Think Before you Ink:

Modeling Laser Tattoo Removal

BEE 453

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Executive Summary

Prior to laser treatment tattoos were removed by destroying the skin containing the ink. The skin would be burned, frozen, or excised surgically. The use of Q-Switched lasers has effectively diminished the abrasive nature of tattoo removal with successful results and is now a commonly used method for tattoo removal. Scientific studies have been conducted that examine the laser intensities and mechanism of removal. These studies have found that the laser selectively heats the thin ink layer beneath the skin, leading to an explosion of the microscopic ink particles. The remnants of these particles, and the cells in which they reside, are subsequently removed by the lymphatic system. The primary aim of this project is to model this laser tattoo removal process. This model uses the heat transfer equation with a laser heat generation term to find the temperature profiles of the ink and surrounding skin layers. Also included in the model are the heat energy effects of evaporation within the tissue as it is heated. A mass transfer equation accounts for the moisture content of the tissue as it is lost to vaporization during heating. Sensitivity analyses performed during the modeling process produced optimal values for the absorptivity of the ink for the Q-Switched Ruby laser, $165 m^{-1}$. They also determined the optimal value for the absorptivity of the skin, $20\ m^{-1}$. The developed model was validated with clinical experimental results which claimed that within one 40 nanosecond laser pulse time, the ink particles reached 900 degrees Celsius while the surrounding skin temperature was between 45 and 55 degrees Celsius. Further applications of this model include optimizing laser intensities and pulsation times to reduce the tissue damage and the pain of the procedure.

Key words: laser, tattoo removal, skin damage

Introduction

Background and Importance

The art of tattooing is by no means new. Tattoos have been traced back to 1300 B.C. when remains from cultures such as Aborigines and ancient Egyptians were found with skin art. Tattooing involves using needles to inject pigment into the skin. The skin itself is composed of two distinct layers. The top layer, the epidermis, is comprised of a variety of cell types from dead cells to epithelial stem cells. This layer acts as a protective barrier and is regenerative. It is only about 0.07-1.4 mm thick. The dermis is the second layer of skin, residing beneath the epidermis. This skin layer is about 0.6-3.0 mm thick and contains mainly fibroblasts and connective tissue (Taylor, 1991). In modern professional tattoos, the ink resides at the top of the dermal skin layer, primarily within the fibroblast cells.
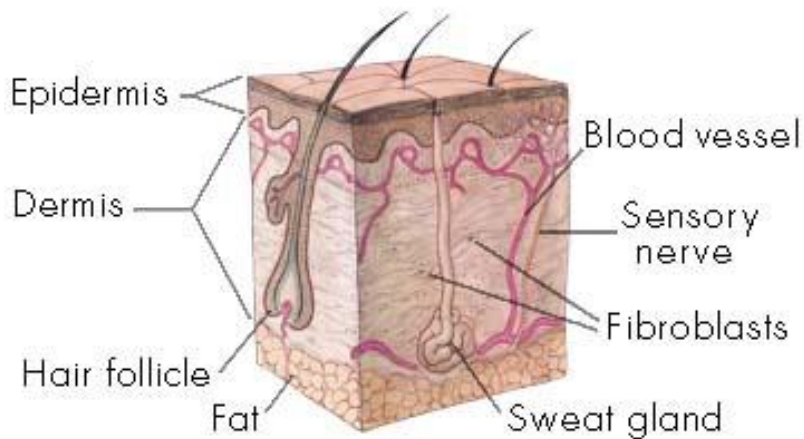

Figure 1: Skin layers. Taken from National Institute of General Medical Sciences.

Since the ink is maintained within these dermal cells, tattoos are permanent skin embellishments. As such, they are difficult to remove. It is estimated that 10% of Americans have at least one tattoo, and it is likely that about half of these people will have one tattoo removed. Judging by these statistics, it is apparent that tattoo removal has become a relatively common dermatological procedure. Effective methods include excision, dermabrasion, and laser removal.

The method of excision for tattoo removal is somewhat primitive relative to more modern and less invasive techniques. In excision, the area of the skin with the tattoo is surgically removed and then stitched together. This can only be done for relatively small tattoos; otherwise a skin graft may be required. In dermabrasion, the area with the tattoo is sprayed with a substance that freezes the skin. The skin containing the tattoo is then sanded off. Other less invasive tattoo removal techniques can involve various creams, lotions and gels, but these are often ineffective.

The most effective and most commonly used method for tattoo removal is laser tattoo removal. Laser removal is optimal because there is a minimal amount of scarring, and while it can be somewhat painful, it is a non-invasive, non-surgical procedure. As a non-invasive process, laser tattoo removal is relatively

fast and convenient in terms of how treatments affect patient lifestyle. Laser tattoo removal works by flashing a Q-switched laser on to the tattoo for a fraction of a second. The wavelength of each laser beam is selectively absorbed by corresponding tattoo pigments. Other examples of lasers include: the Q-Switched Ruby (red light), Q-Switched YAG (infrared& green light) and Q-Switched Alexandrite (purple / red light) (Pfirrmann, 2007). Ink particles burst after they are super heated and reach 900 degrees Celsius. They, and the remnants of the cells they were trapped within, are then removed by the body's lymphatic system (Taylor, 1991). Typically, in order to completely remove a tattoo, between 5 and 15 laser treatments are required (Pfirrmann, 2007). This number is also dependent on whether the tattoo was done professionally (as the ink is in a more uniform layer) as well as on the colors used. Black professionally applied tattoos are the easiest to remove. Non-professional tattoos are harder to remove due to the fact that the pigment granules are not distributed in an even layer within the dermis (Taylor, 1991). In these tattoos, the ink particles are found spread throughout a greater range within the dermis. They also tend to be more condensed when deeper in the dermis layer, making removal especially difficult (Taylor, 1991).

Problem Schematic

The project at hand focused on modeling laser tattoo removal of a professional black tattoo. The tattoo under consideration is of a professional nature so that it could be assumed that the ink was in a uniform layer beneath the skin surface. It is black, because this is the easiest color to remove and because it works exceptionally well with the Q-switched Ruby laser. The following model simulates the heating of the ink layer to 900 degrees Celsius by the laser. The model was used to determine currently unknown experimental properties of the skin and ink, their absorptivities. The basic geometry of the model is shown below in Figure 2.

Convection: T = 298K, h = 25 W/m^2

2×10^{-3} m   Skin

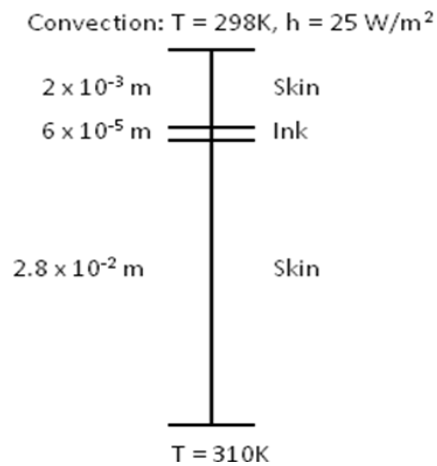6×10^{-5} m   Ink

2.8×10^{-2} m   Skin

T = 310K

Figure 2: Schematic of 1D laser tattoo removal geometry showing the top and bottom skin layers and the ink layer.

Design Objectives

The goals for modeling laser tattoo removal were to:

- Simulate heat transfer due to laser heat generation; model the temperature distribution in the two skin layers and ink layer
    - Ensure that the ink layer reaches 900 degrees Celsius
    - Avoid burning the surrounding skin layers
    - Include evaporation
- Determine appropriate values for currently unknown properties such as the absorptivity of the ink layer and the absorptivities of the skin and ink layers
    - Find numerical values via the iterative method shown below in Figure 3
- Investigate other combinations of laser intensities and pulse times to optimize laser removal therapy

| **Assume Property**
Absorptivity of skin or ink | → | **Run Simulation**
Using COMSOL | → | **Post Process Validation**
Ink layer = 900 °C ?
Skin layers = 45-50 °C? | Yes → | **Final Model**
Validated by experimental and clinical data |

No ↓

Adjust Property
According to Post Processing results (loops back to Run Simulation)
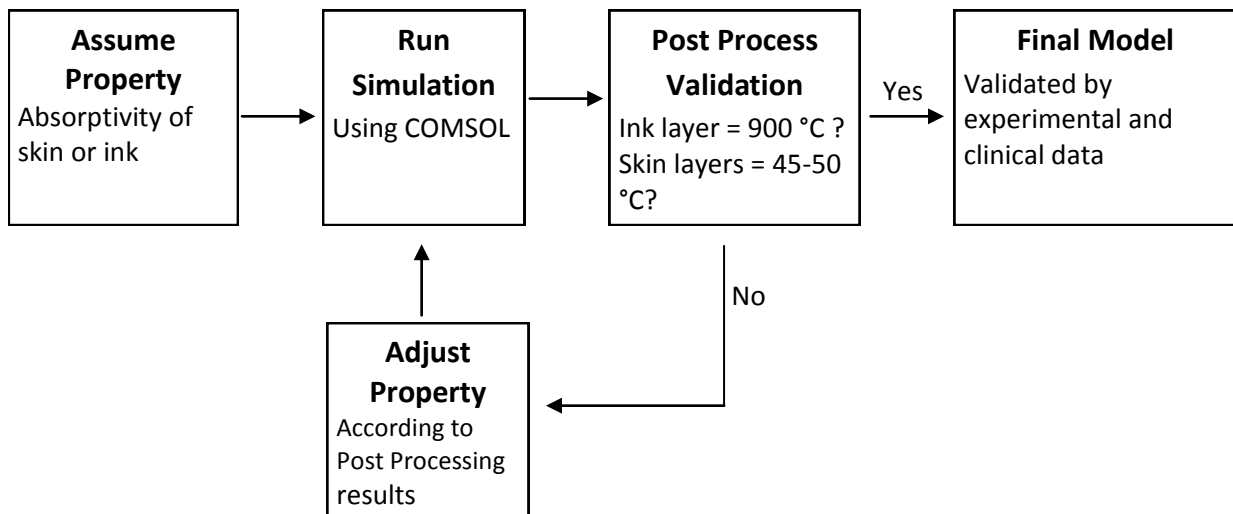
Figure 3: Flow chart of solution process to construct a valid model through the inverse method

Results and Discussion

Before analyzing the simulation results, it is important to understand how various geometries and properties were chosen for the model and the various assumptions that were made. The first section under results and discussion will, therefore, detail the assumptions that were made during the modeling process. This will be followed by the sensitivity analyses, which show how optimal property values were chosen. The simulation results and attempted laser optimization will conclude this section.

Review of Major Assumptions

Initially, a two dimensional geometry was created, however, after incorporating evaporation into the model, the program experienced run time errors and memory issues. By simplifying the model to one dimension, the program was able to run the calculations and produce results. For this reason, we assumed a one dimensional model. This assumption was also made because there was little observable temperature change between the first and second dimension.

The hypothetical tattoo was chosen to be in an area of thick skin. The deeper the ink resides in the skin tissue, the more difficult it will be to cause it to reach 900 degrees Celsius. Thus, this model mimics an extreme case for laser tattoo removal as the ink is exceptionally deep. The upper range of the depth of the epidermis is 1.4 mm. The upper range of the dermis is 3 mm (Revis 2006). As tattoo ink resides at the top of the dermis, a 2 mm depth below the surface of the skin was chosen (Taylor 1991). The total length of the model (top skin layer plus ink layer plus bottom skin layer) is significantly longer than the upper range of epidermal and dermal tissue (4.4 mm versus 30 mm in the model as shown in Figure 2). The bottom layer of tissue was extended in order to mimic a semi-infinite boundary condition so that the temperature at the end could equal that of body temperature.

The properties were maintained throughout the bottom tissue layer. The properties were also assumed to be the same in the top skin layer as the bottom. These properties were considered to be constant and uniform throughout the tissue and throughout temperature and time variations. The inclusion of evaporation in the program was challenging yet essential for a more realistic model. However, variations in the properties due to different tissue compositions or changes in temperature were negligible and would have resulted in increased computation times.

Liberties were taken in assuming the depth of the ink layer. Tattoo ink is marked by a mono-modal distribution of pigment diameter size. Pigment diameters are in a range from 2 to 400 nanometers with the most common diameter size being 40 nanometers. Hundreds of these particles can be found in each cell in the ink layer. From microscopy images of tattooed skin, it appears that thickness of these inked cells within the dermal layer is about 6 micrometers (Taylor 1991). COMSOL has difficulty computing the governing equations at distances this small. Therefore the ink layer was increased by an order of magnitude, making it 60 micrometers, in order to account for stacking of these ink particles.

The density and the thermal conductivity of the ink were assumed to be those of water. The base of the ink is a liquid with properties similar to water, and as there is no information on the density or thermal conductivity of the ink; therefore this is a fairly good approximation. The specific heat of the ink was

assumed to be close to that of water. 5100 J/kg-K was chosen for this property value. Although the ink is well integrated into the tissue, the laser will be heating the individual pigment particles within the cells; thus, separate properties for the ink (not the same as the properties in the tissue) are warranted. The additional properties (absorptivities) are examined later in the sensitivity analysis. The absorptivites determined later become especially important in considering how the laser generation source term is included in the model. Laser heating of the skin/ink construct was assumed to be uniform and a function of depth within the tissue.

Finally, and perhaps most importantly for understanding this particular model is that COMSOL may not work as effectively in calculating governing heat and mass transfer equations at the very small distances and times associated with the ink layer and pulse time respectively. While the ink layer was increased by an order of magnitude to compensate for these problems, the run time remains very small (simulating an actual single pulse time of the Q-Switched Ruby Laser). In fact, the heat and mass transfer equations themselves may break down at distances and times this short. In some of the results, therefore, the effects of these minute properties can be seen as calculation errors. Most of the data still appears accurate, and the temperature profiles correspond to what one would expect from laser heat generation on the skin. Although these errors should be acknowledged, they are unavoidable and can largely be ignored as the rest of the data appears accurate.

Summary of assumptions made:

- Tattoo resides in a region of thick skin
- Constant tissue properties, not varying with time or space
- Depth of ink layer is an order of magnitude larger than appears *in vivo*
- Thermal properties of ink initially assumed to be that of water, or close to that of water
- COMSOL and governing heat and mass transfer equations hold for small time increments over short distances

Sensitivity Analysis

Sensitivity analyses were performed on the material properties for which experimental data could not be found. These properties included the absorptivites of both the skin and the ink layers. For the ink properties, the average temperature in the ink layer was used as an indicator of accuracy. For the skin absorptivity property, the average temperatures in the skin layers were used as accuracy checks. From the sensitivity analyses performed below, optimal values were found which produced the expected results, which were seen in experimental data. The data recorded in the figures below produce results which mimic experimental data (900 degrees Celsius in the ink layer after one pulsation while maintaining lower temperatures, 45-55 degrees Celsius, in the surrounding tissues so as to avoid burning). An additional sensitivity analysis was performed on the specific heat of the ink layer. This was done to examine the sensitivity of the model to changes in the ink's specific heat.

Absorptivity of Ink –

The first sensitivity analysis was performed on the absorptivity of the ink. The average temperatures within the ink layer according to ink absorptivity can be seen below in Figure 4.
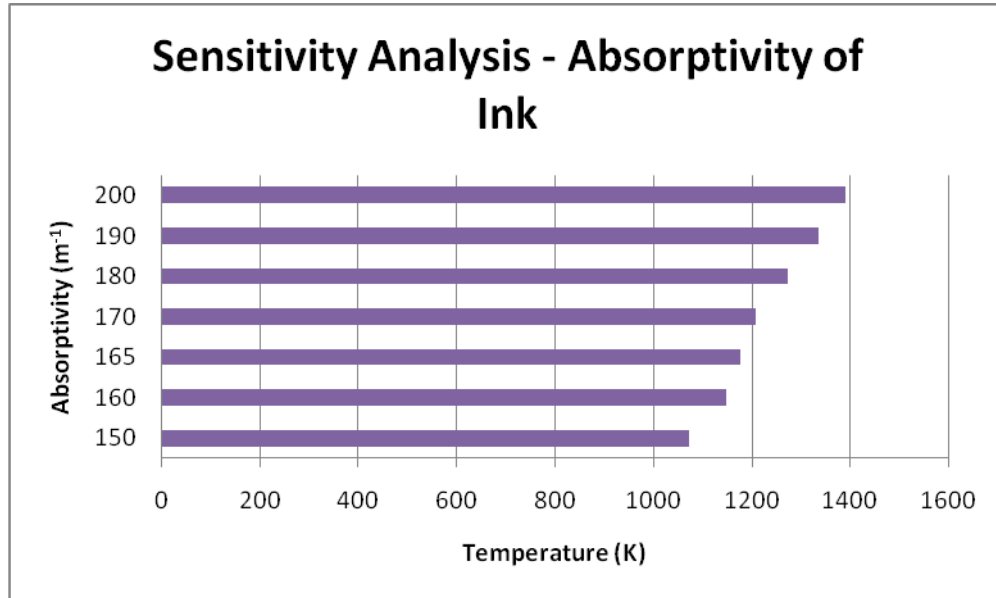

Figure 4: Comparison of the effect of the absorptivity of the ink on ink layer average temperature.

The absorptivity of the ink was a parameter unavailable in tattoo literature. It is known that the laser more selectively heats the ink region, while leaving the skin relatively undamaged. From this, it was deduced that the absorptivity of the ink must be much larger than that of the skin. The original ink absorptivity value was one order of magnitude greater than the original absorptivity value of the skin. The initial value used was 200 m^{-1}, which gave an average temperature in the ink of 1390K. Upon reducing the absorptivity, the average temperature of the ink layer dropped to an appropriate level to mimic temperatures seen in the experimental data. An optimum absorptivity was determined to be 165 m^{-1} because at this absorptivity the average temperature in the ink layer is approximately 900 degrees Celsius.

Absorptivity of Skin –

The second sensitivity analysis was performed on the absorptivity of the skin. The results of this analysis can be seen below in Figure 5.
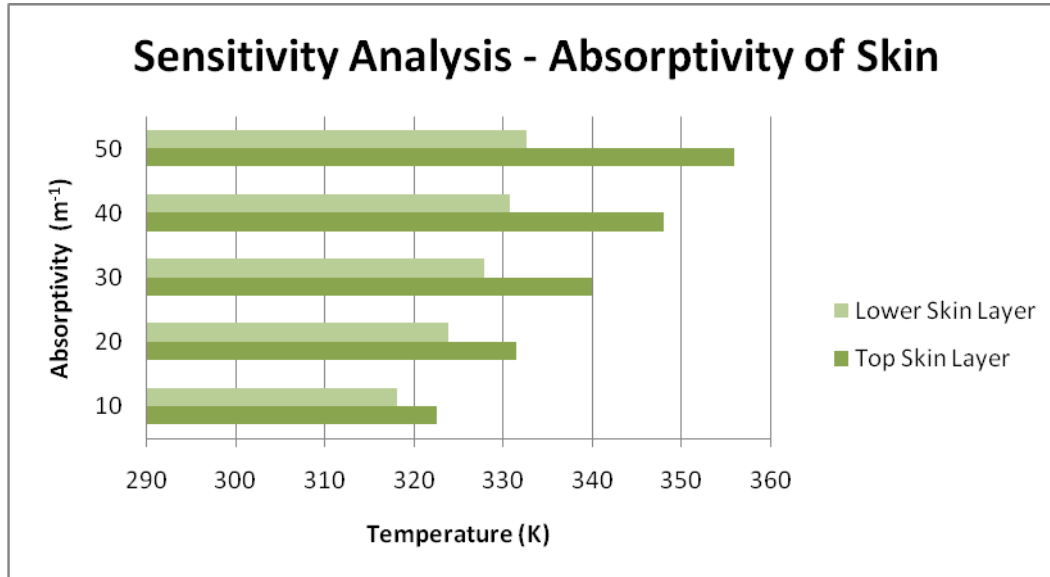

Figure 5: Comparison of the effect of the absorptivity of the skin on skin layers average temperature.

Through a sensitivity analysis examining the absorptivity of the skin, an optimal value was found. The most optimal absorptivity for the skin was determined to be 20 m^{-1}. While the absorptivity of 10 m^{-1} renders lower temperatures for both the top and bottom skin layer, this is not a realistic estimate. Experimental data indicates that pain occurs when skin tissue reaches 45 degrees Celsius (Kamel 2008). Laser tattoo removal causes mild to moderate pain so the surrounding skin layers are probably heated during the process to temperatures around 45-55 degrees Celsius. The skin absorptivity that corresponds best to this temperature range is 20 m^{-1}. Absorptivities of 30 m^{-1} and above make the top skin layer extremely hot; too hot to correspond to the temperatures recorded experimentally. In fact, for some of these high absorptivity values, the top layer of skin would actually begin to burn. The burning of the skin would make the process of laser tattoo removal pointless, as the tattoo would have scarring in its place.

Specific Heat of Ink –

The final sensitivity analysis performed was on the specific heat of the ink. The results of the analysis are below in Figure 6.
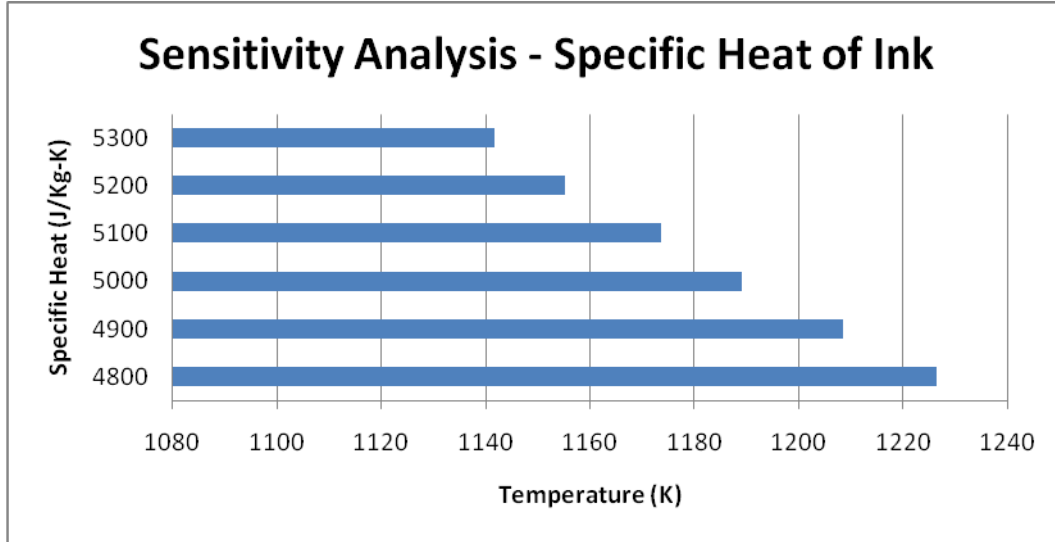

Figure 6: Comparison of the effect of specific heat of ink on ink layer average temperature.

The model showed that over a range of 500 J/kg-K that the average temperature in the ink layer changed by 7.42%. From this sensitivity analysis, it can be concluded that the model is not especially sensitive to ink specific heat values around that of water (4184 J/kg-K).

Simulation Results

The transient analysis of temperature in skin containing a tattoo is shown below in Figure 7 for one laser pulse of 40 nanoseconds. This run was done at the optimal properties as determined through the sensitivity analysis discussed above.
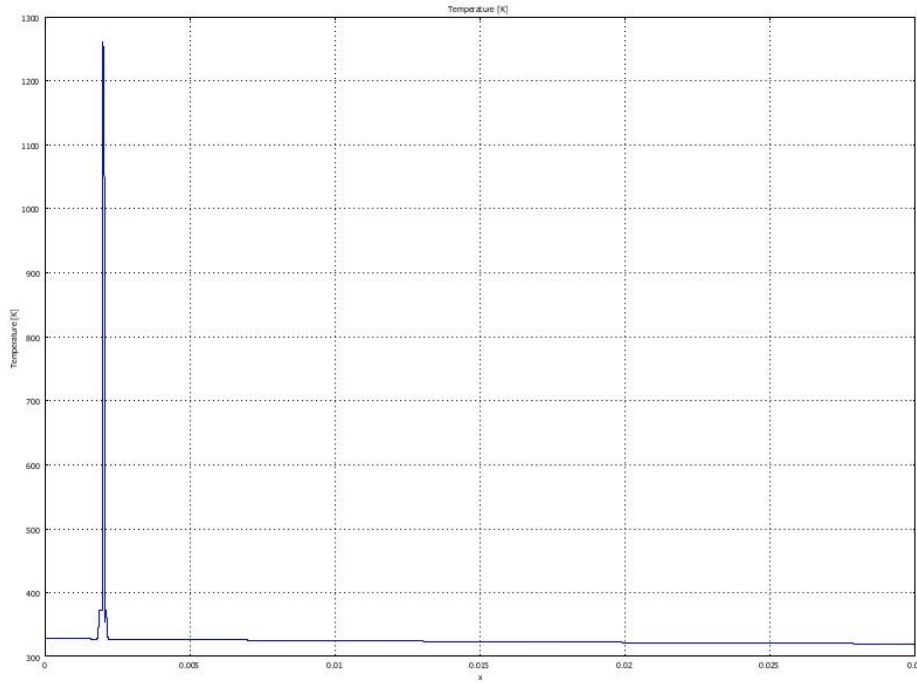
Figure 7: Temperature profile along skin/ink construct at the end of the single 40 nanosecond laser pulsation.

During the 40 nanosecond pulse time, the ink temperature peaks at 1260 K. The average temperature for the ink region, however, is 1176.42 K. This exceeds the targeted temperature for the successful explosion of pigment particles by only 3 K, indicating that the model is a fairly accurate representation of the laser tattoo removal process with the optimal parameters in a 40 nanosecond pulse time. Also, the left side of the temperature profile is level to 310 K, which indicates that the boundary conditions did not force the model to reach body temperature (310 K). The semi-infinite boundary conditions are, therefore, appropriate. Not shown in Figure 7, but significant nonetheless, is that the right convective boundary condition with h = 25 W/m^2, did not affect the temperature of the top skin layer, the epidermis. The model was run for a time period of up to one second, and even then the convection term did not impact the temperature of the epidermal skin layer.

To see the cumulative temperature profile at all of the time steps, Figure 8 was generated. Each color represents a different time step.
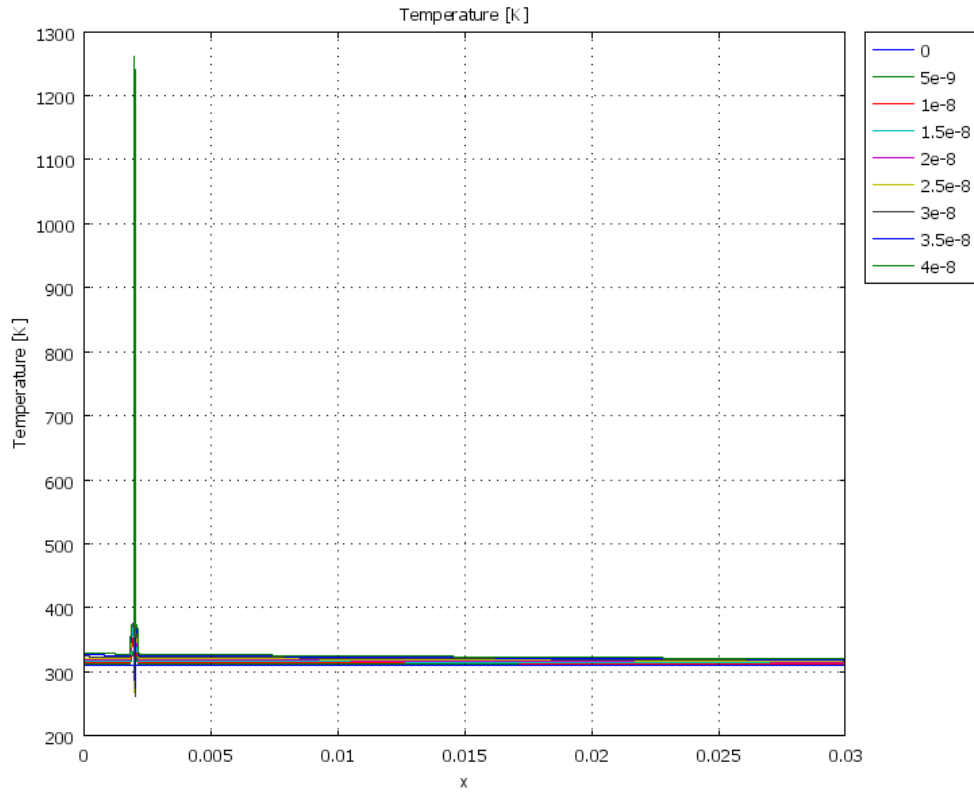
Figure 8: Cumulative run of program using optimal properties for 40 nanoseconds with 5 nanosecond time steps. Temperature as a function of length (y: 200-1300K), (x: 0-0.03m).

Tracing the temperature profiles, at certain time steps COMSOL calculated temperatures in the ink lower than the initial condition. These dips are inconsistent with other calculated time steps and with common heat and mass transfer knowledge (if a body is being heated it should not at any point be colder than the initial temperature). To better visualize this dip at the particular time step, a zoomed image of Figure 8 was generated and is shown in Figure 9 below.
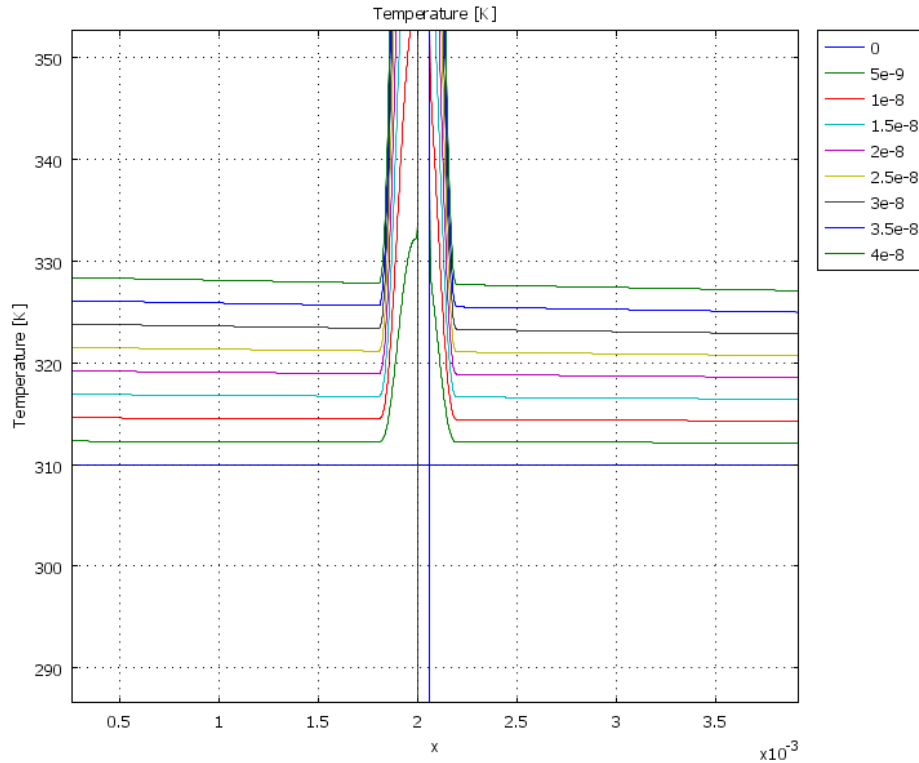
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Figure 9: Zoomed figure of above cumulative run showing error in COMSOL computation from harsh boundary and surrounding skin tissue temperatures in more detail. The figure shows all time steps.

The temperature inconsistencies seen in Figures 8 and 9 can be attributed to problems within the COMSOL software. The program had a difficult time running the model. The difficulty was attributed to the small time step change and sharp boundary changes at the skin/ink interfaces (the properties of the skin and ink were sufficiently different at the interior layer boundaries to cause problems in computation). To help COMSOL, several smoothing functions were included in the model. These functions smoothed the transition of properties between the skin and ink, and helped to remove most of the dips. By adding more smoothing functions, all of the abnormalities could probably be removed. Including more smoothing functions, however, would make the model less realistic as properties of the skin and ink do not change gradually between the layers.

Examining Figure 9 again, it must be noted that the left and right areas bordering the ink layer are nearly 330 K, which is greater than body temperature by about 20 degrees. Sources indicate that skin heated about 45 degrees Celsius cause pain. The regions bordering the ink layer are about 55 degrees Celsius and probably cause a mild amount of pain, which is typical of laser tattoo removal (Kamel 2008). Thus, the model matches up to available experimental data in that the ink layer reaches 900 degrees Celsius and the surrounding skin layers remain cooler. These tissue layers are also heated by the laser generation, but only to the point that they would cause mild to moderate pain, which is common and well documented with this type of procedure. At no point do the surrounding tissue layers reach a temperature that would cause burns and scarring.

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Laser Tattoo Removal Design Optimization

An optimization was conducted to see the effects of changing laser pulsation times and laser intensities on the average temperatures of the three layers (Top, Ink, and Lower). Three different values for each property were chosen, two being extremes and one being the properties for optimal ink absorptivity (as seen in the above simulation results). For the laser intensity, the high value tested was 10^{17} W/m^2 and the low was 10^9 W/m^2. For the pulse times, the high value tested was 60 nanoseconds and the low was 20 nanoseconds. Extremes were chosen to ensure that most other lasers would fall somewhere within the spectrum. Thus, by examining the trends associated with extreme changes in either laser intensity or pulse time, one could better understand how any laser, with properties within the extreme boundaries, would affect skin and ink temperature during tattoo removal.
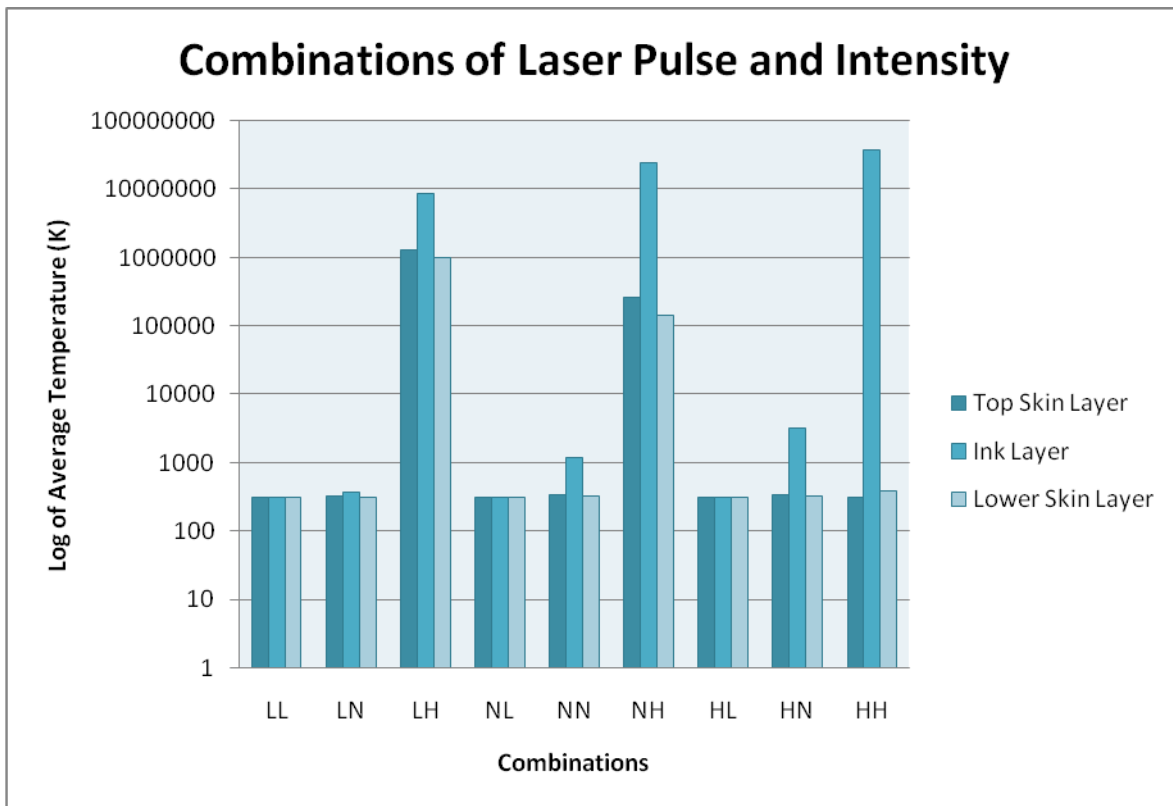

Figure 10: Data showing the average temperature results in each layer for the various laser and pulse combinations. L stands for low and is a lower laser intensity and time pulse than was found in the experimental literature. N is normal and represents the original values used in running the program. H is high and is a higher laser intensity and time pulsation than normal. The first letter in the x-axis symbols refers to pulsation time and the second letter corresponds to laser intensity so LL would be both lower pulse time and lower laser intensity. The actual numerical values for this data can be found in a table in Appendix C.

Based on Figure 10 we can see that changing the laser intensity has a greater effect on the average temperatures than changing the pulse time. Low intensity lasers create flat temperature profiles

throughout all layers. With these low intensities, the average temperature of the ink layer is not increased to the point where the ink particles would explode. Thus, with these lasers, tattoos cannot be removed.   All of the profiles with the normal laser intensity display an increase in the average temperature in the ink layer over the surrounding skin layers. The normal normal (NN) combination seen in the middle of the graph represents the current best laser option for tattoo removal as the temperatures most closely correspond to experimental data. Finally, the high intensity lasers create the largest rise in average temperature in the ink layers. These high intensity lasers also dramatically heat the surrounding skin (Top and Lower layers) to an average temperature that would burn and destroy the tissue. While the ink would certainly explode at every pulse time with these lasers, the tissue damage would be so severe that they would not be used.

Conclusions and Design Recommendations

The model discussed above accurately simulates laser tattoo removal. While it is a very simplified model, it does mimic experimental data and gives indications as to how other lasers, with varying pulse times and intensities, may affect skin and ink layer temperatures. By understanding this, one can anticipate how effective each of these potential lasers may be in removing tattoos. It is important to evaluate whether the original design objectives were met when evaluating the validity of the results.

The first objective was to simulate heat transfer during laser tattoo removal. This model was to ensure that the ink layer reached the experimentally determined 900 degrees Celsius and that the surrounding tissue layers were neither burned nor destroyed within the 40 nanosecond pulse time. Furthermore, the model was to take into account evaporation such that in tissues reaching 100 degrees Celsius, evaporation would occur. Upon complete evaporation the temperature would continue to rise, reaching the aforementioned experimental values without burning the skin. In order to reach the high temperatures within the ink layer while maintaining relatively low temperatures in the surrounding tissue, the second goal of the study, to find the values of various unknown parameters, was accomplished.

The optimal absorptivities for the ink and skin were determined to be 165 m^{-1} and 20 m^{-1} respectively. By determining these unknown values via the sensitivity analyses, the temperature profiles more accurately imitated experimental data. Through sensitivity analyses and comparison with experimental data, the model was validated.

With a working and accurate model, the final objective was accomplished. The aim was to analyze other laser pulse and laser intensity combinations in hopes of optimizing laser tattoo removal. Optimization was defined as reaching 900 degree Celsius in the ink layer while maintaining low surrounding skin temperatures faster. The results from the model show that increasing the pulse time with the existing Q-Switched Ruby Laser may provide the best results in the future.

The COMSOL model is useful for improving future work of laser tattoo removal. The current model mimics the results seen with the laser intensity and pulsation time of 10^{13} W/m^2 and 40 nanoseconds respectively. It provides a framework to evaluate the potential effect of different laser models with varying pulsation times and intensities. Ideally a new model will be made that both brings the ink temperature to the critical value of 1173K and reduces damage of the epidermis and dermis. This will minimize the pain of the procedure and allow for multiple treatments to be performed successively.

Appendix A: Mathematical Model

Geometry

Our project is modeled in one dimension to represent the three dimensional reality of laser tattoo removal. This simplification can be made due to the linear nature of the rapid laser heating. Using the absorption and conductive properties of the epidermis and ink layer, a model for heat transfer was developed using the laser as a generation term dependent on its depth of penetration. Additionally, the effect of evaporation was considered. In order for the computer to process the evaporation, a one dimensional model was necessary. Our model shows heating within the tissue and ink layers as a result of the laser. From the model, we could determine the extent of tissue damage from the laser tattoo removal process. For dimensions see the schematic above.

Governing Equations

$$\underbrace{\rho c \frac{\delta T}{\delta t}}_{Transient} = \underbrace{k\left(\frac{\delta^2 T}{\delta x^2}\right)}_{Conduction} + \underbrace{Q}_{Source\,Term} \qquad \text{(Heat Transfer)}$$

$$\underbrace{\frac{\delta c}{\delta t}}_{Transient} = \underbrace{r_a}_{Source\,Term} \qquad \text{(Mass Transfer)}$$

Where Q is laser generation and latent heat of vaporization, depending on the temperature and concentration of water in the tissue:

$$Q = \begin{cases} \alpha I_0 e^{-\alpha x}, & T < 373K \ \cap c > 0 \ or \ T \geq 373K \ \cap c = 0 \qquad Laser\ Generation \\ -s\Delta H_v, & otherwise \qquad\qquad\qquad\qquad\qquad\quad Latent\ Heat \end{cases}$$

Where, s is the rate of evaporation given by:

$$s = \frac{1}{\Delta t}\left[\frac{\rho c_p (T - 373K)}{\Delta H_v}\right]$$

Initial Conditions

Heat Transfer
All layers are initially at body temperature, 37°C (310K).

Mass Transfer
All layers have an initial water concentration of 960 kg/m^3. Found from the Skin Care Guide which claimed that the moisture content of skin was 80% water: 0.8*(1200 kg/m^3) = 960 kg/m^3

Boundary Conditions

There is a convective boundary condition at the skin-air interface:
$$-k\frac{dT}{dx}\bigg| = h(T - T_\infty)$$

The skin below the ink layer is modeled to be semi-infinite, thus, at x = ∞ the temperature is equal to body temperature (310K).

T(x=∞) = 310 K

Input Parameters

Dimensions:
- Top skin layer: 2E-3 m
- Ink layer: 6E-5 m
- Bottom skin layer: 2.8E-2 m

Table A1: Laser properties (Pfirrmann 2007)

Intensity	W/m^2	1E13
Pulse Time	s	40E-9

Table A2: Boundary properties

Heat Transfer Coefficient (h)	$W/m^{2\text{-}}K$	25
Semi-Infinite Temperature	K	310

Table A3: Subdomain properties (Datta, Rakesh 2008)

		Skin	Ink
Density (ρ)	kg/m^3	1200	1000
Specific Heat (c_p)	J/kg-K	3600	5100**
Thermal Conductivity (k)	W/m-K	0.209	0.65
Absorptivity (α)	m^{-1}	*	*

*NOTE: these values were found by performing a sensitivity analysis
**The value was assumed due to lack of any literature data

Solver

The solver used in this model was COMSOL Multiphysics 3.3.

Time Stepping

Simulations were performed for a 40 nanosecond interval (the time of one laser pulse) and each time step was 5 nanoseconds. Due to the short nature of these time steps, no further reduction was needed to improve the accuracy of the results.

Appendix B: Mesh

Mesh

The mesh was split into three regions: the skin above the ink, the ink layer, and the skin below the ink (see Appendix A for dimensions). As the model was made in 1D, the elements are simply points along the line. There is a higher mesh density moving towards the ink layer than in either skin layer because the temperature gradient is larger in this area. Below is the model meshed with 26 elements. This mesh is shown because one can see the individual points, however, a much finer mesh (where the individual elements are so close together that they cannot be seen) was required to reach convergence. From the mesh convergence (see below), the number of elements used to run the model was 3328.
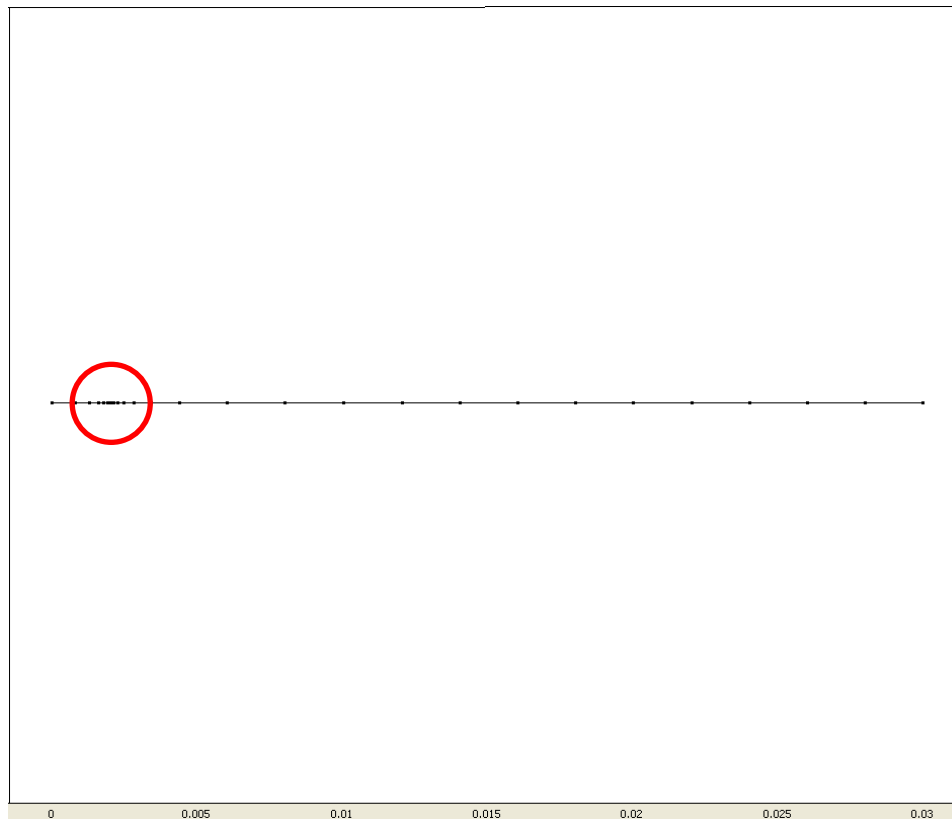

Figure B1: Tissue mesh with 32 elements to show mesh density along the line. Area of increased mesh density in ink layer circled.

Mesh Convergence

A mesh convergence analysis was performed on the model. The geometry was meshed with an increasing number of elements, and the value of the average temperature in the ink after 40 nanoseconds was measured. The initial mesh for the model was 26 elements. Looking at the table, much higher element numbers were needed to reach a consistent average temperature in the ink layer than 26. The element sizes and corresponding average temperatures tested are recorded in Table B1.

Once mesh convergence was reached, no further element numbers were tested. Results are summarized below in Table B1 and Figure B2.

Table B1: Mesh convergence data for laser tattoo removal. Ideal number of elements bolded.

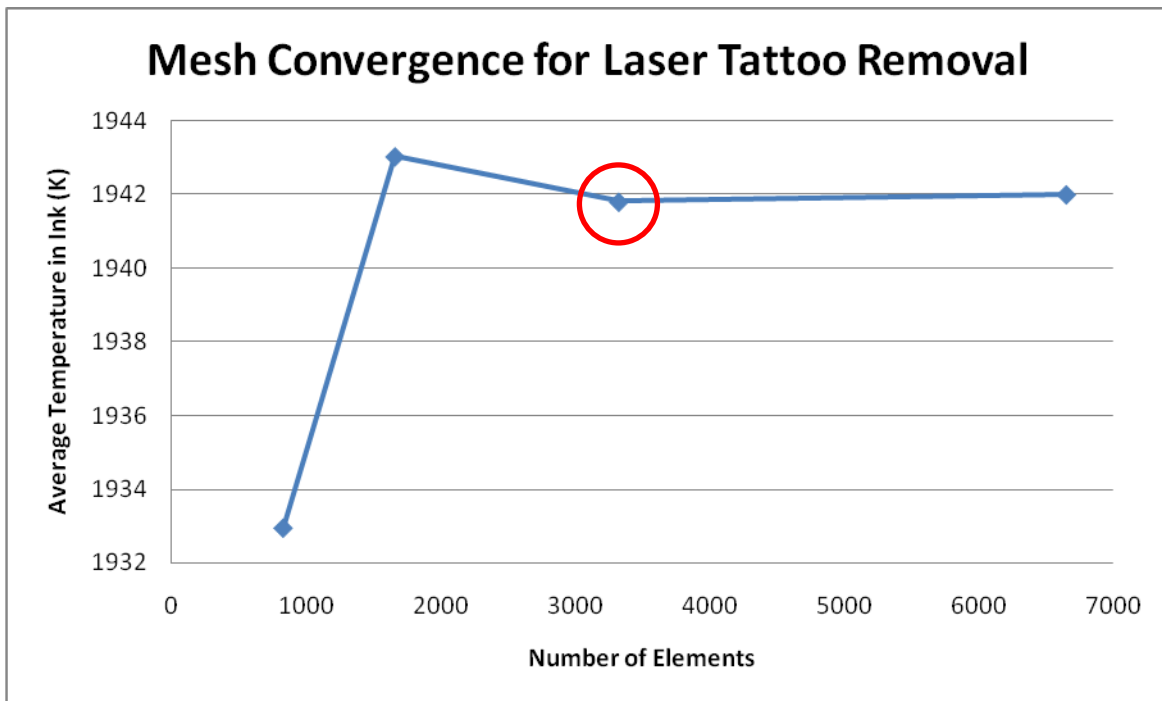| Number of Elements	Average Temperature (K)
832	1932.933
1664	1943.033
3328	**1941.8**
6656	1942

Figure B2: Mesh convergence data. Ideal number of elements circled.

The high average temperatures noted in the mesh convergence data above result from the property values used while performing the mesh convergence. The property values were held constant throughout the convergence testing, so the determined number of elements to use (3328) is valid. Although the property values will be changed for various runs, the mesh convergence has confirmed that 3328 elements should be used.

Appendix C: Additional Material

Laser Tattoo Removal Design Optimization

Table 1: Average temperatures in the ink and skin layers with varying laser intensity and pulsation duration. L stands for low and is a lower laser intensity and time pulse than was found in the experimental literature. N is normal and represents the original values used in running the program. H is high and is a higher laser intensity and time pulsation than normal.

Laser Intensity Laser Pulsation Time	L: 10^9 W/m^2	N: 10^{13} W/m^2	H: 10^{17} W/m^2
L: 20ns	Epidermis: 310.0015 K Ink: 310.08 K Dermis: 310 K	Epidermis: 322.33 K Ink: 376.03 K Dermis: 316.92 K	Epidermis: 1.3 x 10^6 K Ink: 8.4 x 10^6 K Dermis: 6.9 x10^4 K
N: 40ns	Epidermis: 310.0 K Ink: 310.167 K Dermis: 310.001 K	Epidermis: 331.45 K Ink: 1176.42 K Dermis: 323.74 K	Epidermis: 2.6 x 10^5 K Ink: 2.4 x 10^7 K Dermis: 1.4 x 10^5 K
H: 60ns	Epidermis: 310.004 K Ink: 310.367 K Dermis: 310.002 K	Epidermis: 340.15 K Ink: 3157.167 K Dermis:330.53 K	Epidermis: 310.0015 k Ink: 3.65 x 10^7 K Dermis: 393.13 K

Appendix D: References

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