firms.

Figure 1: **Liability Distribution**

(% of total, last survey year)



Source: Own computations using World Bank's Enterprise Surveys. Unlimited liability firms include sole proprietorship and partnerships. Limited liability firms include limited partnerships and shareholding companies. Data is from the last survey round for each country.

What are the aggregate effects of limited liability? To answer this question, I focus on incorporation, a legal status that entrepreneurs can use to obtain limited liability, protecting themselves against downside risk. Under limited liability, entrepreneurs are not personally liable for firms' debts and losses. Thus, in case of default, they only lose the firm (with its assets) while keeping their personal estate intact. The critical trade-off in the incorporation decision is obtaining the insurance provided by limited liability in exchange for higher financing costs. Incorporated firms are more likely to default because the option to do so is less costly for the entrepreneur. As a result, these firms are riskier for lenders and obtain credit at higher interest rates. In addition, incorporating a business typically requires completing a more extensive series of administrative steps at the corresponding government. I study the effects of incorporation on primarily two firm outcomes: the technology entrepreneurs decide to operate and firm size. Entrepreneurs with limited liability have more incentives to operate larger firms and use more profitable but riskier technologies, because they do not fully bear the downside risk. My goal is to understand and quantify the effect of limited liability on firms' outcomes (technology adoption, firm size, and growth) and economic aggregates in a general equilibrium setting.

I proceed in three steps. First, I document that there are systematic empirical differences between incorporated and unincorporated (unlimited liability) firms: incorporated ones are larger and grow more, and at the same time, their productivity is higher and more volatile. I establish this by comparing firms with legal statuses that provide different liability protection in Chile but are otherwise equal: "Incorporated individual" and "Unincorporated individual" firms. Both are single-owned, and they differ only because the first type has limited liability, whereas the second type does not. I leverage the Chilean legislation because most countries do not have a separate legal status for single-owned incorporated firms. Instead, they offer one of limited liability *company*, which can have one or multiple owners. This difference is relevant when analyzing risk-bearing. I use the Chilean survey "Longitudinal survey of firms" (ELE), which includes official (tax) information about firms finances (balance sheet and statement of income), in addition to rich information on other firms' characteristics. With this data, I document first that firms with limited liability are, on average, larger and grow more than their counterparts. In terms of payroll, incorporated firms are 6.2 times larger than unincorporated ones, and they grow 5.8 times faster. Second, that the production processes for these firms are different: the productivity (TFP) of incorporated firms is 4% higher and shows 56% more volatility. Thus, these firms are more productive and riskier. In this part, I do not claim causation because all these variables (legal status, technology, firm size, and personal savings) are equilibrium objects, determined endogenously in the interaction between entrepreneurs, financial intermediaries, and the legal institutions they face. Thus, I take a structural approach and build a model to analyze them.

Second, I build a theory to rationalize these facts. I take a canonical firms dynamics model with limited commitment (Buera, Kaboski, and Shin, 2011; Midrigan and Xu, 2014) and augment it to include incorporation and default in equilibrium as in Herranz, Krasa, and Villamil (2017). In this economy, entrepreneurs are born with some random level of personal wealth, and they must decide what technology to operate and whether to incorporate or not. These characteristics are fixed through the firm's life. There are two possible technologies: *risky*, with high and more volatile TFP, and *safe*, with lower and less volatile productivity. After that, they decide the firm size (inputs) and capital structure (equity/debt) one period in advance before observing the productivity realization. Thus, conditional on everything else, entrepreneurs operating the *risky* technology are subject to more risk in the sense that their productivity and profits are more volatile than for the ones using the safe technology. If they get a sufficiently low productivity shock, they can decide to default. The cost of doing so will depend on the legal status they previously chose. Incorporated entrepreneurs need to surrender only the firm's cash flow, while unincorporated ones also must submit a fraction of their savings. Note that the incorporation and technology decisions are simultaneous and tightly related. Incorporation is more valuable while operating the risky technology because it provides insurance against low productivity shocks, which are more likely to happen under this technology. Consequently, risky entrepreneurs will be more willing to face the increased financing costs and pay some administrative fees in exchange for this insurance than safe ones.

A key outcome of this economy is the sorting of entrepreneurs along the dimensions of technology and legal status, which is driven by the heterogeneous wealth levels of entrepreneurs. In the baseline calibration of the model, (relatively) poor entrepreneurs operate the safe technology and remain unincorporated. For them, the downside risk of using the risky technology has a higher utility cost. At the same time, they prefer to stay unincorporated because they find incorporation relatively more expensive and less valuable given their technology choice. On the other hand, richer entrepreneurs prefer to operate the risky technology because it is more productive and they can bear the risk. In addition, for these entrepreneurs, incorporation is less costly (as a percentage of their wealth) and more valuable because of their preferred technology. Therefore, they choose to use the risky technology and to incorporate. After that, the combination of limited liability and a more productive technology leads them to set up larger firms, accumulate wealth faster and grow more.

Third, I quantify the aggregate effects of incorporation. For this, I calibrate the model for the Chilean economy, using a combination of parameters taken directly from the literature, some obtained from my data work, and others estimated using the Simulated Method of Moments. I find that the calibration successfully generates the mass of entrepreneurs that self-select into different legal statuses and the interest rates these firms face, which speaks directly to how likely are they to default. In addition, it closely replicates some key empirical facts that were not explicitly targeted, such as the size and growth differences between incorporated and unincorporated firms.

Next, I quantitatively evaluate the aggregate consequences of incorporation. To do this, I simulate two counterfactual economies. In the first one, the cost of incorporating a business is infinite, so no one does it. Intuitively, there is less risk-taking in this economy. Regarding technology choices, 3% fewer entrepreneurs operate the risky technology than in the baseline economy. The difference is explained by the marginal (3%) risky entrepreneurs in the baseline economy, who would not run this technology if they cannot insure themselves through limited liability. The remaining risky entrepreneurs, the richest ones, still use the same technology but set up firms 13% smaller because this is the only way they have to limit their losses in a world without incorporation. As a result, aggregate output in this economy is 6.7% smaller than in the baseline. In the second counterfactual, I set the incorporation cost to zero. In this economy, all firms decide to incorporate, regardless of the chosen technology. At the same time, 2% more entrepreneurs operate the risky technology. These are the marginal wealthiest entrepreneurs using safe technology. They were unwilling to run the risky business and pay for incorporation, but having it at no cost induced them to switch technologies, incorporate and increase the firm size. In aggregate terms, this generates an increase in output of approximately 2%. Making incorporation cheaper and extended to a larger fraction of entrepreneurs has unequivocally positive effects on output and productivity, showing that the insurance effect of limited liability is quantitatively more important than the negative effects of the increased financing costs [1]. Transitioning from the first counterfactual experiment to the baseline economy increases the mass of incorporated firms by 28%, and from the baseline to the second experiment by an additional 43%. However, the output gains are lower are inversely proportional to the new mass of incorporated firms. This divergence speaks of decreasing marginal returns of making incorporation cheaper. Every time incorporation is made more affordable marginal entrepreneurs switch from the safe to the risky technology. However, they are poorer than the previous risky entrepreneurs, so they set up smaller firms without contributing as much to total output.

Finally, I also evaluate two policies for the Chilean economy that make default less costly. The first one is an actual policy consisting of a series of measures to reduce the length of the bankruptcy process. The second one is a suggestion consisting of the reduction of its monetary costs. Both are complementary to incorporation because they make default less costly for entrepreneurs, and most of the default is committed by risky incorporated firms. I find that both of them generate output gains, driven mainly by an increase in the fraction of incorporated and risky entrepreneurs. These are individuals for which even with incorporation, default is too costly for their wealth level. When bankruptcy is made less expensive, they are willing to change.

This paper is organized as follows. Section 2 contains the literature review and contributions of this paper. Section 3 investigates the main empirical differences between incorporated and unincorporated firms. Section 4 builds a model to rationalize these facts and evaluate different related policies. Finally, section 5 shows the calibration of the model, its performance and the results of the counterfactual exercises and policies evaluations.

[1] There is debate in the literature on the net effect of limited liability on output, entrepreneurship, and firms' outcomes. Some papers find that it is positive (Fossen, 2014; Jia, 2015), while some others find that it is negative (Akyol et al., 2011; Meh et al., 2008). The key trade-off is always similar: insurance in exchange for higher financing costs. For more details see section 2

2 Related literature and contribution

This paper builds on the literature on the consequences of bankruptcy protection on entrepreneurship, firm financing, and outcomes. Most of these papers have focused on analyzing the effects of different exemption levels considered in personal bankruptcy laws. Typically, when filing for bankruptcy, debtors are allowed to keep some of their assets (exemption), use the rest to pay off debt, and discharge a fraction (not necessarily all) of the outstanding debt. These personal bankruptcy laws also apply to entrepreneurs when they are unincorporated (non-corporate) but do not affect other types of firms like limited liability companies. Similar to this paper, the fundamental trade-off is between the insurance effect of having a higher exemption level and the access to credit. Micro-econometric articles have found mixed results. Berkowitz and White (2004) and Berger, Cerqueiro, and Penas (2011), exploit differences in the exemption levels across US' states and find that a higher exemption level augments the probability of being credit rationed and firms that do get credit do so at a higher interest rate. Fan and White (2003) study the entrepreneurial decision of households and find that living in a state with high exemption levels can significantly increase the probability of starting a business but no effect on the likelihood of ending (exit) one. In contrast, Cerqueiro, Penas, and Seamans (2019) shows that changes in state exemption levels have a small positive effect on firm entry and job creation only in sectors with low start-up capital needs, and non-significant effects in the remaining ones. Cumming and Li (2013) find a negative relationship between exemptions and business births, number, and deaths, in addition to diminishing venture capital. Armour and Cumming (2008) look at a cross-country setting by building a database of fifteen countries during sixteen years and build measures on how forgiving bankruptcy laws are. They find that in countries where regulations are more forgiving, there is more self-employment. In a completely different setting, Koudijs and Salisbury (2020), exploit a change in marital property laws in the US south during 1840-1850 to show that liability protection increases household investment.

The macroeconomic-structural literature on the consequences of bankruptcy protection has also been mostly focused on personal bankruptcy and homestead exemption levels. Akyol and Athreya (2011) build and model of occupational choice over the life cycle with incomplete financial markets (Eaton and Gersovitz, 1981) to analyze the role of household decisions on self-employment, credit, and welfare. They find that when exemptions are increased, the disincentives arising from worsened credit conditions trump the positive effect of insurance, resulting in a decrease in entrepreneurial activity. Similarly, Meh and Terajima (2008) and Li and Sarte (2006) show that eliminating bankruptcy would be welfare decreasing, but diminishing exemptions can increase it.

They conclude that while some level of insurance is optimal (default), it needs to be well designed regarding exemptions. This finding contrasts work by Jia (2015) that, also in a life-cycle model, finds a positive correlation between the exemption level and entrepreneurial activity. On a different vein, Fossen (2014) uses a structural setting to evaluate the introduction of a fresh-start policy in Germany in 1999, finding a positive effect on self-employment. Different from these papers, Mankart and Rodano (2015) study the optimal exemption level in environments with and without secured debt. They find that the presence of secured credit alters the trade-off. When not available, a harsh bankruptcy law maximizes entrepreneurship and welfare. The opposite is true if secured credit is available.

All these previous papers discuss the effects of exemptions levels, which puts limits on entrepreneurial liability. However, they abstract from the legal statuses that provide limited liability, which is standard worldwide (figure 1). Herranz, Krasa, and Villamil (2017) do consider this decision to analyze firm size in the context of entrepreneurs with different risk aversion. In a partial equilibrium setting, they find that less risk-averse entrepreneurs incorporate, while more risk-averse ones do not. They explain this as a commitment device: more risk-averse entrepreneurs prefer to leave some assets exposed as a way of committing to lower default rates while at the same time operating relatively smaller firms. The closest paper to this one is Glover and Short (2018), where the authors use cross-sectional data of the 2003 version of Survey of Small Business Finances in the US to show that incorporated firms are larger and pay higher interest rates (conditional on other observables) than unincorporated ones. They propose a general equilibrium model with endogenous legal status. More productive entrepreneurs decide to incorporate to set up larger firms while protecting themselves against total losses. Through a quantitative evaluation, they find a positive aggregate effect of the option to incorporate.

I make the following contributions to this literature. First, I document novel systematic differences between incorporated and unincorporated firms. I show that incorporated ones not only are larger than their counterparts (Glover and Short, 2018) but they also grow more. In addition, they are more productive and riskier. In line with the micro-econometric literature, I also show that incorporated firms pay higher interest rates than unincorporated ones. This difference is significant even when controlling for other covariates. Second, I take a canonical model of firm dynamics with financial frictions (Buera, Kaboski, and Shin, 2011; Midrigan and Xu, 2014), and augment it with legal status and default in equilibrium. Relative to the literature on liability protection (exemptions and legal status), this model can explain a broader range of empirical relationships between liability protection, entrepreneurial wealth, technology adoption, and firm size and growth. Third, I contribute to the macroeconomic policy evaluation on bankruptcy rules by showing that higher liability protection works around two margins of risk-taking: firm size and technology decision. Thus, incorporation increases firm size and changes the technology distribution of the economy as it incentives some entrepreneurs to operate riskier and more profitable technologies. In addition, I show that making incorporation cheaper and extending it to a more significant fraction of entrepreneurs has marginal decreasing effects over output. In other words, leading new entrepreneurs to incorporate increases output monotonically but does so at a diminishing rate.

I also build on the literature that investigates the effects of risk on entrepreneurial activity and firms outcomes. Bailey Jones and Pratap (2020) show that entrepreneurial risk does not have a first-order impact on firms outcomes but abstain from considering default or from different ways entrepreneurs have to insure themselves against risk (incorporating is one of them). Gottlieb, Townsend, and Xu (2016), Choi et al. (2017) and Ejrnæs and Hochguertel (2014) show that protection against business failure has a positive effect on entrepreneurship, which they study in the case of job-protected leave, the outside option of working as employee and unemployment insurance, respectively. In a different vein, Bianchi and Bobba (2013) show that cash transfers also increase entrepreneurship. They argue that the effect is a consequence of higher willingness to bear risk and not only more relaxed collateral constraints. To these papers, I make two contributions. First, I show empirical regularities of firms with different levels of personal liability or protection against failure. Second, I study and quantify the effects of incorporation, a standard instrument entrepreneurs use to protect themselves against failure.

Papers in the field of macro development, such as Albuquerque and Hopenhayn (2004), Buera, Kaboski, and Shin (2011), Cole, Greenwood, and Sanchez (2016), Midrigan and Xu (2014), Moll (2014), and Verani (2018) have shown how default (limited commitment) can lead to inefficiencies in the financial markets that have considerable effects over output and productivity. In these studies, financial intermediaries will offer entrepreneurs suboptimal financial conditions, leading to inefficient firm sizes and misallocation. Quantitatively, they find significant effects over output, even though their magnitudes differ significantly. To this literature, I contribute by taking a complementary view. I show that default acts as insurance and that making default less costly with incorporation can positively affect economic aggregates.

Finally, I also contribute to the literature that investigates the determinants of heterogeneous outcomes across firms and how they relate to their permanent characteristics. (Belenzon, Chatterji, and Daley, 2017; Sterk, Sedláček, and Pugsley, 2021; Choi, Goldschlag, Haltiwanger, and Kim, 2021) show that firms have different permanent characteristics that determine their outcomes even at an advanced age. I contribute by proposing an empirically motivated mechanism: the interrelation between limited liability, entrepreneurial wealth, and the willingness to operate risky and more profitable technologies.

3 Empirical differences between incorporated and unincorporated firms

The two key facts that motivate this paper are that incorporated firms are larger and grow more than their unincorporated counterparts. Second, their productivity is, on average higher and more volatile. I document these facts using Chilean firm-level data. In this section, I avoid claiming causation because of concerns about self-selection and endogeneity. Entrepreneurs self-select into incorporation depending on non-observable factors, such as technology, wealth, financing opportunities, etc. At the same time, these could determine other firm decisions such as investment and capital structure. Thus, I only establish correlation in this section and address the self-selection issue through a structural model in the next section.

3.1 Data

I use the microdata of the Chilean survey of firms "ELE" (Spanish acronym for Longitudinal Survey of Firms) for 2012-2017[2]. This survey is collected by the Chilean National Institute of Statistics (INE) every two years. It is designed as a rotating panel, where a fraction of the firms are followed from round to round, and the rest are chosen randomly. Each survey version includes approximately 5,000 firms (with unique identifiers) that are nationally representative at industry/size level. Its universe consists of all firms with sales larger than $US\$2,500$, approximately[3] . It is important to highlight that it includes all sectors in the economy. Most of the survey is of public access. However, some key variables have restricted access because of their potential for revealing the real identity of the firms in the dataset. For example, I had to request the economic sector at 2, 3, and 4 digits for this paper[4].

[2]This is from third to fifth round of the survey. Previous two rounds have gap years that prevent observing firms continuously. Data from the sixth round will be released in 2022.

[3]ELE is designed not to include self-employment

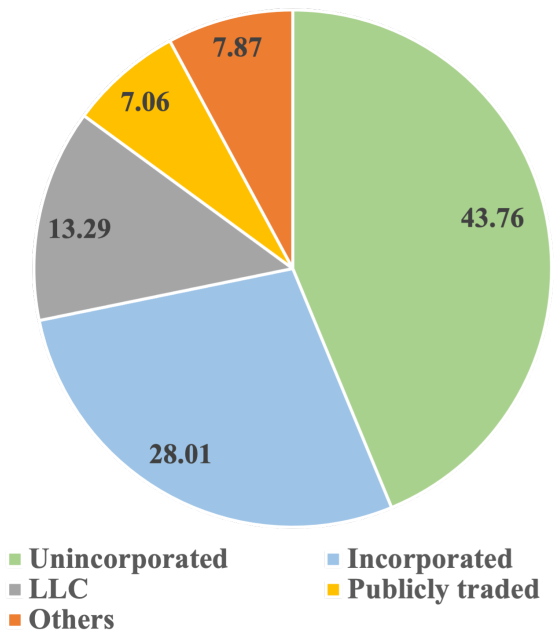
[4]For example: (a) "2-digit" sector 47: Retail sales, except motorized vehicles. (b) "3-digit" sector 472: Retail sales of food, drinks and tobacco in specialized establishments. (c) "4-digit" sector 4721: Retail sales of food in specialized establishments

ELE includes detailed information on firm characteristics and outcomes, such as when the firm was legally constituted, ownership structure, economic activity, labor force, and finances. A key feature for this paper is that official financial statements must back up all financial information. This way, I observe key variables such as revenue, fixed assets, payroll, capital structure, and financial expenditures. In addition, there is some survey financial information: firms are asked about access to different financial instruments, their financing sources, problems meeting their payments, causes of their financial needs, and other variables. Unrelated to this paper, the survey also includes information about various topics, such as general management practices, perception about the economy and government, RD activity, access and use of information technologies, and others.

3.2 Firm size and growth

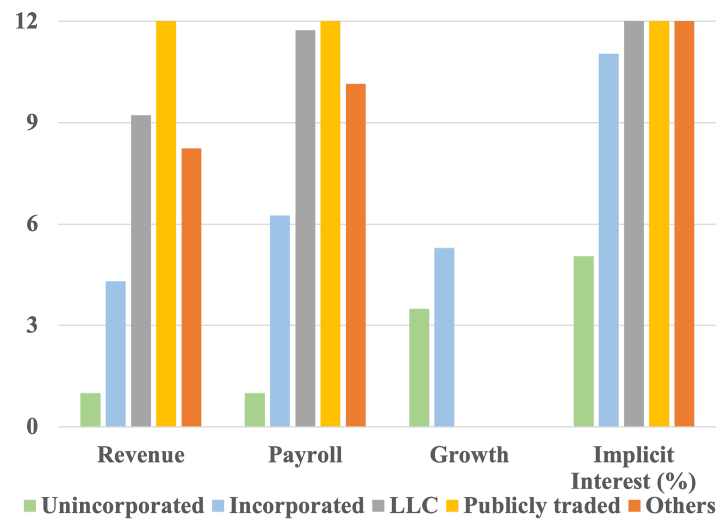
Figure 2 shows the distribution of firms over different legal statuses. Incorporated and unincorporated firms are by a wide margin the two preferred choices by entrepreneurs. At the same time, these firms are of a smaller scale than Limited Liability Companies (LLCs) and Public Corporations. Figure 3 shows three different measures of firm size: revenue, payroll, and fixed assets. Incorporated firms are on average 4.3, 6.3, 6.0 times the size of their unincorporated counterparts. Their differences with LLCs and Public Corporations are much more significant. LLCs have revenue 9.2 times larger than incorporated firms, and Public Corporations 70.6 times. On aggregate, despite comprising almost 50% of the firms in the economy, unincorporated firms obtain only 5% of all revenue (Figure 10). Incorporated ones get 14.2%. Note that these are all endogenous objects and represent the equilibrium of different forces. In this paper, I explain the differences in size and legal status decisions as a consequence of a combination of entrepreneurial wealth, the technologies entrepreneurs operate, and limited liability.

Figure 2: **Types of firms**
(% of total, 2017)



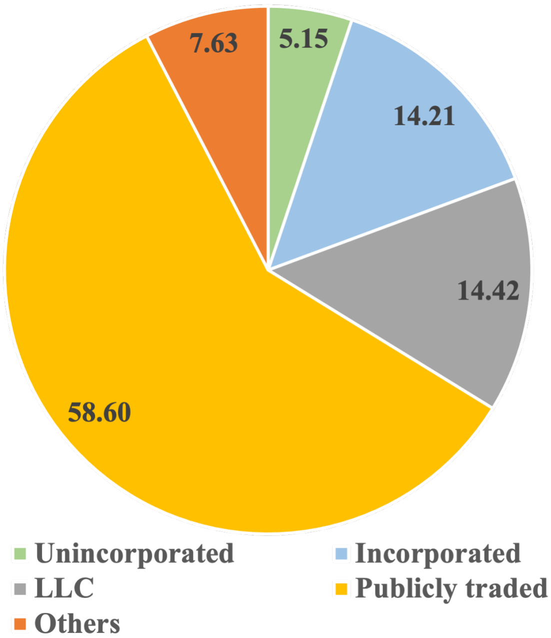
Source: Own computations based on survey "ELE". LLC are limited Liability companies with no majority owner, called "Limited liability partnerships" in the Chilean legislation.

Figure 3: **Summary Statistics**
(averages, 2012-2017)



Source: Own computations based on survey "ELE". LLC are limited Liability companies with no majority owner, called "Limited liability partnerships" in the Chilean legislation. Implicit interest is the ratio total financial expenditures in a year to interest generating liabilities. Revenue, payroll and fixed assets are indexed to unincorporated=1. Values out of the graph for Publicly traded corporations are 70.6 (revenue), 50.9 (revenue), 108.5 (fixed assets).

Figure 4: **Revenue Distribution**
(% of total, 2017)

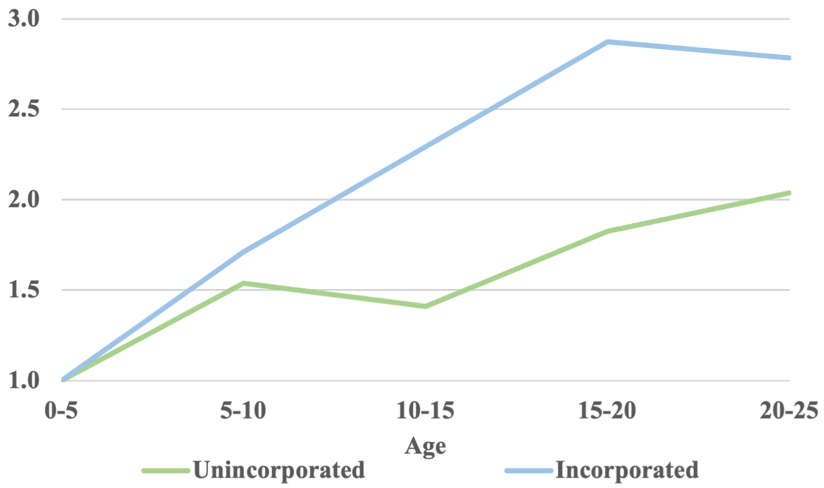


Source: Own computations based on survey "ELE". LLCs are limited liability companies with no majority owner, called "Limited liability partnerships" in the Chilean legislation.

Next, I decompose the size differences between incorporated and unincorporated firms between level and growth. The first component is entirely static and corresponds to the difference in size when firms are set up. The second one is the difference emanating from their respective growth rates. To do this, I follow Hsieh and Klenow (2014) and group firms by age in 5 years groups and compute their average payroll. In the age group of 0-5 years, this is, firms that started operating recently, incorporated firms are 4.1 times larger than unincorporated ones. Thus, the level effect explains 66% of the total size difference, while growth rates explain the remaining 33%. Figure 5 shows the average size for the remaining age groups, conditioning on their starting point. After 20 years, unincorporated and incorporated firms are roughly 2.1 and 2.9 times their initial size, implying an average yearly growth rate of 5.4 and 3.6%.

Then I run two separate regressions of size (revenue) over firm age and sector and year fixed effects, one for each type of firm. In these regressions, the constant term will capture size around the beginning of operations. At the same time, growth will be captured by the age regressor. After running these regressions, I test whether these terms are statistically different. Table 1 shows the results of this exercise. The constant term is 5.5 times larger for unincorporated firms, and this difference is statistically significant. Thus, this confirms that incorporated firms are already larger when they start operating. In addition, by looking at the age term, we see that they

Figure 5: **Payroll growth over life cycle**
(index, synthetic cohorts)



Source: Own computations based on survey "ELE". Datapoints correspond to the average of firms within the age range for the period 2012-2017. Data is indexed to the average size of firms aged 0-5 years, for each category.

also grow faster. Incorporated firms add 7.1 times more revenue each year. These findings are in line with papers such as Sterk, Sedláček, and Pugsley (2021) and Choi, Goldschlag, Haltiwanger, and Kim (2021), which show that firms' persistent, underlying characteristics are an important determinant of their performance. In this case, firms with different legal statuses display different sizes when they are born and growth trajectories afterward.

Table 1: **Regressions on firm size (revenue)**
(pooled regressions, 2012-2017; USD, K)

Firm type:	**Unincorporated**	**Incorporated**	**Statistically ≠? (test p-value)**
Age	7.6***	44.0***	0.00
Level (constant)	118.6***	360.4***	0.00
N	3,428	7,849	–
R^2	0.01	0.02	–

Source: Own computations based on Survey "ELE". *, ** and *** = significant at 10, 5 and 1% level. Third column shows the P-value of t-test testing whether the estimators from both regressions are statistically different. Dollar values where obtained using the average nominal exchange rate during 2012-2017.

3.3 Trade off with financing costs

After a sufficiently negative shock, the option to default yields a higher value for incorporated entrepreneurs because they can keep all of their assets. In contrast, unincorporated ones need to surrender some of their assets to pay off the firm's debt. Thus, incorporated firms have higher incentives to default, conditional on firm size, debt, and entrepreneurs' savings. Moreover, if they decide to do so, the lender can recover a (weakly) lower fraction of the loan. These two effects together make the financing costs of these firms higher (Herranz, Krasa, and Villamil, 2017). To see this, I use the ratio of total financial expenditures to interest generating liabilities, which is a proxy measure of the interest rate (Bai, Lu, and Tian, 2018) that includes all costs of external financing, including interest rate payments and commissions. In Figure 2, we observe that the average implicit interest rate is 5% higher for incorporated firms, in line with the theoretical prediction. Table 2 presents the results of a regression of the interest rate over legal status dummies and a set of other control variables. As expected, the dummy for incorporated firm is positive and statistically significant.

Table 2: **Financing costs regressions**
(pooled, 2012-2017)

Dep. Variable:	**Financing Costs**
Incorporated Dummy	0.035***
N	$7,142$
R^2	0.033

Source: Own computations based on Survey "ELE". *, ** and *** = significant at 10, 5 and 1% level. Control variables include legal satus dummies, age, debt ratio (debt/assets), sector fixed effects and year fixed effects. Baseline category is unincorporated.

3.4 Legal status and differences in productivity

I hypothesize that part of the difference in size and growth between incorporated and unincorporated firms is a consequence of the underlying technologies these firms operate. There are two mechanisms through which this works. First, there is a profitability effect: incorporated firms have higher and more volatile productivity.

Having larger expected productivity leads these firm owners to set up larger firms. At the same time, more volatility makes higher productivity shocks more likely, allowing these firms to grow faster. This volatility effect is asymmetric with low productivity shocks because incorporated entrepreneurs are more protected from the downside risk. In case they suffer a negative productivity shock, they will not lose as much as unincorporated entrepreneurs.

To analyze the productivity of these firms, I compute three different measures. For the first two, I assume that firms in sector i have the following production function

$$Y_i = exp(z)\left(l^{\alpha_i} m^{\beta_i} k^{1-\beta_i-\alpha_i}\right)^{\theta_i}$$

where l, m, and k are labor, materials, and capital, respectively. Note that the inputs shares and the scope parameter are sector-specific. I estimate these parameters in two alternative ways. First, I obtain α and β as the share of labor and expenditure in intermediate inputs over revenue. Similarly, I compute θ_is as ones minus the share of profits over revenue. Second, as a robustness check, I estimate them using the procedure proposed by (Wooldridge, 2009). In both cases, I obtain the productivity, z, as the Solow residual (TFP). The third measure of productivity I consider is value-added per worker. Table 3 shows summary statistics for the three measures. We can see that limited liability firms (incorporated) have larger average and more volatile productivity in all cases. Note that the results of the two methodologies I used to compute TFP are very similar.

Table 3: **Productivity, summary statistics**
(logs, pooled, 2012-2017)

Firm type:	**Unincorporated**	**Incorporated**	**Partnerships**	**Pub. Traded**	**Others**
TFP: Residual					
Mean	-0.0605	-0.004	0.048	0.59	-0.06
SD	0.450	0.516	0.596	0.643	0.609
TFP: Wooldridge					
Mean	-0.073	-0.009	0.043	0.465	-0.099
SD	0.467	0.525	0.604	0.821	0.586
VA/Worker					
Mean	-0.082	-0.013	0.068	0.571	-0.310
SD	0.889	0.960	0.902	1.013	1.094

Source: Own computations based on Survey "ELE". TFP residual is computed using sector specific inputs and profits shares. Wooldridge using the methodology proposed by Wooldridge (2009). Data is detrended using economic sector and year fixed effects. Mean and SD are the pooled data mean and standard deviation, respectively.

Recent literature has shown that firms' productivity has a permanent and a transitory component(Bailey Jones and Pratap, 2020; Sterk, Sedláček, and Pugsley, 2021). Following (Bailey Jones and Pratap, 2020), I estimate the following equation for each sector

$$TFP_{it} = P_i + Y_t + e_{it},$$

where P_i is a firm-level fixed effect that captures the permanent component of TFP, Y_t is a sector-year fixed effect, that captures all shocks affecting firms in the same sector, like a generalized demand or a inputs price shock, and e_{it} is an AR(1) shock. I am isolating the idiosyncratic productivity After that, I estimate the AR(1) process of the shocks to obtain the innovation and the lagged component of the errors using the standard formulation

$$e_{it} = \rho e_{it-1} + \varepsilon_{it}.$$

Note that since I removed sector and year effects in the first stage, the innovation ε_{it} is idiosyncratic to the firm. Table 4 show the average value of the permanent components (P_i) and the standard deviation of the innovations (ε_{it}). We can observe that the unconditional expected productivity of incorporated firms is approximately 4% larger than unincorporated firms. At the same time, their shocks are more volatile, displaying 56% more volatility. In this sense, incorporated firms are more productive and riskier.

Table 4: **TFP components**
(pooled, 2012-2017)

	Innovations (ε_{it}), SD	**Permanent, mean**
Incorporated	0.38	0.02
Unincorporated	0.25	−0.02

To show that these differences are statistically significant, I regress the permanent component of TFP on legal status dummies. To show that they are riskier, I do the same for the absolute value of the innovations of the shocks. Table 5 shows that the results of both regressions confirm my hypothesis.

This relates directly to papers such as Belenzon, Chatterji, and Daley (2017) and Choi, Goldschlag, Haltiwanger, and Kim (2021) and Sterk, Sedláček, and Pugsley (2021) that show that the permanent characteristics of firms have important effects on their outcomes even at an advanced age. In the case of this paper, firms under different legal status show different performance.

Table 5: **Permanent and transitory components of TFP**
(pooled, standarized coeffs.)

Dependent var.:	**Innovations (ε, abs)**	**Permanent TFP (P)**
Incorporated	0.092***	0.130***
Partnership with limited liability	0.064***	0.162***
Publicly Traded	0.002	0.088***
Others	0.015	-0.005
N	8,573	13,261
R^2	0.014	0.058

Source: Own computations based on Survey ”ELE”. *, ** and *** = significant at 10, 5 and 1% level. Baseline category is unincorporated.

3.5 Other differences between Incorporated and Unincorporated firms

So far, I have documented that the TFP processes of incorporated and unincorporated firms are different. In table 6, I show that this is associated with being ”higher-skilled” or more intensive in human capital. Thus, liability protection can play a relevant role in the growth of firms in sectors typically considered to have more advanced technologies. On average, incorporated firms hire 15.22% more workers with tertiary education than unincorporated ones, and spend a larger portion (+16.28%) of their payroll on high skilled workers [5]. In addition, they pay a significantly higher (average) wage (+55%) than their counterparts, signaling a better quality (higher marginal product) of human capital. This difference is consequence of a different composition of their labor force between skilled and unskilled workers, and different wages paid to these groups. The last two rows of table 6 show that the average wage difference is substantially higher for skilled workers than for unskilled ones. Thus, both groups are more productive for incorporated firms, but skilled workers are more so by a considerable margin.

[5] High-Skilled workers include those with tertiary education and directives

Table 6: **Human Capital by Firm Types**
(Averages, 2012-2017)

Firm type:	**Unincorporated**	**Incorporated**
Workers with tertiary education (%)	16.45	31.63
Income Share, high skill workers (%)	22.10	38.38
Wages	1.00	1.55
Wages, high skill workers	1.00	1.44
Wages, low skill workers	1.00	1.22

Source: Own computations based on Survey "ELE". High skilled workers include workers with tertiary education and directives. Wages are averages indexed to the average of unincorporated firms.

In the same line, table 7 analyzes different activities of both types of firms. Panel A shows that Incorporated firms are more active in doing research (of any kind), and are more likely to hire specialized personnel to do it. In addition, in Panel B we can observe that they make a more intensive use of basic information technologies like the internet, websites and any type of software. These two Panels point in the same direction of Incorporated firms being more technologically advanced on average. Panel C shows that the large majority of firms in both groups does not export, which is in line with the fact that they are relatively small compared to the remaining firms. Still, incorporated firms are more likely to do it. Finally, these firms are 26% more likely to use any type of insurance [6], showing that these firm owners are more active in looking for ways to protect themselves from different risk sources.

Table 7: **Other Firm Characteristics**
(%, 2017)

Firm type:	**Unincorporated**	**Incorporated**
Panel A: **Research**		
Does Research	5.83	14.35
Hires Qualified Researchers	42.65	73.32
Panel B: **Information Technologies**		
Uses Internet	87.42	95.66
Has Website	13.71	43.12
Uses (any) Software	87.86	95.97
Panel C: **Other**		
Exports	1.63	3.32
Uses (any) insurance	25.13	51.00

Source: Own computations based on Survey "ELE". Percentages represent the fraction of firms that engage in each of these activities. Research can be basic, applied or experimental development. "Qualified Researchers" are high education workers hired to do research.

[6] Related survey question asks about insurance for fixed assets, products, working capital and others (generically).

4 A model of firm dynamics with legal status and default

I study an economy with risky entrepreneurship, legal status and default. At its core, there are households deciding how much risk to take and how to protect themselves. There are two margins of risk taking: the technology entrepreneurs operate and the size of the firm. There are two available technologies, with one of them yielding higher and more volatile returns. At the same time, larger firms are riskier because they could lead to larger losses.

Entrepreneurs are risk averse. The only instrument available to protect themselves against losses is incorporation. If they decide to incorporate, they are not personally liable for firm's losses. In the case of default, they only need to surrender the firm. If they decide to remain unincorporated, they also need to pledge their personal savings to pay off firms' debt. There are two costs of incorporating a business. First, the entrepreneur must incur in some administrative fees associated with a series of necessary bureaucratic steps. Second, since the option to default is more attractive for incorporated entrepreneurs, they will be riskier for financial intermediaries, and will obtain credit at higher interest rates.

The underlying source of heterogeneity is the wealth with which entrepreneurs are born. They will decide different combinations, legal status, technology and firm size depending on their initial wealth.

4.1 Entrepreneurs

I consider a small open economy, populated by a measure 1 of workers and a measure N of entrepreneurs. Both are born with some initial savings s_I drawn from a distribution $g(s_I)$, with g assumed to be Pareto with scale 1 and shape α_g. They die exogeneously in every period with probability η, and they are replaced next period with a newly born entrepreneur. As in Midrigan and Xu (2014) and Verani (2018), workers can only work at the market wage ω, and entrepreneurs can only create and operate firms. While the occupational choice margin might play and important role, a clear answer on the underlying motives of entrepreneurship and their quantitative relevance is still part of an on going debate in the literature (Moskowitz and Vissing-Jørgensen, 2002; Kartashova, 2014; Jovanovic, 2019). Thus, I have opted for abstaining from such considerations. Entrepreneurs seek to maximize their lifetime utility, $\sum_{t=0}^{\infty}(\beta(1-\eta))^t u(c)$, with $u(c) = \frac{c^{1-\xi}-1}{1-\xi}$. They have two sources of income: savings (s_t), which earn the risk free interest rate r_f, and the profits generated by their firms.

As in Quadrini (2000) there is only one good in the economy that can be produced using different technologies.

Entrepreneurs decide what technology to operate out of two options: *risky* (R) or *safe* (S). Both are standard Cobb-Douglas production functions with decreasing returns to scale, but they differ in that the productivity (TFP) of the risky technology is on average higher and more volatile, and in that operating it requires paying a fixed cost f_R every period. Note that this fixed cost induces a minimum efficient scale of operation, which limits the ability of entrepreneurs of minimizing the risk associated with the risky technology by setting up sufficiently small firms. The production of each type of technology is

$$Y_i(K, L, \varepsilon) = \ exp(\mu_i + \varepsilon)\big(K^\alpha L^{1-\alpha}\big)^\theta, \ \ i = \{R, S\}$$

with $\theta < 1$. μ_i is a permanent component of the productivity, fixed all through firm's life, such that $\mu_R > \mu_S$. ε_{it} is a random shock that follows an AR(1) process

$$\varepsilon_{it} = \rho\varepsilon_{it-1} + e_{it}.$$

The innovations, e_{it} are assumed to be distributed normally, with variance σ_i, this is, $e_{it} \sim N(0, \sigma_i)$, where $\sigma_R > \sigma_S$. The differences in the permanent component of productivity and the variance of the shocks capture the idea that more profitable firms are also riskier.

Firms need to pay for inputs using working capital W_k, which is decided one period in advance, before the realization of the TFP shock in the period. Thus, conditional on everything else, entrepreneurs operating the risky technology (R) bear more risk in the sense that their profits are more volatile. Given the working capital W_k, a non-defaulting firm operating technology i chooses inputs K and L to maximize the expected production Y, this is

$$Y_i^*(W_k) = Max_{\{K,L\}} EY_i(K, L, \varepsilon) - f_i,$$

$$s.t. \ \omega L + rK \leq W_k$$

$$i = \{R, S\}, \ f_S = 0, \ f_R > 0,$$

where ω is the wage and r is the rental rate of capital. The solution to this problem gives the following input demands

$$K^*(W_k) = \alpha W_k / r$$

$$L^*(W_k) = (1 - \alpha) W_k / \omega$$

Working capital can be financed using equity, entrepreneurs own funds, and debt, borrowed from financial intermediaries. I define x_t as the fraction of working capital that is financed with debt. The capital structure decision, x_t, is bounded to the set $(0, \bar{x})$ with $x_t < \infty$, to avoid potential concerns about agents running infinitely leveraged bets with positive expected value. In principle, x_t could be larger than 1 if debt was also used to finance dividend payouts or liabilities, or if the price of this debt were low enough. The associated profits are

$$\pi_i(W_k, x, \varepsilon) = Y^*(W_k, \varepsilon) - W_k x.$$

Entrepreneurs can default after observing the realization of the TFP shock. In this case, the lender seizes all firms' cash-flow and a fraction $(1 - \gamma)$ of the entrepreneur's savings (s_t). The value of γ will depend on the legal status they have previously chosen. If they are incorporated, $\gamma = 1$, thus, personal savings are shielded from the lender. If they are unincorporated, the lender can obtain a fraction $\gamma = \gamma_U \in (0, 1)$. Upon default, the entrepreneur losses control of the firm and cannot set another one immediately. The indicator function $d^N_{i\gamma}(W_k, x, s, \varepsilon)$ takes value 1 if the previously non-defaulting entrepreneur decides to default and 0 otherwise. With probability p_r she leaves the default state at the beginning of next period and can set up another firm.

Entrepreneurs must decide between both legal statuses and blue-print technologies when they are born, and this decision is permanent. Every period t, after observing the productivity shock ε, they decide default, consumption c, savings s, working capital W_k, capital structure x_t, and firms inputs, K and L. The timing is the following

- **(i)** Entrants choose technology and legal status.
- **(ii)** z realized. Inputs are purchased. Production takes place .
- **(iii)** Non-entrants decide default. If so, they surrender $y + (1-\gamma)$ of their savings. $\gamma = \{\gamma_U, 1.0\}$.
- **(iv)** Active entrepreneurs decide c, s', x', W_k'. Inactive ones choose c, s'.
- **(v)** Die exogenously with prob. ν .

Finally, there is a corporate sector of measure N_c that is not subject to any frictions or uncertainty. It produces with blueprint technology $Y = exp(\mu_C)(K^{\alpha}L^{1-\alpha})^{\theta}$.

4.2 Workers

Workers in this economy are standard, they maximize their lifetime utility $\sum_t^{\infty} \beta^t(u(c_t) - A_w(l_t))$, where the utility from consumption is the same as for entrepreneurs, l_t is the labor supplied by entrepreneurs and A_w is the utility cost of labor such that $A_w' > 0$ and $A'' > 0$. They choose consumption c_t, savings s_t and labor supply l_t subject to the following budget constraint:

$$c_t + s_{t+1} \leq \omega l_t + (1 + r_f)s_t$$

4.3 Recursive Problem

Consider an entrepreneur operating technology i under legal status γ. She enters the period with working capital W_k, capital structure x and savings s. She begins drawing a productivity shock e, and must decide whether to default or not. If she does not default, her budget constraint is

$$c + s' \leq (1 + r_f)s + Y_i^*(W_k, x, \varepsilon) - W_k x + qx'W_k' - W_k'$$

and her Bellman equation is

$$V_{i\gamma}^N(W_k, x, s, \varepsilon) = Max_{\{W_k', x', s', c'\}} u(c) + \beta(1-\eta)EV_{i\gamma}(W_k', x', s', \varepsilon')$$

where $V_{i\gamma}(W'_k, x', s')$ is the value of the problem before the default decision. Note that conditional on legal status (γ), technology (i), and the default state, the state space will be given by working capital (W_k), capital structure (x), savings s and productivity (ε). I have done this for expository clarity and it is equivalent to the notation in papers like Quadrini (2000) and Herranz, Krasa, and Villamil (2017) where the state space includes exclusively the total wealth of the entrepreneur [7]. Alternatively, if she decides to default, the budget constraint is

$$c + s' \leq \gamma(1 + r_f)s$$

and the Bellman equation is

$$V^D_{i\gamma}(W_k, x, s, \varepsilon) = Max_{\{s'\}} u(c) + \beta(1-\eta)\big(p_r EV_{i\gamma}(0, 0, s', \varepsilon') + (1-p_r)EV^D_{\gamma}(0, 0, s', \varepsilon')\big).$$

The value of the problem before the default decision is

$$V_{i\gamma}(W_k, x, s, \varepsilon) = max\big(V^N_{i\gamma}(W_k, x, s, \varepsilon), V^D_{i\gamma}(W_k, x, s, \varepsilon)\big)$$

and the default decision is given by the following dichotomy

$$d_{i\gamma}(W_k, x, s, \varepsilon) = \mathbb{1}\big(V^N_{i\gamma}(W_k, x, s, \varepsilon) < V^D_{i\gamma}(W_k, x, s, \varepsilon)\big).$$

Thus, $d^N_{i\gamma}(W_k, x, s, \varepsilon)$ is the associated indicator function that takes value 1 if the entrepreneur decides to default, and 0 otherwise. Note that with probability p_r, the entrepreneur will leave the default state so that default indicator (d') will be 0 next period, and with probability $1 - p_r$ it will remain at 1.

Now consider the problem of a newly born entrepreneur who drew initial savings s_I. She must decide what technology to operate and what legal status to use. I assume she does so before finding out her productivity shock for the period. If the entrepreneur decides to incorporate, she needs to pay an administrative fee IC, which captures the fact that she needs to run some errands at the government, and complete a larger series of steps before operating her business. However, in the baseline calibration and quantitative exercises, this cost is such that all the initial wealth distribution can afford it. The optimal combination of technology and legal

[7] Conditional on the default decision, the policy functions only depend on the total wealth (not savings or working capital in particular) and productivity. However, the default decision will depend on working capital, capital structure and savings. To keep the notation constant across entrepreneur's decision stages, I preferred to make all policy functions depending on $(i, \gamma, W_k, x, s, \varepsilon)$.

status for such entrepreneur is

$$\{i(s_I), \gamma(s_I)\} = argmax_{i,\gamma}\, EV_{i\gamma}(s)$$
$$s.t.\, \gamma \in \{1, \gamma_{unc}\},\ i \in \{S, R\}$$
$$s = s_I - IC \times \mathbb{1}(\gamma = 1)$$

This expression states that the entrepreneur chooses the combination that provides the highest expected value given their initial wealth. For convenience, I have omitted the remaining state variables (W_k, x, ε) from the value function. Since the entrepreneur is not operating the firm yet there is no working capital at the beginning of the period $W_k = x = 0$, and the productivity innovation ε is still unknown. The expected value is taken with respect to the productivity shock. The first constraint describes the available options, and the second one that if the entrepreneur were to choose to incorporate ($\gamma = 1$), she needs to pay IC.

4.4 Financial intermediaries

In this small open economy, the risk-free interest rate r_f will be given by the international financial markets. The domestic lenders are risk neutral, perfectly competitive and have access to all of the entrepreneurs' information, including their personal savings and productivity. After lending qxW_k to the entrepreneur, she either pays back xW_k or defaults. In case of default, the lender seizes all the cash-flow of the firm π and a fraction $1 - \gamma$ of the entrepreneur's savings, or the outstanding debt, whichever is higher. The debt price schedule will reflects the default probabilities and what the lender can recover in case of default. In case of default the lender recovers

$$Rec_{i\gamma}(W_k, x, s, \varepsilon) = min\{(1 - \gamma)s + Y(W_k, \varepsilon), xW_k\}$$

which clearly depends on the firms' technology and legal status. In consequence, firms that differ along these two dimensions will face different debt price schedules. These can be described by the break even condition of the lenders,

$$q_{i\gamma}(W_k, x, s, \varepsilon)xW_k = \frac{E(1 - d_{i\gamma}(R, x, s, \varepsilon))xW_k + E\big(d_{i\gamma}(W_k, x, s, \varepsilon)Rec_{i\gamma}(W_k, s, \varepsilon)\big)}{1 + r_f}.$$

4.5 Equilibrium

Definition: Recursive competitive equilibrium A recursive competitive equilibrium of this economy consists on:

1. Entrepreneurs legal status (γ) and technology (i) decisions.
2. Entrepreneur's policy functions: c, W_k, s, x, d.
3. Workers policy functions: c, s, l.
4. Debt price schedule: q.
5. Wage: ω.
6. Ergodic distribution of firms Υ.

Such that:

1. Given prices (q, r_f, ω), the policy functions $(i, \gamma, c, W_k, x, s, d)$ solve the entrepreneurs' problems.
2. Given prices (r_f, ω), the policy functions (c, s, l) solve the workers' problem.
3. Given the policy functions (W_k, s, d, x), technologies (i) and legal status, (γ) the debt price schedule q is such that lender breaks even in expected value.
4. Labor and goods market clear.

The solution algorithm can be found in the appendix A.

5 Aggregate effects of incorporation

I calibrate the model using a combination of parameters taken from the literature, others directly from the data, and a group calibrated using the simulated method of moments. Then, I show the results of the quantitative evaluation of the model. In equilibrium, wealthier entrepreneurs decide to operate the risky technology and incorporate, while the poorer ones prefer to operate the safe technology and remain unincorporated. In this economy, incorporation induces risk-taking in two dimensions: incentivizing entrepreneurs to operate the riskier and more profitable technology and operate larger firms. In the analysis of the baseline economy, I find that it successfully generates the size and growth differences between incorporated and unincorporated firms, even though these moments are not explicitly targeted. Next, I decompose the size difference into the combination of entrepreneurs with heterogeneous wealth, different underlying technologies, and the insurance effect provided by incorporation. I show that the technology explains roughly 70% of the difference, while the insurance effect another 21%.

Next, I turn to evaluate different policies and experiments. First, I run two counterfactual experiments, where the incorporation cost IC is set to infinite and zero. In the first case, fewer entrepreneurs operate the risky technology, and those who do, set up considerably smaller firms. As a result, the aggregate output is 6.7% lower than in the baseline economy. The opposite happens when it is zero, but the output gains are smaller (2.0%). However, there are decreasing marginal returns of making incorporation cheaper because the additional entrepreneurs adopting the risky technology are relatively poor, so they set up smaller firms and contribute less to output. Second, I also evaluate two policies that make default cheaper and expedited and show that they induce incorporation, with the corresponding effect on technology adoption and output. This complementary follows from the fact that this makes default more attractive, which incorporated entrepreneurs exercise more.

5.1 Calibration

To calibrate the model, I divide the parameters into three sets.

Parameters taken from external sources: This group is composed of mostly standard parameters taken directly from literature or outside sources. Following Atkenson, Khan, and Ohanian (1996) I set $\theta = 0.85$. From Midrigan and Xu (2014) I take and the risk aversion parameter $\xi = 1$, which implies that $u(c) = log(c)$. I choose a relatively low value for risk aversion to avoid possible concerns about the results of this paper depending on assuming particularly high levels of entrepreneurial risk-aversion. Following Verani (2018), I assume that the utility cost of labor for workers is

$$A_w(l) = A_c l^{\frac{1+\nu}{\nu}}.$$

The elasticity of labor, ν, is set to 1 following the same paper. Using the Penn World Table 10.0, I set δ and α to match the average depreciation rate and labor share of income in Chile during the survey period (2012-2017). I set the risk-free interest rate, r_f, to match the average yearly interest rate if the bonuses issued by the Chilean government that had the highest debt qualification and the lowest sovereign risk premium in Latin America during the period. The average bankruptcy process in Chile takes 3.2 years, as reported by the Chilean Ministry of Economy, Development, and Tourism. During this period, pending debts are included in a public registry similar to the credit reports in the US. Once the bankruptcy process finalizes, this information is erased from the system, and the entrepreneur can start with a clean slate. I set p_r, the probability of being reinstated as an entrepreneur after default, to be 1/3.2 so that it matches the expected length of the bankruptcy process. The implicit assumption is that defaulting entrepreneurs have no access to financial markets and cannot set up another firm during this period. The discount rate is set to $\beta = 1/(1 + r_f)$, as standard. The measure of workers in the economy is normalized to 1, and the mass of entrepreneurs, N, is set to 0.086 so that the ratio of entrepreneurs to workers matches the empirical one according to the Chilean IRS. The last parameter of this group is the incorporation cost IC. Intuitively, it corresponds to the additional administrative steps required to incorporate a business. To operate an incorporated firm in Chile is necessary to register it as such at the Chilean IRS. Unincorporated firms do not need to do this, as they can operate as if they were independent workers, which does not require any errand. The World Bank's Doing Business reports that during the analysis period (2012-2017), it took four days to register a business in Chile, which is 1.6% of the annual working time in the country. Thus, I set IC to be 1.6% of the equilibrium wage. Since it is an opportunity cost, I allow for it to change with the market wage.

Parameters taken directly from the data: These parameters are related to the productivity process as described in section 2. To calibrate them, I use the results of the Solow residual estimation of the TFP. First, I set μ_i and σ_i to match the average and standard deviation of the productivity process for incorporated and unincorporated firms. Here, I am following a guess and verify strategy: I assume that in equilibrium, the risky technology firms decide to incorporate and the traditional ones do not. Thus, the expected productivity of the modern sector is the empirical average of incorporated firms; and the same is true for standard deviation and unincorporated ones. Later, after solving the model computationally, I verify that this is the case. It is possible that I do not observe some of the productivity shocks at the very bottom of the distribution because entrepreneurs always prefer to default if they get them, exiting the sample. However, if this were the case, it would be more significant for incorporated entrepreneurs, who already display more variance in the data. In such a case, this calibration would be understating the risks of the risky technology, and the quantitative results of this paper would constitute a lower bound to the actual ones. To obtain a value for ρ, I estimate an $AR(1)$ process for the innovations of all firms. The value obtained (0.74) is within the range found in the literature

(Bai et al 2020). Finally, I set the size of the corporate measure to match the number of all firms that are not incorporated or unincorporated individuals firms.

Parameters calibrated internally: The remaining seven parameters are jointly calibrated with the simulated method of moments because they have no immediate empirical counterpart. These are: (i) A_w, which governs the utility cost of labor, (ii) ψ, the maximum debt to assets ratio allowed by the lenders, (iii) γ_U, the fraction of assets that the unincorporated entrepreneur gets to keep in case of default, (iv) α_g, the shape of the stochastic process for the initial savings of the entrepreneurs, (v) f_R, the fixed cost of operating the risky technology, (vi) η, the exogenous exit probability rate, and (vii) μ_C, the log-productivity of the corporate sector. I consider seven informative moments from the data to match: (1) the equilibrium hours worked in the economy during the period according to the Penn World Table 10.0, (2 & 3) the average interest rate of incorporated and unincorporated firms, (4) the average relative size between entrants and non-entrants, (5) the fraction of incorporated entrepreneurs, (6) the mass of entrants firms, normalized by the total mass of entrepreneurs, and (7) the labor demand of the corporate sector. Moments (2) - (7) are obtained directly from the data work shown in the empirical section of the model, where I use a nationally representative survey of Chilean firms. Thus, the values of these five moments in the data can be considered representative of the national, aggregate ones.

I chose these five moments because they are informative of the main mechanisms of this paper. Moments (2) and (3), the average interest rates, relate directly to how likely entrepreneurs are to default and how much the lender can recover if they do so. Recall that an essential part of the model is the insurance entrepreneurs obtain from limited liability in exchange for higher interest rates due to an increased default probability (and the incorporation cost). Thus, targeting the interest rates relates to matching the empirical relevance of this mechanism. Moment (4), the size of entrants relative to non-entrants, relates directly to average firm growth. I target the average ratio in the economy but let the model determine the different values for incorporated and unincorporated firms. Moment (5), the fraction of incorporated entrepreneurs, captures that entrepreneurs need to choose a technology and a legal status when they are born, before operating firms. In the baseline economy, all entrepreneurs that choose the risky technology will incorporate, and those who pick the safe one will remain unincorporated (figure 9). The decision will depend on the initial wealth of entrepreneurs, which is the only source of heterogeneity at this stage. In consequence, matching the fraction of incorporated/risky entrepreneurs informs the model on what option is more attractive for different entrepreneurs. Moments (1), (6), and (7) are more technical. The first one, the average number of hours worked, is relevant for the policy analysis, as the general equilibrium effects in the labor market can be substantial. The second one, the mass of entrants, helps to match the exogenous exit rate η, which secures the existence of an ergodic distribution of entrepreneurs. Finally, moment (7), the labor demand of the corporate sector, plays a similar role. In the data, this sector explains around 60% of the total labor demand, and ignoring it would lead to overstating the aggregate effects of incorporation and other policies. Table 8 shows the values of these moments in the data and model.

Table 8: **Targeted Moments**

Moment	**Data**	**Model**
Average hours worked	0.33	0.33
Interest rate incorporated (%)	10.11	9.14
Interest rate unincorporated (%)	5.30	4.51
Fraction of incorporated firms	0.28	0.28
Relative size: Entrants/Non-Entrants	0.47	0.47
Mass of entrants	0.071	0.073
Corporate labor demand	0.20	0.20

Working hours are normalized by total available time. Interest rate is total financial expenditures divided by interest generating debt. Firm size is measured by working capital. Firm size is measured by Working Capital. Mass of entrants is normalized by the one of entrepreneurs.

The identification strategy of this group of parameters has the following intuition, where I only discuss each parameter's identification ceteris paribus to make the intuition clear. However, they all are determined jointly because they do have joint effects on the targeted moments. Ψ will be identified by the interest rate of incorporated firms, as long as it is binding for some firms. Conditional on savings and capital, the interest rate will be increasing in the debt undertaken by entrepreneurs, thus movements in Ψ will be reflected in movements in the interest rates of incorporated firms predicted by the model. The effect on the interest rate of unincorporated firms is less significant, because these entrepreneurs take lower debt. At the same time, conditional on the debt limit, a higher (lower) value of γ_U will imply a higher (lower) average interest rate for unincorporated firms. Recall that γ_U determines how much the lender can recover in case the unincorporated entrepreneur defaults. Thus, from the break even condition of the lender I obtain a positive correlation between γ_U and the interest rate of unincorporated firms. α_g is identified from the size difference between entrant and non-entrant firms.

Conditional on the interest rate schedule and the debt limit, the working capital policy function is concave and weakly increasing in the wealth of entrepreneurs. Thus, new-born entrepreneurs having higher initial wealth will result in them setting up firms and lower growth during their life. This parameter is critical for the quantitative evaluation of the model because in this environment, entrepreneurs that are rich enough could potentially self-finance their firms entirely, using no debt, and in consequence never defaulting. η determines the mass of entrants in steady state. In the model, entrant firms are either new-born entrepreneurs, or entrepreneurs that previously defaulted but just regained the possibility to set up firms. Thus, conditional on the fraction on firms that default, variations in η will result in changes in the number of entrants. For the last two parameters, I defined as entrant firms in the data and in the model, those firms that are three years or younger. I do this because of the design of the survey I am working with, which selects the firms in the year prior to interview them. In addition, this increases the number of entrant firms significantly. The fixed cost of operating the risky technology (f_R) is identified from the technology decisions of entrepreneurs. This fixed cost induces a minimum efficient scale that makes it rational to operate this technology only from a certain operation scale. This minimum scale induces poor entrepreneurs to prefer the safe technology over the risky one. Thus, f_R moves to match the mass of incorporated and unincorporated entrepreneurs. The last two parameters, A_c and μ_C, are straightforward. Given prices and the distribution of wealth, they directly determined the labor supplied by workers, and the labor demand of the corporate sector. Table 9 shows the values for all parameters and table.

Table 9: **Parameters**

Parameter		**Value**	**Source/Identification**
Panel A: External sources			
Labor share	α	0.41	PWT 10.0
Scope parameter	θ	0.85	Atkenson et al (1996)
Risk-free rate	r_f	0.041	Central Bank
Depreciation	δ	0.12	PWT 10.0
Discount factor	β	0.92	$1/(1+r_f)$
Measure of entrepreneurs	N	0.08	% in data
Risk Aversion parameter	ξ	1.00	Midrigan & Xu (2014)
Incorporation cost (wages)	IC	0.016	WB's Doing Business
Elasticity of lab. supply	ν	1.00	Verani (2018)
Panel B: Data-work			
TFP permanent component	μ_i	$(0.0, 0.043)$	TFP estimation
TFP shocks variance	σ_i	$(0.25, 0.39)$	TFP estimation
Shock persistence	ρ	0.74	TFP estimation
Size corporate sector	N_c	0.28	% in data
Panel C: Internal Calibration			
Legal status unincorporated	γ_U	0.51	Interest rate (unincorporated)
Debt Limit	Ψ	0.71	Interest rates (incorporated)
Exogenous exit rate	η	0.04	New entrants/Mass of firms
Initial savings	S_I	0.15	Relative size of entrants
Fixed Costs, risky firm	f_R	0.045	% of incorporated firms
Labor utility cost	A_c	1.36	Average hrs. worked
Corporate productivity	μ_C	-0.02	Corp. labor demand

5.1.1 Non-targeted moments

Table 10 shows six relevant non-targeted moments in the data and the baseline calibrated economy. They speak directly to one of the empirical facts that motivate this paper: that incorporated firms are larger and grow more than their counterparts. The model closely matches the average relative size between incorporated and unincorporated firms, which confirms the relevance of the mechanisms proposed in this paper. In the next section, I show that this size difference is explained mainly by the technological difference between firms. In addition, the second and third moments in table 10 show that the model closely predicts the growth of incorporated firms. For this, I provide two different indicators of firm growth. First, how large these firms enter, relative to their average size. Second, the average growth rate over the first 25 years of a firms' life. As in the empirical section, I compute this statistic by comparing the average size of firms aged 0-5 and 20-25 years and calculating the implied growth rate. For both indicators, the model and data are close for incorporated firms. On the other hand, the fourth and fifth moments show that the model can generate the relative size of entrants, but the growth rate is higher in the model than in the data, but the difference is not too significant. In summary, the model does a reasonable job at explaining the size differences in growth patterns for incorporated and unincorporated firms. Finally, the capital-output ratio of the baseline economy is close to the empirical one, which relates directly to the investment rate in this economy.

Table 10: **Non-targeted Moments**
(baseline economy)

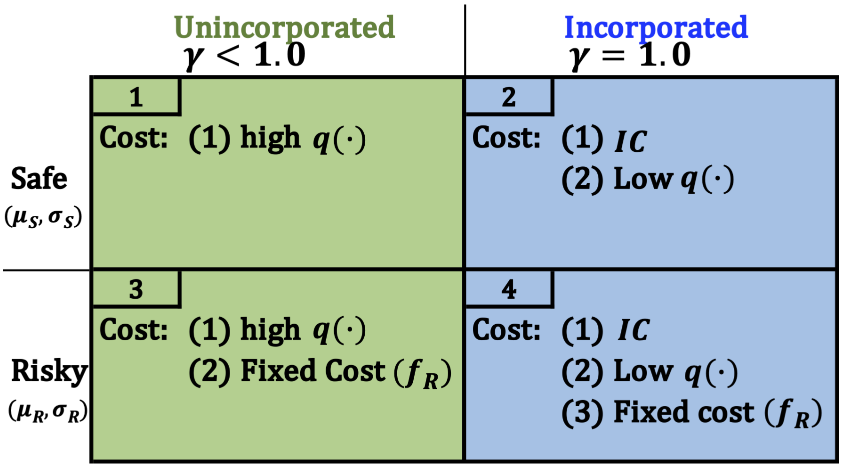
Moment	**Data**	**Model**
Relative size: Unincorporated/Incorporated	0.16	0.21
Relative size Entrants/Non-entrants, Incorporated	0.44	0.43
Growth, Incorporated (%)	5.41	5.77
Relative size Entrants/Non-entrants, Unincorporated	0.59	0.59
Growth, Unincorporated (%)	3.63	4.13
Capital/output	2.10	1.98

Entrants firm are defined as firms that entered the market within the last three periods. Relative firm size is measure by payroll. Data capital-output ratio is obtained from (Ffrench-Davis and Vivanco, 2016).

5.2 Exploring the model's mechanisms

Figure 6 synthesizes the trade-offs of the two decisions entrepreneurs face before operating a firm. On the X axis we find whether to incorporate or not. Conditional on the technology, if they decide to incorporate, they need to pay the incorporation cost IC and will face higher interest rates. The positive aspect of incorporation is obtaining limited liability. On the Y axis, we find the technology decision. And here the main trade-off is between risk and return. By operating the risky technology, entrepreneurs have higher expected profits, but must bear more risk. In addition, the risky technology requires paying a fixed cost f_R, which induces a minimum efficient scale. It is not optimal to operate this technology unless it is done at a sufficiently large scale. This limits the ability of entrepreneurs to minimize their exposure by setting up a sufficiently small firm. At the moment of choosing both characteristics, entrepreneurs are heterogeneous only regarding their personal initial wealth, as they still have not drawn any productivity shock. In consequence, the different decisions will be a function of the initial wealth exclusively.

Figure 6: **Selection Matrix**

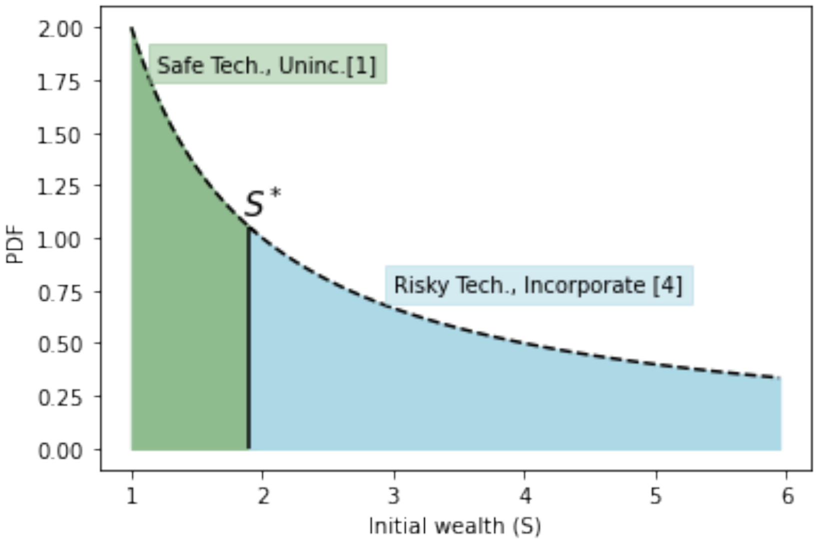


This figure shows the main costs associated with each legal status-technology combination. $q()$ is the price of debt, which is inversely proportional to the interest rate. IC is the administrative cost of incorporating a business. f_R is the fixed cost associated with the risky technology. Outside the box: γ is the fraction of savings the entrepreneur can keep in case of default. μ and σ are the expected value and std. deviation of the TFP processes of each technology. Recall that $\mu_R > \mu_S$ and $\sigma_R > \sigma_S$

In equilibrium, there is a cutoff level of wealth S^*, such that if the entrepreneur's initial wealth S_I is higher than S^*, she will operate the risky technology and incorporate. On the other hand, if it is lower than S^*, she will operate the safe technology and remain unincorporated. To show that this prediction is plausible, in table 13 of appendix B, I show that empirically, the average incorporated entrepreneur has a higher educational level than the average unincorporated one, suggesting a higher wealth and income level. [8]In addition, recall from subsection 3.4 that incorporated firms have higher and more volatile productivity. Figure 7 shows the initial wealth distribution, which was assumed to be Pareto, and pictures the areas of incorporated-risky entrepreneurs (28.01%) and unincorporated-safe ones (43.76%). The remaining mass of entrepreneurs (28.23%) is assigned to the corporate sector.

[8]I use educational level as a proxy for income because the ELE survey of firms does not include information about entrepreneurs wealth or income other than the firm itself.

Figure 7: **Initial wealth distribution**



This figure shows the sorting of entrepreneurs into safe and risky technology, and into incorporation depending on their initial wealth. Entrepreneurs with initial wealth larger than S^* (blue area) operate the risky technology and incorporate (quadrant 4 in figure 6). Entrepreneurs with initial wealth lower than S^* do the exact opposite (quadrant 1 in 6).

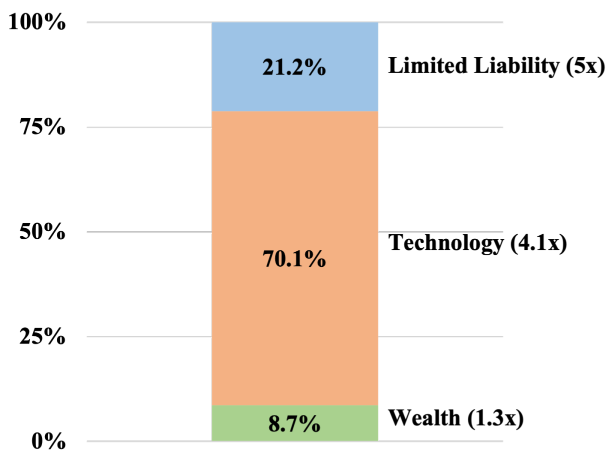
A key characteristic of the equilibrium is that all risky entrepreneurs decide to incorporate, and all safe ones do not. Thus, the technology and legal status decisions cannot be considered separately. Entrepreneurs operating the safe technology always prefer to remain unincorporated, regardless their initial wealth, because they are not willing to pay the incorporation cost. If incorporation were free, safe entrepreneurs would also incorporate, but they prefer not to do so if they have to pay IC. Risky entrepreneurs on the other side would always incorporate, regardless the initial wealth. The difference is in the technology the operate. For every possible value of S_I, the value of incorporating is higher for risky entrepreneurs. Intuitively, the risky technology makes incorporation more valuable for two reasons. First, the volatility of TFP shocks make profits more volatile. Second, the fixed cost f_R induces a minimum efficient scale, which implies that there is a minimum level of risk entrepreneurs need to undertake to operate the technology. Thus, entrepreneurs cannot limit their risk exposure by operating sufficiently small firms, which is particularly relevant for relatively poor entrepreneurs. They have to risk larger fraction of their wealth than they would want to. It is important to keep in mind that the incorporation cost is low enough so that all entrepreneurs have the resources to pay for it, although, it does represent a higher fraction of income for poorer ones.

Conditional on the technology decision and wealth, incorporation leads entrepreneurs to operate larger firms. Since they do not bear all downside risks under this legal status, they are willing to face more considerable losses potentially. The combination of larger productivity, limited liability, and higher entrepreneurial wealth leads risky-incorporated entrepreneurs firms to be larger when they are set up, which generates faster wealth accumulation and firm growth.

5.2.1 Decomposing the size difference between incorporated and unincorporated firms

The model predicts that incorporated firms are roughly five times larger than unincorporated ones (table 10). Figure 8 shows the decomposition of this size difference into three simultaneous effects, intending to understand their quantitative importance. In this exercise, I keep the distribution of firm types constant. Thus, incorporated entrepreneurs (28.01% of the total) are born with wealth $S^I > S^*$, while the opposite is true for unincorporated ones (43.76%). The goal is to analyze what leads incorporated and unincorporated firms to display these size differences, conditional on their chosen types. Consequently, there are no changes in sorting or aggregate prices (wage). The following subsections will relax these assumptions.

Figure 8: **Composition of size differences**
(incorporated-unincorporated, baseline economy)



This figure decomposes the size difference between incorporated and unincorporated firms. I compute three channels cumulatively. First, the wealth effect is the average size difference if they only differ in their initial wealth. Second, the technology effect assumes, in addition, that individuals with initial wealth higher (lower) than S^* operate the risky (safe) technology. Finally, I compute the limited liability effect assuming that risky (safe) entrepreneurs decide to incorporate (remain unincorporated). Numbers besides legend show the size of incorporated relative to unincorporated firms generated by that particular effect.

First, the wealth effect exclusively captures that entrepreneurs with different initial wealth operate these firms. I assume that both types of entrepreneurs operate the safe technology and have unlimited liability to compute it. If this were the only difference, incorporated firms would be 1.34 times unincorporated ones. Thus, this effect explains 8.7% of the total difference, a relatively minor share. Second, I compute the additional effect generated by the different technologies they operate. I compute it assuming that incorporated entrepreneurs operate the risky technology but have unlimited liability in addition to the heterogeneity in initial wealth. I find that in this case, incorporated firms would be 4.1 times larger than unincorporated ones. This number represents 70.1% of the total. Limited liability explains the remaining 21.2%. This simple exercise highlights that the technology decision is crucial in generating the differences between incorporated and unincorporated firms. The following sections show that the technology adoption margin is affected by the cost or availability of incorporation, which has relevant aggregate consequences. In a sense, they are complementary: making incorporation available or more attractive incentives entrepreneurs to operate the risky technology, generating sizable output gains. Even relatively small changes in the number of risky entrepreneurs have potentially noticeable effects on output because this technology change generates significant increases in firm size and output.

5.3 The value of incorporation

In this section, I study the effects of incorporation by varying the incorporation cost (IC) and compare the economic outcomes to the baseline economy. I examine three cases, two counterfactual, and one real reform. The counterfactual ones consist in setting IC to take extreme values: first, I let it converge to infinity, so that no one chooses to incorporate, regardless of their technology decision; and second, I set it equal to zero, so that all entrepreneurs decide to incorporate. The real reform I analyze consists of a series of measures undertaken by the Chilean government to simplify the procedures necessary to constitute an incorporated firm. I report the results in table 11. In figure 9, I show the cutoff wealth (sorting) of entrepreneurs that determines who operates what technology in equilibrium, for the three alternative policies. The green and blue areas represents entrepreneurs that always choose to operate the safe and risky technology respectively, regardless of the incorporation cost. In contrast, the grey area is composed by those entrepreneurs that can switch technologies depending on IC.

5.3.1 Infinite incorporation cost ($IC \Rightarrow \infty$)

Consider the first counterfactual where the incorporation cost is infinite ($IC \rightarrow \infty$). In this scenario, all entrepreneurs will set up unincorporated firms, regardless of their technology decision. In Panel A of table 11, we can see that this leads to a lower fraction of entrepreneurs using the Risky (high risk/high return) technology. At the same time, the entrepreneurs that use it are relatively wealthier on average. The reason behind this is that the cutoff wealth is higher in this world. In figure 9 I denominate it as S^{∞}, it is located to the right of the baseline one S^*. Those entrepreneurs born with wealth between S^∞ and S^* (in the grey area) are the fraction that operates the risky technology in the baseline scenario but not in this case. Note that now they would not pay the incorporation cost and would face lower interest rates, allowing them to invest more intensively in their firms at a cheaper cost. However, the potential losses in case of a bad productivity shock are large enough to prefer the safer and less profitable technology. The average wealth of entrepreneurs who still operate both technologies is larger because of the composition effect. Both sets include wealthier individuals on average.

In Panel B, I report the effects on firm size. I compare firm size using capital and not payroll, not to include the direct impact of potential wage changes over the size measure. Even though wealthier entrepreneurs operate these firms, the average risky firm is now 13.7% smaller than before. Without the ability to incorporate and protect themselves against low productivity shocks, these individuals decide to operate smaller firms and limit their exposure this way. In this scenario, risky entrepreneurs pay lower interest rates (ceteris paribus) and face lower wages, positively affecting firm size and output. Still, it is not enough to outweigh the harmful effects of unlimited liability. On the opposite side, safe firms are 12.3% larger because wealthier entrepreneurs operate them and lower wages. In terms of production, Risky and Safe firms now produce 87.7 and 102.2% of their baseline output, respectively. In aggregate terms, the increase of safe firms is not enough to compensate for the losses of the risky one: production is 6.3% lower than in the baseline economy. In synthesis, the option to incorporate at the specified cost is only valuable for entrepreneurs wealthy enough to operate high-risk/high-return technologies, which corresponds to the top 39% of the wealth distribution in the baseline scenario. It induces risk-taking as a more significant fraction of entrepreneurs are willing to operate riskier and more profitable technologies. At the same time, it fosters firm size and output because incorporated entrepreneurs operate larger firms.

5.3.2 Free incorporation ($IC = 0$)

Consider now the scenario where incorporation is free ($IC = 0.0$). In this world, all entrepreneurs incorporate regardless of their technology decision, and at the same time, it increases the percentage of entrepreneurs operating the risky technology by 2%. In figure 9, I denominate the new cutoff wealth as S^0 and it can be found to the left of S^*. The intuition is the opposite of the previous case: when incorporation is free the cutoff wealth S^0 is lower than the baseline economy. In the baseline scenario, entrepreneurs born with wealth between S^0 and S^* are unwilling to accept the risk from the risky technology or pay for the insurance provided by incorporation. However, when it is free, they do incorporate and operate the Risky technology. In consequence, the average wealth of entrepreneurs operating both types of firms is lower due to the change in relative shares of the technologies.

In Panel B, we see that, unlike the previous case, the wealth effect trumps the insurance effect. The average firm is smaller in both sectors, even though they have limited liability, which results from these firms being operated by poorer entrepreneurs on average. In addition, they face higher financing costs due to incorporation, which pushes for even smaller firms. However, the composition effect leads the average firm to be larger than in the baseline scenario: more entrepreneurs are operating risky firms at a larger scale. As a result, the impact on aggregate output is positive (+2.0%), although smaller (in absolute value) than in the first counterfactual. The increase is also less significant for the mass of entrepreneurs operating the risky technology. This asymmetry in the effects of incorporation speaks of decreasing marginal returns from making it cheaper. As IC decreases, the extra mass of entrepreneurs operating the risky technology is smaller and poorer every time. Consequently, they set up smaller firms and contribute relatively less to aggregate output.

5.3.3 An actual policy: lower incorporation cost

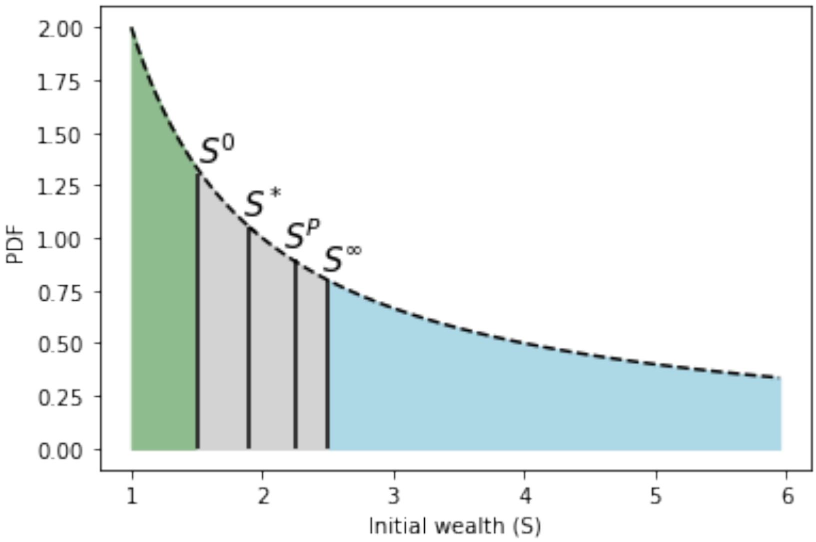
Finally, I also evaluate the effects of a realistic policy undertaken by the Chilean government that made incorporation cheaper and more accessible. It is not one specific policy but a series of measures designed to simplify setting up an incorporated firm. As a result, the World Bank's Doing Business reports that the time required to set up a firm decreased from 7.5 days during 2012-2017 to 4 in 2020. I calibrated the model considering four days, and now I evaluate the alternative scenario pre-reform, assuming a time requirement of 7 days. This situation is between the first counterfactual ($IC \Rightarrow \infty$) and the baseline scenario, and its results are also in the middle. In figure 9, we can fin the cutoff wealth for this scenario, S^P, in the middle of S^* and S^∞. The fraction of entrepreneurs operating the risky technology is 1.8% smaller than the baseline economy, roughly 70% of the respective number for the first counterfactual. However, the size and aggregate differences are much smaller. Risky firms operate at a very similar scale, and safe ones are only 1% larger. As a result, the output is 1.8% smaller. Note that in this pre-reform scenario, the incorporation cost is 87.5% larger than in the baseline, and the economy had already reaped most of the benefits from offering incorporation. Part of the explanation is that even before the reform, the incorporation cost was affordable for most entrepreneurs willing to operate the Risky technology. At the same time, the 1.8% of entrepreneurs adopting the risky technology after the reform are poorer than the remaining risky entrepreneurs, so they set up smaller firms, contributing less to the aggregate output. As a result, incorporation is more valuable for wealthier individuals, as they are more willing to operate the more profitable technologies. From an aggregate point of view, it is also true that providing incorporation to these individuals has a higher impact on output.

Table 11: **Effects of changes in IC**

	$IC \to \infty$	**Pre-reform**	**Baseline**	$IC \to 0$
Panel A: Selection				
Fraction of risky firms	25.37	26.21	28.01	30.05
Fraction of safe firms	46.39	45.55	43.76	41.22
Wealth of risky entrepreneurs	106.69	104.88	100.0	92.85
Wealth of safe entrepreneurs	132.19	111.55	100.0	74.01
Panel B: Firm Outcomes				
Risky firm output	87.68	99.67	100.0	97.94
Safe firm output	106.19	101.68	100.0	98.5
Risky firm interest (%)	4.67	9.02	9.14	9.56
Safe firm interest (%)	4.51	4.51	4.51	4.52
Panel C: Aggregate effects				
Hours Worked	97.91	99.56	100.0	100.47
Agg. Capital	94.30	98.54	100.0	101.11
Agg. Output	93.72	98.28	100.0	102.05

All entries, except for the fraction of firms and interest rates, are averages indexed relative to the baseline economy. Pre-reform economy has an incorporation cost 87.5% higher.

Figure 9: **Changes in the incorporation cost and sorting**



This figure shows how the mass of entrepreneurs using the risky technology changes with the incorporation cost IC. Green and blue areas are entrepreneurs that always choose to operate the safe and risky technology, respectively. Grey area are entrepreneurs that switch. The lower the incorporation cost IC, the higher percentage of entrepreneurs use the risky technology and incorporate. This is captures by a movement

5.4 Changes in the cost of default

In this section, I evaluate two policies. The first one is an actual policy undertaken by the Chilean government that made the default process faster, decreasing the average time taken from 4.5 to 3.2 years. In the model, this is the probability of being reinstated as an entrepreneur after default (ε) which took the value 1/3.2. I evaluate the situation pre-reform, with $\varepsilon = 1/4.5$. Thus the reform allows entrepreneurs to set up another firm after defaulting faster. The second one is a proposal, where the default process is made cheaper. In order to default, entrepreneurs need to hire lawyers, auctioneers and engage in different transactions that can cost a sizable amount of resources. According to the Chilean Ministry of the Economy, defaulting costs on average 15% of the entrepreneur's estate. At the same time, according to the Doing Business report, it costs 9.2% on the average OECD country, reaching a minimum of 1% in Germany. Through this paper, I have abstained from such a cost, so I compare a scenario where defaulting costs are 15% of the personal savings to the baseline one to investigate the potential gains of reducing it.

We can observe the results of both policies in table 12. Both policies generate output gains, incentive entrepreneurs to set up larger firms, and lead a fraction of them to switch to risky technology. This last effect is because both policies increase the value of defaulting, which complements the effect of limited liability. Consequently, they limit the potential utility losses from operating the risky technology, making it more attractive. Note, however, that their magnitudes are different: diminishing the defaulting costs can generate output gains of 5.9% while making default more expedite adds 1.3%. In both cases, Risky firms would be smaller before the reform, even though their average owner is wealthier than in the baseline economy. The reason is that in both cases, defaulting is more expensive in utility terms, so entrepreneurs look for different ways to limit their exposure or potential losses, operating different firms.

Table 12: **Changes in the cost of default**

Economy:	**Baseline**	**Before Time Reduction**	**Before Cost Reduction**
Panel A: Selection			
Fraction of risky firms	28.01	26.55	25.03
Fraction of safe firms	43.76	45.21	46.73
Av. wealth of risky entrepreneurs	100.0	103.66	107.87
Av. wealth of safe entrepreneurs	100.0	112.42	136.80
Panel B: Firm Size			
Risky firm output	100.0	99.87	90.00
Safe firm output	100.0	102.18	105.97
Risky firm interest (%)	9.14	9.03	4.60
Safe firm interest (%)	4.51	4.51	4.51
Panel C: Aggregate effects			
Hours worked	100.0	98.12	97.62
Agg. Capital	100.0	98.88	94.68
Agg. Output	100.0	98.74	94.15

All entries, except for the fraction of firms and interest rates, are averages indexed relative to the baseline economy. Before time reduction economy considers an average bankruptcy process of 4.5 years, instead of 3.2 as in the baseline. Before Cost reduction economy considers a monetary cost of processing default of 15% of entrepreneur's savings.

6 Conclusion

The results of this paper highlight the aggregate consequences of entrepreneurs' willingness to undertake risky investments. In line with recent findings in the literature, I find that when entrepreneurs can insure themselves against business losses, they take more risks, fostering firms' outcomes and economic aggregates (Bianchi and Bobba, 2013; Gottlieb, Townsend, and Xu, 2016; Choi et al., 2017). This study shows the relevance of bankruptcy rules in providing such insurance. Incorporation provides insurance against downside risk at the expense of higher financing costs and administrative fees. The insurance effect induces risk-taking in two dimensions: it incentivizes entrepreneurs to use more profitable and riskier technologies and operate larger firms. In aggregate terms, it generates sizable output gains. In this sense, this paper also contributes to the literature on the aggregate effects of liability protection by showing that it fosters firm size and growth and the adoption of more profitable technologies. These results contribute to the debate on the aggregate effects of liability protection by showing that the insurance mechanism is quantitatively more important than the increase in financing costs, which is part of an ongoing debate in the literature Fossen (2014), Jia (2015), and Akyol and Athreya (2011).

However, there is a limitation to the potential gains from this insurance. Through a series of policy evaluations, I show that making incorporation cheaper always leads to a larger mass of incorporated entrepreneurs, more use of the risky technology, and higher output. Still, it does so at a decreasing rate because the new incorporated entrepreneurs are relatively poor, operate smaller firms, and contribute less to aggregate production. These experiments show that this insurance needs to be considered in the context of other determinants of risk-taking. For example, extending it to entrepreneurs operating relatively safe or small firms might not have the intended consequences. At the same time, individuals in the bottom or top of the wealth distribution, might not be affected by incorporation as much as those in the middle. In the context of this paper, entrepreneurs in both tails would not change their technology decision depending on the availability of incorporation, yielding a smaller (and sometimes negligible) contribution to aggregate output. From a broader perspective, the evaluation of policies aimed at inducing entrepreneurial risk-taking needs to consider the heterogeneity among entrepreneurs. As Schoar (2010) points out, to reap the benefits of policies aimed at fostering entrepreneurship and firms' outcomes, it is crucial to recognize that entrepreneurs are indeed different and react in different ways to these policies. In this study I show that heterogeneous wealth levels lead entrepreneurs to varying reactions in terms of technology adoption and firm size. Other sources of heterogeneity, not considered in this paper, can also have important policy implications. Thus, economic policies that work along these two margins need to be carefully designed and targeted.

This paper has important policy implications, especially for developing economies with less advanced financial and legal environments. The paper shows the aggregate effects of several alternative policies: making incorporation cheaper, and making default cheaper and more expedited. Their results show that aggregate effects are the largest when they induce more risk-taking, particularly on the technology adoption margin. In all these cases, the production gains from larger and more productive firms were quantitatively more important than worsened credit conditions. However, the equilibrium of both forces will depend on the actual context where these policies are evaluated. For example, in countries with more fragile financial systems, or lower quality of financial information, incorporation and more generally, limited liability, would need to be considered in conjunction with a broader set of measures designed to address these fundamental issues.

Further research could advance this paper in different directions. First, limited liability is available in most countries, but its utilization rate shows significant cross-country variation. In addition, the percentage of firms that use it is associated with higher GDP (see appendix C). Understanding the sources of this relationship can have important implications for macroeconomic policy design. One candidate worth considering is the institutional framework these countries have, meaning judiciary system, public sector efficiency, or quality of the financial information. Another one is the characteristics of the economic activities they perform. Second, this paper can be extended to consider a broader range of risk-sharing agreements, particularly public companies. These explain a more significant portion of output than single-owned firms and have more complex risk-sharing contracts. They have limited liability, delegate management on a board, and imply some risk-sharing level among stockholders, generating a different incentives scheme. Third, another promising research vein would be to analyze the effects of limited liability on the financial sector. In this paper, financial intermediaries are modeled in a standard way that does not capture their willingness to bear risk. Considering either regulatory o preventive reasons to prefer safer to riskier loans could make a big difference for policy. Finally, this paper also has important implications for the relationship between inequality, entrepreneurship and firms' outcomes that are relevant in the global context of increasing inequality and would constitute and impactful vein of research.

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A Solution Algorithm

The solution algorithm relies on the general idea proposed proposed by Arellano (2008) and Sargent and Stachurski (2021) in the context of sovereign default.

1. Guess a value for the wage ω

2. Guess: value functions (i) $V_{i\gamma}(W_k, x, s, \varepsilon)$ and (ii) $V^D_{i\gamma}(W_k, x, s, \varepsilon)$, (iii) default $di\gamma(W_k, x, s, \varepsilon)$ and (iv) debt price schedules $q_{i\gamma}(W_k, x, s, \varepsilon)$; for $i = \{R, S\}$ and $\gamma = \{\gamma_U, 1\}$ and default decisions for the entire state space.

3. At each point of the state space, update $V^N_{i\gamma}(W_k, x, s, \varepsilon)$ and $V^D_{i\gamma}(W_k, x, s, \varepsilon)$

4. Update $V_{i\gamma}(W_k, x, s, \varepsilon)$, $d_{i\gamma}(W_k, x, s, \varepsilon)$ and $d_{i\gamma}(W_k, x, s, \varepsilon)$. In this step, I obtain the policy functions for technology and legal status (i and γ).

5. Check for convergence of the value functions. If no convergence, repeat steps 3-4.

6. Check the market clearing condition of the labor market. If it is not satisfied, repeat 1-5.

B Income and firm types

Unfortunately, the Longitudinal survey of firms does not include information regarding the wealth or other sources of income for entrepreneurs. However, the third round of the survey, collected in 2013, includes information about the educational level of managers, which can be the owner or not. In this appendix, I use the educational level of entrepreneurs as a proxy measure of their wealth or income before setting up firms.

The first line of table 13 shows that the vast majority of both types of firms are operated directly by their owners. The next three rows show the educational attainment of firm owners. We can observe that incorporated firms are more likely to be set up by entrepreneurs with at least tertiary education, while the majority of entrepreneurs operating unincorporated firms have at most a high-school diploma. The last row of this table shows that the average age and work experience are very similar for both types of firms.

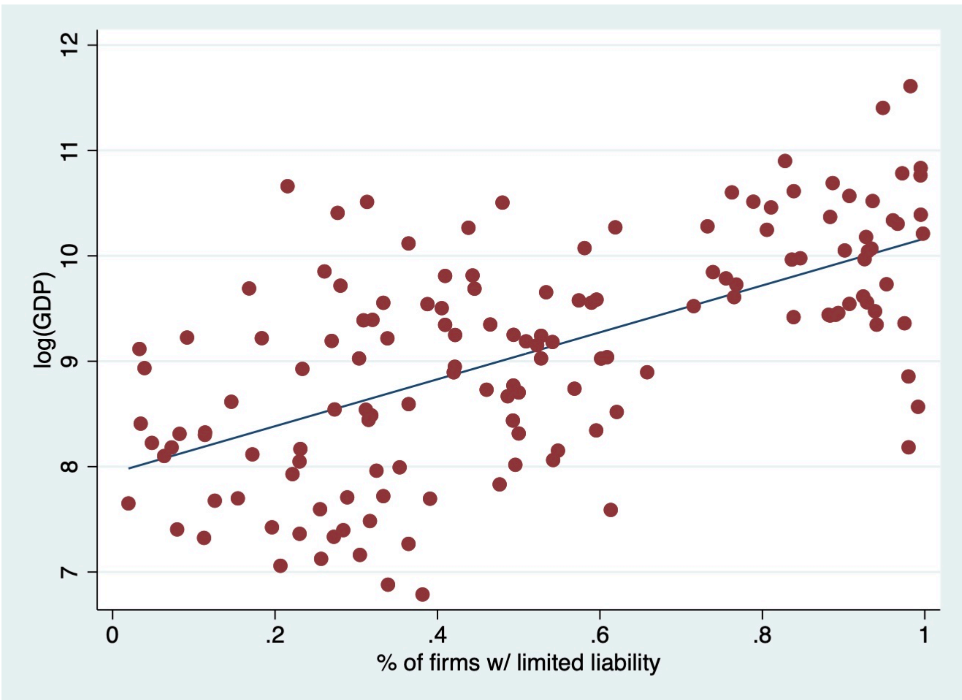
Table 13: **Educational level and firm type**

Firm type:	**Unincorporated**	**Incorporated**
Managed by owners (%)	90.53	66.47
Panel A: Educational Attainment		
High-school or less (%)	70.91	33.25
Tertiary education (%)	27.34	61.52
Post-graduate education (%)	1.75	5.22
Panel B: Average Age/Experience		
Age (years)	55.68	53.49
Experience (years)	23.27	21.26

Educational attainment entries consider only those firms managed by their owners. Experience of entrepreneurs are years working in the same economic activity as the firm.

C GDP and Limited Liability

Figure 10: **GDP P/C and Limited liability**
(% of total, constant 2017 international USD)



Source: % of firms with limited liability comes from World Banks's Enterprise Survey, and GDP per capita from World Bank's Databank. Enterprise survey are conducted in different years for each country. I selected the last version for each one of them. The correlation is 0.78 and statistically significant at 1% level.

Chapter 3: **Aggregate effects of wealth redistribution on technology adoption and productivity**

1 Introduction

What are the aggregate effects of wealth redistribution on risk taking and firm outcomes? To answer this question, I focus on firm size and technology adoption. Recent literature has shown that providing entrepreneurs with insurance against business risk can foster entrepreneurship and firm outcomes. However, these papers have largely abstained from the *positive* or *transformational* aspect of risky entrepreneurship. Investments that entail substantial risk, can also have high potential productivity, and could generate positive returns on aggregate. In this paper I show that the willingness of entrepreneurs to operate riskier but also more productive technology has positive effects on the economic aggregates, and that a benevolent government looking to incentivize the adoption of such technologies, can do so through wealth redistribution.

I proceed in two steps. First, I take the baseline economy from the second chapter of this dissertation, with the corresponding calibration. In this economy, entrepreneurs are born with some level of wealth, and before operating a firm they must decide what technology to operate (and whether to incorporate or not). They have two options, either *risky*, which implies high risk and high return, or *safe*, which is low risk and low return. In equilibrium, (relatively) poor entrepreneurs operate the safe technology because the downside risk of using the risky technology has a higher utility cost. On the other hand, richer entrepreneurs prefer to operate the risky technology because it is more productive and they can bear the risk.

In this economy there is scope for redistribution policies with positive aggregate effects on firm size and output

(Buera, Moll, and Shin, 2013) because the underlying source of heterogeneity, explaining the sorting between technologies is the wealth of entrepreneurs. To show this, I evaluate taxing the top 10% of the initial wealth distribution with a lump sum tax equivalent to 1% of their initial wealth and experiment with transferring it to different population groups. I find that output increases the most when the transfers are directed towards the "marginally safe" entrepreneurs, who are just poor enough to operate the safe technology but would switch if they were slightly richer. The reason is that they will increase output by taking more risk in the two margins of interest: operating a more profitable, albeit riskier technology and setting up larger firms. In contrast, transferring it to the bottom of the wealth distribution does not substantially change the sorting or generate considerable output gains. In this sense, these experiments suggest that if the objective is to increase output by inducing individuals to operate more productive but riskier technologies, the redistribution policies should be targeted towards the "middle class". To quantify the role of risk-taking on the technology adoption margin, I rum the same policy experiment, but fixing the technology decision of entrepreneurs. This is, not allowing those safe entrepreneurs that receive a transfer from the government to adopt a risky technology. I find that in this world, these entrepreneurs invest almost as much as before, but the effect on output is much lower, because they operate less productive firms. Thus, the ability to take risks is crucial for the effectiveness of this policy. Finally, I also show that this redistribution is complementary to the insurance provided by incorporation: if we were to implement it in a scenario without it, the total effect over output would be 50% lower.

This paper is organized as follows. Section 1.1 contains the literature review and contributions of this paper. Section 2 summarizes the key ingredients of the model used in the policy experiments. Finally, section 3 shows the results of the counterfactual exercises and policies evaluations.

1.1 Related literature and contribution

This paper builds on the literature that investigates the effects of risk on entrepreneurial activity and firms outcomes, which is a relatively new topic. Bailey Jones and Pratap (2020) argue that entrepreneurial risk does not have a first-order impact on firms outcomes. They run a counterfactual experiment where the productivity of entrepreneurs displays less variance, and find that the life-cycle of firms remain unaltered. However, they abstain from considering the different ways entrepreneurs have to insure themselves against risk. In addition, this paper uses a database of farm owners in the state of New York, and in consequence, the external validity of their results needs to be carefully considered. An opposite conclusion has been found by papers that show

show that protection against business failure has a positive effect on entrepreneurship. Gottlieb, Townsend, and Xu (2016) shows that post-natal job-protected leave fosters entrepreneurship among women, especially in sectors where there are low entry barriers. The firms they form are more likely to hire workers and incorporate than the average firm in the economy, signaling they are more productive. From a similar perspective Hombert, Schoar, Sraer, and Thesmar (2020) and Ejrnæs and Hochguertel (2014) show that unemployment insurance fosters entrepreneurship. Hombert, Schoar, Sraer, and Thesmar (2020) shows that the newly created firms are more productive that the average firm in the economy, and end up driving the least productive ones out of the market. In a vein more similar to this study, Choi et al. (2017) argues that individuals with a higher outside option (measures as the value of working as an employee) are more likely to start a firm, and to engage in risky experimentation, leading to higher productivity on average. Bianchi and Bobba (2013) show that cash transfers also increase entrepreneurship. They argue that the effect is a consequence of higher willingness to bear risk and not only more relaxed collateral constraints. To this literature I contribute by taking a somewhat different perspective. With the exemption of Choi et al. (2017), these papers largely focus on the occupational choice margin, abstaining from insurance and technology adoption. Risk is mostly understood as a deterrent, and not as a necessary condition of productive investments. In addition, they all focus on partial equilibrium effects, not considering the general equilibrium ones that could be potentially relevant. In this study I show that the ability to take risk in the form of more productive and riskier investments is a crucial determinant of the aggregate output. These results are in line with the seminal work of Schoar (2010) on the different characteristics and aggregate relevance of *transformational* and *subsistence* entrepreneurs. I also contribute to this literature by showing how can wealth distribution can induce entrepreneurs to take more risk, engaging in more productive activities

This paper also contributes to the literature on the effects of wealth changes on entrepreneurship and firm dynamics. Papers such as Chaney, Sraer, and Thesmar (2012) and Buera, Moll, and Shin (2013) show that wealth increases can foster entrepreneurship and firm outcomes because it relaxes the collateral constraints of entrepeneurs. However, this literature abstains from technology adoption, and its relationship with risks, which I show is a key determinant of the aggregate effects. Finally, just like the previous chapter, this study also contributes to the literature that investigates the determinants of heterogeneous outcomes across firms and how they relate to their permanent characteristics (Belenzon, Chatterji, and Daley, 2017; Sterk, Sedláček, and Pugsley, 2021; Choi, Goldschlag, Haltiwanger, and Kim, 2021). I contribute by the relevance of the interrelation between entrepreneurial wealth, and the willingness to operate risky and more profitable technologies. Finally, I propose that wealth redistribution is a powerful policy tool in this sense.

2 A model of firm dynamics with legal status and default

The economy is identical to the second chapter of this dissertation. In this world, entrepreneurship is risky because investing in firms can potentially yield negative returns. Risk-taking has three dimensions: the technology entrepreneurs operate, the size of the firm and whether to incorporate or not. There are two available technologies, with one of them yielding higher and more volatile returns. At the same time, larger firms are riskier because they could lead to larger losses. Finally, entrepreneurs are allowed to incorporate, limiting their exposure to these losses. In this study, I am interested in analyzing the sorting along the two dimensions (technology and incorporation), as I showed in the previous chapter that they have relevant effects on output.

Entrepreneurs are born with heterogeneous levels of wealth, and before operating their firm must decide what technology to operate and whether to incorporate or not. Since the only source of heterogeneity is the wealth with which entrepreneurs are born, this variable will generate the sorting of entrepreneurs in the aforementioned dimensions. They will decide different combinations, legal status, technology and firm size depending on their initial wealth. This economy is also populated by workers, who are standard (for more details see the second chapter).

2.1 Entrepreneurs

Entrepreneurs are born with initial savings s_I drawn from distribution $g(s_I)$, with g assumed to be Pareto with scale 1 and shape α_g. They die exogeneously in every period with probability η, and they are replaced next period with a newly born entrepreneur. They cannot work, but they can only create and operate firms. I omit the occupational choice because its quantitative relevance is still unclear in the literature. Entrepreneurs maximize their lifetime utility, $\sum_{t=0}^{\infty}(\beta(1-\eta))^t u(c)$, with $u(c) = \frac{c^{1-\xi}-1}{1-\xi}$. They have two sources of income: savings (s_t), which earn the risk free interest rate r_f, and the profits generated by their firms.

Before operating a firm, entrepreneurs decide what technology to operate out of two options: *risky* (R) or *safe* (S). They differ in that the productivity (TFP) of the risky technology is on average higher and more volatile, and in that operating it requires paying a fixed cost f_R every period. The production of each type of technology is

$$Y_i(K, L, \varepsilon) = \ exp(\mu_i + \varepsilon)\big(K^\alpha L^{1-\alpha}\big)^\theta, \ \ i = \{R, S\}$$

with $\theta < 1$. μ_i is a permanent component of the productivity, fixed all through firm's life, such that $\mu_R > \mu_S$. ε_{it} is a random shock that follows an AR(1) process

$$\varepsilon_{it} = \rho\varepsilon_{it-1} + e_{it}.$$

The innovations, e_{it} are assumed to be distributed normally, with variance σ_i, this is, $e_{it} \sim N(0, \sigma_i)$, where $\sigma_R > \sigma_S$. The differences in the permanent component of productivity and the variance of the shocks capture the idea that more profitable firms are also riskier.

Firms need to pay for inputs using working capital W_k, which is decided one period in advance, before the realization of the TFP shock in the period. Entrepreneurs operating the risky technology (R) bear more risk in the sense that their profits are more volatile. Given the working capital W_k, a non-defaulting firm operating technology i chooses inputs K and L to maximize the expected production Y, this is

$$Y_i^*(W_k) = Max_{\{K,L\}} EY_i(K, L, \varepsilon) - f_i,$$

$$s.t. \ \omega L + rK \leq W_k$$

$$i = \{R, S\}, \ f_S = 0, \ f_R > 0,$$

where ω is the wage and r is the rental rate of capital. Working capital can be financed using equity and debt. x_t is the fraction of working capital that is financed with debt. x_t, is bounded to the set $(0, \bar{x})$ with $x_t < \infty$, to prevent infinitely leveraged bets with positive expected value. The associated profits are

$$\pi_i(W_k, x, \varepsilon) = Y^*(W_k, \varepsilon) - W_k x.$$

Entrepreneurs can default after observing the realization of the TFP shock. In this scenario, the lender seizes all firms' cash-flow and $(1-\gamma)$ of the entrepreneur's savings (s_t). If they are incorporated, $\gamma = 1$ and if they are unincorporated, $\gamma = \gamma_U \in (0, 1)$. In consequence, incorporation provides insurance to entrepreneurs because it limits their exposure to losses. Upon default, the entrepreneur losses control of the firm and cannot set another

- **(i)** Entrants choose technology and legal status.
- **(ii)** z realized. Inputs are purchased. Production takes place .
- **(iii)** Non-entrants decide default. If so, they surrender $y + (1-\gamma)$ of their savings. $\gamma = \{\gamma_U, 1.0\}$.
- **(iv)** Active entrepreneurs decide c, s', x', W_k'. Inactive ones choose c, s'.
- **(v)** Die exogenously with prob. ν .

one immediately. With probability p_r she leaves the default state at the beginning of next period and can set up another firm.

Entrepreneurs decide between both legal statuses and technologies before operating a firm, and this decision is permanent. The timing is the following

2.2 Recursive Problem

An entrepreneur operating technology i under legal status γ enters the period with working capital W_k, capital structure x and savings s. After drawing a productivity shock e, and must decide whether to default or not. If decides against it her budget constraint is

$$c + s' \leq (1 + r_f)s + Y_i^*(W_k, x, \varepsilon) - W_k x + qx'W_k' - W_k'$$

and her continuation value is

$$V_{i\gamma}^N(W_k, x, s, \varepsilon) = Max_{\{W_k', x', s', c'\}} u(c) + \beta(1-\eta)EV_{i\gamma}(W_k', x', s', \varepsilon')$$

where $V_{i\gamma}(W_k', x', s')$ is the value of the problem before the default decision. Note that after a low enough productivity shock, entrepreneurs operating larger firms (larger W_k can potentially yield larger losses. In other words, they would run higher risk. If she decides to default, the budget constraint is

$$c + s' \leq \gamma(1 + r_f)s$$

and her continuation value is

$$V_{i\gamma}^{D}(W_k, x, s, \varepsilon) = Max_{\{s'\}} u(c) + \beta(1-\eta)\big(p_r EV_{i\gamma}(0,0,s',\varepsilon') + (1-p_r) EV_{\gamma}^{D}(0,0,s',\varepsilon')\big).$$

The value of the problem before the default decision is

$$V_{i\gamma}(W_k, x, s, \varepsilon) = max\big(V_{i\gamma}^{N}(W_k, x, s, \varepsilon), V_{i\gamma}^{D}(W_k, x, s, \varepsilon)\big)$$

and the default decision is given by the following dichotomy

$$d_{i\gamma}(W_k, x, s, \varepsilon) = \mathbb{1}(V_{i\gamma}^{N}(W_k, x, s, \varepsilon) < V_{i\gamma}^{D}(W_k, x, s, \varepsilon)\big).$$

Where $d_{i\gamma}^{N}(W_k, x, s, \varepsilon)$ is the associated indicator function that takes value 1 if the entrepreneur decides to default, and 0 otherwise.

2.2.1 Legal status and technology decision

Now consider an entrepreneur who was just born with savings s_I. Before operating a firm, she must decide what technology and what legal status to use. There is and administrative cost IC of incorporation. The optimal combination of technology and legal status for such entrepreneur is

$$\{i(s_I), \gamma(s_I)\} = argmax_{i,\gamma}\ EV_{i\gamma}(s)$$
$$s.t.\ \gamma \in \{1, \gamma_{unc}\},\ i \in \{S, R\}$$
$$s = s_I - IC \times \mathbb{1}(\gamma = 1)$$

For convenience, I have omitted the remaining state variables (W_k, x, ε) from the value function. Note that both decisions, technology and legal status, depend uniquely on the initial wealth of the entrepreneur, which is the only source of heterogeneity in this model. In equilibrium, entrepreneurs with different levels of initial savings will choose differently.

2.3 Financial intermediaries

Financial intermediaries are competitive and have access to all the information of the entrepreneur. The debt price schedule will reflects the default probabilities and what the lender can recover in case of default. After lending her qxW_k, she either pays back xW_k or defaults. In case of default the lender recovers

$$Rec_{i\gamma}(W_k, x, s, \varepsilon) = min\{(1-\gamma)s + Y(W_k, \varepsilon), xW_k\}$$

which depends on the firms' technology and legal status. Lenders price debt according to the following break even condition,

$$q_{i\gamma}(W_k, x, s, \varepsilon)xW_k = \frac{E(1 - d_{i\gamma}(R, x, s, \varepsilon))xW_k + E\big(d_{i\gamma}(W_k, x, s, \varepsilon)Rec_{i\gamma}(W_k, s, \varepsilon)\big)}{1 + r_f}.$$

2.4 Equilibrium

Definition: Recursive competitive equilibrium A recursive competitive equilibrium of this economy consists on:

1. Entrepreneurs legal status (γ) and technology (i) decisions.
2. Entrepreneur's policy functions: c, W_k, s, x, d.
3. Workers policy functions: c, s, l.
4. Debt price schedule: q.
5. Wage: ω.
6. Ergodic distribution of firms Υ.

Such that:

1. Given prices (q, r_f, ω), the policy functions $(i, \gamma, c, W_k, x, s, d)$ solve the entrepreneurs' problems.
2. Given prices (r_f, ω), the policy functions (c, s, l) solve the workers' problem.
3. Given the policy functions (W_k, s, d, x), technologies (i) and legal status, (γ) the debt price schedule q is such that lender breaks even in expected value.

4. Labor and goods market clear.

The solution algorithm can be found in the appendix A.

3 Aggregate effects of wealth redistribution

In this section I present the results of the quantitative evaluation of the model. I show that there is scope for output enhancing redistribution policies and that they have the largest effect when targeted towards the middle of the wealth distribution or middle class because these entrepreneurs are willing to change from the safe to the risky technology. Thus, when this channel explain the vast majority of the output gains. In it absence (if entrepreneurs are not allowed to adopt the risky technology), output gains are almost negligible. Thus, the ability to take risks play a key role in the success of this policy. Another way to see this, is related to the insurance provided by incorporation. This legal status fosters the effectiveness of this policy on the firm size and technology adoption margins.

3.1 Parametrization

Given that the economy is identical to the second chapter of this dissertation, I use the same values for the parameters. Table 1 shows the values chosen. Panel A contains the parameters taken from external sources (literature and databases), which in general take relatively standard values. Panel B those computed in the previous chapter either by measuring directly using Chilean Data, or estimating them directly using the simulated method of moments.

Crucial for this study is the technological difference between risky and safe firms, which is measured directly in the data. However, recall that this is done using the observed distribution of shocks, which might be prone to bias as firms suffering sufficiently low shocks might decide to close down or default, and leave the sample. This would imply, that the risky technology would be even riskier, and the results of this paper should be interpreted as an upper bound. If the risky technology implies relatively more volatile productivity shocks, poor individuals would be weakly less willing to adopt it after the government transfers. Notice that how much a technology implies for the entrepreneur also depends on the legal status, because incorporation provides some insurance against losses, limiting the risks bore by firm owners/

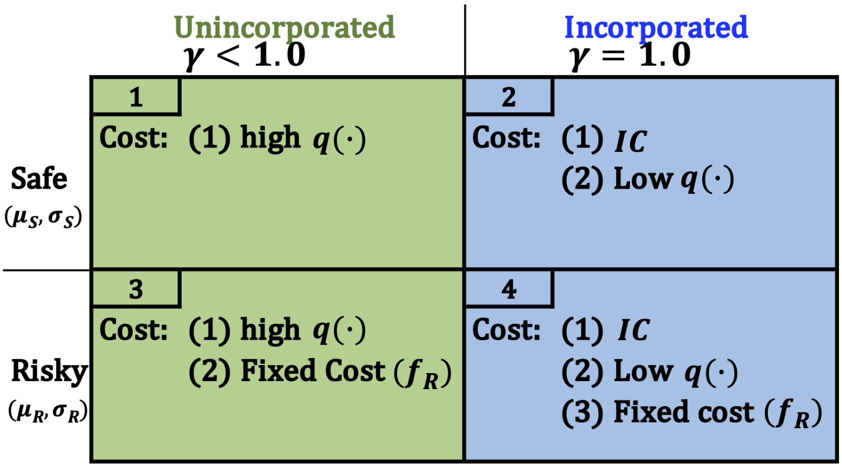
Table 1: **Parameters**

Parameter		Value	Source/Identification
Panel A: External sources			
Labor share	α	0.41	PWT 10.0
Scope parameter	θ	0.85	Atkenson et al (1996)
Risk-free rate	r_f	0.041	Central Bank
Depreciation	δ	0.12	PWT 10.0
Discount factor	β	0.92	$1/(1+r_f)$
Measure of entrepreneurs	N	0.08	% in data
Risk Aversion parameter	ξ	1.00	Midrigan & Xu (2014)
Incorporation cost (wages)	IC	0.016	WB's Doing Business
Elasticity of lab. supply	ν	1.00	Verani (2018)
Panel B: Second Chapter			
TFP permanent component	μ_i	$(0.0, 0.043)$	TFP estimation
TFP shocks variance	σ_i	$(0.25, 0.39)$	TFP estimation
Shock persistence	ρ	0.74	TFP estimation
Size corporate sector	N_c	0.28	% in data
Legal status unincorporated	γ_U	0.51	Interest rate (unincorporated)
Debt Limit	Ψ	0.71	Interest rates (incorporated)
Exogenous exit rate	η	0.04	New entrants/Mass of firms
Initial savings	S_I	0.15	Relative size of entrants
Fixed Costs, risky firm	f_R	0.045	% of incorporated firms
Labor utility cost	A_c	1.36	Average hrs. worked
Corporate productivity	μ_C	-0.02	Corp. labor demand

3.2 Sorting Equilibrium

In this subsection I summarize the characteristics of the sorting equilibrium that are relevant for the analysis of the policy in question. Figure 1 synthesizes the trade-offs of the two decisions entrepreneurs face before operating a firm. On the X axis we find whether to incorporate or not. If they decide to incorporate, they need to pay the incorporation cost IC and face higher interest rates. On the plus side, they get limited liability. On the Y axis, we find the technology decision. By operating the risky technology, entrepreneurs have higher expected profits and bear more risk. In addition, the risky technology requires paying a fixed cost f_R. Since the only source of heterogeneity at this stage is initial wealth, entrepreneurs will decide on different options depending on its value. Thus, redistribution policies that increase or decrease the wealth of entrepreneurs at this point have the potential to alter the sorting.

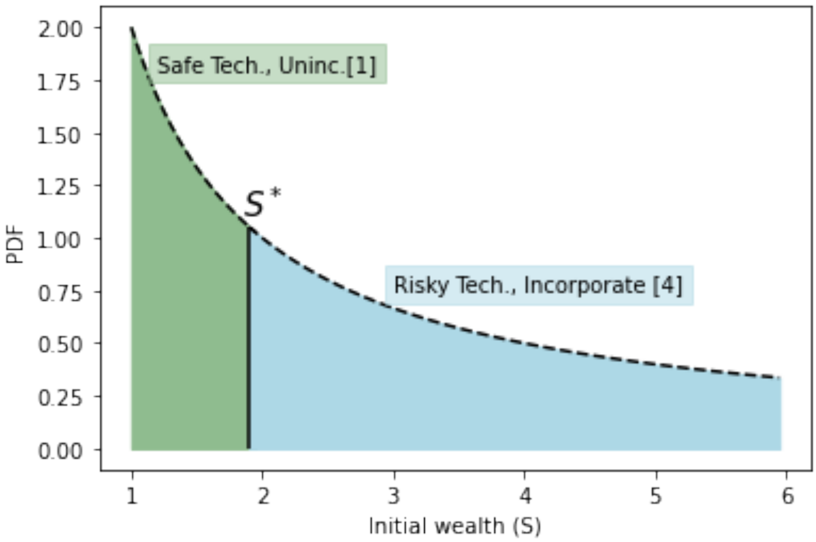
Figure 1: **Selection Matrix**



This figure shows the main costs associated with each legal status-technology combination. $q()$ is the price of debt, which is inversely proportional to the interest rate. IC is the administrative cost of incorporating a business. f_R is the fixed cost associated with the risky technology. Outside the box: γ is the fraction of savings the entrepreneur can keep in case of default. μ and σ are the expected value and std. deviation of the TFP processes of each technology. Recall that $\mu_R > \mu_S$ and $\sigma_R > \sigma_S$

In equilibrium, there is a cutoff level of wealth S^*, such that if the entrepreneur's initial wealth S_I is higher than S^*, she will operate the risky technology and incorporate. On the other hand, if it is lower than S^*, she will operate the safe technology and remain unincorporated. Figure 2 shows the initial wealth distribution and the areas of incorporated-risky entrepreneurs (28.01%) and unincorporated-safe ones (43.76%). The remaining mass of entrepreneurs (28.23%) is assigned to the corporate sector. As we will see, this cutoff plays a crucial value in determining the effectiveness of redistribution in fostering technology adoption because it can lead entrepreneurs to move from one side of the distribution to the other.

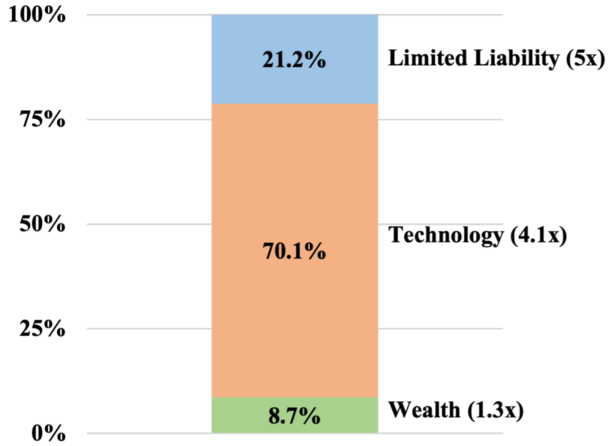
Figure 2: **Initial wealth distribution**



This figure shows the sorting of entrepreneurs into safe and risky technology, and into incorporation depending on their initial wealth. Entrepreneurs with initial wealth larger than S^* (blue area) operate the risky technology and incorporate (quadrant 4 in figure 1). Entrepreneurs with initial wealth lower than S^* do the exact opposite (quadrant 1 in 1).

Recall from the previous chapter that the model predicts that incorporated firms are roughly five times larger than unincorporated ones. 8.7% of the total difference, a relatively minor share, is explained by the wealth effect, which is the fact that risky firms are operated by relatively richer entrepreneurs. 70.1% of the total is consequence of these firms operating different technologies and limited liability explains the remaining 21.2%. Redistribution policies can potentially work through these three channels simultaneously. Entrepreneurs receiving transfers from the government could increase their investment in their firms just because their income would be higher. At the same time, this new level of income might be high enough to lead them to adopt the risky technology and to obtain limited liability. In light of the previous results, most of the impact of the policy will come from the technology adoption channel.

Figure 3: **Composition of size differences**
(incorporated-unincorporated, baseline economy)



This figure decomposes the size difference between incorporated and unincorporated firms. I compute three channels cumulatively. First, the wealth effect is the average size difference if they only differ in their initial wealth. Second, the technology effect assumes, in addition, that individuals with initial wealth higher (lower) than S^* operate the risky (safe) technology. Finally, I compute the limited liability effect assuming that risky (safe) entrepreneurs decide to incorporate (remain unincorporated). Numbers besides legend show the size of incorporated relative to unincorporated firms generated by that particular effect.

3.3 The effects of wealth redistribution on the technology decision, firms' outcomes and output

Suppose a government implements a tax over the top 10% of the initial wealth distribution and uses it to subsidy a different group to be defined. The transfers occur only for newly born entrepreneurs, and there is no tax or subsidy once they have been operating for more than one period. The government keeps the fiscal balance intact with this policy, so the revenue obtained with the tax matches exactly the total subsidies distributed. In table 2 I show the results of different potential recipients of the subsidy. I start by considering only the poorest decile of the population, as they have the highest marginal utility of consumption and less initial wealth to invest in their firm. Then, I also consider giving this transfer to the marginal decile, those with wealth immediately less than S^*, and successively expand it to include the 20% and 30% at the border.

Compare the first and second policies that correspond to subsidize the bottom decile and the marginal decile (columns 2 and 3 of table 2). In the first case, we observe that this increases the average wealth of the entrepreneurs choosing the safe technology but fails to induce more of them to adopt the risky one. Even more, there is a marginal decrease in the usage of this technology, attributable to the fact that wages are slightly higher. In figure **??** this would be represented by a decrease in the mass of entrepreneurs at both tails of the wealth distribution. In addition, firm size does not change on average. Safe firms are (2.7%) larger because their owners have more capital to invest, but risky firms are 1% smaller because of the opposite reason. Note in the third and fourth lines of this table, that when the transfer is targeted towards the marginal 10%, the average wealth of the risky entrepreneur decreases by 19% because new entrepreneurs adopt the risky technology, and they are poorer than the previous risky entrepreneurs. At the same time, the average wealth of safe entrepreneurs decreases because the richest among the safe entrepreneurs in the baseline economy, now adopt the risky technology. In contrast, when the transfers are targeted towards the bottom 10%, there is basically no change in the technology selection margin, so the average wealth of entrepreneurs in both categories change only because of the transfers. In aggregate terms, firm size and output remain unchanged. It is worth noticing, that the target entrepreneurs in this case are not only the most capital deprived, but also the ones with the highest marginal utility of consumption, so this policy is welfare increasing in spite of having no effect on output.

In contrast, the effects of subsidizing the marginal decile are of much more relevant magnitudes. It induces an extra 7.2% of entrepreneurs to adopt the risky technology (and incorporate) and fosters firm size by 4.1%. As a result, a fraction of entrepreneurs now operates a more productive technology with limited liability. In addition, since these individuals have a lower marginal utility of consumption than those at the bottom of the wealth distribution, the effects of the wealth increase over investment will be more significant. This policy generates an increase in total output of 5.1%. Thus, if the policy's objective is to increase or maximize production, it is clear that this should be targeted towards the middle of the distribution or "middle class." More specifically, towards those individuals with the potential to change from safer and less profitable technologies to riskier and more profitable ones if they were to become marginally wealthier.

Table 2: **Wealth Redistribution**

Subsidy target:	**Baseline**	**Mg 10%**	**Bottom 10%**	**Mg 20%**	**Mg 30%**
Panel A: Selection					
Fraction of risky firms	28.01	35.18	27.90	34.87	30.789
Fraction of safe firms	43.76	36.58	43.86	36.90	40.98
Av. wealth of risky entrep.	100.00	81.67	98.62	81.88	90.76
Av. wealth of safe entrep.	100.00	55.68	116.67	67.72	98.10
Panel B: Aggregate effects					
Av. Working Capital	100.00	105.12	100.0	104.97	101.73
Agg. Output	100.00	105.12	100.0	104.97	101.73

All entries, except for the fraction of firms, are indexed relative to the baseline economy. Marginal (Mg) $X\%$ corresponds to the $X\%$ entrepreneurs with wealth marginally lower than cutoff S^* (see figure **??**). Bottom 10% is the 10% of entrepreneurs with the lowest initial wealth.

The remaining policy options, subsidizing the marginal 20, 30%, and so on, suffer from a different phenomenon. Even though they increase the wealth of those individuals at the border, they fail to generate effects of the same magnitude as subsidizing just the marginal 10 percent. The reason is that, in this case, the resources have to suffice for a larger share of the population, giving each entrepreneur less money than before. Thus, some individuals immediately under the cutoff S^* do not receive enough wealth to incorporate and adopt the risky technology. In consequence, the effects on firm size and output are minor. Moreover, extending the subsidy to a larger share of the population monotonically decreases the output gains. We can observe this phenomenon in columns 4 and 5 of 2. Other policies, such as subsidizing a fraction x_0 under s^* and a fraction x_1 above S^*, generate smaller output gains than the marginal 10% and are not reported here for convenience. In conclusion, these results reaffirm that the main difference between the two types of firms is a technological one, and in consequence, redistribution policies (and in the previous chapter, insurance policies) are relatively more effective when they induce technology adoption. In the next subsection I will quantify this specific channel.

3.4 Wealth redistribution and the willingness to take risks

To quantify how much of the output increase can be explained by risk taking in the technology adoption channel, I run a counterfactual experiment where I implement the same redistribution policies as before, but assuming the technologies entrepreneurs choose are fixed. This is, recipients of government transfers cannot adopt the risky technology. Given that subsidizing the marginal 10% yields the highest output gains, I will use this policy in the forthcoming analysis.

Table 3 shows the effects of the redistribution policy in both worlds. Note in the first two rows, that the equilibrium mass of firms using both technologies is the same in the world without technology adoption and in the baseline, and it is 7% lower than in the scenario with adoption. This ends up generating output gains that are a fraction of the potential ones. Interestingly, the average working capital is almost the same in the economy with and without adoption, which means entrepreneurs that are receiving the transfers are capital deprived and they will invest it in the firm even if it is using the safe technology. However, because now these individuals are using a less productive technology, the total effect on output is negative as we can see in the last row of this table.

Table 3: **Wealth Redistribution with and without technology adoption**

	Baseline	**With tech. adopt.**	**Without tech. adopt.**
Panel A: Selection			
Fraction of risky firms	28.01	35.18	28.01
Fraction of safe firms	43.76	36.58	43.76
Av. wealth of risky entrepreneurs	100.00	81.67	120.86
Av. wealth of safe entrepreneurs	100.00	55.68	195.54
Panel B: Aggregate effects			
Av. Working Capital	100.00	104.12	99.01
Agg. Output	100.00	105.12	96.84

All entries, except for the fraction of firms, are indexed relative to the baseline economy.

3.5 Wealth redistribution and insurance

The success of this policy depends crucially on the insurance provided by incorporation. Table 4 shows that if this option were not available, the output gains from redistribution towards the marginal 10% would be 2.7%, roughly half the 5.1% in the baseline scenario. The difference is also noticeable in the technology decision of firms. While in the baseline case, the policy can induce an extra 7.2% of the population to operate the risky technology, it does so for 6.5%. These policies, redistribution and incorporation, are complementary because both of them foster risk-taking in this economy. The wealth obtained through the redistribution makes the benefited entrepreneurs more willing to operate the risky technology and set up larger firms. It also gives them more resources to invest. In addition, the option to incorporate allows them to obtain insurance against downside risks.

Table 4: **Wealth Redistribution with and without incorporation**

	With Incorporation		**Without Incorporation**	
	Baseline	**Mg 10%**	**Baseline**	**Mg 10%**
Panel A: Selection				
Fraction of risky firms	28.01	35.18	25.37	31.84
Fraction of safe firms	43.76	36.58	46.39	39.92
Av. wealth of risky entrepreneurs	100.00	81.67	106.69	87.77
Av. wealth of safe entrepreneurs	100.00	55.68	132.69	102.5
Panel B: Aggregate effects				
Agg. Output	100.00	105.12	93.72	96.43

All entries, except for the fraction of firms, are indexed relative to the baseline economy. Pre-reform economy has an incorporaton cost 87.5% higher.

4 Conclusion

The results of this paper highlight the aggregate consequences of entrepreneurs' willingness to undertake risky investments. In line with recent findings in the literature, I find that when entrepreneurs can insure themselves against business losses, they take more risks, fostering firms' outcomes and economic aggregates (Bianchi and Bobba, 2013; Gottlieb, Townsend, and Xu, 2016; Choi et al., 2017). Unlike previous papers in this nascent literature, I focus on the *transformational* aspect if risk. This is, that firms or investment project with larger potential returns, entail more risk. I show that the willingness of entrepreneurs to undertake such projects has important aggregate effects. In addition, I show that wealth redistribution can incentivize some entrepreneurs to adopt more advanced and riskier technologies, generating sizable output gains. It is essential that these transfers are targeted towards the middle of the wealth distribution (and not the bottom) for this policy to be successful. I also show that this distribution is complementary to the insurance against losses provided by incorporation.

This paper has important policy implications, especially for economies in need to foster technology adoption and foster firms outcomes. The results show that aggregate effects are the largest when they induce more risk-taking, particularly on the technology adoption margin. Specific policies, such as rebates or providing in kind transfers towards the middle class should be carefully designed and targeted to be successful. For example, in this study I consider a one sector economy but it is plausible that the effects might differ depending on the characteristics of the main economic sectors in the economy. Barriers to entry or financial constraints might affect the quantitative results of this paper. In addition, not all sources of risk are the same. The optimal policies designed to provide insurance to entrepreneurs will depend on the main sources of uncertainty. In this sense, the policies prescribed in this paper are somewhat abstract.

Future research could advance this paper in different directions. First, by measuring empirically the response of entrepreneurs to different transference's policies. Second, by analyzing a wider set of entrepreneurial decisions: are the economic sectors chosen affected? is there a role for the substitution between workers and capital?, among other questions. And finally, by looking deeper into the sources of uncertainty and how entrepreneurs react. Different sources of risk might generate different protection strategies on the entrepreneurial side.

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A Solution Algorithm

The solution algorithm relies on the general idea proposed proposed by Arellano (2008) and Sargent and Stachurski (2021) in the context of sovereign default.

1. Guess a value for the wage ω

2. Guess: value functions (i) $V_{i\gamma}(W_k, x, s, \varepsilon)$ and (ii) $V^D_{i\gamma}(W_k, x, s, \varepsilon)$, (iii) default $di\gamma(W_k, x, s, \varepsilon)$ and (iv) debt price schedules $q_{i\gamma}(W_k, x, s, \varepsilon)$; for $i = \{R, S\}$ and $\gamma = \{\gamma_U, 1\}$ and default decisions for the entire state space.

3. At each point of the state space, update $V^N_{i\gamma}(W_k, x, s, \varepsilon)$ and $V^D_{i\gamma}(W_k, x, s, \varepsilon)$

4. Update $V_{i\gamma}(W_k, x, s, \varepsilon)$, $d_{i\gamma}(W_k, x, s, \varepsilon)$ and $d_{i\gamma}(W_k, x, s, \varepsilon)$. In this step, I obtain the policy functions for technology and legal status (i and γ).

5. Check for convergence of the value functions. If no convergence, repeat steps 3-4.

6. Check the market clearing condition of the labor market. If it is not satisfied, repeat 1-5.