STRATEGIC PLANNING TO ALLEVIATE OPERATIONAL, FINANCIAL AND CLIMATE RISK IN SUPPLY CHAIN MANAGEMENT

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The supply chain is a complex system with the participation of various parties. Due to its complexity , supply chain management not only involves the regular uncertainty from the demand and supply side, but also the specific risks depending on the operational context and the industry. The regular risk has long been studied in the literature. How to use operational strategy to alleviate the unconventional risk remains less explained.

In this dissertation, we investigate different sources of supply chain risk: (1) operational risk, (2) financial risk and (3) climate risk. We develop linear programming and dynamic programming models to draw insight of how the firm can proactively prepare for those risks, either under operational constraints, third party intermediary, or government policy restriction. We also conduct numerical studies both using data of the real firm and simulation to validate the conclusion from the mathematical models.

The dissertation consists of three main chapters.

- Chapter 2: Mitigating Supply Chain Operational Risk Using Part Inventory Portfolios. This is joint work with Professor William Schmidt.

- Chapter 3: Mitigating Supply Chain Financial Risk Using Reverse Factoring. This is joint work with Professor Li Chen.

- Chapter 4: Mitigating Climate Risk Using Strategic Growing Area Planning in Agricultural Supply Chain. This is joint work with Professor Nagesh

Gavirneni.

The three chapters are independent but compliment each other. Chapter 2 uses the linear programming model to show how the firm reroutes its production plan upon disruption, and how to proactive change in inventory policy to mitigate the firm's disruption exposure in a cost effective way using company data. Chapter 3 uses the dynamic programming cash management model to study how Reverse Factoring creates value for both a small supplier and a big buyer, and how the liquidity provider designs the optimal term. Chapter 4 builds stylized model to explore how the agricultural firm's fixed investment decision is impacted under yield uncertainty with or without the government restriction on carrying over products. All these chapters commonly focus on different sources of risk in the supply chain. Therefore, this dissertation aims to contribute to strategically alleviating the risk in supply chain management by incorporating specific context of the supply chain.

BIOGRAPHICAL SKETCH

Xiaobo Ding received a Bachelor's degree in Business Administration, second Bachelor's degree in Applied Mathematics, and Master's degree in Management Science, from Tsinghua University. He received his Master's and doctoral degree in Operations, Technology and Information Management from SC Johnson College of Business at Cornell University.

This document is dedicated to all Cornell graduate students.

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CHAPTER 1

INTRODUCTION

1.1 Overall Introduction

The supply chain is a complex system with the participation of various parties. Due to its complexity , supply chain management not only involves the regular uncertainty from the demand and supply side, but also the specific risks depending on the operational context and the industry. The disruption risk has long been studied in the literature and has received more attention in the global pandemic situation. How to use operational strategy to alleviate the disruption risk of the supply chain now lies in the key interest of many firms. Nevertheless, the earlier financial crisis in 2008 has caused the tight credit rating for small business and the consequence of high borrowing cost for small business is exacerbated in the current pandemic situation as well since firms call for more need of liquidity. How supply chain can be configured to solve this problem and facilitate the cooperation among supply chain parties also becomes the key to the success of business. Last, specifically for the food supply chain in the agricultural context, the climate risk impacts the profitability hence the investment decision of the firms. In some agricultural sectors, the climate risk has posed challenge for the regulators to balance the risk-bearing capability of the supply chain and the control of the market. How to design the regulation to provide the right incentives for the agricultural supply chain participants under the climate risk, has an important implication for the long-term development for the agricultural industry of a country.

1.2 Summary of Chapter 2

High impact / low probability supply chain disruptions pose a major challenge for firms. Such disruptions represent a form of bottleneck shifting because the firm's constrained resource changes over time. When the firm can backlog some portion of its unmet customer orders, its disruption exposure is driven by (i) lost orders in the disruption stage due to part constraints and (ii) lost orders in the recovery stage due to capacity constraints.

Although holding inventory has been shown to mitigate a firm's disruption exposure, it can be a costly approach. We address this challenge by showing that the firm's disruption exposure is (i) decreasing at a decreasing rate with the inventory quantity of the disrupted part and (ii) decreasing at a decreasing rate with the inventory quantity of *non-disrupted* parts. We examine the practical implications of these effect using the detailed supply chain and operational data from our research partner (anonymized as DMF), a large manufacturer of material handling equipment.

We show how targeted changes to a firm's inventory policies across a set of parts can cost-effectively achieve a significant reduction in its disruption exposure. In DMF's case, disruption exposure is reduced by 52.6% to 55.4% while aggregate inventory holding and ordering costs are reduced by 1.7% to 8.1%. We further show that the firm can materially reduce its disruption exposure across *all* parts by holding a small amount of incremental inventory of selected parts.

This "strategic portfolio" of parts serves as a means for the firm to effectively store its otherwise unused disruption period production capacity.

1.3 Summary of Chapter 3

Small suppliers are often cash-strapped due to long payment collection cycles and high credit rates. Supply chain finance, also known as reverse factoring (RF), is a popular instrument used in practice to address this problem. Specifically, SCF enables a supplier to collect outstanding payments from a buyer earlier at a discount through a third-party intermediary, usually a bank. In this paper, we study the supplier's cash management problem under RF, and also quantify the value of RF to the supplier. We show that the optimal policy for the multi-period cash management problem is a modified (L_n, U_n) policy, with the (borrow-up-to) L_n threshold increasing over time and the (invest-down-to) U_n threshold remaining constant. The value of SCF to the supplier is shown to increase in the outstanding payment as well as in the credit rating disparity between the supplier and the buyer. We explain how RF creates a win-win-win situation for the supplier, the buyer and the bank. Moreover, our model can be used to inform the design of the optimal term for supply chain finance.

1.4 Summary of Chapter 4

INAO (French version of FDA) decided to relax its regulation (June 20, 2018) on the AOC wine due to the bad weather condition of the year to help the wine makers to better hedge the risk. Previously, the wine makers are permitted 10% of the annual yield as VCI and a total VCI of 30% in three years. The new regulation allows 20% VCI annually and 50% in three years.

Extended from the wine regulation practice, we look at how the regulation can impact the agricultural firm's selling and stocking decision in different circumstances. Specifically, we want to look at two dimensions: (1) whether the selling decision for the agricultural product is made before the market price shock; (2) whether the product can be carried over across periods. Both the yield and market price are considered uncertain for the agricultural firm, but the realization of the price uncertainty may depend on the specific agricultural market: it could be realized before selling quantity is decided or the agricultural firm can make responsive selling quantity decision after observing the market price. The agricultural firm's decision involves its growing area decision beforehand and its selling quantity with or without the presence of the regulation, where the regulation could be the permission of carrying over products in a regulated market, i.e., the wine case.

CHAPTER 2

MITIGATING SUPPLY CHAIN OPERATIONAL RISK USING PART INVENTORY PORTFOLIOS

2.1 Introduction

High impact, low probability supply chain disruptions represent a critical and persistent business risk [14]. Examples include supplier outages [37], floods [57], industrial accidents [30], and labor disputes [54], among others. Our research focuses on mitigating the impact of such disruptions using part inventory. The academic and practitioner literature has long recognized that holding inventory can reduce the firm's exposure to a supply chain disruption [58, 59, 53]. However, holding inventory can be expensive. As [8] neatly summarize, "Holding inventory in this situation can get very costly. The reason is simple: while holding inventory costs are incurred continuously, the inventory would be used only in the rare event of a disruption."

We consider the disruption exposure (*DE*) of a supply chain disruption to a multi-product firm that can backlog a portion of its unsatisfied customer order commitments. We measure *DE* as the impact of the disruption on the firm's gross profits. In this setting, a supply chain disruption can evoke two distinct stages – a disruption stage, during which a supplier is unable to deliver a needed part to the firm, and a recovery stage, during which the firm is working to satisfy any backlogged orders. The firm faces a bottleneck shifting problem across the two stages. In the disruption stage, the firm's *DE* is driven by unmet orders due to part constraints. In the recovery stage, the firm's *DE* is driven by unmet orders due to capacity constraints. The capacity constraints in the recovery stage are a consequence of the unused production capacity in the disruption stage, and the resulting backlog of orders. This backlog can overwhelm the firm's production capacity in the recovery stage, and prevent the firm from recovering all of the backlogged orders from the disruption stage. Our research accounts for these dynamic changes in the firm's constraining resource.

Our objective is to examine how part inventory can be used to cost effectively reduce the firm's *DE*. An important feature of our analysis is accounting for the fact that a supply chain disruption can occur at any time during the consumption replenishment cycle for a part. In both our analytical and empirical analyses, we account for this variation in the amount of disrupted part inventory that the firm has available.

Using a restricted 3-product, 2-part optimization model, we analytically show that a firm's *DE* to a part is decreasing at a decreasing rate with the amount of inventory of that part. We refer to the reduction in the firm's *DE* with increases in disrupted part inventory as the *primary effect* of inventory. We also analytically show that the firm's *DE* is decreasing at a decreasing rate with the amount of inventory of other, *non-disrupted* parts. We refer to the reduction in the firm's *DE* with increases in non-disrupted part inventory as the *secondary effect* of inventory. The secondary effect of inventory raises the possibility that more inventory for non-disrupted parts can theoretically be used to mitigate the firm's *DE* for *all parts*. The intuition behind the secondary effect is that additional non-disrupted part inventory allows the firm to avoid idling their production capacity during the disruption stage by instead overproducing products that are not reliant on the disrupted part. This overproduction frees up production capacity in the recovery stage that the firm can deploy to meet its backlog of disrupted products.

Our analytical results cannot inform whether the primary and secondary effects of part inventory are economically material, or even present, in practical settings. For that, we turn to the detailed operational and supply chain data from our research partner, a large diversified manufacturing firm (hereafter anonymized as DMF). DMF is a make-to-order manufacturer with over \$1.5 billion in annual sales and a range of market-leading product offerings. The company maintains a pipeline of committed customer orders, representing approximately 8 to 13 weeks of production time. It regularly updates its production plan, and uses it to make delivery commitments to its customers. Minor delivery delays are tolerable. For longer delays, however, DMF is exposed to losing a portion of its late customer orders.

To quantify our analyticl insights, we develop a linear programming model that estimates DMF's *DE* to part outages in its supply chain. The firm must reallocate its parts and production capacity to minimize the impact of the disruption on its total lost gross profit across both the disruption and recovery stages. Other performance measures are trivial to implement, including lost sales or lost units of production. We represent a disruption as an outage of a single part from a single supplier for a fixed period of time (our main results are presented using a 5-week disruption). In the case of DMF, there are 8,832 unique part-supplier pairs (hereafter, disruption scenarios). We estimate the *DE* under different amounts of disrupted part inventory by setting six equally-spaced inventory levels for each disrupted part based on the firm's inventory policy for that part. We apply our model to analyze each disruption scenario at each inventory level for the disrupted part.

Using our model, we first confirm that the firm's *DE* decreases at a decreasing rate with the amount of disrupted part inventory, as predicted by our restricted model. This marginal effect of inventory on the firm's *DE* varies widely, not only across parts but also within a part. Based on a disruption duration of 5 weeks, the average range of the firm's *DE* for a disrupted part over its normal inventory replenishment cycle is $484K, with a minimum range of $0 and a maximum range of $5.54M. We demonstrate the practical relevance of these insights by showing that DMF can achieve a significant reduction in its *DE* at a low-cost or even no cost by making targeted changes to its inventory policies. Depending on the part, these changes take two general forms – holding more safety stock and / or reducing the order up to amount. We provide implementation examples that reduce the firm's *DE* between 52.6% and 55.4% while simultaneously reducing its inventory holding and ordering costs between 1.7% and 8.1%. There are many operational factors beyond disruption risk mitigation that a firm must consider before making such policy changes. Therefore, this portion of our analysis can be interpreted as an example of how risk-mitigating inventory policy changes can be identified. Such options can then be balanced with other operational considerations before implementation.

Next, we find that the secondary effect of part inventory is absent at DMF for *individual* non-disrupted parts. We provide intuition for why this is likely to be the case in many practical settings. The firm can, however, create a "strategic portfolio" of parts to unlock the secondary effect. Establishing a strategic portfolio involves coordinating the inventory level across all of the parts needed to build a particular product. We demonstrate that a well-chosen strategic portfolio can have the dual benefit of (i) generating a large primary effect for some disruption scenarios by meeting more of the production requirements in the disruption stage and (ii) generating a large secondary effect for other disruption scenarios by using idle production capacity in the disruption stage to meet a portion of the production requirements of the recovery stage. For some strategic portfolios, the secondary effect benefit can exceed that of the primary effect. The *DE* reduction from a well-chosen strategic portfolio is also materially larger than what can be achieved by naively increasing inventory levels across all parts.

Our analysis of strategic portfolios is limited to sets of parts that comprise the bill-of-materials for a single product. We provide an example that the firm can further reduce its *DE* by allocating the same budget to a strategic portfolio that includes the bill-of-materials for more than one product. Identifying the optimal strategic portfolio is computationally expensive, however. For practical settings in which this may be an issue, we introduce a heuristic that can materially reduce the processing time. In our practical setting, we find that the heuristic estimate deviates from the full model in fewer than 0.2% of the disruption scenarios, and results in a *DE* estimate that, on average, is less than 0.5% larger than the full model.

Our findings show that to more effectively mitigate *DE* using part inventory, firms must account for (1) cross-part dependencies, (2) the primary and secondary effects of part inventory, and (3) the related decreasing marginal benefit of incremental inventory. By doing so, a firm can inexpensively achieve a material reduction in its *DE* through targeted modifications to its inventory policies and coordinating its inventory levels for a strategic portfolio of parts.

2.2 Literature

Our research is built upon three streams of operations management literature. The first deals with mitigating the operational performance impact of low probability, high impact operational disruptions. Firms can consider a variety of proactive strategies to mitigate their disruption exposure [34], including holding more inventory [41, 23, 61], establishing multiple sources of supply [58, 12, 63, 21], business interruption insurance [16], and investing in process flexibility [60, 2]. Using a principal-agent contracting framework, [28] investigate the incentives and outcomes under performance based contracts with restoration service providers that are intended to help a company recover from disruptions. [58] study the contingent rerouting of suppliers as a component of the optimal disruption management strategy to reduce the firm's cost. [55] theorize strategies that may be cost-effective and time-efficient, including storing inventories at strategic locations that can proxy for carrying more safety stock. [47] examines service level mitigation strategies under a disruption in a two-echelon system consisting of multiple plants and distribution centers. The focus is on echelon-level disruptions (as opposed to part-level disruptions) and mitigation strategies that include drawing finished products from alternative, non-disrupted facilities and distribution centers. [15] provide insights on how to develop a portfolio of suppliers when the yields from the suppliers' unreliable production processes are correlated. [36] propose an (s, S) production-inventory policy with random disruption in an unreliable bottleneck production system. We add to this literature by examining how part-level disruptions can be mitigated in a production system with backlogging. Although other research acknowledges that it can be costly to use inventory buffer as a hedge against disruption risk [61, 51], we show how to address this challenge by taking advantage of the primary and secondary effects of inventory with targeted changes to part-level inventory policies.

We also build on the relatively smaller literature that explicitly accounts for how firms operationally recover from a disruption. Much of this research is focused on the transportation industry generally, and within that, the commercial airline industry. In this context, the firm's recovery problem is formalized similar to its original planning problem, but with fewer available resources due to the disruption. Applications include aircraft re-routing [46], crew-flight pairing [66], and train re-scheduling [62]. These papers focus on formalizing the problem and developing a general approach to solve them. Less attention is given to quantifying the impact or efficiency of the solution. Various strategies are used in airline disruption recovery planning, including redistributing existing slack [1], adding extra buffers to the schedule, and avoiding operational complexity [29]. We refer the reader to [9] for an overview of the disruption research in the airline industry. While the transportation-related literature deals with recovering from a service delivery disruption, other work examines disruption recovery in the context of manufacturing a physical product. [42] develop a recovery plan for disruptions in a two-stage production inventory system with consideration for process reliability. [56] propose a heuristic for recovery in a machine breakdown problem with two products. [25] exogenously set the duration of the recovery stage in an analysis of disruptions in a manufacturing setting. [64] analyze recovering from disruptions in a two-stage coordinated supply chain. We contribute to this stream of literature by identifying how firms can implement changes that exploit the decreasing marginal impact of part inventory on the firm's disruption exposure, and quantifying the practical impact of those changes using data from our research partner.

Last, we contribute to the resurgence of practice-based research in manufacturing and service operations management. Examples of such practiced-based research include[27], [3], [49], and [45]. [27] deals with the optimization of the loading and routing decisions for a manufacturing firm's supply chain. [3] implements a collective decision support system of an automotive company. [49] describes a solution approach for the scheduling problem of a manufacturing firm. [45] develops a simulator to aid in planning, preparation, and evacuation decisions of a military installation in response to tropical storm forecasts. We apply the optimization methodology to study the disruption risk mitigation strategy using inventory policy, in the context of a large materials handling equipment manufacturer. Our approach for measuring the impact of a disruption is inspired by that of [50], but with four critical extensions. First, we generalize our model to allow for backlogging of customer orders. This results in a more complex, albeit realistic problem with shifting production constraints across periods. Second, we account for variation in the firm's disrupted part inventory by running each disruption scenario across representative ranges of inventory levels. Third, we account for the production of all of the firm's products, not just those that rely directly on the disrupted part. Fourth, we explicitly model a recovery stage and account for the production capacity constraints faced by the firm.

2.3 Research Setting and General Problem Description

We implement our general model and conduct our research in collaboration with DMF, a large diversified manufacturing firm. DMF is a leader in its industry with annual revenue of approximately $1.5 billion. It primarily serves North American customers, but augments this with exports to Central and South America, Australia, and the Middle and Far East.

We consider a common make-to-order manufacturing system in which the firm receives committed orders from customers in advance. The firm manufactures multiple products with multiple parts from multiple suppliers. Many of the firm's parts are standardized and used across multiple products. Based on inbound customer order commitments, the firm periodically updates its production plan, schedules part orders with its suppliers, and commits to delivery dates with its customers. The firm's production capacity is scheduled and allocated across products during the firm's production planning process. The firm has reasonably stable capacity over the short term, with variations over time that are due to things such as scheduled maintenance, holidays, allowed overtime, and planned capacity changes. Production plans and customer delivery commitments must be balanced such that there is sufficient production capacity to meet committed customer deliveries, with a small buffer to accommodate minor delays. DMF's committed production plan represents approximately 90% of its maximum production capacity in each period.

DMF's production plan is also the basis for its rolling order purchase commitments with its suppliers. These commitments provide reasonable assurance that the firm will have sufficient parts to meet its production plan. DMF can deviate from its supplier commitments, but there are guard rails that constrain the magnitude of the deviations. For instance, DMF shares its production plans with suppliers on a rolling 13-week basis, and its purchase commitments can be modified by ±15% at 9 weeks and ±10% at 6 weeks. The firm's materials requirement planning (MRP) system continuously reviews the inventory balances against the planned consumption for each part, and makes order recommendations based on the inventory policies in place for the part and the firm's purchase commitments. Part inventory levels vary over time and across parts as those parts are consumed in the manufacturing process and replenished by suppliers. The firm does not maintain any uncommitted finished goods inventory.

The firm employs flexible production processes, allowing it to switch over production lines from products with low orders to products that are experiencing high orders. Reallocations of flexible capacity can be done in the production planning process, or in response to a disruption.

We obtained detailed operational data from DMF, including production plans, product bill-of-materials, part inventory policies, suppliers information, and product financial information. The firm has 327 suppliers that provide it with 8,616 unique parts. None of the parts have more than two suppliers and 8,400 out of 8,616 parts have a single supplier. Consequently, DMF has 8,832 unique supplier-part pairs.

DMF has 20 products, representing a wide range of customer order quantities. The average weekly production for each product ranges from 4 to 246 units, and there is no discernable seasonality. The number of unique parts used in each product ranges from 241 to 2,014. Minor deviations in the timing of cus-

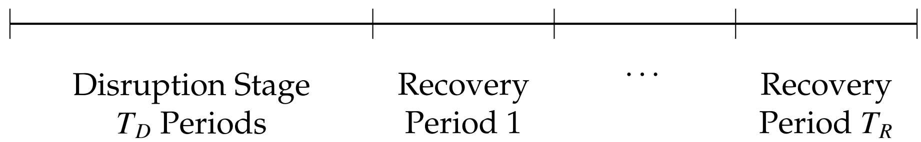


Figure 2.1: Timeline for Supply Chain Disruption

tomer deliveries are acceptable, but DMF tries to minimize them. For longer delays, the company estimates that 90% of its unmet customer orders can be backlogged into the next delivery period.

As shown in Figure 2.1, our analysis spans two stages. The disruption stage starts when a supplier is unable to deliver a part, and it ends when the disrupted part arrives at the firm. The recovery stage starts once the disruption period ends (i.e. once the disrupted parts are delivered), and it ends when the firm no longer has any backlogged production. We consider an all-or-nothing disruption such that the supplier cannot meet any of the firm's orders for the disrupted part during the disruption period, and can meet all of the firm's current and backlogged orders at the conclusion of the disruption period. We define the *DE* of a disruption scenario as the total lost gross profit that the firm incurs as the result of the disruption. This is measured as the lost gross profit in the disruption and recovery stages. When a disruption occurs, the firm can adjust its production plan over the disruption and recovery stages to minimize its *DE*.

In addition to our general problem, we analyze a restricted problem with two parts and three products. This simplification of the general problem allows us to build mathematical insights that we can test in our numerical analysis of the general problem.

2.4 The Model

2.4.1 Assumptions

We assume that the firm has planned its part orders and original production plan without errors. This assumption allows us to focus on a disruption to the focal part, and maintain that the firm will have enough production capacity and inventory of non-disrupted parts to meet its original production plan. We assume that the firm's production resources are sufficiently flexible so as not to constrain revisions to the original production plan. In a disruption, the firm can modify its original production plan, given its production capacity and the pool of parts required to meet the original production plan. We ignore certain costs associated with the firm's response to the supply chain disruption, including the holding cost of inventory, expedited shipment costs, or higher part costs. The cost of ordering and holding inventory is accounted for in our analysis of potential inventory policy changes, however. During a disruption, such costs can be minor relative to the potential lost profits due to lost orders. For instance, when asked by one of the authors whether inventory holding costs, transportation costs, or higher part prices factored into their disruption response planning, a senior executive at Nissan Motor Corporation (not affiliated with DMF) made clear – "We are not worried about saving pennies ... Instead, we are focused on returning to normal production levels." This sentiment was shared by the managers at DMF.

2.4.2 Notation

We summarize our notation in Appendix A.1. The duration of the disruption stage is T_D and the duration of the recovery stage is T_R. T_R is endogenously determined by the model for each disruption scenario. As a result, we conduct our analysis by pooling the periods in the disruption stage, but we must analyze the recovery stage period-by-period. We index time periods with t, where $t \in \{T_D, \cdots, T_D + T_R\}$.

Let $\mathcal{S}$, $\mathcal{F}$, and $\mathcal{P}$ denote the set of tier-1 suppliers, the set of finished products, and the set of parts, respectively. Let $\mathcal{S}^D$, $\mathcal{F}^D$, and $\mathcal{P}^D$ denote the set of disrupted suppliers, the set of finished products that rely on the disrupted part, and the set of disrupted parts. We use index i, j, k to indicate a supplier i in $\mathcal{S}$, a finished product j in $\mathcal{F}$ and a part k in $\mathcal{P}$. Product j has gross profit of g_j. Producing one unit of product j requires w_j of production capacity and r_{kj} units of part k. The portion of the unmet orders of product j that can be backlogged into the next period is γ_j. Supplier i can deliver a maximum c_{ik}^t units of part k in period t. The firm has a maximum production capacity of H^t in period t. The original planned production of product j in period t is d_j^t.

In each period t, the firm decides the amount of each part k that will be sourced from each supplier i (u_{ik}^t), the allocation of each part k to each product j (y_{kj}^t), and the production quantity of each product j (p_j^t). The shortfall in the production of product j in period t compared to the original plan is l_j^t. The inventory of part k at the beginning of period t is q_k^t. The inventory of product j at the beginning of period t is s_j^t.

2.4.3 General Model

The general model can be described with the following objective and constraints.

The firm's objective function is to allocate its part inventory and production capacity to minimize its DE, which is expressed as:

$$\sum_{t=T_D}^{T_D+T_R} \sum_{j \in \mathcal{F}} (1-\gamma_j) g_j l_j^t.$$

The solution is subject to the following constraints:

- **Inventory Allocation Constraint**: The units of part k allocated to the production of product j is equal to the production of product j times the units of part k needed to produce one unit of product j:

$$y_{kj}^t = p_j^t \cdot r_{kj}^t, \quad \forall k \in \mathcal{P}, \forall j \in \mathcal{F}, t \in \{T_D, \cdots, T_D + T_R\}$$

- **Inventory Balance Constraint**: The ending part inventory for part k is the beginning inventory, plus total supply of part k, less the total allocation of part k in the production process:

$$q_k^{t+1} = q_k^t + \sum_{i \in \mathcal{S}} u_{ik}^t - \sum_{j \in \mathcal{F}} y_{kj}^t, \quad \forall k \in \mathcal{P}, t \in \{T_D, \cdots, T_D + T_R\}$$

- **Supplier Capacity Constraint**: The part supply is constrained by c_{ik}^t when part k from supplier i is not disrupted. The part supply is zero when part k from supplier i is disrupted.

$$\begin{cases} u_{ik}^t = 0 & \text{if } i \in \mathcal{S}^{\mathcal{D}}, k \in \mathcal{P}^{\mathcal{D}} \text{ and } t = T_D, \\ u_{ik}^t = \sum_{t=T_D}^{T_D+1} c_{ik}^t & \text{if } i \in \mathcal{S}^{\mathcal{D}}, k \in \mathcal{P}^{\mathcal{D}} \text{ and } t = T_D + 1, \\ u_{ik}^t \le c_{ik}^t, & \text{otherwise.} \end{cases}$$

The second term in this constraint set reflects our modeling assumption that the disrupted supplier can meet all of the firm's backlogged orders for the disrupted part once the disruption stage ends. More relaxed or constraining assumptions can also be implemented.

- **Firm Capacity Constraint**: The production of finished products is constrained by the firm's production capacity:

$$\sum_{j \in \mathcal{F}} w_j p_j^t \leq H^t, \quad t \in \{T_D, \cdots, T_D + T_R\}$$

- **Unmet Production Constraint**: The unmet production for each product j is the planned production plus the backlogged production, plus the change in product inventory, minus the production amount. The firm enters the disruption without any backlogged orders.

$$l_j^t = d_j^t + \gamma_j l_j^{t-1} + (s_j^{t+1} - s_j^t) - p_j^t, \quad \forall j \in \mathcal{F}, t \in \{T_D, \cdots, T_D + T_R\}$$

- **Non-negativity Constraints**: $y_{kj}^t, u_{ik}^t, p_j^t, q_k^t, s_j^t, l_j^t \geq 0$ ($\forall i \in \mathcal{S}, \forall k \in \mathcal{P}, \forall j \in \mathcal{F}, t \in \{T_D, \cdots, T_D + T_R\}$)

2.4.4 Three-product Model

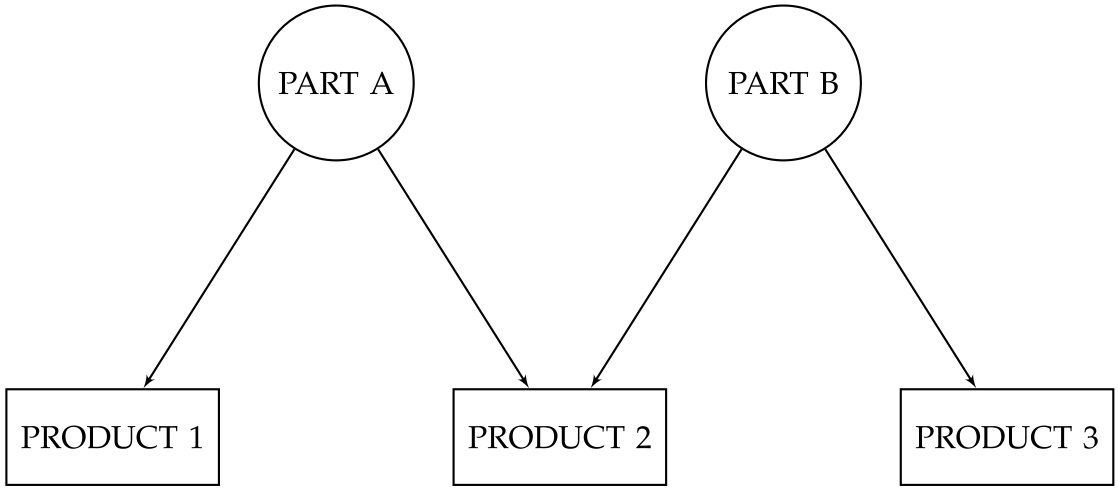
Mathematically expressing the optimal solution to the general model in a practical setting is analytically tedious due to the high dimensionality of the problem. To develop mathematical insights, we analyze a reduced form of this model. We then apply the general model to data from our research partner to validate these insights and quantify the practical results. We utilize two terms in our analysis. **Gross Profit per Part (GPP)** is the ratio of a product's gross profit and the number of units of part k needed to produce one unit of that product, i.e. $\frac{g_j}{r_{kj}}$. **Gross**

Profit per Capacity (GPC) is the ratio of a product's gross profit and production capacity needed to produce one unit of that product, i.e. $\frac{g_j}{w_j}$. GPC is used only in the appendix.

In the reduced setting, $\mathcal{F} = \{1, 2, 3\}$ and $\mathcal{P} = \{A, B\}$, where A is the disrupted part and B is the non-disrupted part. To simplify our notation, we assert that the firm sources each part from one supplier. This allows us to remove the supplier's index from all of our notation. To simplify our analysis, we also assert that the disruption and the recovery stages each last one period, i.e. $T_D = 1$ and $T_R = 1$, and any unmet orders at the end of the recovery stage are lost. Finally, we assume an equivalent backlogging rate (γ) across products, i.e. $\gamma = \gamma_1 = \gamma_2 = \gamma_3$. To avoid trivial outcomes, we consider the case where the starting inventory for the disrupted part is not sufficient to meet the combined orders for both dependent products in the disruption stage.

The bill-of-materials structure is illustrated in Figure **??**. Products 1 and 2 each use one unit of part A. Products 2 and 3 each use one unit of part B. The suppliers have sufficient capacity to meet the firm's part requirements when they are not disrupted. The disrupted supplier cannot deliver any units of part A in the disruption stage, but will make up for the backlogged order in the recovery stage. Thus, $c_A^1 = 0$, $c_A^2 \geq d_1^1 + d_2^1 + \gamma \cdot (d_1^2 + d_2^2)$, $c_B^1 \geq d_2^1 + d_3^1$ and $c_B^2 \geq d_2^2 + d_3^2$. The firm has just enough production capacity to cover the original planned production in each period, i.e., $H^1 = \sum_{j=1}^{3} d_j^1 \cdot w_j$ and $H^2 = \sum_{j=1}^{3} d_j^2 \cdot w_j$.

We represent the firm's production decision as $\mathbf{p} = (\mathbf{p^1}, \mathbf{p^2}) = (p_1^1, p_2^1, p_3^1, p_1^2, p_2^2, p_3^2)$, and the production plan for product j as $\mathbf{p}_j = (\mathbf{p_j^1}, \mathbf{p_j^2})$, $j = 1, 2, 3$ (the slight variation between these two notational forms is to ease the presentation of our proofs in the appendix). Based on the simplifications of



the three-product model, we describe it with the following objective and constraints:

$$\min(1-\gamma)\cdot\sum_{j=1}^{3} g_j l_j^1 + \sum_{j=1}^{3} g_j l_j^2$$

subject to

$$\begin{cases} q_A^2 = q_A^1 - p_1^1 - p_2^1 & \textbf{(Inventory Balance - Part A)} \\ q_B^2 = q_B^1 + u_B^1 - p_2^1 - p_3^1 & \textbf{(Inventory Balance - Part B)} \\ u_A^1 = 0; u_B^1 \le c_B^1 & \textbf{(Supplier Capacity)} \\ \sum_{j=1}^3 w_j \cdot p_j^1 \le H^1 & \textbf{(Firm Capacity - Disruption)} \\ \sum_{j=1}^3 w_j \cdot p_j^2 \le H^2 & \textbf{(Firm Capacity - Recovery)} \\ l_j^1 = d_j^1 + s_j^2 - p_j^1 & \textbf{(Unmet Production - Disruption)} \\ l_j^2 = d_j^2 + \gamma\cdot l_j^1 - s_j^2 - p_j^2 & \textbf{(Unmet Production - Recovery)} \\ u_B^1, q_A^2, q_B^2, p_j^1, p_j^2, l_j^1, l_j^2, s_j^2 \ge 0, \forall j \in \{1,2,3\} & \textbf{(Non-negativity)} \end{cases}$$

The feasible region **F** of the firm's decision problem can be reduced to the subspace of $\{0 \le p_j^1 \le d_j^1 + d_j^2, 0 \le p_j^2 \le [d_j^2 + \gamma\cdot l_j^1 - l_j^1 + (d_j^1 - p_j^1), j = 1,2,3\}$, since it is never optimal to produce more than the overall orders for a product in the disruption and recovery stages. The firm's adjusted production plan is constrained by different resources in the disruption and recovery stages. In the disruption stage, the disrupted part inventory is the primary constraint, although the non-disrupted part inventory can also serve to constrain over-production of certain products. In the recovery stage, the recovery capacity is the only constraint as it is not useful to overproduce any products.

2.5 Analytical Results for 3-Product Model

The 3-product model allows us to develop analytical insights that can be tested using our general model and data from our research partner. The proofs for our propositions are in the Appendix. As with our general model, we must account for both the disruption stage and a recovery stage in our analysis. In the disruption stage, the potential constraining resources are: (1) part A inventory, (2) part B inventory, and (3) production capacity. In the recovery stage, the only constraining resource is the production capacity. The problem satisfies strong duality, thus the KKT (Karush-Kuhn-Tucker) condition [31] is sufficient and necessary for the solution. This problem represents two sub-LP problems based on the range of l_j^1 and s_j^2 that can be characterized as:

- **Product j Under-produced.** $l_j^1 \geq 0$ and $s_j^2 = 0$.
- **Product j Over-produced.** $l_j^1 = 0$ and $s_j^2 > 0$.

Whether product j is under-produced or over-produced is determined by the parameter set $\mathbf{R} = \{q_A^1, q_B^1, g_j, w_j, d_j^1, d_j^2\}$. The optimal production plan $\mathbf{p}^*$ is the optimal solution of at least one LP problem. In each optimal solution, there is a product that is allocated the last unit of available disrupted part inventory. For expository convenience, we refer to this product as the **marginal product**.

Let $\mathbf{\Omega}$ represent a basis in our linear program, or a set of binding constraints with rank equal to the number of decision variables [13]. In our problem, Ω can be formally described by two characteristics: (1) the resources that are binding and (2) the actual production versus the original production plan. In the disruption stage, the actual production of a product can meet, exceed, or fall short of the original production plan. In the recovery stage, the actual production of a product can meet or fall short of the original production plan plus any backlogged production from the disruption stage. The parameter set $\mathbf{R}$ determines the optimal LP basis, and therefore the optimal disrupted part allocation decision. The complete partition of the parameter space and the optimal LP basis in each region can be analyzed with the following process: (1) propose the binding constraints that compose the corresponding solution basis Ω; (2) calculate the corresponding solution $\mathbf{p_\Omega} = \mathbf{A_\Omega^{-1}} \cdot \mathbf{b_\Omega}$; (3) validate that the solution is feasible, i.e., $\mathbf{p_\Omega} \in \mathbf{F}$; (4) confirm the parameter settings for the proposed solution to be optimal, i.e., $\mathbf{A_\Omega^{-T}} \cdot \mathbf{c_\Omega} \geq 0$, where A_Ω and c_Ω are the constraint matrix and objective function coefficients corresponding to Ω. To avoid trivial cases, the disrupted part (part A) inventory and the recovery period capacity must be binding. Other potentially binding constraints are the non-disrupted part (part B) and the upper and lower bounds constraints in $\mathbf{p} \in \mathbf{F}$.

The optimal production plan can be used to determine the marginal benefit of the binding resources. There are eight subspaces that must be analyzed. We provide the details of the optimal production plan and the marginal benefit of the part inventory for one subspace in the Appendix. The details of the remaining subspaces follow a very similar process, and are available from the authors.

For a given parameter set $\mathbf{R}$ and base $\mathbf{\Omega}$, we can express the firm's *DE* as a function of $\mathbf{R}$, i.e. $DE^{\Omega}(q_A^1, q_B^1, \cdots)$. The function is a linear combination of all of the terms of the binding constraints. The coefficients are the corresponding shadow price of the binding resources [5], i.e. $A_{\Omega}^{-T} \cdot c_{\Omega}$. We refer to the change in the firm's *DE* with respect to changes in disrupted part inventory ($\frac{\partial DE^{\Omega}(q_A^1, q_B^1, \cdots)}{\partial q_A^1}$) as the *primary effect* of inventory. We refer to the change in the firm's *DE* with respect to changes in non-disrupted part inventory ($\frac{\partial DE^{\Omega}(q_A^1, q_B^1, \cdots)}{\partial q_B^1}$) as the *secondary effect* of inventory. The primary and secondary effects of inventory remain constant as long as the change in q_A^1 and q_B^1 does not cause a change in $\mathbf{\Omega}$. When $\mathbf{\Omega}$ changes, the marginal impact of the disrupted and non-disrupted part inventories can also change. This means that the marginal effects of part A inventory and part B inventory are both piece-wise linear.

We show that the firm's *DE* decreases with increases in disrupted part inventory, and the rate of decrease weakly decreases when $\mathbf{\Omega}$ changes. This finding is captured in Proposition 1. The *DE* is piece-wise linear decreasing at a decreasing rate in the amount of disrupted part inventory. Each linear piece represents the optimal production plan represented by a specific base of the LP problem.

Theorem 1. *The* DE *weakly decreases in q_A^1 at a weakly decreasing rate.*

Similarly, the firm's *DE* decreases with non-disrupted part inventory at a decreasing rate. In the three-product model, additional inventory of part B allows the firm to over-produce product 3 in the disruption stage. Doing so preserves production capacity that may otherwise be lost in the disruption stage, and alleviates the production requirements in the recovery stage. This finding is captured in Proposition 2.

Theorem 2. *The* DE *weakly decreases in* q_B^1 *at a weakly decreasing rate.*

Propositions 1 and 2 reflect the primary and secondary effect of part inventory, respectively. These propositions hint at the complexity of the firm's optimal allocation decision, and the need to account for both disrupted and non-disrupted part inventory. To shed light on this complexity, we start by further simplifying the analysis. If we assume that the firm is unable to backlog customer orders, then the firm can resume normal operations when the disruption ends, and there is no recovery stage. Intuitively, this means that secondary effect of non-disrupted inventory is zero and the firm is solely interested in reducing its *DE* in the disruption stage. There may be other circumstances in which the firm is solely interested in minimizing its *DE* in the disruption stage. For instance, if the disruption is so great that it may jeopardize the firm's survival during the disruption stage. This intuition leads to the following Lemma.

Theorem 3. *The firm can minimize the* DE *in the disruption stage by allocating disrupted part inventory to meet the production requirements of disrupted products in descending order of the products' GPP.*

This simplification of our problem serves as a convenient heuristic that can be applied to both the three product model and the general model. Under this heuristic, the firm's updated production plan is straightforward: (1) allocate sufficient disrupted part inventory to the disrupted products with the highest GPP, up to the point that customer orders for those products are met in the disruption stage; (2) produce all non-disrupted products according to the original production plan. It can be advantageous to deviate from this heuristic if the firm is capacity constrained in the recovery stage. In that case, the firm faces a bottleneck shifting problem, and it may benefit by producing products in the disruption stage that require a higher production capacity per allocated part. This can include producing a disrupted product with a lower GPP, or overproducing disrupted or non-disrupted products. Doing so allows the firm to avoid producing those capacity-hungry products in the recovery stage. Intuitively, this is akin to "storing" production capacity from the disruption stage that will otherwise be unutilized. Deviations from the heuristic can come with costs, however. Overproducing a disrupted or non-disrupted product may require under-producing products that share a part with the overproduced product. If the firm produces a disrupted product with a lower GPP, it loses orders of the under-produced, higher GPP product. These tradeoffs are complex in practical settings because there are many products, and parts are often shared across those products. To understand how pervasive deviations from the heuristic are in practice, and to test the prevalence and relevance of our analytical results, we turn to the data from our research partner.

2.6 Practical Evidence

We conduct an analysis using industry data to achieve three objectives – validate our analytical results, quantify the primary and secondary effects of part inventory for individual parts, and provide examples of how the firm can exploit these effects by making targeted changes to its part inventory policies. The last objective demonstrates how appropriate inventory policy changes can materially and cost effectively reduce DMF's *DE*. Based on input from DMF, we present the results from our analysis of disruptions with durations of 5 weeks. Results using other disruption durations are available from the authors. All of our results are generated by solving the model defined in Section 2.4.3 with ILOG CPLEX (version 12.6).

Recall that the supply of all non-disrupted parts is sufficient to match the production requirements from the firm's original production plan in each period. We do not have records for DMF's part order commitments to suppliers. Unless otherwise stated, we therefore assume that existing part inventory and supplier part order commitments are just enough to meet the original production plan in each period. We examine whether having additional inventory of any particular part has any bearing on the firm's *DE* in our analysis of Proposition 2.

The provision of the disrupted part from the disrupted supplier is interrupted for the duration of the disruption, leaving only the firm's on-hand inventory and deliveries from alternative suppliers (when present). The disruption can start at any point in the disrupted part's consumption-replenishment cycle. To capture this, we construct six evenly spaced levels of inventory for each disrupted part that range from its minimum (level 1) to its maximum order-up-to amount (level 6). While this aligns with a periodic review (s,S) inventory policy common at DMF and other firms, the specifics are not central to our analysis. What is important, however, is that we capture that inventory positions are fluctuating, and that a disruption to a part can strike when the firm's inventory for that part is low or high. This allows us to examine how DMF's *DE* changes over the normal inventory range of each disrupted part.

2.6.1 Impact of Disrupted Part Inventory on *DE*

From Proposition 1, we anticipate that the firm's *DE* weakly decreases at a weakly decreasing rate with more inventory of the disrupted part. This implies that the *DE* can be higher when the firm has little inventory of the disrupted part (level 1) compared to when it has abundant inventory of the disrupted part (level 6). To check this using DMF's data, we run the general model for each of the six inventory levels of each disrupted part to calculate the *DE*. Table 2.1 provides summary statistics of DMF's *DE* by inventory level across all 8,832 disruption scenarios. It is evident from this table that there are material changes in DMF's *DE* over its normal inventory consumption and replenishment cycle. The average *DE* drops from $503K to $19K as inventory of the disrupted parts increases from level 1 to level 6. Within each inventory level, the range in the *DE* over the parts is also material. At each inventory level, several hundred to several thousand disrupted parts have a *DE* of $0, while the maximum *DE* is $5.6M for level 1 and $1.4M for level 6.

Table 2.1: Summary Statistics for *DE* by Part Inventory Level (5-week Disruption Duration)

Level	Mean	SD	1-Quartile	Median	3-Quartile	Max	*DE* = $0
1	$503K	$729K	$4K	$181K	$1,017K	$5,584K	542
2	$192K	$298K	$2K	$53K	$236K	$3,692K	845
3	$91K	$199K	$0K	$1K	$74K	$3,042K	3,472
4	$52K	$136K	$0K	$0K	$11K	$2,429K	4,545
5	$32K	$90K	$0K	$0K	$7K	$1,881K	5,597
6	$19K	$58K	$0K	$0K	$4K	$1,391K	5,824

Note: There are 8,832 disruption scenarios for each inventory level; "DE = $0" identifies the number of disruption scenarios with $0 *DE*.

The change in *DE* with inventory varies by part. Some parts continue to have a large *DE* at inventory level 6, while some parts maintain low *DE* across all inventory levels (for instance, 542 scenarios have zero *DE* at all inventory levels). In Figure 2.2, we order all scenarios based on the difference in the scenario *DE* between inventory levels 1 and 6. As reflected in this figure, there is a wide dispersion across parts in the difference of their *DE* over their normal inventory range – the largest difference is $5.54M and the smallest is $0. On the right side of the figure, 30% of disrupted parts (approximately 2,000) have a *DE* difference in excess of $1M from inventory level 1 to 6. On the left side, more than 40% of the disrupted parts have a *DE* difference less than $0.1M from inventory level 1 to 6.

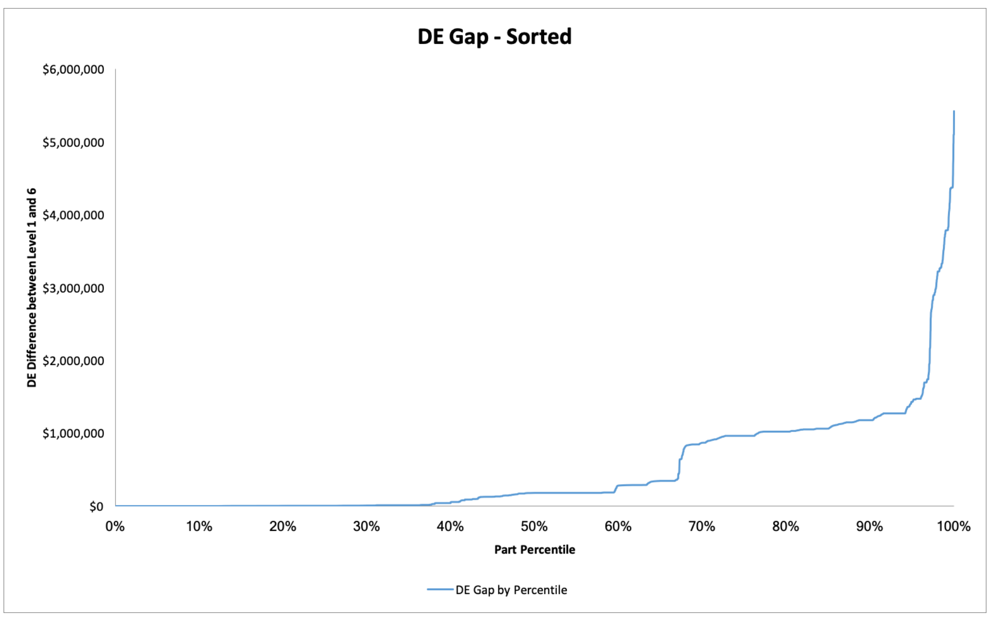


Figure 2.2: The Gap of *DE* between Inventory level 1 and 6

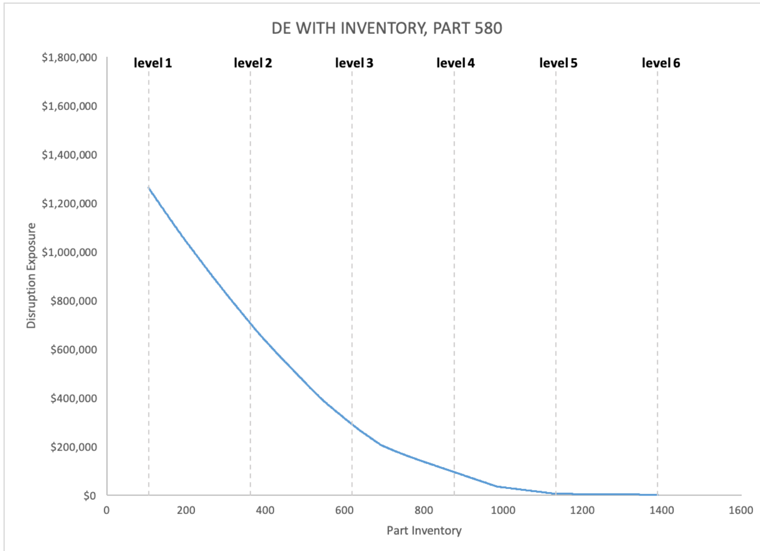
To illustrate how *DE* changes with inventory of a disrupted part, we estimate the *DE* over all possible inventory positions (as opposed to the six discrete levels) between levels 1 and 6 for two parts – 580 and 5736. These parts are used in integer quantities by all of the products that depend on them. Part 580 has 1,284 units of inventory between levels 1 (104 units) and 6 (1,388 units). Part

5736 has 5,326 units of inventory between levels 1 (0 units) and 6 (5,326 units). Therefore, to estimate the *DE* for each inventory position across these two products requires 6,610 runs of the model. The *DE* results by inventory for each part are displayed in Figure 2.3, and the pattern for both parts corresponds to relationship described by Proposition 1. We check the slope of *DE*-inventory line for each inventory position and confirm that the *DE* weakly decreases at a weakly decreasing rate with more disrupted part inventory.

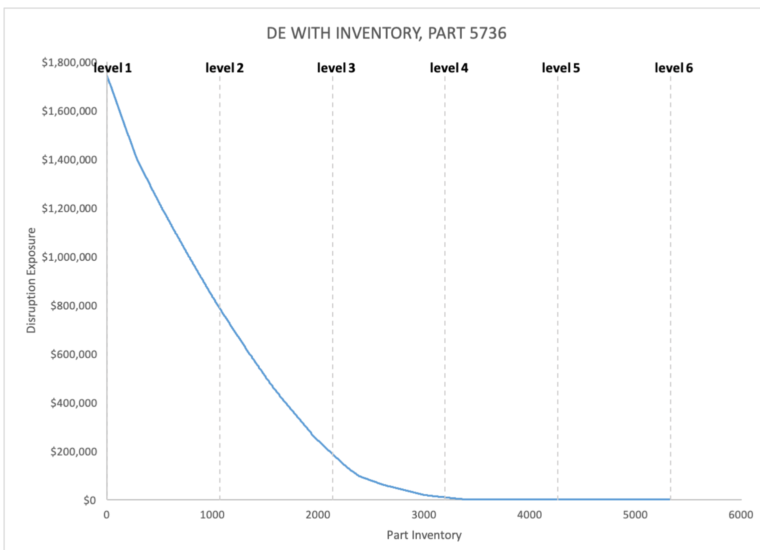
To more clearly establish the practical evidence supporting Proposition 1, we turn to Table 2.2. This table presents summary statistics over each disruption scenario for the *DE* differences by inventory level. The average reduction in *DE* from inventory level 1 to level 2 is $310K, and the magnitude of the reduction weakens for each subsequent inventory level. For each individual scenario, we confirm that the *DE* for a part is weakly decreasing at a weakly decreasing rate in the disrupted part inventory. We also conduct a perturbation analysis at each inventory level and determine the marginal decrease in *DE* at those points. For each scenario we find that the marginal decrease in *DE* is weakly decreasing at higher inventory levels. This provides ample evidence that Proposition 1 holds in practical settings, and no evidence that contradicts it.

2.6.2 Impact of Non-Disrupted Part Inventory on *DE*

From Proposition 2, we anticipate that the firm's *DE* weakly decreases at a weakly decreasing rate with more inventory of a non-disrupted part. Checking every non-disrupted part for every scenario is impractical as it would require running each of the 8,832 scenarios 8,831 times to check a single inventory



(a) Part 580



(b) Part 5736

Figure 2.3: Examples of *DE* change with inventory.

position for each non-disrupted part. Instead, we take two samples of the non-disrupted parts and conduct our analysis on those. The first is a convenience sample of the 5 parts that exhibit the largest primary effect when their inventory level is increased from 1 to 2. The second is a random sample of 5 parts.

Table 2.2: Summary Statistics for *DE* Differences by Part Inventory Level (5-week Disruption Duration)

Level	Mean	SD	Min	1-Quartile	Median	3-Quartile	Max	ΔDE = \$0
1 → 2	\$310K	\$567K	\$0K	\$2K	\$73K	\$401K	\$5,098K	592
2 → 3	\$101K	\$145K	\$0K	\$1K	\$38K	\$181K	\$1,633K	894
3 → 4	\$39K	\$78K	\$0K	\$0K	\$1K	\$33K	\$766K	3,566
4 → 5	\$20K	\$51K	\$0K	\$0K	\$0K	\$9K	\$591K	4,595
5 → 6	\$14K	\$38K	\$0K	\$0K	\$0K	\$2K	\$527K	5,640

Note: There are 8,832 disruption scenarios for each inventory level; "ΔDE = \$0" identifies the number of disruption scenarios with \$0 change in *DE* across adjacent levels.

We look at the effect on the firm's *DE* from a disruption to each of the ten in-sample parts after increasing the availability of a non-disrupted part by 1% beyond what is just enough to meet the original production plan. This involves running the model 88,310 times (8,831 non-disrupted parts times 10 disrupted parts). Our analysis is conducted at inventory level 2 for the disrupted parts. The *DE* is unchanged in each case, meaning the secondary effect of incremental non-disrupted part inventory is zero for all of the 88,310 model runs.

These results do not contradict the claim of Proposition 2, nor do they support a practically meaningful effect. In the simple 3-product model, extra inventory of the single non-disrupted part B allows the firm to overproduce non-disrupted product 3. This saves the recovery period capacity from producing product 3 and allows that capacity to instead be directed toward producing the backlogged production of the disrupted products. In practical settings, however, the bill of material (BOM) for a product can be much more complex than a single part. For DMF, an average of 867 unique parts (a range from 241 to 2,014 parts) are needed to build a product. Overproducing any product requires under-producing multiple other products unless there is excess inventory of *ev-*

ery part needed by the over-produced product. This means that additional inventory of any single non-disrupted part is very unlikely to mitigate the *DE* for another part. Obvious exceptions to this are if the non-disrupted part is a substitute for the disrupted part or if additional inventory is available for all other parts needed to produce a product. In Section 2.6.3, we revisit how DMF can exploit these otherwise tepid findings for Proposition 2

2.6.3 Developing Proposed Inventory Changes to Reduce *DE*

By understanding which parts have material changes in *DE* across inventory levels, firms can identify (1) opportunities to reduce their *DE* through inventory policy changes to targeted parts and (2) the appropriate adjustments to the inventory policies of those parts. Other considerations enter into a firm's inventory decisions beyond *DE* mitigation, such as fixed order increments, minimum order quantities, limited delivery schedules, supplier service levels, economic order quantity guidelines, etc. We relax these constraints in our analysis. Instead, our objective is to demonstrate how firms can develop a set of proposed adjustments and quantify the value of those adjustments. Such proposed adjustments can be then be weighed against these other inventory adjustment considerations before implementation.

We include two inventory-related costs – inventory holding cost and ordering cost. Recall that we ignored these costs when quantifying the *DE* for DMF, arguing that they are minor considerations during the disruption and recovery stages compared to the cost of lost production. These costs are much more salient in our analysis of inventory policy changes because these costs would now be ongoing. Our analysis is based off of an annual inventory holding cost rate for DMF of 6% and a fixed order cost of $100. We focus on 5,608 parts (out of 8,616) for which DMF provided unit cost information. Using all 8,616 parts would afford additional opportunities for *DE* mitigation and cost savings.

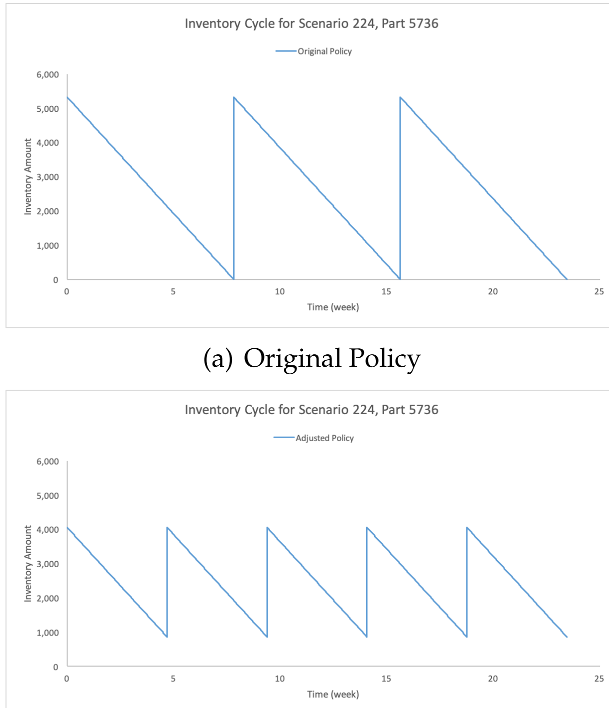
Inventory Changes to One Part

We first consider how the firm can change it's *DE* for a single part. Such an analysis is relevant in practical settings when a specific part is deemed to have a high probability of disruption, such as when the part's production is transferred to a new supplier facility or even a new supplier.

In this analysis, we take advantage of Proposition 1 to identify inventory policy changes that reduce the range of *DE* for the part, while maintaining the current inventory cost of the part. The strategy is simple. First, take advantage of the primary effect of inventory and hold more safety stock. Doing so will reduce the firm's highest *DE* for the part, but it will increase the inventory holding and ordering costs for the part. Second, take advantage of the primary effect of inventory and reduce the order-up-to amount for the part. Doing so will marginally increase the firm's lowest *DE* for the part and further increase the ordering cost, but it will reduce the inventory holding cost. Although this strategy may not achieve cost neutrality in all cases, it can be very effective at reducing *DE* for parts that have a large maximum *DE* that is sensitive to additional inventory, and a low minimum *DE* that is insensitive to less inventory.

As an illustration, we apply this process to part 5736 and limit our inventory choices to the 6 inventory levels. Expanding our choice set of inventory positions can yield further benefits. Figure 2.3(b) provides the *DE*-inventory curve. The inventory of part 5736 fluctuates between 0 units (level 1) and 5,326 units (level 6). The average weekly demand for Part 5736 is 681 units, which implies that an order is place approximately once every 7.82 weeks ((5,326-0)/681). The per-unit cost for this part is \$35.95. Using our cost assumptions, the firm's current inventory policy costs are approximately \$6,409 annually. The annual ordering cost is \$665 (52 / 7.82 $\times$ \$100), the annual inventory holding cost is \$5,744 (0.06 $\times$ (0+5,326)/2 $\times$ \$35.95). DMF's *DE* for part 5736 ranges from \$1,740,227 (level 1) to \$0 (level 6), and an average *DE* of \$347,705 over all inventory positions between levels 1 and 6. DMF can consider two changes – (1) increase the base stock to level 2 (1,065 units) and (2) reduce the order up to amount to level 5 (4,261 units). The first change reduces DMF's maximum *DE* for this part to \$785,693, but it involves a material increase in the average inventory. The second change leverages the findings from Proposition 1 and allows DMF to offset its inventory holding cost while, in this case, keeping its minimum *DE* at \$0. As a result, the average *DE* for this part is reduced from \$347,705 to \$176,213 over the remaining inventory range. Figure 2.4 shows the inventory cycles for the original policy and the proposed policy with the two changes. Figure 2.5 shows the *DE* under the original and proposed policies.

The average inventory is the same under the proposed inventory policy and the original policy, as is the inventory holding cost. However, the part is ordered more often – once every 4.69 weeks compared to once every 7.82 weeks. This yields an incremental ordering cost of \$444 annually. Increased cost = (52/4.69 - 52/7.82) $\times$ \$100 = \$444, new annual cost=\$6,409 + \$444 = \$6,853. To offset this cost, DMF could reduce its inventory holding costs by shifting down its safety stock and order-up-to amount to 859 units and 4,055 units respectively (not



(a) Original Policy

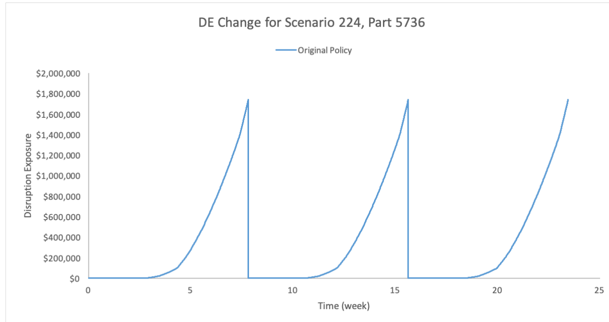
(b) Adjusted Order Policy

Figure 2.4: Inventory Cycle for Different Policies, Part 5736

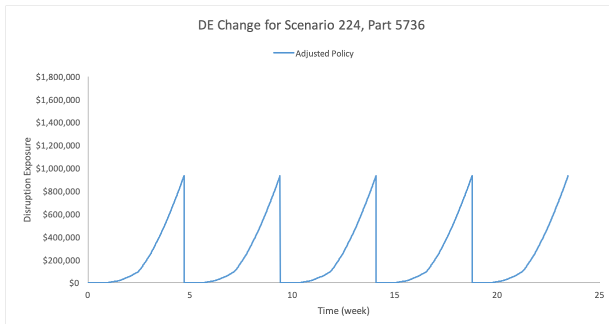
shown). Doing so yields a cost-neutral policy with an average *DE* of $231,593, still far below the average *DE* under the original inventory policy.

Simultaneous Inventory Changes to Multiple Parts

DMF need not restrict itself to changing the inventory policy for a single part. Instead, it can take a portfolio approach to minimize its aggregate *DE* exposure across all parts. By decreasing the order-up-to amount of some parts, DMF can free up budget at the cost of increasing the *DE* of these parts. However, the budget savings can be redeployed to increase the safety stock level of other parts to reduce their *DE*. By leveraging the intuition of Proposition 1, DMF can identify those parts that incur a small increase in *DE* from an order-up-to amount



(a) Original Policy



(b) Adjusted Inventory Policy

Figure 2.5: DE for the Original and Adjusted Inventory Policies, Part 5736

decrease, and those parts that yield a large decrease in *DE* from a safety stock increase.

Reducing the order-up-to level and increasing the safety stock level is akin to DMF selling and buying disruption insurance. Under this framework, the implied insurance rate is the ratio of the cost impact and the *DE* impact due to either a decrease in the order-up-to amount or an increase in the safety stock for a part, i.e. $-\Delta Cost/\Delta DE$. DMF can consider decreasing a part's order-up-to amount when the rate is high and increasing a part's safety stock when the rate is low.

We apply this portfolio approach to DMF, and limit our inventory choices for each part to the 6 inventory levels. Once again, expanding the set of allowable inventory positions can yield further benefits. We present our results based on DMF decreasing a part's order-up-to amount if the implied insurance rate is greater than 10% (i.e., the cost savings exceeds 10% of the increase in the average *DE*), and increasing its safety stock if the implied insurance rate is below 2% (i.e., the cost incurred is below 2% of the decrease in the average *DE*). Other rates are trivial to implement, and should be based on the target firm's *DE* mitigation strategy.

We require that for any inventory policy change, at least one inventory level separates the safety stock and order up to amount for each part. At each inventory level for each part, we calculate the impact on DMF's inventory costs and *DE* by moving up or down one inventory level. Starting at level 6 for each part, if the insurance rate from moving down one level is above 10%, we move the order-up-to amount for that part down one level. We repeat this process until reaching level 2 or until the insurance rate for the next change in level drops below 10%. We next turn to evaluating the firm's safety stock by part. Starting at level 1 for each part, if the insurance rate from moving up one level is below 2%, we move the safety stock for that part up one level. We repeat this process until reaching level 5, reaching one level below DMF's new order-up-to amount, or until the insurance rate rises above 2%. Note that this process can result in a change to both the safety stock and order-up-to amount for the same part. While we restrict our analysis to levels 1 through 6, the firm may also use this framework to analyze inventory quantities above a part's current order-up-to amount or below its safety stock.

Table 2.3 summarizes the total annual inventory cost and average *DE* before and after the inventory policy changes. Before the implementation, the average *DE* is \$153K across all inventory levels and parts with cost information. After the implementation, the average *DE* is reduced to \$68.3K, a 55.4% reduction. The annual inventory cost drops from \$23.6 M to \$23.1 M, a 2.1% reduction.

Table 2.3: Implementations of Multiple Part Inventory Policy Changes

Inventory Policy	Total Annual Cost	Average *DE*
Original	\$23.6M	\$153K
Order-up-to at 10%, Safety stock at 2%	\$23.1M	\$68.3K

Note: Inventory changes are restricted to six levels for each part. The analysis is applied to the 5,637 parts with cost information.

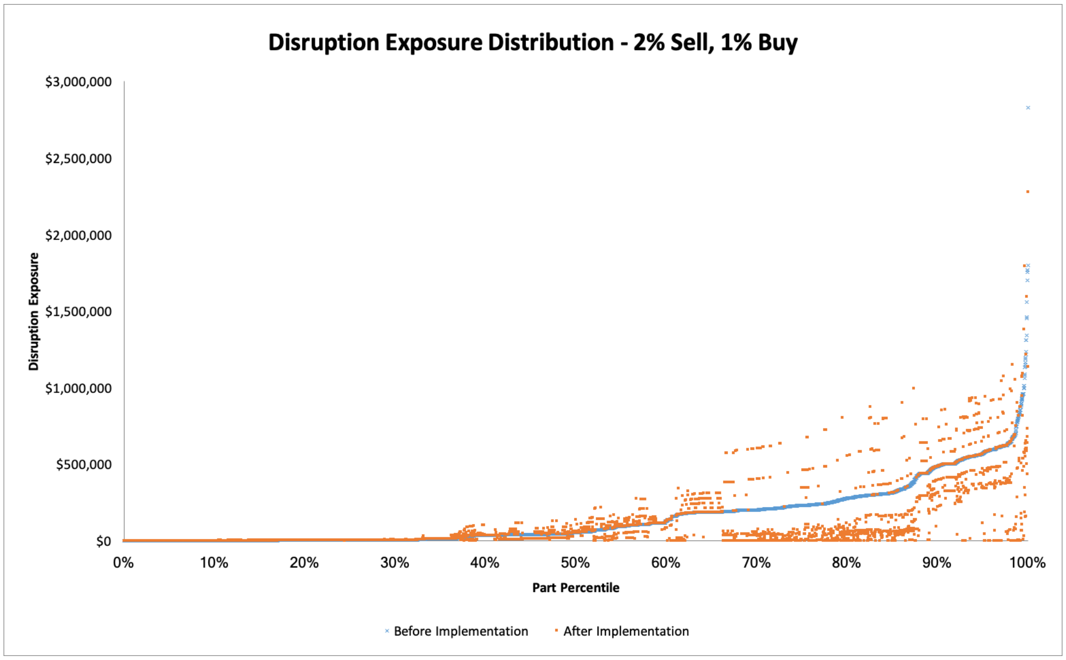
The approach is similar when using other assumptions, although the outcomes will vary. In Table A.2 of the Appendix, we include results based on (1) an insurance rate threshold of 5% for changes to the order-up-to amount and 2% for changes to the safety stock, and (2) switching the sequence to change the safety stock first and the order-up-to amount second. These results are substantively similar to our main results.

For each of the 5,608 parts with cost information (spanning 5,637 scenarios), we calculate the average *DE* over the part's inventory levels, before and after implementation of the proposed policy changes. Figure 2.6(a) presents the average *DE* for each part pre-implementation (blue) and post-implementation (orange) sorted from left to right in order of increasing pre-implementation average *DE*. This figure makes clear that while the policy implementation leads to a net decrease in the average *DE* for some parts, it also yields a net increase in the average *DE* for other parts. The order-up-to amount is reduced for 1,148 (20%) parts, the safety stock is raised for 1,846 (33%) parts, the order-up-to amount is reduced and the safety stock is raised for 120 (2%) parts, and 2,494 (44%) parts are left unchanged. Figure 2.6(b) presents the same results, but with the post-implementation *DE* sorted separately in increasing order. This figure shows that the post-implementation distribution of average *DE* by part does not stochastically dominate the pre-implementation distribution, despite having an overall decrease in the *DE* for the firm.

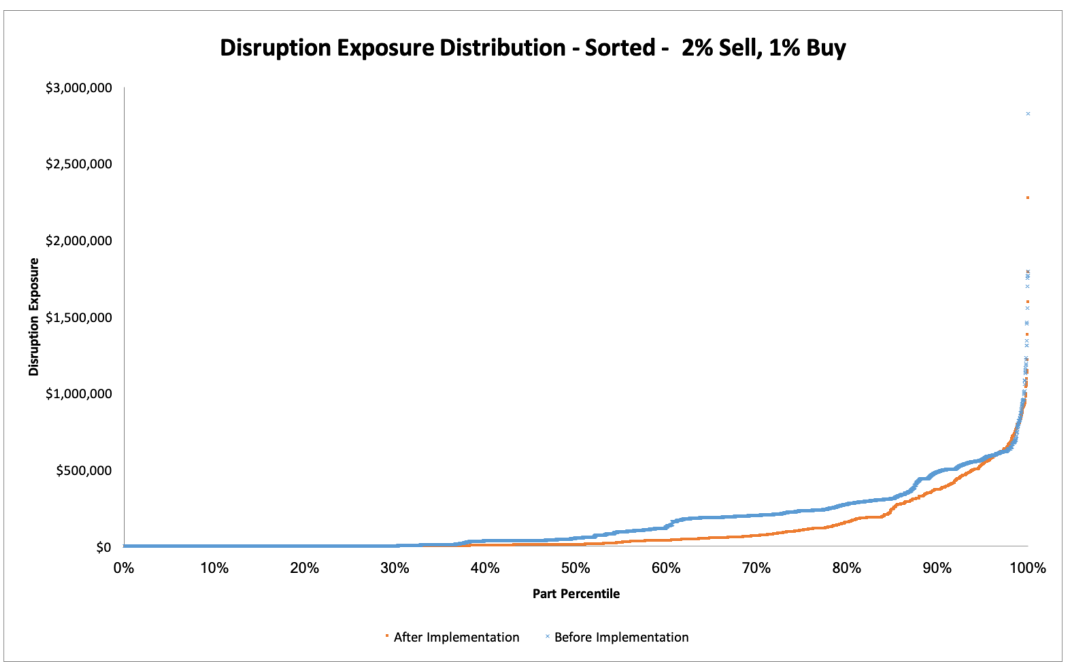
As described earlier, companies should weigh these recommendations against other operational constraints and produce a list of implementable adjustments. Even then, it is unlikely that a firm can implement a large number of changes in a short time period. Instead, the final set of adjustments may need to be prioritized and implemented over time. Because of these practical constraints, our analysis likely overstates the number of adjustments that will be implemented, and therefore overstates the realizable expected benefits, at least in the short term.

Strategic Portfolios of Part Inventory

Recall from Section 2.6.2 that we found no evidence of a positive secondary effect when analyzing inventory changes to individual non-disrupted parts. We reexamine this issue in the context of a "strategic portfolio" of parts. In particular, we consider holding additional inventory for *all* parts that compose the BOM of a single product. Such a strategic portfolio will allow DMF to overproduce the strategic portfolio product during the disruption stage, effectively storing production capacity that can then meet a portion of the production requirements in the recovery stage. This process, unlike the process outlined in Section 2.6.3, is simple to implement as it does not require changes in the part order frequencies. Once the strategic portfolio is in place, the order frequencies and order amounts for the affected parts revert to normal levels. However, holding inventory in a strategic portfolio incurs an ongoing inventory holding cost.



(a) DE Distribution, scenarios sorted on the x-axis by increasing order of pre-implementation *DE*



(b) DE Distribution, scenarios sorted on the x-axis by increasing order of *DE*, independently for pre-implementation and post-implementation

Figure 2.6: The Change of Risk Profile after the Implementation

We generate a strategic portfolio for each of DMF's 20 products using the same inventory holding cost budget, and compare the results to a benchmark case that instead allocates the budget to increasing the inventory for all of DMF's parts. As Propositions 1 and 2 make clear, the largest reduction in *DE* due to incremental inventory is realized when inventory positions are low. For this reason, it is not clear a priori whether a larger *DE* reduction can be achieved through a strategic portfolio, and if so, which strategic portfolio will be most effective.

We consider four annual budgets for the inventory holding cost – \$25,000, \$50,000, \$100,000 and \$200,000. Although we do not have the cost for each of DMF's parts, we do have the total part cost for each of its products. Using the holding cost of 6% per year, we translate each budget amount into the equivalent annualized increase in inventory for each product-centric strategic portfolio. For each strategic portfolio, we adjust the inventory levels based on the strategic portfolio inventory, and we re-run all of the disruption scenarios. The inventory levels for the disrupted, strategic portfolio parts are shifted up by the quantity in the strategic portfolio for those parts. The availability of the non-disrupted, strategic portfolio parts is increased by the quantity in the strategic portfolio for those parts beyond what is just enough to meet the original production requirements. The inventory for all disrupted, non-portfolio parts follows the original six inventory levels. The availability of the non-disrupted, non-portfolio parts is maintained at just enough to meet the original production requirements. We repeat this process for each budget. This entails running the model 4,451,328 times (8,832 scenarios $\times$ 21 strategic portfolios $\times$ 4 budget levels $\times$ 6 inventory levels).

Table 2.4 summarizes the impact of each strategic portfolio on DMF's total *DE* (the results for a $200,000 budget is in Table A.3 of the Appendix). The presented results are based on inventory level 2 (results for the other levels are available from the authors). The strategic portfolios are numbered from 1 to 20, corresponding to the 20 DMF products in the analysis. The row labeled "All" is our benchmark case and corresponds to increasing the inventory for all parts. By considering the values in the "Total" columns for the different strategic portfolios, it is apparent that while several strategic portfolios have a larger *DE* reduction than the benchmark case, many others lag. Characteristics that intuitively make a strategic portfolio perform poorly include low orders for the strategic portfolio product during the recovery stage, and a low gross profit margin for the strategic portfolio product. The first indicates that DMF can quickly saturate the production requirements for a strategic portfolio product when orders for that product are low, thereby gaining little from further over production. The second implies that the firm does not generate much value from over-producing low-margin strategic portfolio product in the disruption stage compared to simply writing off the orders for that product in the recovery stage.

To better understand what may lead some strategic portfolios to perform so much better than the benchmark strategic portfolio, we disambiguate the total *DE* impact of each strategic portfolio in Table 2.4 into the primary effect of additional disrupted part inventory and the secondary effect of additional non-disrupted part inventory. In line with our definition of the primary effect of inventory, we classify the reduction in *DE* as a primary effect when the disrupted part is included in the strategic portfolio and the part is used to produce more units of a disrupted product in a disruption scenario. We classify the re-

Table 2.4: Reduction in DMF's *DE* Due to the Adoption of Strategic Portfolio Inventory

	Budget: $25,000			Budget: $50,000			Budget: $100,000		
Portfolio	Total	Primary	Secondary	Total	Primary	Secondary	Total	Primary	Secondary
1	$14.5M	$1.7M	$12.8M	$23.5M	$3.2M	$ 20.3M	$29.3M	$4.6M	$24.7M
2	$34.6M	$15.8M	$18.8M	$55.8M	$30.3M	$25.5M	$83.5M	$56.0M	$27.6M
3	$121.4M	$38.0M	$83.4M	$218.3M	$71.9M	$146.4M	$323.8M	$109.2M	$214.6M
4	$77.9M	$32.5M	$45.4M	$151.6M	$63.8M	$87.8M	$287.0M	$123.3M	$163.7M
5	$15.4M	$15.3M	$0.0M	$28.5M	$28.5M	$0.0M	$51.0M	$50.9M	$0.0M
6	$19.4M	$12.5M	$6.9M	$30.0M	$23.1M	$6.9M	$48.2M	$41.3M	$6.9M
7	$26.6M	$5.1M	$21.4M	$51.1M	$9.9M	$41.2M	$85.1M	$19.0M	$66.1M
8	$56.5M	$32.1M	$24.3M	$111.0M	$63.7M	$47.3M	$214.3M	$124.6M	$89.7M
9	$52.1M	$31.4M	$20.7M	$101.7M	$61.3M	$40.4M	$195.1M	$117.7M	$77.5M
10	$46.8M	$18.8M	$28.0M	$83.3M	$36.5M	$46.8M	$133.7M	$69.1M	$64.6M
11	$58.9M	$12.0M	$46.9M	$115.1M	$23.4M	$91.7M	$221.0M	$45.3M	$175.7M
12	$27.7M	$6.4M	$21.3M	$54.6M	$12.4M	$42.2M	$103.2M	$23.9M	$79.3M
13	$21.8M	$6.6M	$15.2M	$30.2M	$12.1M	$18.0M	$40.7M	$22.6M	$18.1M
14	$5.4M	$0.8M	$4.6M	$6.0M	$1.5M	$4.6M	$6.8M	$2.2M	$4.6M
15	$0.8M	$0.6M	$0.2M	$1.5M	$1.2M	$0.3M	$2.7M	$2.3M	$0.5M
16	$11.0M	$0.8M	$10.2M	$13.6M	$1.5M	$12.1M	$14.3M	$2.1M	$12.2M
17	$0.7M	$0.6M	$0.2M	$1.4M	$1.1M	$0.3M	$2.5M	$2.0M	$0.5M
18	$3.1M	$3.1M	$0.0M	$6.0M	$6.0M	$0.0M	$11.5M	$11.5M	$0.0M
19	$1.1M	$1.1M	$0.0M	$2.2M	$2.1M	$0.0M	$3.9M	$3.9M	$0.0M
20	$1.0M	$1.0M	$0.0M	$1.9M	$1.9M	$0.0M	$3.5M	$3.4M	$0.0M
All	$51.1M	$21.8M	$29.4M	$100.6M	$43.4M	$57.3M	$195.2M	$85.8M	$109.3M

Note: Results are based on inventory level 2. Results for other inventory levels are available from the authors.

duction in *DE* as a secondary effect when the disrupted part is not included in the strategic portfolio and the strategic portfolio inventory is instead used to produce more units of a non-disrupted product. When a strategic portfolio has a large total reduction in *DE*, it tends to have a large secondary effect. In many cases, the magnitude of the secondary effect dominates that of the primary effect.

The primary and secondary effects of inventory are complementary. The primary effect works through reducing lost orders in the disruption stage, and the secondary effect works through applying otherwise wasted production capacity in the disruption stage to reduce lost orders in the recovery stage. Recall that Propositions 1 and 2 contend that the *DE* weakly decreases at a weakly decreasing rate with increased inventory. This is captured in Table 2.4 for the total, primary (Proposition 1), and secondary (Proposition 2) effects of inventory. As the budget doubles from $25K to $50K to $100K, the total, primary and secondary effects of inventory decrease DMF's *DE*, but at a decreasing rate. The presence of this pattern for the secondary effect provides our first support for Proposition 2, and highlights that the secondary effect of inventory can have a material impact on a firm's *DE*. The secondary effect provides the mechanism for a strategic portfolio to mitigate the firm's *DE* across a wider range of disruption scenarios. For instance, Strategic Portfolio 3 reduces DMF's *DE* for all 5,146 disruption scenarios with a positive *DE* and production capacity constraints in the recovery stage. This includes 3,745 scenarios involving disruptions to parts *not* in the strategic portfolio.

The *DE* impact of a strategic portfolio *relative* to other strategic portfolios depends on the budget. For instance, while strategic portfolio 3 has the largest *DE* impact using a budget of $25K, $50K, and $100K, strategic portfolio 4 has the largest *DE* impact using a budget of $200K (refer to Table A.3 of the Appendix for the results using a $200K budget). The firm can further improve the *DE* impact per dollar of strategic portfolio inventory by combining parts for multiple products in its strategic portfolio. For example, using a strategic portfolio that is based on allocating a $100K budget equally to the BOM for products 3 and 4 generates a total *DE* reduction of $351.9M (not tabled). From Table 2.4, this a larger *DE* reduction that what can be achieved by allocating the full $100K to a strategic portfolio that consists only of either product 3 or product

4. We leave optimizing the composition of a firm's strategic portfolio to future research. Collectively, our results demonstrate that (1) unlocking the secondary effect of inventory can require a well-chosen portfolio of parts and (2) such a strategic portfolio can reduce the firm's *DE* over a wide range of parts, not just those parts included in the strategic portfolio.

2.6.4 Heuristic Performance

In practice it may be useful to reduce the time required to complete this analysis, provided the results are commensurate with the full model. We show that this is possible in the case of DMF. It requires over 112 hours of processing time to run the 741,888 models that generate the results in Table 2.4 (and comensurably longer for the 4,451,328 scenarios across all of the analyses in Section 2.6.3. By using the heuristic described in Lemma 3, the processing time is reduced to less than 37 hours, a reduction of 67%. This proportional reduction in run time is consistent for other parts of the analysis in the paper. To check the accuracy of the heuristic, we regenerate the results from Table 2.1 using the heuristic and summarize them in Table 2.5. The heuristic results compare favorably to the results from the full model. The last two columns in Table 2.5 identify that results from a small proportion of the scenarios differ between the full model and the heuristic, and that the difference in the average DE between the full model and the heuristic is slight. The results of the strategic portfolio analysis using the heuristic (Table A.4 of the Appendix) are also very similar to those presented in Tables 2.4 and A.3 using the full model.

Table 2.5: Inventory Level Sensitivity by Parts - Heuristic (Disruption Duration 5, Backlogging All 0.9)

Level	DE mean	DE sd	1-quartile	median	3-quartile	DE max	DE= \$0	Diff≠ 0	Diff mean
1	\$503K	\$730K	\$4K	\$181K	\$1,017K	\$5,584K	542	157	\$213
2	\$193K	\$300K	\$2K	\$53K	\$239K	\$3,834K	845	282	\$741
3	\$91K	\$201K	\$0K	\$1K	\$74K	\$3,512K	3,472	147	\$421
4	\$53K	\$138K	\$0K	\$0K	\$11K	\$2,970K	4,545	93	\$295
5	\$32K	\$92K	\$0K	\$0K	\$7K	\$2,350K	5,597	78	\$225
6	\$19K	\$60K	\$0K	\$0K	\$4K	\$1,769K	5,824	83	\$165

Note: "Diff≠ 0" identifies the number of scenarios in which the full model and the heuristic provide different results. "Diff mean" identifies the difference in the mean DE between the full model (Table 2.4) the heuristic results.

2.7 Discussion and Managerial Implications

In this paper, we demonstrate how a firm can make cost-neutral changes to its part inventory policies to achieve significant reductions in its disruption exposure. We accomplish this by first providing analytical support that a firm's *DE* is weakly decreasing at a weakly decreasing rate with the quantity of both the disrupted and non-disrupted part inventories. We combine this with the simple observation that part inventories are continuously changing over their normal consumption and replenishment cycles, to highlight that a firm's *DE* for a part can be highly variable over time. For instance, using the data from our research partner, we observe that the variation in the firm's *DE* (coefficient of variation = 1.218) and is indeed much more variable than the variation in its part inventories (coefficient of variation = 0.669). The magnitude of DMF's *DE* variation is also economically material, from an average of \$503K per part at the safety stock inventory levels to an average of \$19K per part at the order-up-to inventory levels (based on a 5-week disruption).

We characterize the primary effect of inventory as the reduction in the *DE* of a part due to more inventory of that part, and the secondary effect of inventory as the reduction in the *DE* of a part due to more inventory of other parts. The latter raises the possibility that a small set of parts can be used to mitigate the risk of all parts. Motivated by our analytical findings that both the primary and secondary effect of inventory are weakly decreasing at a weakly decreasing rate, we show how DMF can better exploit the part-level relationship between *DE* and inventories to inexpensively mitigate its *DE* through a combination of reductions in the order-up-to amount and increases in the safety stock across many parts. The firm's portfolio of parts can be viewed as a means for self-insurance – "buying" insurance by holding more parts at inventory levels that have the largest reduction in *DE* with additional inventory, and "selling" insurance by holding fewer parts at inventory levels that have the smallest reduction in *DE* with additional inventory. This complements existing literature by demonstrating how inventory can be an inexpensive means to reduce a firm's *DE*.

Although we observe substantial evidence for the primary effect of inventory using DMF data, we find little evidence for the secondary effect of inventory. In practice, having a secondary effect from more inventory of a single part is rare because of the complexity of most product BOMs. To address this challenge, we introduce the concept of a "strategic portfolio" of inventory. A strategic portfolio consists of all of the composing parts for a product's BOM. Using DMF data, we show there is wide variation in the *DE* mitigation of strategic portfolios based on the product BOMs that compose those portfolios. Assessing the efficacy of prospective strategic portfolios is computationally resource intensive using practical data, however. In response, we propose a heuristic, and show that it performs well using the data from our research site. Our heuristic minimizes the firm's *DE* in the disruption stage, but it can lead to higher *DE* in the recovery stage. While the heuristic may provide suboptimal results across the combined disruption and recovery stages, it may be operationally attractive if, for instance, the firm wants to prioritize its cash flow during the disruption stage.

The insights from our research can be creatively applied to accommodate other practical constraints. For instance, in some settings it may be impractical to maintain a physical strategic portfolio of inventory onsite. The firm may still unlock the secondary effect of the strategic portfolio, which we find can be larger than the primary effect, by ensuring expedited delivery of the parts included in the strategic portfolio during the disruption stage. Our findings on strategic portfolios also imply that there are benefits from coordinating inventory levels, safety stock, and even part deliveries across a set of parts such that synchronized levels of those parts are available. Finally, implementing inventory policy changes can be restricted due to a variety of operational realities, such as fixed order increments, minimum order quantities, limited delivery schedules, supplier service levels, and economic order quantity guidelines. A strategic portfolio, on the other hand, is simple to implement and effectively bypasses many of these complexities.

CHAPTER 3

MITIGATING SUPPLY CHAIN FINANCIAL RISK USING REVERSE FACTORING

3.1 Introduction

Access to financing is a challenge for many small suppliers, especially when high inflation and interest rates are present. In particular, many firms find it difficult to finance their production cycle, because most buyers demand 30 to 90 days to pay after goods are delivered. The unpaid invoices are recorded as account receivables in the supplier's balance sheet, which are not liquid assets. Since the small suppliers already have large amounts of capital tied up by their inventory, work-in-process and raw materials, delayed payment could raise severe financial issues during operations. Most small suppliers face high borrowing rates from the bank due to short credit histories and unstable financial status, which results in their need of cheaper financing sources. As another player in the supply chain, the buyers are usually relatively bigger and have better credit ratings, but they still wish to have payment terms to free up cash while keeping the suppliers in good shape for a cooperative relationship.

To alleviate such problems and resolve the conflicting interests of each party concerning payment periods, Reverse Factoring (RF) has become increasingly popular as a solution to take advantage of the credit arbitrage between buyers and suppliers. Reverse Factoring, also referred as Supply Chain Finance (SCF), is a form of supplier financing in which firms sell their credit-worthy accounts receivable at a discount (equal to interest plus service fees) and receive immediate cash. It is not a loan and there are no additional liabilities on the firm's balance sheet, although it provides functional capital financing. A third party financial institution, usually a bank, is involved when implementing the SCF program. Figure 3.1 outlines the detailed process of carrying out Reverse Factoring: the buyer transmits its account payable file with approved invoices to the platform, and once the supplier chooses to execute the RF option for immediate cash, the bank pays the discounted amount and request payment from the buyer on the payment date. If RF option is not used, the buyer pays directly to the supplier on the payment date.

Those parts of the industry who have been practicing supply chain finance have reported its cost savings. [39] reported in the Wall Street Journal that P&G, who spends more than $50 billion each year on procurement, could free up as much as $2 billion in cash by extending its payment terms from 45 days to 75 days. From our conversations with the supply chain finance manager of the Brazilian Food Corporation, we learned that by launching RF programs with its suppliers, the BRF is in the position to extend average payment terms from 90 to 190 days. Demica's survey in 2010 also reported a typical savings between 1 and 4 percentage points on borrowing cost by joining SCF programs for the small suppliers.

Regardless of its adoption in the industry, there is a lack of analytical tools and models for the value of supply chain finance, and academic literature is falling behind. Reverse factoring can benefit both parties in a supply chain with a small supplier and a larger buyer. The bank also earns an interest premium by providing the service, which makes supply chain finance a win-win-win situation for all participants. The buyer enjoys payment extension and more operational flexibility. The supplier can save cash flow cost by claiming its account receivables earlier, at a lower cost, and with more flexibility. The value of the reverse factoring largely comes from its value to the supplier's cash flow management. The bank is also challenged with choosing an appropriate rate for reverse factoring to make it beneficial for all entities. Thus, we want to build an analytical model to quantify the value of reverse factoring and give support for banks and suppliers when designing terms.

Regardless of the growing trend, only a small fraction of financial payables are currently being financed using SCF. As estimated by McKinsey in 2015, this fraction is \$200 billion out of \$2 trillion worth of financial payables, which is around 10%.

It is essential for both parties to understand the nature of RF and how RF adds to their value, in order for them to choose the appropriate contract terms that fit their needs. On the other hand, the banks, as facilitators, are actively participating in supply chain finance to serve the financial needs of the supply chain, as well as earning interest premium to serve their own needs. While there are various financial institutes willing to offer supply chain finance service, small suppliers should be able to decide whether the contract terms are acceptable and which specific reverse factoring service to go for, depending on their current financial status.

The value of the reverse factoring for the supplier comes from the liquidity value in the supplier's cash management. The supplier's cash management strategies will differ with reverse factoring to best capture its value. Another difficulty is that the supplier's financial status is constantly changing and its amount of outstanding account receivable with the buyer also dynamically changes, while the bank is the one to choose the interest rate for reverse factoring. Thus the supplier needs to carefully value the supply chain finance contracts and decide whether to accept or not.

Our goal in this paper is to provide insights into (1) the mechanisms of reverse factoring and how they impact the firm's decision; (2) frame the decision problem for the supplier from the cash flow management perspective and what is the optimal decision; (3) elaborate how the cash flow management model could be utilised in other related problems. To do this, we consider a stochastic dynamic model as a framework for understanding the decision making process and the nature of Reverse Factoring. This paper is structured as follows. Section 2 reviews relevant literature on both Cash Management and Interface of operations and finance. Section 3 makes assumptions and establishes the basic model. In Section 4, we carry out analytics for the model and then run a numerical experiment in Section 5. We concludes the paper in Section 6.

3.2 Literature Review

Although there is no model in the literature addressing both cash management and supply chain finance issues, there is substantial literature covering both cash management and inventory decision making with financial constraint. Cash management has long been a concern in the literature for both management and economics, while supply chain finance has emerged as a relatively new concept as the business world calls for more sources and options to manage their working capital. Despite the well-developed literature of cash models for production and inventory management, little exploration is done for the interplay between cash management and the build-in financing tools such as supply chain finance (as opposed to using external source for financing).

[26] gives a review of much of the early literature on how a firm can best balance the sources and uses of funds (i.e., cash management problem). Early literature on the economics of cash management take the view of estimating the cash demand for households and firms. [4] frames a deterministic model with a fixed transfer cost, which is quite similar to the EOQ model in the traditional operations literature. More often, stochastic programming models are established to characterize optimal cash control policies ([17], [35], [38]). Similar policies for inventory are derived in [18] for fashion products. Linear programming is another way to capture the nature of a cash problem as [40] used an unequal-period mode.

An important issue in supply chain management is the coordination of material flows, information flows and financial flows. Cash management problems interact with these three flows as well. [33] integrates material flows with cash flows in a multi-divisional supply chains. Recently more research has been focused on the joint decision of inventory and finance, among which trade credit is receiving great attention. See [43] for a detailed review. [22] studies the impact of trade credit on supply chain contracting and inventory management and comes up with an optimal stock policy. [32] and [65] view trade credit as a tool for risk sharing and supply chain coordination. As opposed to reverse factoring, trade credit favors buyers by giving them the additional option of early payment at a discounted price, but [52] suggests that it also benefits the supplier by serving as a screening mechanism for suppliers to evaluate the risk of their customers. There are also other recent works at the interface of operations and finance with similarities to our concern of reverse factoring. [6] interrelates the financial capacity and operational decisions of a retailer in an asset-based financing setting. [11] models the borrowing behavior of a capital-constrained newsvendor.

As mentioned in the introduction, the feasibility of reverse factoring lies in the fact that there is credit arbitrage between small suppliers and buyers. Research on small business finance confirms that large corporations have significantly lower financing costs than their SME suppliers. [24] estimates that marginal equity flotation costs for large firms start at 5.0%, while the corresponding figure for small firms is 10.7%; bankruptcy costs amount respectively to 8.4% or 15.1% of capital. [20] argues that a bankruptcy cost of 5% adds 1.2% to the equity premium, while [10] points out that the SMEs take on more severe information asymmetry and transaction cost.

Unfortunately, the mechanism of Reverse Factoring receives little attention in the research community compared with Trade Credit. A few studies that specifically looks into RF usually restrict themselves to carrying out high-level management assessments (e.g., [48]), and use simple financial ratios to conclude the benefits for the RF participants (e.g., [44]). We contribute to the literature by developing a rigorous framework that integrates both the cash management decision and the financial decision to reveal the value of RF and understand how RF creats a win-win situation for the supply chain as a financing tool.

Thus, despite the plentiful literature discussing the interface of operations and finance, there does not appear to be much specifically addressing underlying mechanisms and the impact of reverse factoring as a financial tool for cash management.

3.3 Problem Formulation and Analysis

Consider a supply chain with a small supplier and a large buyer. The buyer, being larger and more solvent, enjoys a higher credit rating than does the supplier. Under the common fixed-term trade credit arrangement, when the buyer orders from the supplier goods worth W dollar value, the buyer agrees to pay the supplier in M periods after receiving the goods. In this case, while the buyer has the option to pay the supplier sooner, the rational response is always to pay the supplier at the agreed payment due date because of interest discounting.

An RF arrangement can help improve the fixed-term arrangement for both parties. Specifically, the RF arrangement offers the supplier an option to receive all or a portion of the payment W from a third party financier (henceforth referred to as a bank) anytime before the payment due date, and at the same time enables the buyer to extend the payment due date from M periods to N periods, with $N > M$ (See Figure 3.2 for an illustration). In return for the service, the bank charges the supplier an immediate interest payment by paying the discounted amount of the requested payment at interest rate r for each period before the payment due date.

Figure 3.1 illustrates the sequence of events under the RF arrangement: 1) after the supplier delivers goods and an invoice to the buyer, the buyer sends the approved invoice with amount W and due date N to a selected technology platform; 2) the supplier views the approved invoice on the platform and decides whether or not to request an early payment of all or a portion of W; 3) if an early payment is requested by the supplier, the bank will review and approve the payment request; 4) upon approval of the request, the bank sends the discounted payment amount to the supplier at interest rate r, i.e., charges the supplier an interest payment; and finally, 5) at the payment due date N, the buyer remits the financed portion of W to the bank and the remaining portion (if any) to the supplier.

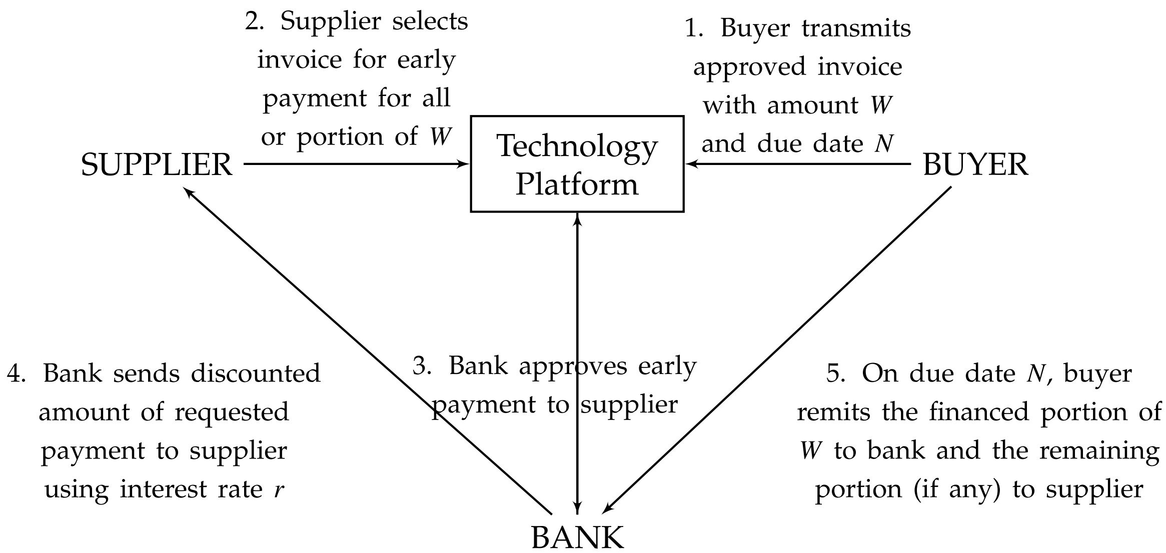


Figure 3.1: Sequence of Events under Reverse Factoring

Clearly, the premise for the RF arrangement to work is that the interest rate r charged by the bank under the RF arrangement should be lower than the supplier's own per period bank borrowing rate r_S, i.e., $r \leq r_S$; otherwise, the supplier has no incentive to participate. On the other hand, since the buyer is liable to the bank for the financed portion of W at the payment due date N, it would be profitable for the bank to offer the service as long as $r \geq r_B$, where r_B is the buyer's bank borrowing rate per period. Taking these two observations together, we arrive at the first condition for RF to work: $r_B \leq r \leq r_S$. In other words, an RF arrangement would work only if $r_B < r_S$. Therefore, the buyer needs to have better credit rating than the supplier.

Another point of interest is to determine the optimal range of r that will ensure the participation of both the supplier and the bank under RF. To facilitate our subsequent analysis, we need to define the supplier's discounting interest rate as r_C per period, which can be interpreted as the risk-free interest rate the supplier can earn from its savings account. To rule out arbitrage, we assume that $r_C < r_B < r_S$. Otherwise, the supplier could make a risk-free profit by borrowing at rate r_S and investing to earn the difference $r_C - r_S$. The same applies to the buyer. Thus, the supplier's discount factor is given by $\delta = 1/(1 + r_C)$.

Let us first evaluate the RF arrangement through the lens of net present value (NPV) analysis. Suppose that the supplier decides to withdraw an early payment z (with $z \leq W$) at the beginning of period n (with $n \leq N$). The supplier will receive the discounted amount $z(1 + r)^{-(N-n+1)}$, equivalently as paying an interest cost of $z[1 - (1 + r)^{-(N-n+1)}]$. Thus, the NPV of the interest cost incurred by the supplier is $\delta^{n-1} z[1 - (1 + r)^{-(N-n+1)}]$. Hence, the NPV of withdrawing z at the beginning of period n is

$$\delta^{n-1} z - \delta^{n-1} z[1 - (1 + r)^{-(N-n+1)}] = \delta^{n-1} z(1 + r)^{-(N-n+1)} < \delta^{n-1} z(1 + r_C)^{-(N-n+1)} = \delta^N z,$$

where the inequality follows from the fact that $r > r_C$. The above analysis implies that the NPV of withdrawing z at the beginning of period n is strictly less than the NPV of receiving the payment at the end of period N (the payment due date). The following lemma summarizes the result:

Lemma 1. *From the NPV perspective, it is not optimal for the supplier to withdraw early payment under the RF arrangement.*

The above result is clearly at odds with the popularity of RF arrangements observed in practice. This is because the NPV analysis fails to capture the value of cash flow liquidity offered by the RF arrangement for the supplier. In order to evaluate the liquidity value of RF, we need a more sophisticated dynamic cash management model, which is introduced and studied below.

3.3.1 Dynamic Cash Management Model

Consider an infinite-horizon cash management problem for the supplier. The periods are numbered forward as $n = 1, 2, ..., \infty$. In each period n, the supplier experiences an i.i.d. random cash shock ξ_n, where $\xi_n > 0$ represents a net cash outflow and $\xi_n < 0$ a net cash inflow. Let $f(\cdot)$ and $F(\cdot)$ denote the probability density function (PDF) and cumulative distribution function (CDF) of the random cash shock ξ_n, respectively. Also let $\xi_i^j = \sum_{n=i}^{j} \xi_n$ denote the cumulative random cash shock from period i to period j. The supplier's objective is to minimize the expected total discounted cost over the infinite horizon. (For ease of reference, definitions of variables and parameters used in our model are listed in Appendix B.1.)

Let x_n be the supplier's cash level at the beginning of period n. Before the cash shock realization, the supplier can adjust the cash level x_n up or down to a new level y by either taking a short-term loan (paying interest rate r_S) or making a short-term investment (earning interest rate r_C). We assume that the cash adjustment is achieved immediately and there is no fixed transaction fee. To keep things simple, we further assume that the time frame of the short-term loan or investment is one period, that is, the principal of the loan or investment will be repaid in one period.

At the end of each period, if the cash position is negative, then the supplier pays an interest cost with rate r_S (e.g., interest charged on the business revolver account). If the cash position ends positive, there is no cost incurred, nor is interest earned (the supplier needs to make an investment decision at the beginning of a period to earn interest). Because the interest cost of an ending negative cash position is the same as the interest cost of the short-term loan for the supplier, it effectively rules out the option of taking a short-term loan at the beginning of a period as the supplier can always do better without it.

The RF arrangement provides a new option for the supplier to raise its cash level at the beginning of a period. Let w_n be the available RF funds that can be withdrawn at the beginning of period n. As discussed earlier, if the supplier decides to withdraw $z = (y - x_n)^+ \le w_n$ from the RF funds at the beginning of period n, it will receive the discounted amount $(y - x_n)^+(1 + r)^{-(N-n+1)}$, equivalent to paying an interest cost of $c_n(r)(y - x_n)^+$, where $c_n = 1 - (1 + r)^{-(N-n+1)}$ and $(\cdot)^+ = \max\{\cdot, 0\}$. Therefore, under an RF arrangement with rate r and due date N, given the cash level x_n, available RF funds w_n, and cash level decision y with $y \le x_n + w_n$, the supplier's single-period cost function can be written as

$$R(y, x_n, w_n \mid r, N) = -r_C(x_n - y)^+ + c_n(r)(y - x_n)^+ + r_S E\left[(\xi_n - y)^+\right], \qquad (3.1)$$

where $-r_C(x_n - y)^+$ represents the single-period income from short term investment, $c_n(r)(y-x_n)^+$ represents the interest cost incurred from early withdrawal of the RF funds, and $r_S E\left[(\xi_n - y)^+\right]$ represents the expected interest payment owed to the bank at the end of the period. Note that the investment income is expressed as negative cost, i.e., $-r_C(x_n - y)^+$, since our objective is minimizing cost.

Let $V_n(x_n, w_n \mid r, N)$ denote the cost-to-go function from period n onward with the cash level x_n and available RF funds w_n under the RF arrangement with rate r and due date N. To reduce notation, we shall suppress the parameters (r, N) in $R(y, x_n, w_n \mid r, N)$ and $V_n(x_n, w_n \mid r, N)$ whenever there is no confusion.

Therefore, the dynamic cash management problem under the RF arrangement can be formulated as the following infinite-horizon dynamic programming: for $n = 1, 2, \ldots,$

$$V_n(x_n, w_n) = \min_{y \le x_n + w_n} \left\{ R(y, x_n, w_n) + \delta E \left[V_{n+1}(x_n + (y - x_n)^+ - \xi_n, w_n - (y - x_n)^+) \right] \right\}, \quad (3.2)$$

with $w_1 = W$, $x_{N+1} = x_1 + W - \xi_1^N$, and $w_{N+1} = 0$. This is because the supplier will receive the whole amount of RF funds W by the end of period N, and thus there will be zero available RF funds from period $N + 1$ onward.

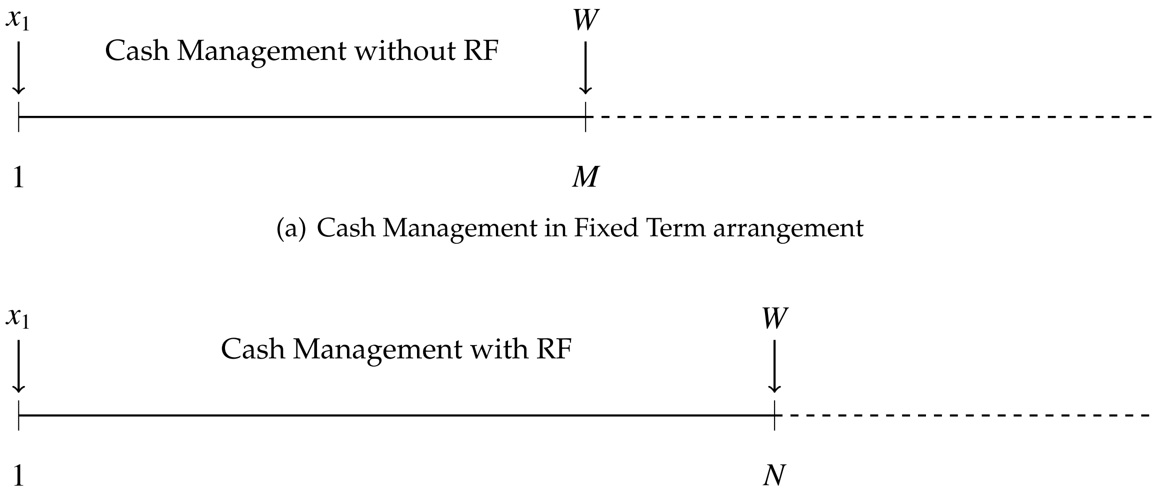
In the fixed term arrangement, the supplier has no option to receive early payment of its AR, and it can be treated as a special case for Problem (3.2) when $r = \infty$. This is because the supplier will never execute its option to request early payment of AR due to the infinite cost involved, thus it is equivalent to the fixed term arrangement where no early payment request is allowed. The cost-to-go function for the fixed term arrangement is hence $V_n(x_n, w_n \mid \infty, N)$.

Given the same amount of AR with the same due date, the supplier's cost is lower in the RF arrangement than the fixed term arrangement because of the additional flexibility provided by RF. This observation can actually be generalized. In the RF arrangement, the supplier will always have lower costs if the RF rate offered by the bank is lower, because it is less costly for the supplier to take the same action of changing its cash level when r is smaller. The RF arrangement converges to the fixed term arrangement when $r = \infty$. However, the due dates are in general different in the RF arrangement and the fixed term arrangement, since the buyer will ask for an extension of the payment period in return. With the same amount of AR, the supplier's cost is higher for a longer payment period assuming a fixed term arrangement. This is because the longer payment period postpones the collection of the supplier's AR, which will be permanently added to its cash pool, therefore adding flexibility to cash management onward. This observation can be generalized as well. For a RF arrangement with fixed r, the supplier's cost is smaller if the payment period is shorter, following the same argument as the fixed term arrangement. These observations are summarized in the following lemma.

Lemma 2. *(1)$V_1(x_1, W \mid r, N)$ increases in r, specifically, $V_1(x_1, W \mid r, N) \leq V_1(x_1, W \mid \infty, N)$; (2)$V_1(x_1, W \mid r, N)$ increases in N, specifically, $V_1(x_1, W \mid \infty, M) \leq V_1(x_1, W \mid \infty, N)$.*

Now we have defined the supplier's cash management problems in the RF arrangement and the fixed term arrangement. They are illustrated in Figure 3.2. In the fixed term arrangement, the supplier has its AR due in M periods without option to request cash payment early, and all AR will be collected at the end of period M; in the RF arrangement, the supplier has its AR due in N periods ($N > M$), during which the supplier can request early payment of a portion or all of its AR at a cost, and the unclaimed AR will be paid at the end of period N. The RF interest rate r is determined by the bank. The supplier has no control over r, but it knows well its own financial condition, namely, (x_1, W). It is at the supplier's will to decide whether or not to accept the reverse factoring arrangement that extends payment terms from M to N and has RF rate r. The supplier should accept the RF arrangement if the expected cash flow cost is lower than the original fixed term arrangement.

We have learned from Lemma 2 that a lower r will make reverse factoring more attractive to the supplier. Hence, for an RF arrangment that extends payment terms from M to N, if the supplier is willing to accept an RF rate r, it will accept any RF rate below r. There will be a breakeven point that makes the



(a) Cash Management in Fixed Term arrangement

(b) Cash Management in Reverse Factoring arrangement

Figure 3.2: Fixed Term versus Reverse Factoring

supplier indifferent between the fixed term arrangement and the RF arrangement. Let's denote this breakeven point of RF rate as $\bar{r}(x_1, W \mid M, N)$ because it depends on the supplier's condition (x_1, W) and the payment term extension (M, N); it is also the maximal RF rate the supplier could accept. We will use $\bar{r}$ for short when there is no confusion. The condition for the supplier's acceptance of the RF arrangement is summarized in the following proposition.

Proposition 1. *The maximal RF rate the supplier can accept for extending the payment term from M to N is $\bar{r}$, where $\bar{r}(x_1, W, M, N)$ is a function of x_1, W, M and N:*

$$\bar{r}(x_1, W, M, N) = \max\{r_B \leq r \leq r_S \mid s.t.\ V_1(x_1, W \mid r, N) \leq V_1(x_1, W \mid \infty, M)\}$$

and the supplier will accept any RF rate smaller than $\bar{r}$.

Recall that the bank's offer of the RF rate falls in the range $[r_B, r_S]$. If $\bar{r} \geq r_S$, the supplier will accept all RF rates offered by the bank; if $\bar{r} < r_B$, the supplier will never accept the RF arrangement; if $r_B \leq \bar{r} < r_S$, the supplier will be willing to accept any RF rate in the range $[r_B, \bar{r}]$. The actual rate r in practice depends on the bargaining power of the supplier, buyer and bank .

Now we turn to determining what factors affect $\bar{r}$. As we have already discussed, by switching from a fixed term arrangement to an RF arrangement with payment extension, the supplier faces a tradeoff. On one hand, the RF option adds some liquidity value in cash flow management. On the other hand, the extended payment terms lead to further delay in AR payment and the NPV of its AR is lowered as well. When the supplier operates well with significant positive net cash inflow, it does not need to use the RF option to claim its AR in advance. Hence, the supplier incurs a cost for receiving its AR later. To ease our analysis, we further propose a payment arrangement where the supplier will receive its AR at the end of period N without the RF option, therefore the expected cost in this hypothetical arrangement is $V_1(x_1, W \mid \infty, N)$. We can then separate and quantify the liquidity value of RF and the cost of delayed payment. The liquidity value of RF with payment period N is $V_1(x_1, W \mid \infty, N) - V_1(x_1, W \mid r, N)$, and the cost of delaying payment from M to N without RF option is $V_1(x_1, W \mid \infty, N) - V_1(x_1, W \mid \infty, M)$. Note that the cost of delayed payment is fixed given x_1, W, M and N, while the liquidity value of RF decreases with r. The breakeven point $\bar{r}$ will be reached with the result that these two effects cancel each other.

For the supplier, the maximal acceptable RF rate hinges on its current financial status as it has little control over the payment terms M and N. The bank can profit most from the reverse factoring service by designing the optimal payment terms such that the RF rate offered is pushed close to $\bar{r}$.

3.4 Optimal Cash Management

To have the maximal acceptable RF rate $\bar{r}$, we need to know the value functions V_1^0 and V_1 for the No RF and RF cases respectively. We examine the supplier's optimal cash flow decisions in both cases.

3.4.1 Optimal Policy with RF

In this case, the expected cost-to-go from period $N+1$ onward is $E\left[V_{N+1}(x_1 + W - \xi_1^N, 0)\right]$. Further notice that, for $n \geq N+1$,

$$V_n(x_n, 0) = \min_{y \leq x_n} \Big\{ R(y, x_n, 0) + \delta E\left[V_{n+1}(x_n - \xi_n, 0)\right] \Big\},$$

where it is clear that $V_{n+1}(\cdot, 0)$ is independent of y. Hence, a myopic policy that minimizes $R(y, x_n, 0)$ is optimal. Let $R^*(x_n, 0) = \min_{y \leq x_n} R(y, x_n, 0)$. It follows that we can write

$$E\left[V_{N+1}(x_1 + W - \xi_1^N, 0)\right] = \sum_{i=0}^{\infty} \delta^i E\left[R^*\left(x_1 + W - \xi_1^{N+i}, 0\right)\right], \tag{3.3}$$

which is a constant that can be predetermined based on $x_1 + W$ and N. Therefore, the infinite-horizon dynamic program (3.2) is effectively reduced to an N-period finite-horizon dynamic program with an expected terminal value specified in (3.3).

By solving Problem 3.2, we determine the optimal cash policy for the supplier to be the following:

Proposition 2. *The optimal cash policy when the reverse factoring option is present is a modified (L_n, U_n) policy ($L_n < U_n$) i:*

(1) *When the cash level* $x_n > U_n$*, invest money down to* U_n*;*

(2) *When the cash level* $x_n < L_n$*, borrow money up to* $\min\{x_n + w_n, L_n\}$*;*

(3) *When the cash level falls into* $[L_n, U_n]$*, make no change*

Proposition 3. *The borrow-up-to threshold* L_n *is monotonously increasing in n under the condition* $r_C \geq \frac{1}{1+(r+1/r)}$ *or* $r \leq \left[(1-r_C) - \sqrt{1-2r_C-3r_C^2}\right]/(2r_C)$ *(*$r_C \leq \frac{1}{3}$ *needs to hold);and the invest-down-to threshold is a constant with* $U_n \equiv U = F^{-1}(\frac{r_S - r_C}{r_S})$.

Figure 3.3 illustrates the modified (L_n, U_n) policy. The (L_n, U_n) policy structure can be found in inventory management literature, such as [7]. The inventory model with limited capacity is also well studied ([19]). The modified (L_n, U_n) policy for the supplier has the simple (L_n, U_n) type structure while capacitated by the supplier's current outstanding account receivables. The inherent (L_n, U_n) bounds are decided regardless of the supplier's current outstanding AR. Within the scope of the supplier's cash management, the current available w_n only acts as a capacity for the borrowing-up limit. When offered reverse factoring, the supplier is trading off the benefit of an earlier claim of AR and the loss of cash management flexibility in subsequent periods. Hence, the supplier is less aggressive in using the RF option in early periods since RF has a higher flexibility value, while it uses the RF option more aggressively as it approaches the due date of AR since there are less periods left and the flexibility value of RF matters less.

As Proposition 2 suggests that (L_n, U_n) does not depend on the current state (x_n, w_n), we can calculate L_n by specifically setting $W \to \infty$. The L_n for Problem (3.2) is therefore the same as the L_n for the following problem:

$$V_n^{\infty}(x_n) = \min_y \left\{ \begin{array}{c} -r_C \cdot (x_n - y)^+ + c_n(r) \cdot (y - x_n)^+ \\ +r_S \cdot E(\xi_n - y_n)^+ + \delta \cdot E[V_{n+1}^{\infty}(y \vee x_n - \xi_n)] \end{array} \right\} \tag{3.4}$$

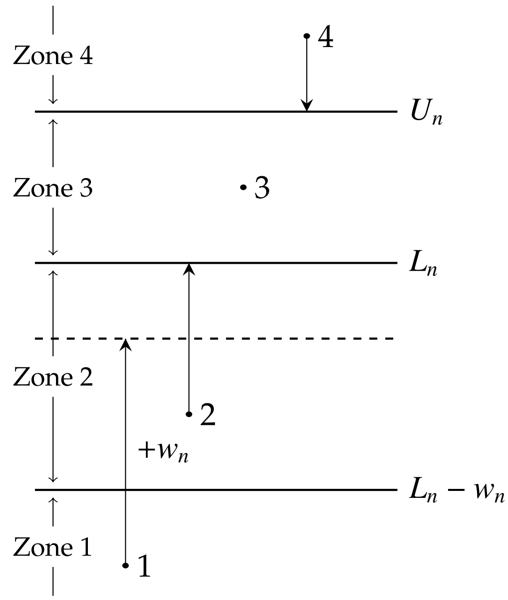


Figure 3.3: Illustration of Optimal Policy with RF

with $V^{\infty}_{N+1}(\cdot) = 0$.

Since $W \to \infty$, we have in each period $w_n \to \infty$, and the state variable for available account receivables can be dropped. Thus, Problem (3.4) is a reduced form problem of Problem (2) with only one state variable. The optimal cash policy of this new problem is simply the non-modified (L_n, U_n) policy. Thus, it is sufficient to only solve the reduced form problem to get the optimal policy for the original problem.

3.4.2 Benchmark: Optimal Policy with No RF

To solve Problem (3.2), we start from the supplier's cash management problem when there is no RF option, which serves as the benchmark when we quantify the value of reverse factoring. This problem is a special case of Problem (3.2) by setting $r = \infty$. The value function $V_n(x_n, w_n \mid \infty, N)$ can be shortened to $V^0_n(x_n, w_n \mid$

N) as mentioned above.

There is only one real state variable for the No RF case, namely, the cash position at the beginning of each period x_n. Since there is no early cash claim through the RF option, w_n is a constant and thus a pseudo state variable, i.e., $w_n \equiv W$. $V_n^0(x_n, W \mid N)$ is the value function that measures the expected total cash flow cost valued at the beginning of period n with initial cash x_n, and with amount W of AR due at the end of period N. Since the state transition does not depend on the control y taken, this dynamic programming problem can be decoupled and is equivalent to solving a single period problem. The optimal policy for each period can be summarized as Proposition 4.

Proposition 4. *The optimal cash policy when there is no reverse factoring option is a "U" policy with* $U^0 \equiv U = F^{-1}(\frac{r_S - r_C}{r_S})$*, and* $V_n^0 \geq V_n$*:*

(1) *when* $x_n > U^0$*, bring the cash level down to* U^0*;*

(2) *when* $x_n \leq U^0$*, keep the initial cash level unchanged*

This is a "U" policy with an upper bound for the initial cash at the beginning of each period. It implies that the supplier should never borrow. This should be the case for most suppliers if their borrowing rate r_S is greater than the opportunity cost r_C. Borrowing from the bank does not lower the cash flow cost if the supplier has a low cash level. On the other hand, holding too much cash is not favorable even for a small supplier and it is beneficial for them to invest the extra cash once the cash amount exceeds a certain threshold. For those suppliers with low cash, cash management does not add any value. Instead, they should seek to improve their cash generating activities to have better net cash flow in subsequent operations.

3.4.3 Comparison with Myopic Policy

To obtain optimal cash management for the case with RF, the supplier trades off the immediate benefit of changing the cash level in the current period with the potential cost in future periods. The analytical form of the optimal policy bounds are difficult to characterize. However, an intuitive sub-optimal policy can be easily obtained. This is the myopic policy. In the myopic policy the supplier only tries to minimize its cash flow cost in the current period, and the resulting cost function is an upper bound for the actual cost function. Proposition 5 gives the myopic policy for Problem (3.2) and compares it with the optimal policy.

Proposition 5. *The myopic cash policy when there is reverse factoring is a modified* (L_n^m, U_n^m) *policy, where in period n the invest-down-to bound is* $U_n^m = F^{-1}(\frac{r_S - r_C}{r_S})$ *and the borrow-up-to bound is* $L_n^m = F^{-1}(\frac{r_S - c_n(r)}{r_S})$ *with the capacity limit* w_n*. Compared with the optimal policy:*

(1) $L_n^m \leq L_n \leq U_n$;

(2) $U_n = U_n^m \equiv U$;

(3) $V_1^m(x_1, W) \geq V_1(x_1, W)$

Here $V_n^m(x_n, w_n)$ denotes the cost function under the myopic policy. The effective upper bound U_n^m remains unchanged for all n, while the effective lower bound L_n^m changes with the period n. Since $c_n(r)$ is decreasing in n, L_n^m are increasing in n with $L_N^m = F^{-1}(\frac{r_S - r}{r_S})$. Specifically for $r_S < c_n(r)$, we have $L_n^m \to -\infty$ and the myopic policy converges to the "U" policy. The myopic policy tends to use the RF option more aggressively in later periods due to the lower cost of RF borrowing. In a myopic policy, the supplier ignores the future impact of current decisions and only considers the cash flow cost in the current period when minimizing its cash flow cost at the beginning of period n. The firm acts as if the problem is decoupled.

The myopic policy shares the same policy structure with the optimal policy, but the myopic policy bounds differ from the optimal policy bounds: the upper bounds are the same while the optimal lower bound is above its counterpart in the myopic policy for each period. The invest-down-to bounds are the same, because only the market return counts when making cash investment decisions, and no future impact needs to be considered because all investments are short-term. For the borrow-up-to bound in a given period, the supplier tends to be too conservative in early claims of its AR when myopic sighted. This lies in the fact that the supplier is afraid of the high cost of early AR claims as the accrued RF interest rate $c_n(r)$ is greater for smaller n in a myopic sense. However, there will be more cash management flexibility due to the increased cash availability by early claims of AR, despite the decreased outstanding AR available. Thus, there are additional future benefits of early claims of AR in situations where the supplier needs to borrow. The myopic decision neglects these future benefits and hence results in a lower borrow-up-to bound than what should be optimal.

Specifically, in the two period case, we have described the optimal policy as the following corollary.

Corrollary 1. *When $N = 2$, the supplier's optimal cash policy is a modified (L_n, U_n) policy: for the "U" bound, $U_1 = U_2 = F^{-1}(\frac{r_S - r_C}{r_S})$; for the "L" bound, $L_2 = F^{-1}(\frac{r_S - r}{r_S})$ and $F^{-1}(\frac{r_S - (1+\delta)\cdot r + \delta \cdot r_C}{r_S}) \leq L_1 \leq F^{-1}(\frac{r_S - r}{r_S})$.*

Note that the myopic borrow-up-to bound for period 1 is $F^{-1}(\frac{r_S - (1+\delta)\cdot r}{r_S})$. The supplier can actually borrow more aggressively than its myopic decision when in a low cash position, because the execution of the RF option can leverage its cash pool in the future as well. The worst consequence the supplier will have for borrowing with the RF option is that it has too much cash on hand in the future and incurs an opportunity cost at the rate of r_C. The worst scenario happens when there is a large cash inflow in the future after the supplier uses its RF option to borrow money. The borrow-up-to bound implied by the worst scenario is $F^{-1}(\frac{r_S-(1+\delta)\cdot r+\delta\cdot r_C}{r_S})$, since the effective cash holding cost in $L(y,\xi)$ function is $r_S+\delta\cdot r_C$. The supplier can borrow more aggressively than this scenario, and the actual optimal L_1 should be greater than (or equal to) $F^{-1}(\frac{r_S-(1+\delta)\cdot r+\delta\cdot r_C}{r_S})$.

3.5 Numerical Results

Now we will numerically solve the optimal cash policy and see how the bank will choose the optimal RF rate for extending payment terms, while making both the supplier and the buyer better off. We use a normal distribution for the iid cash shocks. The market risk free rate is drawn from the "LIBOR" rate, which is an annualized rate. The USD LIBOR for one month is 0.99% on May 2nd in 2017, according to *www.global-rates.com*. The spread of an "*A+*" credit rating firm with an interest coverage ratio around 6 is about 1%. Both the "LIBOR" and interest rate spread are annualized. In our paper, we are looking at a period base of 15 days, meaning one period equals 15 days. All parameters need to be translated into a monthly basis. We thus approximately use $r_C = 0.1\%$ as the market risk free return and $r_B = 0.2\%$ for a well credit rated large buyer. However, the credibility of a cash constrained small supplier is bad and varies greatly, hence its short-term borrowing rate is quite high and differs a lot from firm to firm. We use 2% as a benchmark monthly borrowing rate for the small supplier in our model, i.e., $r_S = 2\%$. For the iid random cash shock ξ_i, we use the normal distribution $N(-1, 2)$. Here we normalize the expected cash inflow of the supplier to \$1 per period and the per period cash inflow is highly volatile (2 times of the expected cash inflow) due to the nature of a small supplier's operations activity. The parameters of the base case are summarized in Table 3.1.

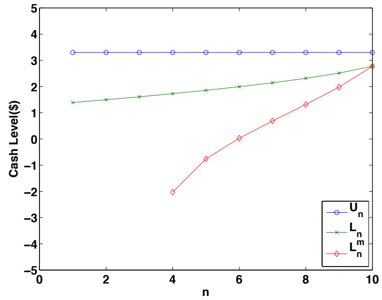
Table 3.1: Base Case Parameters

Interest Rates			Normal Cash Shock	
r_C	r_B	r_S	μ	σ
0.1%	0.2%	2.0%	-1	2

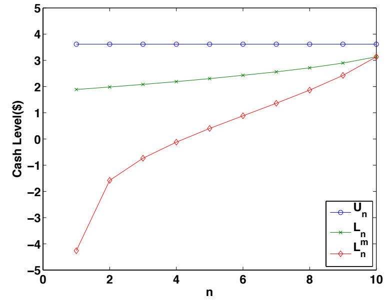
3.5.1 Optimal Cash Policy

To show how the (L_n, U_n) policy changes for suppliers with different credit ratings, we change the r_S while keeping all other paremeters fixed. Specifically, we will look at two more cases in addition to the base case: one with $r_S = 2.5\%$ referred to as "High r_S Case" and one with $r_S = 1.5\%$ referred to as "Low r_S Case". The U_n bound, L_n bound and the myopic L_n^m bound for the three cases are shown in Figure 3.4.

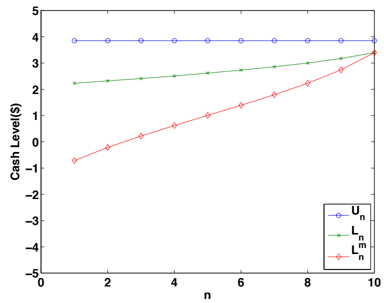
The U_n bound remains constant in all periods, while the L_n lies in between the U_n bound and the myopic bound L_n^m. The mismatch between the optimal and myopic lower bounds is greater in early periods and they converge towards the end of the payment term. The opitmal cash policy in the No RF case is a "U_n" policy, and thus can be interpreted as a special (L_n, U_n) policy with $L_n = -\infty$. Since the U_n bound is the same in the No RF and RF cases, the greater difference in L_n results in more aggressive use of the RF option, thus leading to a greater



(a) $r_S = 1.5\%$



(b) $r_S = 2.0\%$



(c) $r_S = 2.5\%$

Figure 3.4: (L_n, U_n) Bounds, $r_C = 0.1\%$, $r_B = 0.2\%$, $\xi_n \sim N(-1, 2)$, $N = 10$

difference in the expected cash flow cost. Figure 3.4 indicates greater L_n for greater r_S, hence a supplier with a lower credit rating will enjoy the benefit of more cash cost savings with an RF contract. In other words, the value of RF increases as the credit disparity between the supplier and the buyer broadens.

3.5.2 Value of Reverse Factoring

For the base case, we will now fix the initial cash $x_1 = 1$ and see how the cash flow cost changes with W when employing the optimal policy and the myopic policy. The cash flow cost in the No RF case is set as the benchmark and all cost is expressed as a percentage of the benchmark cash cost. 3.5 shows the cash flow cost for the No RF case, the RF with myopic policy case, and the RF with optimal policy case. Both the optimal and myopic policies result in significant cost reduction compared with the No RF case, and the cash cost reduction becomes more significant as the outstanding amount of AR increases. In practice, suppliers with bounded rationality may not perfectly evaluate their decisions and may end up with a policy between the optimal and myopic policies. However, reverse factoring is still quite attractive to the supplier for cash flow management purposes even with a sub-optimal policy. The added liquidity and flexibility in managing cash allows the supplier to accept the offer of a reverse factoring contract from the buyer.

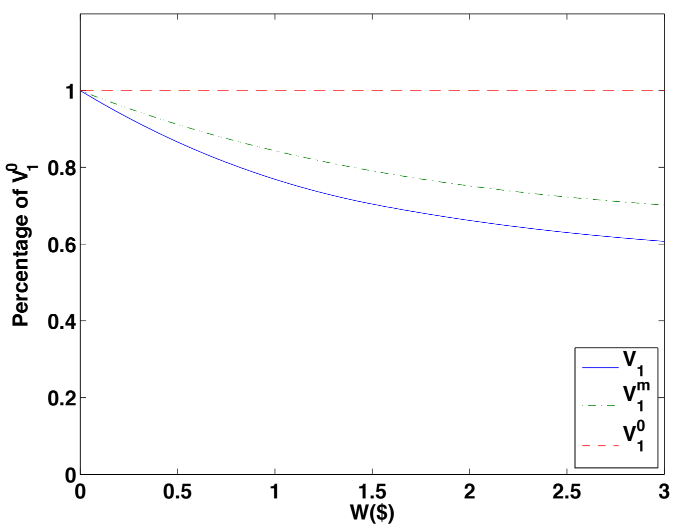


Figure 3.5: Total Cash Flow Cost for $x_1 = 1$, $N = 5$

3.5.3 Maximal Acceptable RF Rate

We will now look at the case where the supplier is originally offered a No RF contract with payment term $M = 2$ and the RF contract targets to extend the payment term to $N = 4$. We look at different initial cash levels for the supplier ranging from \$0 to \$2, to represent different financial conditions the supplier may be facing. We also look at different amounts of outstanding account receivables for the supplier. Figure 3.6 shows how the maximal acceptable RF rate $\bar{r}$ changes with x_1 for different W.

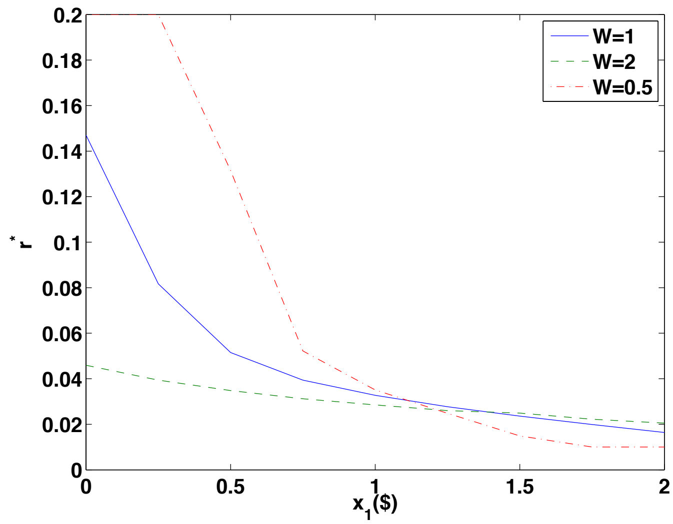
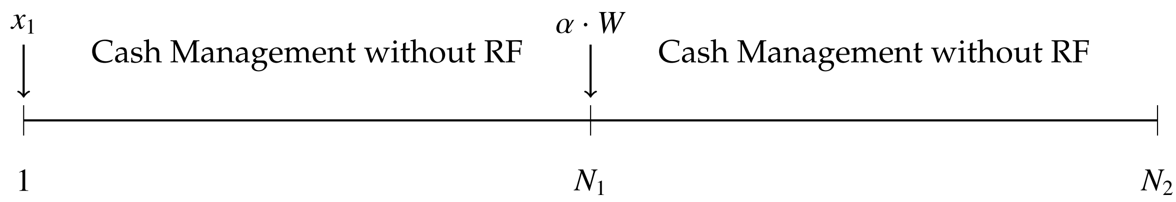
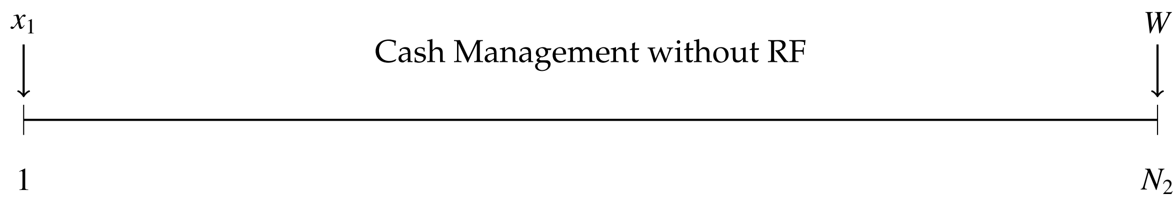


Figure 3.6: Maximal Acceptable RF Rate for $M = 2$, $N = 4$

The bank will offer an RF contract with effective borrowing rate $r \in [r_B, r_S]$. The supplier will accept any RF rate below $\bar{r}$. The actual RF rate also depends on the bargaining power between the parties. For a given W, the supplier tends to accept a wider range of offered r, or equivalently a higher $\bar{r}$, while it has more cash on hand. The acceptable range of r is more sensitive to the supplier's cash on hand when there is a smaller amount of outstanding account receivables. When the amount of outstanding account receivables is relatively small, the supplier does not suffer much from the cost of delayed payment; thus the supplier's maximal acceptable r largely relies on its available cash on hand, as it impacts the liquidity value of reverse factoring. As the figure illustrates, for an extremely cash constrained supplier, a larger W leads to a greater $\bar{r}$ due to the increasing liquidity value of RF while the cost of delayed payment is not significant; for a supplier with abundant cash, on the contrary, a larger W leads to a smaller $\bar{r}$ due to the increasing cost of delayed payment while the liquidity



(a) Early Payment of Account Receivables



(b) No Early Payment of Account Receivables

Figure 3.7: Cash Management with Two-term Trade Credit

value of RF is largely insignificant.

3.6 Two-term Trade Credit From A Cash Flow Perspective

Under the cash management framework, we can also explain two-term trade credit. The supplier can decide the discount acceptable for early payment using the cash flow management model. Two-term trade credit is equivalent to two cash management problems without RF, but with different initial cash levels, as illustrated in 3.7.

The smallest discount factor the supplier is willing to accept is the minimal α such that

$$V_1^0(x_1 \mid N_1) + \delta^{N_1} \cdot EV_1^0(x_1 + \alpha \cdot W + \sum_{i=1}^{N_1} \xi_i \mid N_2 - N_1) \leq V_1^0(x_1, W \mid N_2)$$

which we denote as $\underline{\alpha}$. From the NPV view, the two-term trade credit discount factor α^{NPV} is set such that

$$\frac{W}{(1+r_C)_2^N} = \frac{\alpha^{NPV} \cdot W}{(1+r_C)^{N_1}}$$

Compared with the discount solely determined from the NPV view, the supplier is willing to accept a deeper discount of early cash payments because of added liquidity.

Theorem 4. *lemma:* $\underline{\alpha} \leq \alpha^{NPV}$

This lemma explains the case as found in the real world and can assist both the supplier and the buyer in optimal contract design.

3.7 Conclusion

In this paper, we deal with a supply chain composed of a small supplier and a big buyer. Specifically, we explore the cash management problem of the supplier and frame it as a dynamic programming model. We study the case where there is no RF option and also the case where the supplier is myopic when using the RF option.

We found the optimal cash policy for the supplier with the RF option to be the modified (L_n, U_n) policy and the amount is bounded by the current outstanding AR. The two bounds are shown not to rely on the states of the supplier and they can be solved in a degenerated problem with an infinite amount of AR. We also show that the borrow-up-to bound L_n increases in n and it is greater than that of the myopic policy. The RF option adds liquidity value to the supplier, and the value of reverse factoring increases as the credit disparity between the supplier and buyer widens. By helping the supplier lower its cost of cash flow management, the buyer is better in a better position to extend the payment terms when designing the contract. The maximal payment term that can be achieved results from the tradeoff between the added liquidity value and cost of delayed payment. Our numerical result shows that the maximal payment term achievable first increases and then decreases with the outstanding AR, and a more cash constrained supplier is less reluctant to accept payment term extension. Our research explains how RF works and creates a win-win situation for the supply chain. It also assists the managers in designing the contract terms.

CHAPTER 4

MITIGATING CLIMATE RISK USING STRATEGIC GROWING AREA PLANNING IN AGRICULTURAL SUPPLY CHAIN

4.1 Introduction

The agricultural industry is a special industry in that the selling season and the production (growing) season usually have a significant time interval in between, and that the growing season is highly impacted by weather conditions, an uncontrollable external risk. The storage of finished goods to smooth the supply-demand mismatch is widely used in the industry, but there are some particularity in carrying agricultural products across seasons to smooth the yield uncertainty caused by the different weather condition each year: (1) the agricultural products are perishable goods and have a limited shelf-life, thus the firm either incurs a significant cost in carryover or needs to process the fresh materials into non-fresh products for further sales; (2) agricultural products are widely regulated by the government or related institutes, with policies such as yield control, price restrictions and subsidy. It is interesting to see how agricultural regulations impact the firm's ability to operate with regards to the risk posed by weather conditions.

The wine industry in France is a good example, especially AOC wine. AOC (Appellation d'Origine Contrôlée), or "protected designation of origin" in English, is the French certification granted to certain French agricultural products such as wines, cheeses, butters, and others, that indicates their geographical origin, all under the auspices of the government bureau Institut National Des Appellations d'Origine, now called Institut National de l'Origine et de la Qualité

(INAO). This is based on the concept of terroir. (From Wiki "AOC Wine") There is a ceiling for the yield per hectare for the wine makers. In most of Europe, yield is measured in hectoliters per hectare, i.e., by the volume of wine. In most of the New World, yield is measured in tonnes per hectare (or short tons per acre in the USA), i.e. by the mass of grapes produced per unit area. For the part harvested in excess of the ceiling, it can be stored as VCI for future use. VCI (Volume Complémentaire Individuel), is the individual supplement volume of wine yield in excess of the annual yield allowed, which the wine makers can voluntarily stock to hedge against weather risks that can lead to quantity and quality problem of wine output in future years. VCI is used for several purposes: (1) to guarantee the quality of the production, (2) to help the wine maker to meet demand during years of low yield. It serves as an insurance for the wine maker against the risk posed by weather. Management of the VCI is performed by AVA, and the wine makers bear the relevant costs.

The VCI cannot be commercialized and sold to the consumers or the wine dealers under normal conditions. The wine makers can make a claim to use it in the following circumstances: (1) when there is a deficit in the harvest and the annual maximum yield is not attained, (2) when there is a quality deficit in the output and VCI is needed as a substitute. The wine maker can choose to replace the VCI from the previous year with an equivalent amount of VCI in the new year if applicable. In addition, if the VCI is not used as DREV in the year following, it needs to be destroyed under the title of DRA.

Due to the increasing weather uncertainty, the National Institute of Origin and Quality (INAO) of France has relaxed the use of VCI for wine makers in the past years, by lowering the upper bound of VCI each year. It set new rules in

Figure 4.1: Timeline of Events - Upfront Selling Quantity Decision

June, 2018 that allow the producers of red and white wines to stock 20% of their annual output, for a total of 50% over a 3-year period. Previously, they were only allowed to stock 10% of their output. This decision was made to reinforce the resilience of vineyards against recurrent weather hazards after a series of fierce hailstorms battered fields across France in May, 2018.

In this chapter, we study how the stocking of finished agricultural products such as VCI impacts an agricultural firm's growing decision given the varying nature of weather risks, and what their optimal stocking decisions are. We will also numerically show how the restrictive government regulation on carrying over influences the operations of firms.

4.2 The Model

4.2.1 Single period, one market, planned selling quantity upfront

The risk-neutral agriculture firm grows a crop that is turned into one final agricultural product. At the beginning of each season, the firm determines its growing area x (hectares), and plans for the selling q units. The firm occurs the opportunity cost c (dollars) for each unit of growing area. Due to the weather risk, the yield rate Y is random with cumulative distribution function F and density function f. Assume f is continuous and has first order derivatives almost everywhere. When the harvest season comes, the realized yield rate y will be observed and the actual selling amount $\overline{q}$ will be the minimum of the planned amount to be sold and the realized total harvest, i.e., $\overline{q}(q, x, y) = \min\{q, y \cdot x\}$. The selling season follows the harvest season and the firm's agricultural product is sold on the market where all producers compete on the amount of quantity available to sell. The market price is revealed when the actual amount to be sold is decided and the market shock is observed. The market price is linear in the actual quantity the firm will sell according to the Cournot model, along with the random shock due to the fluctuating market condition ξ, where $\xi \sim H$ and ξ is independent of y. Without loss of generality, $E\xi = 0$. Thus the market price is $p(\overline{q}, \xi) = a - b \cdot \overline{q} + \xi$. Here, $a + \xi$ represents maximal willingness-to-pay of the consumers. Any leftover quantity will be destroyed with no salvage value at the end of the selling season. The time-line of events is described in Figure 4.1. The firm's expected profit function is expressed as

$$
\begin{aligned}
\pi(q, x) &= E_{y,\xi}\left[\overline{q} \cdot p(\overline{q}, \xi)\right] - c \cdot x \qquad (4.1)\\
&= E_y\left\{E_\xi\left[\overline{q} \cdot p(\overline{q}, \xi) \mid y\right]\right\} - c \cdot x\\
&= E_y\left\{E_\xi\left[\overline{q} \cdot (a - b \cdot \overline{q} + \xi) \mid y\right]\right\} - c \cdot x\\
&= E_y\left[\overline{q} \cdot (a - b \cdot \overline{q}) + \overline{q} \cdot E_\xi(\xi \mid y)\right] - c \cdot x\\
&= E_y\left[\overline{q} \cdot (a - b \cdot \overline{q}))\right] - c \cdot x\\
&= [1 - F(\frac{q}{x})] \cdot (a \cdot q - b \cdot q^2) + F(\frac{q}{x}) \cdot E_y[a \cdot (y \cdot x) - b \cdot (y \cdot x)^2 \mid y < \frac{q}{x}] - c \cdot x
\end{aligned}
$$

Note that the random price shock does not matter in the expected firm profit due to the independence between ξ and y. It is obvious that the planned selling amount is chosen in the range $[0, \frac{b}{a}]$. On one hand, given the growing area x, the firm's best corresponding planned selling amount $q^*(x)$ is obtained by setting the FOC with respect to q to zero, i.e.,

$$\frac{\partial\pi(q,x)}{\partial q} = [1 - F(\frac{q}{x})] \cdot (a - 2b \cdot q)$$

By checking the second order derivative, we note that concavity is not guaranteed. However, it is easy to show that the best planned selling quantity is always $\frac{a}{2b}$, i.e., $q^*(x) = \frac{a}{2b}$.

On the other hand, we can also obtain the optimal responsive function of x with respect to q by taking the FOC with respect to x, i.e.,

$$\frac{\partial\pi(q,x)}{\partial x} = \int_0^{\frac{q}{x}} f(y) \cdot (a \cdot y - 2by^2 \cdot x)dy - c$$

Using the optimal responsive planned selling amount, we get

$$\frac{\partial\pi(q^*(x),x)}{\partial x} = \int_0^{\frac{a}{2b}\cdot\frac{1}{x}} f(y) \cdot (a \cdot y - 2by^2 \cdot x)dy - c \tag{4.2}$$

It is natural to assume that $a \cdot Ey > c$, which means that the maximal expected revenue of a unit of land exceeds the cost of a unit if land. Hence, the optimal growing area x^* solves the equation (4.2). In sum, with no harvest storage and only one possible market, the agricultural firm will maximize its profit by targeting its selling quantity at $q^* = \frac{a}{2b}$ and growing on x^* hectares of land, where x^* sets the equation (4.2) equal to zero.

Without the loss of generality, we assume that $\frac{a}{b}$ is sufficiently large, which implies that the market size for the agricultural product is large enough, as the average quantity to make the market price zero is $\frac{a}{b}$ in the pricing line $p = a - b \cdot q$. We further restrict the yield distribution to the uniform distribution to derive the closed form formula for the optimal growing area, i.e., $Y, Y' \sim U[\underline{Y}, \overline{Y}]$. So we now have $[\underline{Y}, \overline{Y}] \in (-\infty, \frac{a}{2b} + q - Y]$. The FOC becomes

$$\frac{1}{\overline{y} - \underline{y}} \cdot (\frac{a}{2} \cdot y^2 - \frac{2b}{3} \cdot y^3 \cdot x) \Big|_{y=\underline{y}}^{\overline{y}} - c = 0$$

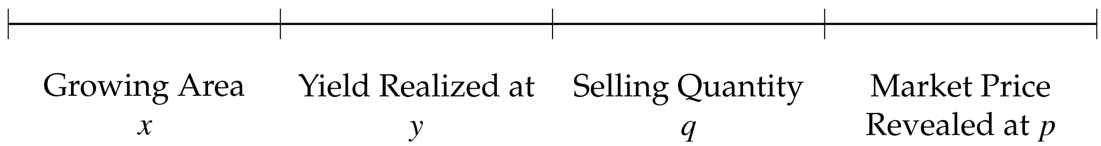


Figure 4.2: Timeline of Events - Post Harvest Selling Quantity Decision

optimal growing area x^* becomes

$$x^* = \frac{1}{\tilde{y}} \cdot \frac{-\frac{1}{\tilde{y}} \cdot c + a}{b \cdot (2 - \frac{2}{3} \cdot R^2)}$$

4.2.2 Single period, one market, selling quantity decided after observing yield rate, before price realization

Let us consider another scenario all conditions equal, except that the selling quantity is decided after the harvest amount is observed. The time-line is described in Figure 4.2. Given x and y, the firm decides its responsive selling quantity which is a function of x and y. Given x, the firm's expected profit upon the realization of yield y is

$$\begin{aligned} \pi(q \mid x, y) &= E_{\xi}[q \cdot p(q)] - c \cdot x \qquad (4.3) \\ &= E_{\xi}[q \cdot (a - b \cdot q + \xi)] - c \cdot x \\ &= q \cdot (a - b \cdot q) + q \cdot E\xi - c \cdot x \\ &= q \cdot (a - b \cdot q) - c \cdot x \end{aligned}$$

, where $q < y \cdot x$.

Note that the random shock does not impact the expected profit here, which is the same as in the case of upfront selling quantity. The firm's problem in the harvest season is maximizing $\pi(q \mid x, y)$ with respect to q. The optimal responsive selling quantity is solved to be $q^*(x, y) = \min\{\frac{a}{2b}, y \cdot x\}$. Thus, considering

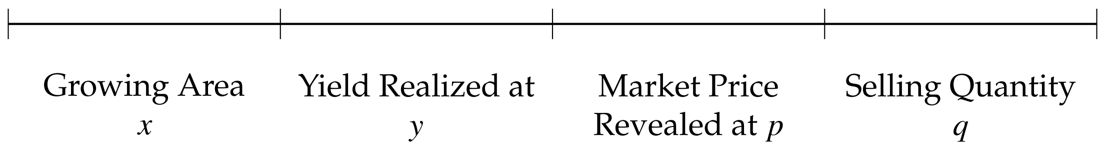


Figure 4.3: Timeline of Events - Post Harvest Selling Quantity Decision

the optimal responsive selling quantity, the expected profit is a function of the growing area x as the following,

$$\begin{aligned}\pi(x) &= E_y \pi(q^*(x,y) \mid x, y) \qquad (4.4)\\ &= [1 - F(\frac{a}{2b} \cdot \frac{1}{x})] \cdot \frac{a^2}{4b} + F(\frac{a}{2b} \cdot \frac{1}{x}) \cdot E_y[(y \cdot x) \cdot (a - b \cdot (y \cdot x)) \mid y < \frac{a}{2b} \cdot \frac{1}{x}] - c \cdot x\end{aligned}$$

Taking the first order derivative, we have

$$\frac{d\pi(x)}{dx} = \int_0^{\frac{a}{2b} \cdot \frac{1}{x}} f(y) \cdot (a \cdot y - 2by^2 \cdot x) dy - c \qquad (4.5)$$

$\pi(x)$ is concave in x, as the second order derivative is negative. Note that equation (4.5) is the same as (4.2), thus the optimal growing area x^* and selling amount q^* is the same as the case where q is decided upfront. The only difference is that here $q^*(x, y)$ is a responsive selling amount bounded by the available harvest amount $y \cdot x$. In sum, with no harvest storage and only one possible market, the agricultural firm will maximize its profit by setting selling quantity at $\frac{a}{2b}$ if the realized harvest amount is greater than that, and setting the selling quantity at $y \cdot x^*$ otherwise. The optimal growing area x^* is solved by setting equation (4.5) equal to zero.

4.2.3 Single period, one market, selling quantity decided after observing yield rate, after price realization

In this scenario, q is decided based on x, and the realization of y and ξ. Given (x, y, ξ), the firm's profit function with respect to q is

$$\begin{aligned} \pi(q \mid x, y, \xi) &= q \cdot p(q) - c \cdot x \\ &= q \cdot (a - b \cdot q + \xi) - c \cdot x \end{aligned} \tag{4.6}$$

, where $q < y \cdot x$.

Given x, and upon the realization of y and ξ, the firm will choose q to maximize $\pi(q \mid x, y, \xi)$. The optimal responsive selling quantity is solved to be $q^*(x, y, \xi) = \min\{\frac{a+\xi}{2b}, y \cdot x\}$. Thus, considering the optimal responsive selling quantity, the firm's ex-ante expected profit function with respect to x is

$$\begin{aligned} \pi(x) &= E_{y,\xi}\pi(q^*(x, y, \xi) \mid x, y, \xi) \\ &= E_y E_\xi \left[q^* \cdot (a - b \cdot q^* + \xi) - c \cdot x \mid y\right] \\ &= E_y \left[\int_{-\infty}^{2y \cdot x \cdot b - a} \frac{(a + \xi)^2}{4b} \cdot f(\xi) d\xi + \int_{2y \cdot x \cdot b - a}^{\infty} (y \cdot x) \cdot (a - b \cdot (y \cdot x) + \xi) \cdot h(\xi) d\xi\right] - c \cdot x \end{aligned} \tag{4.7}$$

Note that unlike the previous two scenarios, the random price shock will matter when the firm chooses the optimal growing area. Let $g(y, x) := \int_{-\infty}^{2y \cdot x \cdot b - a} \frac{(a+\xi)^2}{4b} \cdot f(\xi) d\xi + \int_{2y \cdot x \cdot b - a}^{\infty} (y \cdot x) \cdot (a - b \cdot (y \cdot x) + \xi) \cdot h(\xi) d\xi$. We then have

$$\pi(x) = E_y g(y, x) - c \cdot x$$

Take the first order derivative of $\pi(x)$, we have

$$
\begin{aligned}
\frac{d\pi(x)}{dx} =& E_y \frac{\partial g(y,x)}{\partial x} - c \qquad (4.8)\\
=& E_y[2y \cdot b \cdot b \cdot (y \cdot x)^2 \cdot h(2y \cdot x \cdot b - a) - 2y \cdot b \cdot b \cdot (y \cdot x)^2 \cdot h(2y \cdot x \cdot b - a)\\
& + \int_{2y\cdot x\cdot b-a}^{\infty} ((a+\xi) \cdot y - 2b \cdot y^2 \cdot x) \cdot h(\xi) d\xi] - c\\
=& E_y \left\{ \int_{2y\cdot x\cdot b-a}^{\infty} \left[(a+\xi) \cdot y - 2b \cdot y^2 \cdot x \right] \cdot h(\xi) d\xi \right\} - c\\
=& \int_{-\infty}^{\infty} \left\{ \int_{2y\cdot x\cdot b-a}^{\infty} \left[(a+\xi) \cdot y - 2b \cdot y^2 \cdot x \right] \cdot h(\xi) d\xi \right\} \cdot f(y) dy - c\\
=& \int_{-\infty}^{\infty} \left\{ \int_{-\infty}^{\frac{a+\xi}{2b\cdot x}} \left[(a+\xi) \cdot y - 2b \cdot y^2 \cdot x \right] \cdot f(y) dy \right\} \cdot h(\xi) d\xi - c\\
=& \phi(\tilde{\xi}) > \phi(\bar{\xi}) = \phi(0)
\end{aligned}
$$

, where $\tilde{\xi} > 0$.

To compare the x solved by Equation 4.8 with that solved by Equation 4.5, we want to show the concavity of $\int_{-\infty}^{\frac{a+\xi}{2b\cdot x}} \left[(a+\xi) \cdot y - 2b \cdot y^2 \cdot x \right] \cdot f(y) dy$ as the function in ξ. Denote this as $\phi(\xi)$, we show $\phi(\xi)$ to be an increasing function, and

$$\frac{d\phi(\xi)}{d\xi} = \int_{-\infty}^{\frac{a+\xi}{2b\cdot x}} a \cdot f(y) dy$$

,which proves that $\phi(\xi)$ is convex in ξ and Equation 4.8 is greater than Equation 4.5 for all x by Jensen's Inequality. Equation (5) = $\phi(\bar{\xi})$, Equation (9) = $\int_{-\infty}^{\infty} \phi(\xi) h(\xi) d\xi$. Hence the optimal growing area is larger when the selling quantity is determined after the random price shock.

4.2.4 Analog of the current model with Newsvendor

Before we go on to the discussion of how the optimal growing area changes with the parameter settings for the three cases given specific distributions of yield and price shock, we need to look at the expected amount of yield that will be left over with the optimal selling quantity. We can express the expected leftover inventory of the case in Section 4.2.2 and Section 4.2.3 as the following. We ignore the case when selling quantity is decided upfront before yield and price realization, because the result is identical to the case in Section 4.2.2. We denote the case in Section 4.2.2 as Case 1 and the case in Section 4.2.3 as Case 2. Let $LO(x)$ be the expected left over inventory given growing area x.

$LO_1(x)$ can be expressed as

$$\begin{aligned} E(y \cdot x - q^*)^+ &= \int_{\frac{a}{2b}\cdot\frac{1}{x}}^{\infty} (y \cdot x - \frac{a}{2b}) \cdot f(y)dy \\ &= x \cdot \int_{\frac{a}{2b}\cdot\frac{1}{x}}^{\infty} y \cdot f(y)dy - \frac{a}{2b} \cdot F(y > \frac{a}{2b} \cdot \frac{1}{x}) \end{aligned}$$

$LO_2(x)$ can be expressed as

$$E(y \cdot x - q^*)^+ = \int_{-\infty}^{\infty} \left[\int_{\frac{a+\xi}{2b}\cdot\frac{1}{x}}^{\infty} (y \cdot x - \frac{a+\xi}{2b}) \cdot f(y)dy \right] h(\xi)d\xi$$

Let $k(x,\xi) = \int_{\frac{a+\xi}{2b}\cdot\frac{1}{x}}^{\infty} (y \cdot x - \frac{a+\xi}{2b}) \cdot f(y)dy$, then $LO_1(x) = k(x,0) = k(x,E\xi)$ and $LO_2(x) = Ek(x,\xi)$. We have

$$\frac{\partial k(x,\xi)}{\partial \xi} = \frac{1}{4b^2} \cdot \frac{1}{x} \cdot F(y > \frac{a+\xi}{2b} \cdot \frac{1}{x})$$

Since $\frac{\partial k(x,\xi)}{\partial \xi}$ decreases in ξ, we observe $k(x,\xi)$ to be a concave function. Hence, by Jensen's inequality, we have $LO_2(x) < LO_1(x)$. We also know that the optimal growing area for Section 4.2.2 is greater than that for Section 4.2.3, i.e., $x_1^* > x_2^*$. We then need to compare $LO_1(x_1^*)$ and $LO_2(x_2^*)$. It is obvious that $k(x,\xi)$ is increasing in x, so $LO_1(x)$ and $LO_2(x)$ will be increasing in x as well. We then have

$$LO_2(x_2^*) < LO_2(x_1^*) < LO_1(x_1^*)$$

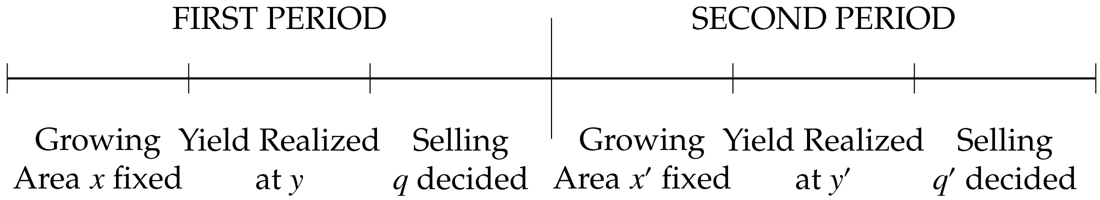


Figure 4.4: Timeline of Events - Two Periods, One Market

Therefore, we know that when the firm can decide the quantity to be sold after yield and price realization, and decide its optimal growing area, it will result in less expected left over agricultural products.

4.2.5 Two period, one market, planned selling quantity upfront

Now the agricultural firm has the option to reserve some extra yield of its product without selling it in the current period, but uses it to hedge the risk of decreased yield in the future. The timeline is illustrated in Figure 4.4.

For the simple case, assume the agricultural firm sells its product in two periods. The firm sets the selling quantity after observing the yield in each period; it can choose not to sell all the products in the first period when yield permits, and instead carry over some portion of the products to the second period. The unsold products have zero salvage value in the second period. We also assume a fixed growing area and zero price shock for the first-cut analysis. [So Y and Y' stands for the total yield in the first and second periods respectively, $Y, Y' \sim h(\cdot)$, i.i.d.]

The profit function for the second period (as a function of selling quantityq'), given (q, Y, Y') is

$$\pi(q' \mid q, Y, Y') = -c \cdot x + (a - b \cdot q') \cdot q'$$

with $q' \leq Y' + (Y - q)$. The optimal responsive q' is $q'(q, Y, Y') = \min\{\frac{a}{2b}, Y' + (Y - q)\}$. The total profit of the first and second periods is (as a function of q) given Y is

$$\begin{aligned}
&\pi^T(q \mid Y) \\
&= -c \cdot x + (a - b \cdot q) \cdot q + E_{Y'}[\pi(q'(q, Y, Y') \mid q, Y, Y')] \\
&= -c \cdot x + (a - b \cdot q) \cdot q + \int_{\frac{a}{2b}+q-Y}^{\infty} \frac{a^2}{4b} h(Y')dY' \\
&\quad + \int_{-\infty}^{\frac{a}{2b}+q-Y} [(a - b \cdot (Y' + Y - q)) \cdot (Y' + Y - q)]h(Y')dY'
\end{aligned}$$

The concavity can be easily verified. So the optimal selling quantity decision is determined by the following FOC (under the requirement of $q \leq y$):

$$\frac{d\pi^T(q \mid Y)}{dq} = a - 2b \cdot q - \int_{-\infty}^{\frac{a}{2b}+q-Y} (a - 2b \cdot (Y' + Y - q))h(Y')dY'$$

The term $\int_{-\infty}^{\frac{a}{2b}+q-Y} (a - 2b \cdot (Y' + Y - q))h(Y')dY'$ represents the downside risk of yield in the second period, and will determine how the optimal q in the first period will deviate from the one period model in section 3.2, i.e, $q^* = \min\{\frac{a}{2b}, Y\}$. Obviously, we have $\int_{-\infty}^{\frac{a}{2b}+q-Y} (a - 2b \cdot (Y' + Y - q))h(Y')dY' > 0$ for any $q < Y$.

Without the loss of generality, we assume that $\frac{a}{b}$ is sufficiently large, which implies that the market size for the agricultural product is large enough, as the average quantity to make the market price zero is $\frac{a}{b}$ in the pricing line $p = a - b \cdot q$. We further restrict the yield distribution to the uniform distribution to derive the closed form formula for the optimal growing area, i.e., $Y, Y' \sim U[\underline{Y}, \overline{Y}]$. So we have $[\underline{Y}, \overline{Y}] \in (-\infty, \frac{a}{2b} + q - Y]$. The FOC becomes

$$\frac{d\pi^T(q \mid Y)}{dq} = a - 2b \cdot q - \int_{-\infty}^{\frac{a}{2b}+q-y} (a - 2b \cdot (Y' + Y - q))h(Y')dY' = -4b \cdot q + 2b \cdot (\overline{Y} + \underline{Y})$$

We then have $q^* = (\overline{Y}+\underline{Y})/2$ when $q < Y$ is guaranteed, i.e., $q^* = \min\{(\overline{Y}+\underline{Y})/2, Y\}$. Compared with the one-period case, the agricultural firm now is enabled to target at a much more balanced production level. In the one-period case, the firm targets $\frac{a}{2b}$, which is unrealistically high. Now, let's look at what the optimal growing area is in the two-period case.

We assume that the growing area decision x is made upfront, thus Y' and Y are i.i.d. random variables. We have already characterized the firm's optimal selling quantity in the first period, given x, i.e., $q^* = \min\{(\overline{Y}+\underline{Y})/2, Y\}$.

Proposition 6. *When $y > \frac{a}{b}$, the firm can allocate at least $\frac{a}{2b}$ for both the first and second period to achieve the global optimal. When $Y < \frac{a}{b}$, the trade-off is made between the two periods and is yield distribution dependent.*

Now, adding back the growing area decision x, we have $\pi(q \mid y, x)$ as the following,

$$
\begin{aligned}
&\pi^T(q \mid y, x) \\
=& - c \cdot x + (a - b \cdot q) \cdot q + \int_{\frac{1}{x}\cdot(\frac{a}{2b}+q)-y}^{\infty} \frac{a^2}{4b} f(y')dy' \\
&+ \int_{-\infty}^{\frac{1}{x}\cdot(\frac{a}{2b}+q)-y} [(a - b \cdot (y' \cdot x + y \cdot x - q)) \cdot (y' \cdot x + y \cdot x - q)] f(y')dy'
\end{aligned}
$$

We have already characterized the optimal selling quantity given (y, x), i.e., $q^*(y, x)$. The profit function given (y, x) is thus $\pi(q^* \mid y, x)$, where q^* is the abbreviated notation for $q^*(y, x)$. We are interested in the FOC of the expected profit with respect to x, i.e., $\frac{dE_y \pi(q^*|y,x)}{dx}$.

$$
\begin{aligned}
&\frac{d\pi^T(q^* \mid y, x)}{dx} \\
=& - c + a \cdot \frac{dq^*}{dx} - 2b \cdot q^* \cdot \frac{dq^*}{dx} + \int_{-\infty}^{\frac{1}{x}\cdot(\frac{a}{2b}+q^*)-y} [a - 2b \cdot (y' \cdot x + y \cdot x - q^*)] \cdot (y' + y) f(y')dy'
\end{aligned}
$$

With a uniform distribution for y and y', i.e., $y, y' \sim U[\underline{y}, \overline{y}]$, i.i.d., and sufficiently large market size, we can further simplify the expression. First, we have $q^*(y, x) = x \cdot min\{(\overline{y} + \underline{y})/2, y\}$, and let $\tilde{y} = (\overline{y} + \underline{y})/2$ and $\Delta = (\overline{y} - \underline{y})/2$. Multiply both side of FOC by Δ^2, we thus have

$$
\begin{aligned}
&\Delta^2 \cdot \frac{dE_y \pi^T(q^* \mid y, x)}{dx} \\
&= -\Delta^2 \cdot c + \Delta^2 \cdot \frac{1}{2} \cdot (a \cdot \tilde{y} - 2b \cdot \tilde{y}^2 \cdot x) + \Delta \cdot (\frac{1}{2} \cdot a \cdot y^2 - \frac{2b}{3} \cdot x \cdot y^3) \mid_{y=\underline{y}}^{\tilde{y}} \\
&\quad + \frac{a}{6} \cdot (y + \overline{y})^3 \mid_{y=\underline{y}}^{\overline{y}} - \frac{a}{6} \cdot (y + \underline{y})^3 \mid_{y=\underline{y}}^{\overline{y}} - \frac{b}{6} \cdot x \cdot (y + \overline{y})^4 \mid_{y=\underline{y}}^{\overline{y}} + \frac{b}{6} \cdot x \cdot (y + \underline{y})^4 \mid_{y=\underline{y}}^{\overline{y}} \\
&\quad + \frac{b}{3} \cdot x \cdot \tilde{y} \cdot (y + \overline{y})^3 \mid_{y=\tilde{y}}^{\overline{y}} + \frac{b}{4} \cdot x \cdot (y + \overline{y})^4 \mid_{y=\underline{y}}^{\tilde{y}} - \frac{b}{3} \cdot x \cdot \overline{y} \cdot (y + \overline{y})^3 \mid_{y=\underline{y}}^{\tilde{y}} \\
&= -\Delta^2 \cdot c + b \cdot x \cdot (8\tilde{y}^3 \cdot \Delta - 29\tilde{y}^2 \cdot \Delta^2 + 4\tilde{y} \cdot \Delta^3 - \frac{73}{12} \cdot \Delta^4) + \frac{25}{6} \cdot a \cdot \tilde{y} \cdot \Delta^2 - \frac{1}{2} \cdot a \cdot \Delta^3
\end{aligned}
$$

The FOC gives us the optimal growing area decision upfront:

$$
x^* = \frac{-c \cdot \Delta^2 + \frac{25}{6} \cdot a \cdot \tilde{y} \cdot \Delta^2 - \frac{1}{2} \cdot a \cdot \Delta^3}{b \cdot (-8\tilde{y}^3 \cdot \Delta + 29\tilde{y}^2 \cdot \Delta^2 - 4\tilde{y} \cdot \Delta^3 + \frac{73}{12} \cdot \Delta^4)}
$$

Denote the ratio $\frac{\Delta}{\tilde{y}}$ as R, which measures the dispersion of the uncertain yield. Dividing both the numerator and the denominator by $\tilde{y}^4$, we have

$$
x^* = \frac{1}{\tilde{y}} \cdot \frac{-\frac{1}{\tilde{y}} \cdot c \cdot R^2 + \frac{25}{6} \cdot a \cdot R^2 - \frac{1}{2} \cdot a \cdot R^3}{b \cdot (-8R + 29R^2 - 4R^3 + \frac{73}{12} \cdot R^4)}
$$

Both the numerator and the denominator become polynomials of R, and x^* is also scaled by the inverse of the average yield. The optimal growing area x^* is solely determined by the average yield and the dispersion of yield relative to the average yield, under the uniform distributed yield. The analytical results for other general distributions are less likely to be obtained, thus we will focus on the discussion of the uniform distributed yield. The larger the average yield, the less the optimal growing area, which is very intuitive. However, it is not clear how x^* changes with R as $x^*(\tilde{y}, R)$ is not necessarily a monotonous function. It is generally multimodal. The cost factor c and the market-pricing factor (a, b)

will impact how the firm reacts in choosing the optimal growing area when its yield uncertainty distribution is changed, either due to weather conditions or a change in the level of available technology.

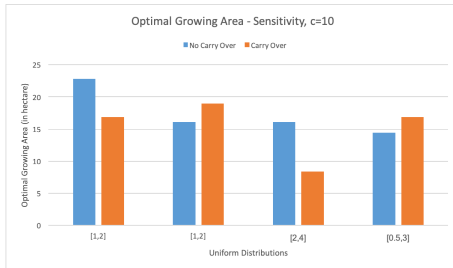
4.3 Numerical Experiment

We compare the optimal growing area decision for all cases under uniform distribution in the following table.

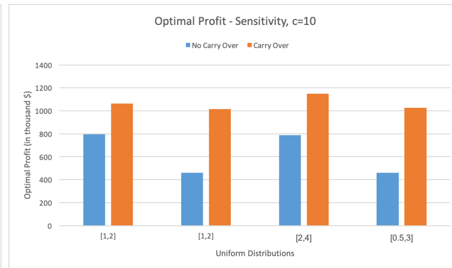
Table 4.1: Parameters and Optimal Growing Area

Market Condition			Random Yield		No Carryover		Carryover	
a	b	c	$\underline{y}$	$\bar{y}$	x^*	π^*	x^*	π^*
50	1	10	1	2	22.82	793.56	16.84	1064.7
50	1	10	0.5	2.5	16.14	458.50	18.95	1014.0
50	1	10	2	4	16.14	786.37	8.42	1150.5
50	1	10	0.5	3	14.43	461.32	16.84	1027.3
50	1	2	1	2	51.03	727.06	18.95	1206.8
50	1	2	0.5	2.5	36.08	636.91	25.26	1189.4
50	1	2	2	4	36.08	697.17	10.53	1228.1
50	1	2	0.5	3	32.27	620.90	25.26	1188.5

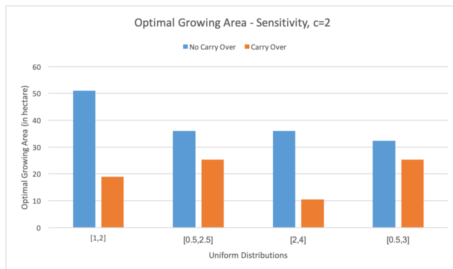
The numerical results attest to our analytic analysis for the uniform distribution, that by allowing the carryover of the finished products, the optimal growing area can either be greater or smaller than a case without carryover. This depends on the nature of the yield uncertainty. When the random yield is more dispersed, the agricultural firm tends to be more conservative in planning its growing area. In addition, when the average yield is low, the firm is more aggressive in planning the growing area to hedge against the potential downturns in the future period. The effect of the carryover policy also depends on the cost per hectare of the growing area. In our simulation, when the growing area cost is low, the carryover policy is more likely to result in a smaller growing area. In



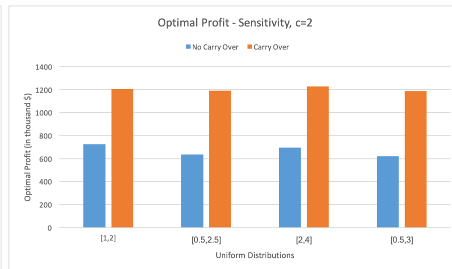
(a) Optimal Growing Area, c=10



(b) Optimal Profit, c=10



(c) Optimal Growing Area, c=2



(d) Optimal Profit, c=2

Figure 4.5: Sensitivity Analysis for Optimal Growing Area and Optimal Profit

both parameter settings, the firm has higher profits when carryover is allowed.

The results are rather complicated and do not seem to be monotonic, which is mainly due to the multiplicative feature of the growing area to the random yield. The results for the additive random yield form are much simpler but do not represent the context we are studying here.

4.4 Managerial Insight

We have studied the agricultural firm's growing area decision under the presence of risks posed by weather, while facing other constraints: (1) market price is uncertain; (2) carrying over finished products is allowed. It is clear that, the growing area in the multiperiod carrying case is not necessarily smaller than the one-period no carrying over case. The area is parameter dependent, based on factors such as the market pricing line, the cost of land, and the distribution of random yield.

APPENDIX A

APPENDIX OF CHAPTER 2

A.1 Summary of Symbols

Table A.1: Symbol Definition

Symbol	Explanation
$\mathcal{F}$	Set of all finished products
$\mathcal{S}$	Set of all tier-1 suppliers
$\mathcal{P}$	Set of all parts
$\mathcal{F}^D$	Set of disrupted products
$\mathcal{S}^D$	Set of disrupted suppliers
$\mathcal{P}^D$	Set of disrupted parts
H^t	Capacity of DMF in period t
r_{kj}	Units of part k required to produce unit product j
T_D	Duration of disruption period
T_R	Number of recovery periods
d_j^t	Planned production amount for product j in period t
q_k^t	Inventory of part k at the beginning of period t
s_j^t	Inventory of product j at the beginning of period t
u_{ik}^t	Total delivery quantity of part k from supplier i in period t
p_j^t	Total production quantity of product j in period t
y_{kj^t}	Allocation of part k to product j in period t
l_j	Unfulfilled production plan of product j in period t
w_j	Capacity occupation of DMF for one unit of product j
g_j	Gross profit for one unit of product j
c_{ik}	Capacity of part k for supplier i
γ_j	Backlogged rate for lost sales for product j

A.2 Proof of Propositions 1 and 2

For ease of exposition in our proof, we transform the formulation of the 3-product model by (1) replacing l_j^t with its component parts, (2) applying the inequality instead of the equality that connect l_j^{t-1} and l_j^t, (3) removing the s_j^t and (4) applying the inequality instead of the equality in the related constraints. The production equilibrium in the original formulation is removed because the slack variables l_j^t are removed, and $(d_j^1 - p_j^1)^+$ and $(d_j^2 - p_j^2 - (p_j^1 - d_j^1)^+ + \gamma \cdot (d_j^1 - p_j^1)^+)^+$ represent the original l_j^t and implicitly contains the production equilibrium. The marginal impact of the Part A and Part B stays the same for a given optimal production plan in this new formulation, compared with the original formulation. This alternative formation of the 3-product model serves to simplify our notation and the presentation of our proof.

The objective function is also changed to a convex function. The problem can then be represented as:

$$\min_{p_j^1, p_j^2} (1-\gamma) \cdot \sum_{j=1}^{3} g_j \cdot (d_j^1 - p_j^1)^+ + \sum_{j=1}^{3} g_j \cdot (d_j^2 - p_j^2 - (p_j^1 - d_j^1)^+ + \gamma \cdot (d_j^1 - p_j^1)^+)^+$$

subject to

$$\begin{cases} p_1^1 + p_2^1 \leq q_A^1 & (\textbf{Part A}) \\ p_2^1 + p_3^1 \leq q_B^1 + c_B^1 & (\textbf{Part B}) \\ \sum_{j=1}^{3} w_j \cdot p_j^1 \leq H^1 & (\textbf{Disruption Capacity}) \\ \sum_{j=1}^{3} w_j \cdot p_j^2 \leq H^2 & (\textbf{Recovery Capacity}) \\ \mathbf{p} \in \mathbf{F} & (\textbf{Feasible Production}) \end{cases}$$

A.2.1 Proof of Proposition 1 and 2

In the transformed 3-product model, the optimal production plan is the optimal solution of at least one of the sub-LP problems and thus we have effectively 10 constraints all together: "Part A", "Part B", "Disruption Capacity", "Recovery Capacity" and the 6 constraints from production, either from "Product j Under-produced" or "Product j Over-produced" (for each product, both "Product j Under-produced" and "Product j Over-produced" involve 2 constraints). Since we have 6 decision variables $\{p_1^1, p_2^1, p_3^1, p_1^2, p_2^2, p_3^2\}$, the optimal LP base consists of 6 binding constraints.

"Part A" and "Recovery Capacity" are assumed to be binding because (1) when "Part A" is not binding, the objective function will be zero and the proposition is obvious and (2) when "Recovery Capacity" is not binding, the problem becomes a disruption-stage only problem as in [50] and the part A allocation will be in the order of GPP, hence the proposition is obvious. Of the other 8 constraints, there are 4 more binding constraints from $S = \{Part_B, S_j^1, S_j^2, j = 1, 2, 3\}$. For the 4 remaining constraints, "Disruption Capacity" must be loose for "Recovery Capacity" to be binding, and the other 3 constraints in S are in general not binding.

For a given parameter setting $\{q_A^1, q_B^1, H^1, H^2\}$, let $\{S', S'', S''', S''''\}$ represent the four binding constraints, in addition to "Part A" and "Recovery Capacity" for the corresponding optimal base Ω. Since $\{q_B^1, H^1, H^2\}$ does not change, we can treat Ω as a function of q_A^1 when analyzing the marginal benefit of part A inventory, i.e. $\Omega(q_A^1)$. The corresponding solution $p_{\Omega(q_A^1)}$ with Part A inventory level at q_A^1 satisfies $\{S', S'', S''', S''''\}$. When we increase q_A^1, holding everything else equal, the optimal base will stay the same until one of the constraints $S''''' \in S - \{S', S'', S''', S''''\}$ becomes binding. At that point, one of the $\{S', S'', S''', S''''\}$ constraints will be replaced by S'''''. Without loss of generality, we assume that S' is replaced. When q_A^1 ($q_A^1 < \overline{q}_A^1$) is increased to the extreme point of the convex hull (feasible region) $\overline{q}_A^1$, $\{S', S'', S''', S''''\}$ and S''''' (i.e. all five) are binding, along with the two resource constraints "Part A" and "Recovery Capacity". We want to prove that increasing Part A inventory from q_A^1 to $q_A^1 + \epsilon$ has more value compared with increasing Part A inventory from $\overline{q}_A^1$ to $\overline{q}_A^1 + \epsilon$, where ϵ is a perturbation. Let Ω' represent the new optimal LP based for with Part A inventory at $\overline{q}_A^1 + \epsilon$. The corresponding optimal solutions are $p_{\Omega'(\overline{q}_A^1+\epsilon)}$ and $p_{\Omega'(\overline{q}_A^1)}$ for Part A inventory at $\overline{q}_A^1 + \epsilon$ and $\overline{q}_A^1$. The difference of these two solutions are $\Delta_{\Omega'} = p_{\Omega'(\overline{q}_A^1+\epsilon)} - p_{\Omega'(\overline{q}_A^1)}$. $\Delta_{\Omega'}$ makes "Part A", "Recovery Capacity" and $\{S'', S''', S'''', S'''''\}$ zero, and has non-zero values for constraints S'. $f(p_{\Omega'(\overline{q}_A^1+\epsilon)}) - f(p_{\Omega'(\overline{q}_A^1)}) = f(\Delta_{\Omega'}) = \epsilon \cdot \lambda_{\Omega'}$, where $f(\cdot)$ is the objective function and $\lambda_{\Omega'}$ is the marginal benefit of Part A inventory further increasing from $\overline{q}_A^1$. If $\lambda_{\Omega'} < \lambda_{\Omega}$, i.e., the marginal DE reduction benefit of Part A inventory is higher for LP base Ω' compared with Ω, we can construct a contradiction. Let's look at the optimization problem with Part A level at $q_A^1 + \epsilon$ and check the feasible solution $p_{\Omega(q_A^1)} + \Delta_{\Omega'}$. This solution has two interior points for constraints S' and S''''', and does not correspond to the optimal LP at $p_{\Omega(q_A^1+\epsilon)}$. However, $f(p_{\Omega(q_A^1)} + \Delta_{\Omega'}) = f(p_{\Omega(q_A^1)}) + f(\Delta_{\Omega'}) = f(p_{\Omega(q_A^1)}) + \xi \cdot \lambda_{\Omega'} < f(p_{\Omega(q_A^1)}) + \epsilon \cdot \lambda_{\Omega} = f(p_{\Omega(q_A^1+\epsilon)})$. This is lower (better) than the optimal value indicated by the optimal LP base Ω, so $\lambda_{\Omega'} < \lambda_{\Omega}$ cannot be true, and we have proved Proposition 1.

The proof of Proposition 2 follows the same logic, except that now "Part A", "Part B" and "Recovery Capacity" are assumed to be binding, and the other 3 binding constraints are from $\{S_j^1, S_j^2, j = 1, 2, 3\}$.

A.2.2 Definition of Production Plans

We define firm's reactive production plans that correspond to the extreme points in the convex hull, which are the potential optimal production plans. They can be characterized by the priority levels of each product in the disruption and recovery periods. In the disruption period, each product has eight regions of priority. We describe only four of them because the other four are symmetric. The four regions are (1) prioritized to be produced for the disruption-period demand and overproduced (if any) for the recovery-period demand, denoted as O (totally prioritized to overproduction); (2) prioritized to be produced for the disruption-period demand and overproduced with an amount such that there will be no unmet demand for the other product in the recovery period; this is a modification of O when the recovery period capacity is not fully consumed with priority O, thus denoted as O^H (modified for recovery period capacity); (3) prioritized to be produced for disruption-period demand but not overproduced, denoted as U (disruption period prioritized, underproduction of just enough); (4) prioritized to be produced for the disruption-period demand to the point where there will be no unmet demand for the prioritized product in the recovery period; this is a modification of U where the recovery period capacity would be used to produce both products if prioritized as U, thus we denote it as U^H (modified for recovery period capacity). The remaining four priority levels $\{\overline{U}, \overline{U^H}, \overline{O}, \overline{O^H}\}$ are symmetric: the corresponding priority levels of the other product are $\{U, U^H, O, O^H\}$ respectively. Note that part inventory is very low, there is no possibility for overproduction, hence O is the same as U and O^H does not exist. In the recovery period, each product has two levels of priority: (1) prioritized to be produced for recovery-period demand , denoted as R (recovery period prioritized); (2) produced after the recovery-period demand of the other product is fulfilled, denoted as $\overline{R}$ (the complement set for R, equivalent to having the other product in priority level R).

We use Ω_{ab} to denote the production plan defined above, where a and b denote the priority level of product 1 in the disruption period and recovery period respectively. We have $a \in \{O^H, O, U^H, U, \overline{U^H}, \overline{U}, \overline{O}, \overline{O^H}\}$ and $b \in \{R, \overline{R}\}$. There are 16 production plans defined altogether.

A.3 Supporting Results

Table A.2 summarizes the cost and *DE* impacts of other multi-part implementations using different thresholds of the insurance rate for the order-up-to decrease and the safety stock increase, as well as changing whether modifications are first made to the order-up-to amount or the safety stock.

Table A.2: Implementations of Multiple Part Inventory Policy Changes

Inventory Policy	Total Annual Cost	Average DE
Original	\$23.6M	\$153K
Order-up-to at 10%, Safety stock at 2%	\$23.1M	\$68.3K
Order-up-to at 5%, Safety stock at 2%	\$21.8M	\$70.4K
Safety stock at 2%, Order-up-to at 10%,	\$23.2M	\$68.7K
Safety stock at 2%, Order-up-to at 5%,	\$21.7M	\$72.6K

Note: Inventory changes are restricted to six levels for each part. The analysis is applied to the 5,637 parts with cost information.

Table A.3 provides the *DE* reduction for different strategic portfolios with a $\$200,000$ budget. It supplements the results of Table 2.4.

Table A.4 summarizes the overall disruption exposure reduction for different

Table A.3: Reduction in DMF's *DE* Due to the Adoption of Strategic Portfolio Inventory with Budget $200K

Portfolio	Budget: $200,000		
	Total	Primary	Secondary
1	$31.3M	$6.4M	$24.9M
2	$127.0M	$99.4M	$27.6M
3	$375.7M	$134.6M	$241.1M
4	$518.5M	$230.7M	$287.8M
5	$86.3M	$86.3M	$0.0M
6	$76.4M	$69.6M	$6.9M
7	$121.3M	$35.4M	$85.8M
8	$398.8M	$240.0M	$158.8M
9	$336.5M	$212.5M	$124.0M
10	$191.8M	$121.4M	$70.3M
11	$375.1M	$83.8M	$291.3M
12	$167.9M	$44.5M	$123.4M
13	$58.2M	$40.1M	$18.1M
14	$8.0M	$3.4M	$4.6M
15	$4.2M	$3.7M	$0.6M
16	$15.3M	$3.3M	$12.2M
17	$3.9M	$3.2M	$0.7M
18	$21.4M	$21.4M	$0.0M
19	$7.0M	$7.0M	$0.1M
20	$6.3M	$6.2M	$0.1M
All	$366.3M	$168.4M	$197.9M

Note: Results are based on inventory level 2. Results for other inventory levels are available from the authors.

strategic portfolios when we run all scenarios using the heuristic solution.

Table A.4: Reduction in DMF's *DE* Based on Heuristic Modeling of the Adoption of Strategic Portfolios

Portfolio	Budget: $25,000	Budget: $50,000	Budget: $100,000	Budget: $200,000
	Total	Total	Total	Total
1	$14.6M	$23.7M	$29.6M	$31.6M
2	$34.4M	$55.6M	$83.4M	$126.8M
3	$117.0M	$210.8M	$313.4M	$364.7M
4	$76.9M	$149.7M	$283.2M	$511.8M
5	$15.4M	$28.7M	$51.1M	$86.3M
6	$19.4M	$30.0M	$48.2M	$76.5M
7	$26.2M	$50.5M	$84.2M	$120.2M
8	$54.8M	$107.7M	$207.9M	$387.4M
9	$51.2M	$99.9M	$191.6M	$330.8M
10	$45.5M	$81.2M	$131.0M	$189.2M
11	$58.9M	$115.2M	$221.1M	$375.6M
12	$27.3M	$53.9M	$101.9M	$166.2M
13	$21.6M	$30.1M	$40.6M	$58.3M
14	$5.5M	$6.1M	$6.8M	$8.1M
15	$0.8M	$1.6M	$2.8M	$4.3M
16	$11.1M	$13.6M	$14.5M	$15.7M
17	$0.7M	$1.4M	$2.6M	$4.0M
18	$3.1M	$6.1M	$11.6M	$21.4M
19	$1.1M	$2.1M	$3.7M	$6.8M
20	$1.0M	$1.9M	$3.5M	$6.3M
All	$50.2M	$98.9M	$191.7M	$359.9

Note: Results are based on inventory level 2.

APPENDIX B

APPENDIX OF CHAPTER 3

B.1 Symbol Definition

Table B.1: Symbol Definition

Symbol	Explanation
r_C	The opportunity cost of holding cash for the supplier
r_S	The borrowing cost of the supplier
r_B	The borrowing cost of the buyer
r	The borrowing rate set by the bank for reverse factoring
δ	The discount factor, i.e., $\delta = \frac{1}{1+r_C}$
N	Number of payment periods
N_1	Number of payment periods in the fixed payment contract
N_2	Number of payment periods in the reverse factoring contract
W	Total amount of outstanding Account Receivables for the supplier
w_n	The Account Receivables available for the supplier at the beginning of period n
x_n	The cash position of the supplier at the beginning of period n before decisions
y	The cash position of the supplier at the beginning of period n after decisions
ξ_n	The random cash shock during period n with i.i.d. density function $f(\xi_n)$ and cdf $F(\xi_n)$

B.2 Proof of Proposition 2

B.2.1 Two Period Problem

In the two period problem, $N = 2$, $c_1(r) = \frac{2r+r^2}{(1+r)^2}$ and $c_2(r) = \frac{r}{1+r}$. From the optimal cash policy under No RF case, we know that it is never wise for a firm to borrow on its own rate compared with staying at the current cash level. Then the only wise choice possible to increase cash level is by using RF options. The myopic policy says we should use RF up to a certain amount, we then want to to prove it is optimal.

We can write the two period model as

$$V_2(x_2, w_2) = \min_{y_2 \le x_2 + w_2} \begin{Bmatrix} -r_C \cdot (x_2 - y_2)^+ + c_2(r) \cdot (y_2 - x_2)^+ + L(y_2, \xi_2) \\ +\delta \cdot E[V_3(x_1 + W - \xi_1 - \xi_2, 0)] \end{Bmatrix}$$

$$V_1(x_1, w_1) = \min_{y_1 \le x_1 + w_1} \begin{Bmatrix} -r_C \cdot (x_1 - y_1)^+ + c_1(r) \cdot (1+\delta)(y_1 - x_1)^+ \\ +L(y_1, \xi_1) + \delta \cdot E[V_2(y_1 \vee x_1 - \xi_1, w_1 - (y_1 - x_1)^+)] \end{Bmatrix}$$

$$x_2 = y_1 \vee x_1 - \xi_1$$

$$w_2 = w_1 - (y_1 - x_1)^+, \quad w_1 = W$$

The optimal policy for V_2 is the myopic policy. Once the cash decision y_1 is made as well as the random shock ξ_1 is realized in period 1, we can rewrite the expression for $V_2(x_2, w_2)$ by substituting the expression for x_2, w_2. The value function V_2 in period 2 is a function of (y_1, ξ_1), written as $V_2(y_1, \xi_1)$. Hence EV_2 is a function of y_1, written as $EV_2(y_1)$.

We further plug in the choice of y_2 according to the optimal policy (myopic policy in this case) in period 2 and we can have the expression for $V_2(y_1, \xi_1)$ as the following. (We intentionally leave out EV_3 for simplicity since it does not impact the decisions for period 1 and 2.) For $x_2 \le \underline{y}_2 - w_2$, i.e., $\xi_1 \ge x_1 + w_1 - \underline{y}_2$, the optimal $y_2 = x_1 + w_1 - \xi_1$ and we have $V_2 = c_2(r) \cdot (x_1 + w_1 - y_1 \vee x_1) + L(x_1 + w_1 - \xi_1, \xi_2)$; for $\underline{y}_2 - w_2 < x_2 \le \underline{y}_2$, i.e., $y_1 \vee x_1 - \underline{y}_2 \le \xi_1 < x_1 + w_1 - \underline{y}_2$, the optimal $y_2 = \underline{y}_2$ and we have $V_2 = c_2(r) \cdot (\underline{y}_2 - y_1 \vee x_1 + \xi_1) + L(\underline{y}_2, \xi_2)$; for $\underline{y}_2 < x_2 \le \overline{y}$, i.e., $y_1 \vee x_1 - y_1^* \le \xi_1 < y_1 \vee x_1 - \underline{y}_2$, the optimal $y_2 = x_2 = y_1 \vee x_1 - \xi_1$ and we have $V_2 = L(y_1 \vee x_1 - \xi_1, \xi_2)$; for $x_2 > \overline{y}$, i.e. $\xi < y_1 \vee x_1 - \overline{y}$, the optimal $y_2 = \overline{y}$ and we have $V_2 = -r_C \cdot (y_1 \vee x_1 - \overline{y} - \xi_1) + L(\overline{y}, \xi_2)$.

So the expression for $EV_2(y_1)$ is

$$
\begin{aligned}
EV_2(y_1) = & \int_{x_1+w_1-\underline{y}_2}^{\infty} \left[c_2(r)\cdot(x_1+w_1-y_1\vee x_1)+L(x_1+w_1-\xi_1,\xi_2)\right]\cdot f(\xi_1)d\xi_1 \\
& + \int_{y_1\vee x_1-\underline{y}_2}^{x_1+w_1-\underline{y}_2} \left[c_2(r)\cdot(\underline{y}_2-y_1\vee x_1+\xi_1)+L(\underline{y}_2,\xi_2)\right]\cdot f(\xi_1)d\xi_1 \\
& + \int_{y_1\vee x_1-\overline{y}}^{y_1\vee x_1-\underline{y}_2} L(y_1\vee x_1-\xi_1,\xi_2)\cdot f(\xi_1)d\xi_1 \\
& + \int_{-\infty}^{y_1\vee x_1-\overline{y}} \left[-r_C\cdot(y_1\vee x_1-\overline{y}-\xi_1)+L(\overline{y},\xi_2)\right]\cdot f(\xi_1)d\xi_1
\end{aligned}
$$

Thus, Problem for V_1 incorporating the optimal decision in period 2 becomes

$$
V_1(x_1,w_1) = \min_{y_1\le x_1+w_1} \left\{
\begin{array}{c}
-r_C\cdot(x_1-y_1)^+ + r\cdot(1+\delta)(y_1-x_1)^+ + L(y_1,\xi_1) \\
+\delta\cdot\{\int_{x_1+w_1-\underline{y}_2}^{\infty}\left[c_2(r)\cdot(x_1+w_1-y_1\vee x_1)+L(x_1+w_1-\xi_1,\xi_2)\right]\cdot f(\xi_1)d\xi_1\} \\
+\delta\cdot\{\int_{y_1\vee x_1-\underline{y}_2}^{x_1+w_1-\underline{y}_2}\left[c_2(r)\cdot(\underline{y}_2-y_1\vee x_1+\xi_1)+L(\underline{y}_2,\xi_2)\right]\cdot f(\xi_1)d\xi_1\} \\
+\delta\cdot\{\int_{y_1\vee x_1-\overline{y}}^{y_1\vee x_1-\underline{y}_2} L(y_1\vee x_1-\xi_1,\xi_2)\cdot f(\xi_1)d\xi_1\} \\
+\delta\cdot\{\int_{-\infty}^{y_1\vee x_1-\overline{y}}\left[-r_C\cdot(y_1\vee x_1-\overline{y}-\xi_1)+L(\overline{y},\xi_2)\right]\cdot f(\xi_1)d\xi_1\}
\end{array}
\right\}
$$

Let $G_1(x_1,w_1,y_1)$ be the NPV of cost when choosing cash level y_1 in period 1 and optimizing choice in period 2. Since x_1 and w_1 are constants given. G_1 is a function of y_1. We can write $G_1(y_1)$ (short for $G_1(x_1,w_1,y_1)$) as $G_1(y_1) = -r_C\cdot(x_1-y_1)^+ + c_1(r)\cdot(y_1-x_1)^+ + L(y_1,\xi_1) + \delta\cdot EV_2(y_1)$. We want to investigate when $G_1(y_1)$

is minimized by choosing y_1. For the case $y_1 > x_1$,

$$
\begin{aligned}
\frac{dEV_2(y_1)}{dy_1} =& \int_{x_1+w_1-\underline{y}_2}^{\infty} (-c_2(r)) \cdot f(\xi_1) d\xi_1 \\
&+ \int_{y_1-\underline{y}_2}^{x_1+w_1-\underline{y}_2} (-c_2(r)) \cdot f(\xi_1) d\xi_1 - L(\underline{y}_2, \xi_2) \cdot f(y_1 - \underline{y}_2) \\
&+ \int_{y_1-\overline{y}}^{y_1-\underline{y}_2} [-r_S \cdot Pr(y_1 - \xi_1 < \xi_2)] \cdot f(\xi_1) d\xi_1 \\
&+ L(\underline{y}_2, \xi_2) \cdot f(y_1 - \underline{y}_2) - L(\overline{y}, \xi_2) \cdot f(y_1 - \overline{y}) \\
&+ \int_{-\infty}^{y_1-\overline{y}} (-r_C) \cdot f(\xi_1) d\xi_1 + L(\overline{y}, \xi_2) \cdot f(y_1 - \overline{y}) \\
=& \int_{y_1-\underline{y}_2}^{\infty} (-c_2(r)) \cdot f(\xi_1) d\xi_1 \\
&+ \int_{y_1-\overline{y}}^{y_1-\underline{y}_2} [-r_S \cdot (1 - F(y_1 - \xi_1))] \cdot f(\xi_1) d\xi_1 \\
&+ \int_{-\infty}^{y_1-\overline{y}} (-r_C) \cdot f(\xi_1) d\xi_1 \\
=& \int_{y_1-\underline{y}_2}^{\infty} (-c_2(r)) \cdot f(\xi_1) d\xi_1 + \int_{y_1-\overline{y}}^{y_1-\underline{y}_2} (-r_S) \cdot f(\xi_1) d\xi_1 \\
&+ \int_{y_1-\overline{y}}^{y_1-\underline{y}_2} (r_S) \cdot F(y_1 - \xi_1) \cdot f(\xi_1) d\xi_1 + \int_{-\infty}^{y_1-\overline{y}} (-r_C) \cdot f(\xi_1) d\xi_1 \\
=& (-c_2(r)) \cdot [1 - F(y_1 - \underline{y}_2)] + (-r_S) \cdot [F(y_1 - \underline{y}_2) - F(y_1 - \overline{y})] \\
&+ (-r_C) \cdot F(y_1 - \overline{y}) + r_S \cdot \int_{y_1-\overline{y}}^{y_1-\underline{y}_2} F(y_1 - \xi_1) \cdot f(\xi_1) d\xi_1 \\
=& - c_2(r) + (r - r_S) \cdot F(y_1 - \underline{y}_2) + (r_S - r_C) \cdot F(y_1 - \overline{y}) \\
&+ r_S \cdot \int_{y_1-\overline{y}}^{y_1-\underline{y}_2} F(y_1 - \xi_1) \cdot dF(\xi_1) \\
=& - c_2(r) + (r - r_S) \cdot F(y_1 - \underline{y}_2) + (r_S - r_C) \cdot F(y_1 - \overline{y}) \\
&+ r_S \cdot [F(\underline{y}_2) \cdot F(\overline{y} - \underline{y}_2) - F(\overline{y}) \cdot F(y_1 - \overline{y})] \\
&- r_S \cdot \int_{y_1-\overline{y}}^{y_1-\underline{y}_2} F(\xi_1) dF(y_1 - \xi_1) \\
=& - c_2(r) + r_S \cdot \int_{\underline{y}_2}^{\overline{y}} F(y_1 - t) \cdot f(t) dt
\end{aligned}
$$

The last step holds because we have $F(\underline{y}_2) = \frac{r_S - c_2(r)}{r_S}$, $F(\overline{y}) = \frac{r_S - r_C}{r_S}$. Moreover, we are replacing $y_1 - \xi_1$ with t in the last step. Hence we have for $y_1 \in [x_1, x_1 + w_1]$

$$\frac{dEV_2(y_1 - \xi_1, x_1 + w_1 - y_1)}{dy_1} = -c_2(r) + r_S \cdot \int_{\underline{y}_2}^{\overline{y}} F(y_1 - t) \cdot f(t)dt \tag{B.1}$$

More generally, for any random variable $\tilde{\xi}$ with cdf $\tilde{F}(\cdot)$, we have

$$\frac{dEV_2(y_1 - \tilde{\xi}, x_1 + w_1 - y_1)}{dy_1} = -c_2(r) + r_S \cdot \int_{\underline{y}_2}^{\overline{y}} \tilde{F}(y_1 - t) \cdot f(t)dt \tag{B.2}$$

Thus, take derivative of $G_1(y_1)$ with respect to y_1, we have

$$\begin{aligned}\frac{dG_1(y_1)}{dy_1} =& c_1(r) - r_S \cdot Pr(y_1 < \xi_1) + \delta \cdot \frac{dEV_2(y_1)}{dy_1} \\ =& c_1(r) - r_S \cdot (1 - F(y_1)) + \delta \cdot \frac{dEV_2(y_1)}{dy_1} \\ =& c_1(r) - \delta \cdot c_2(r) - r_S + r_S \cdot F(y_1) + \delta \cdot r_S \cdot \int_{\underline{y}_2}^{\overline{y}} F(y_1 - t) \cdot f(t)dt\end{aligned}$$

To understand the convexity of $G_1(y_1)$ with respect to y_1, we need to know the convexity of $EV_2(y_1)$ because the the rest part is already convex in y_1. Take the second derivative of $EV_2(y_1)$ with respect to y_1, we easily have $\frac{d^2EV_2(y_1)}{dy_1^2} = r_S \cdot \int_{\underline{y}_2}^{\overline{y}} f(y_1 - t) \cdot f(t)dt \geq 0$. Thus $EV_2(y_1)$ is convex in y_1.

For G_1, we then have

$$\frac{d^2G_1}{dy_1^2} = r_S \cdot f(y_1) + \frac{r_S}{1 + r_C} \cdot \int_{\underline{y}_2}^{\overline{y}} f(y_1 - t) \cdot f(t)dt \geq 0$$

and G_1 is proved to be convex in y_1.

Now we turn back to calculate the optimal y_1 (denote as $\underline{y}_1^*$) that sets $\frac{dG_1}{dy_1} = 0$. We have the expression for $\frac{dG_1}{dy_1}$ as the following:

$$\frac{dG_1}{dy_1} = c_1(r) - \delta \cdot c_2(r) - r_S + r_S \cdot F(y_1) + \delta \cdot r_S \cdot \int_{\underline{y}_2}^{\overline{y}} F(y_1 - t) \cdot f(t)dt$$

Since we have $F(\underline{y}_2) = \frac{r_S - c_2(r)}{r_S}$, $F(\overline{y}) = \frac{r_S - r_C}{r_S}$, and $F(y_1 - t) \in [0, 1]$. We have $0 \leq r_S \cdot \int_{\underline{y}_2}^{\overline{y}} F(y_1 - t) \cdot f(t)dt \leq c_2(r) - r_C$. We have two bounds for $\frac{dG_1}{dy_1}$:

$$c_1(r) - \delta \cdot c_2(r) - r_S + r_S \cdot F(y_1) \leq \frac{dG_1}{dy_1} \leq c_1(r) - \delta \cdot c_2(r) - r_S + r_S \cdot F(y_1) + \delta \cdot (c_2(r) - r_C)$$

Assume $\underline{y}_1^*$ is the value of y_1 that sets the derivative $\frac{dG_1}{dy_1}$ to be zero. By valueing at $y_1 = \underline{y}_1^*$, we have

$$c_1(r) - \delta \cdot c_2(r) - r_S + r_S \cdot F(\underline{y}_1^*) \leq \frac{dG_1}{dy_1} \big|_{y_1 = \underline{y}_1^*} = 0 \leq c_1(r) - \delta \cdot c_2(r) - r_S + r_S \cdot F(\underline{y}_1^*) + \delta \cdot (c_2(r) - r_C)$$

which results in a range for $\underline{y}_1^*$ (also as L_1)

$$L_1^m < F^{-1}(\frac{r_S - c_1(r) + \delta \cdot r_C}{r_S}) \leq L_1 \leq F^{-1}(\frac{r_S - (c_1(r) - \delta \cdot c_2(r))}{r_S}) > L_2 = F^{-1}(\frac{r_S - c_2(r)}{r_S})$$

Thus, the optimal policy for period one is also a modified (L_n, U_n) policy with upper bound $\overline{y}$ (denoted as U), lower bound $\underline{y}_1^*$ (denoted as L_1) and the capacity $w_1 = W$.

B.2.2 Multi Period Problem

For the two period problem, we have proved the optimal policy in the first period to be a modified (L_n, U_n) cash policy. Now we want to generalize it to the multi-period case. For each period, the decision problem can be formulated as a simplified Problem. Let $c_n(r) = r \cdot \frac{1-\delta^{N-n+1}}{1-\delta}$, and we have $c_N(r) = r$. We define

$$\begin{aligned} G_n(y_n, x_n, w_n) = & - r_C \cdot (x_n - y_n)^+ + c_n(r) \cdot (y_n - x_n)^+ + L(y_n, \xi_n) \\ & + \delta \cdot E[V_{n+1}(y_n \vee x_n - \xi_n, w_n - (y_n - x_n)^+)] \end{aligned} \tag{B.3}$$

Thus for $n = 1, \cdots, N$, the simplified Problem becomes (B.4)

$$V_n(x_n, w_n) = \min_{y_n \leq x_n + w_n} \{G_n(y_n, x_n, w_n)\} \tag{B.4}$$

In the n^{th} period problem, we can follow the same logic in the two-period problem to obtain that the "U" bound is still $F^{-1}(\frac{r_S - r_C}{r_S})$. Now we will look at the "L" bound and focus on $y_n \in [x_n, x_n + w_n]$. We want to show that $G_n(y_n)$ (short for

$G_n(y_n, x_n, w_n)$) is convex in $y_n \in [x_n, x_n + w_n]$ given any pair of (x_n, w_n). The first part of G_n $-r_C \cdot (x_n - y_n)^+ + c_n(r) \cdot (y_n - x_n)^+ + L(y_n, \xi_n)$ is convex in y_n, and we only need to know the convexity of $E[V_{n+1}(x_{n+1}, w_{n+1})] = E[V_{n+1}(y_n \vee x_n - \xi_n, w_n - (y_n - x_n)^+)]$ in y_n. In the two-period problem, we have proved that $E[V_2(y_1 \vee x_1 - \xi_1, w_1 - (y_1 - x_1)^+)]$ is convex in $y_1 \in [x_1, x_1 + w_1]$ given any (x_1, w_1).

Now we assume that for period $n+1$, $E[V_{n+1}(y_n \vee x_n - \xi_n, w_n - (y_n - x_n)^+]$ is convex in $y_n \in [x_n, x_n + w_n]$ given any (x_n, w_n). This is true for $n + 1 = N$, i.e., $n = N - 1$. Then the optimal cash policy in period n is the modified (L_n, U_n) policy structure with upper bound $\bar{y}$, lower bound $\underline{y}_n^*$ and the capacity w_n. This is derived from the convexity of G_n. We want to show that this condition holds for period n as well, which will guarantee the modified (L_n, U_n) policy structure in period $n - 1$, so the induction could continue backward. This condition for period n is that $E[V_n(y_{n-1} \vee x_{n-1} - \xi_{n-1}, w_{n-1} - (y_{n-1} - x_{n-1})^+]$ is convex in $y_{n-1} \in [x_{n-1}, x_{n-1} + w_{n-1}]$ given any (x_{n-1}, w_{n-1}).

We can write the expression of $EV_n(y_{n-1} \vee x_{n-1} - \xi_{n-1}, w_{n-1} - (y_{n-1} - x_{n-1})^+)$ for $y_{n-1} \in [x_{n-1}, x_{n-1} + w_{n-1}]$ as the following:

$$
\begin{aligned}
& EV_n(y_{n-1} - \xi_{n-1}, x_{n-1} + w_{n-1} - y_{n-1}) \qquad\qquad \text{(B.5)} \\
= & \int_{x_{n-1}+w_{n-1}-\underline{y}_n^*}^{\infty} [c_n(r) \cdot (x_{n-1} + w_{n-1} - y_{n-1}) + L(x_{n-1} + w_{n-1} - \xi_{n-1}, \xi_n) \\
& \qquad + \delta \cdot EV_{n+1}(x_{n-1} + w_{n-1} - \xi_{n-1} - \xi_n, 0)] \cdot f(\xi_{n-1}) d\xi_{n-1} \\
& + \int_{y_{n-1}-\underline{y}_n^*}^{x_{n-1}+w_{n-1}-\underline{y}_n^*} [c_n(r) \cdot (\underline{y}_n^* - y_{n-1} + \xi_{n-1}) + L(\underline{y}_n^*, \xi_n) \\
& \qquad + \delta \cdot EV_{n+1}(\underline{y}_n^* - \xi_n, w_{n-1} - \underline{y}_n^* + x_{n-1} - \xi_{n-1})] \cdot f(\xi_{n-1}) d\xi_{n-1} \\
& + \int_{y_{n-1}-\bar{y}}^{y_{n-1}-\underline{y}_n^*} [L(y_{n-1} - \xi_{n-1}, \xi_n) + \delta \cdot EV_{n+1}(y_{n-1} - \xi_{n-1} - \xi_n, w_{n-1} + x_{n-1} - y_{n-1})] \cdot f(\xi_{n-1}) d\xi_{n-1} \\
& + \int_{-\infty}^{y_{n-1}-\bar{y}} [-r_C \cdot (y_{n-1} - \bar{y} - \xi_{n-1}) + L(\bar{y}, \xi_n) + \delta \cdot EV_{n+1}(y_{n-1} - \xi_{n-1} - \xi_n, w_{n-1} + x_{n-1} - y_{n-1})] \cdot f(\xi_{n-1}) d\xi_{n-1}
\end{aligned}
$$

For the simplicity in notation, we use $V_n(y_{n-1})$ as the short form for $V_n(y_{n-1} - \xi_{n-1}, x_{n-1} + w_{n-1} - y_{n-1})$ in the following. Given the different initial cash level $x_n = y_{n-1} - \xi_{n-1}$ (equivalently as different random shock ξ_{n-1} since x_{n-1} is given) at the beginning of period n, we can employ the optimal cash policy in period n to value $V_n(y_{n-1} - \xi_{n-1}, x_{n-1} + w_{n-1} - y_{n-1})$. For $x_n \leq \underline{y}_n^* - w_n$, i.e., $y_{n-1} - \xi_{n-1} \leq \underline{y}_n^* - (x_{n-1} + w_{n-1} - y_{n-1}) \Leftrightarrow \xi_{n-1} \geq x_{n-1} + w_{n-1} - \underline{y}_n^*$, we have $V_n(y_{n-1} - \xi_{n-1}, x_{n-1} + w_{n-1} - y_{n-1}) = G_n(x_n + w_n)$; for $\underline{y}_n^* - w_n < x_n \leq \underline{y}_n^*$, i.e., $\underline{y}_n^* - (x_{n-1} + w_{n-1} - y_{n-1}) < y_{n-1} - \xi_{n-1} \leq \underline{y}_n^* \Leftrightarrow y_{n-1} - \underline{y}_n^* \leq \xi_{n-1} < x_{n-1} + w_{n-1} - \underline{y}_n^*$, we have $V_n(y_{n-1} - \xi_{n-1}, x_{n-1} + w_{n-1} - y_{n-1}) = G_n(\underline{y}_n^*)$; for $\underline{y}_n^* < x_n \leq \overline{y}$, i.e., $\underline{y}_n^* < y_{n-1} - \xi_{n-1} \leq \overline{y} \Leftrightarrow y_{n-1} - \overline{y} \leq \xi_{n-1} < y_{n-1} - \underline{y}_n^*$, we have $V_n(y_{n-1} - \xi_{n-1}, x_{n-1} + w_{n-1} - y_{n-1}) = G_n(x_n)$; for $x_n > \overline{y}$, i.e., $y_{n-1} - \xi_{n-1} > \overline{y} \Leftrightarrow \xi_{n-1} < y_{n-1} - \overline{y}$, we have $V_n(y_{n-1} - \xi_{n-1}, x_{n-1} + w_{n-1} - y_{n-1}) = G_n(\overline{y})$. The expression for $V_n(y_n)$ depending on ξ_{n-1} can be summarized as the following:

$$V_n(y_{n-1}) = \begin{cases} G_n(x_n + w_n) & \text{if } \xi_{n-1} > x_{n-1} + w_{n-1} - \underline{y}_n^* \\ G_n(\underline{y}_n^*) & \text{if } y_{n-1} - \underline{y}_n^* < \xi_{n-1} \leq x_{n-1} + w_{n-1} - \underline{y}_n^* \\ G_n(x) & \text{if } y_{n-1} - \overline{y} < \xi_{n-1} \leq y_{n-1} - \underline{y}_n^* \\ G_n(\overline{y}) & \text{if } \xi_{n-1} \leq y_{n-1} - \overline{y} \end{cases}$$

For future convenience, we denote the four pieces of $V_n(y_{n-1})$ as V_n^1, V_n^2, V_n^3, V_n^4 (for given $x_{n-1}, w_{n-1}, \xi_{n-1}$), corresponding to $G_n(x_n + w_n)$, $G_n(\underline{y}_n^*)$, $G_n(x)$, $G_n(\overline{y})$ respectively. The detailed expressions are:

$$V_n^1(y_{n-1}) = c_n(r) \cdot (x_{n-1} + w_{n-1} - y_{n-1}) + L(x_{n-1} + w_{n-1} - \xi_{n-1}, \xi_n) + \delta \cdot EV_{n+1}(x_{n-1} + w_{n-1} - \xi_{n-1} - \xi_n, 0)$$

$$V_n^2(y_{n-1}) = c_n(r) \cdot (\underline{y}_n^* - y_{n-1} + \xi_{n-1}) + L(\underline{y}_n^*, \xi_n) + \delta \cdot EV_{n+1}(\underline{y}_n^* - \xi_n, w_{n-1} - \underline{y}_n^* + x_{n-1} - \xi_{n-1})$$

$$V_n^3(y_{n-1}) = L(y_{n-1} - \xi_{n-1}, \xi_n) + \delta \cdot EV_{n+1}(y_{n-1} - \xi_{n-1} - \xi_n, w_{n-1} + x_{n-1} - y_{n-1})$$

$$V_n^4(y_{n-1}) = -r_C \cdot (y_{n-1} - \overline{y} - \xi_{n-1}) + L(\overline{y}, \xi_n) + \delta \cdot EV_{n+1}(y_{n-1} - \xi_{n-1} - \xi_n, w_{n-1} + x_{n-1} - y_{n-1})$$

We will show that for each specific ξ_{n-1}, $V_n(y_{n-1} - \xi_{n-1}, x_{n-1} + w_{n-1} - y_{n-1})$ is convex in $y_{n-1} \in [x_{n-1}, x_{n-1} + w_{n-1}]$. For a given ξ_{n-1}, $V_n(y_{n-1} - \xi_{n-1}, x_{n-1} + w_{n-1} - y_{n-1})$ is a function

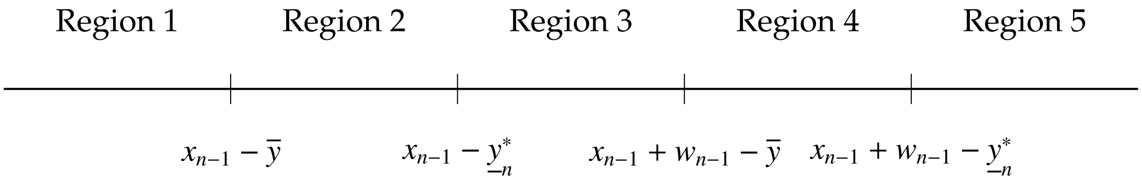


Figure B.1: Critical Points of ξ_{n-1} in Scenario (A)

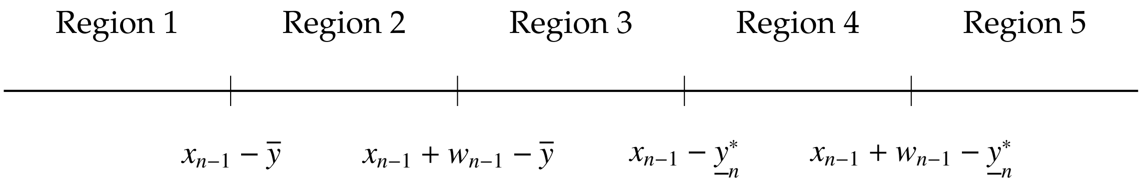


Figure B.2: Critical Points of ξ_{n-1} in Scenario (B)

in y_{n-1} and it may be piecewise, i.e., for different range of ξ_{n-1}, $V_n(y_{n-1})$ can be in different forms. Depending on the order of $x_{n-1} - \underline{y}_n^*$ and $x_{n-1} + w_{n-1} - \bar{y}$, we have two scenarios: (A) $x_{n-1} - \underline{y}_n^* < x_{n-1} + w_{n-1} - \bar{y}$, i.e., $w_{n-1} > \bar{y} - \underline{y}_n^*$; (B)$x_{n-1} + w_{n-1} - \bar{y} < x_{n-1} - \underline{y}_n^*$, i.e., $w_{n-1} < \bar{y} - \underline{y}_n^*$.

In scenario (A) and (B), the form of $V_n(y_{n-1})$ changes in the value of ξ_{n-1} with the following critical points:

In Scenario (A), the form of $V_n(y_{n-1})$ is then

$$V_n(y_{n-1}) = \begin{cases} V_n^4(y_{n-1}) & \text{if } \xi_{n-1} \le x_{n-1} - \bar{y} \\ \begin{cases} V_n^3(y_{n-1}) & \text{for } y_{n-1} \in [x_{n-1}, \bar{y} + \xi_{n-1}] \\ V_n^4(y_{n-1}) & \text{for } y_{n-1} \in (\bar{y} + \xi_{n-1}, x_{n-1} + w_{n-1}] \end{cases} & \text{if } x_{n-1} - \bar{y} < \xi_{n-1} \le x_{n-1} - \underline{y}_n^* \\ \begin{cases} V_n^2(y_{n-1}) & \text{for } y_{n-1} \in [x_{n-1}, \underline{y}_n^* + \xi_{n-1}] \\ V_n^3(y_{n-1}) & \text{for } y_{n-1} \in (\underline{y}_n^* + \xi_{n-1}, \bar{y} + \xi_{n-1}] \\ V_n^4(y_{n-1}) & \text{for } y_{n-1} \in (\bar{y} + \xi_{n-1}, x_{n-1} + w_{n-1}] \end{cases} & \text{if } x_{n-1} - \underline{y}_n^* < \xi_{n-1} \le x_{n-1} + w_{n-1} - \bar{y} \\ \begin{cases} V_n^2(y_{n-1}) & \text{for } y_{n-1} \in [x_{n-1}, \underline{y}_n^* + \xi_{n-1}] \\ V_n^3(y_{n-1}) & \text{for } y_{n-1} \in (\underline{y}_n^* + \xi_{n-1}, x_{n-1} + w_{n-1}] \end{cases} & \text{if } x_{n-1} + w_{n-1} - \bar{y} < \xi_{n-1} \le x_{n-1} + w_{n-1} - \underline{y}_n^* \\ V_n^1(y_{n-1}) & \text{if } \xi_{n-1} > x_{n-1} + w_{n-1} - \underline{y}_n^* \end{cases}$$

In Scenario (B), the form of $V_n(y_{n-1})$ is then

$$V_n(y_{n-1}) = \begin{cases} V_n^4(y_{n-1}) & \text{if } \xi_{n-1} \le x_{n-1} - \overline{y} \\ \begin{cases} V_n^3(y_{n-1}) & \text{for } y_{n-1} \in [x_{n-1}, \overline{y} + \xi_{n-1}] \\ V_n^4(y_{n-1}) & \text{for } y_{n-1} \in (\overline{y} + \xi_{n-1}, x_{n-1} + w_{n-1}] \end{cases} & \text{if } x_{n-1} - \overline{y} < \xi_{n-1} \le x_{n-1} + w_{n-1} - \overline{y} \\ V_n^3(y_{n-1}) & \text{if } x_{n-1} + w_{n-1} - \overline{y} < \xi_{n-1} \le x_{n-1} - \underline{y}_n^* \\ \begin{cases} V_n^2(y_{n-1}) & \text{for } y_{n-1} \in [x_{n-1}, \underline{y}_n^* + \xi_{n-1}] \\ V_n^3(y_{n-1}) & \text{for } y_{n-1} \in (\underline{y}_n^* + \xi_{n-1}, x_{n-1} + w_{n-1}] \end{cases} & \text{if } x_{n-1} - \underline{y}_n^* < \xi_{n-1} \le x_{n-1} + w_{n-1} - \underline{y}_n^* \\ V_n^1(y_{n-1}) & \text{if } \xi_{n-1} > x_{n-1} + w_{n-1} - \underline{y}_n^* \end{cases}$$

To prove that for each ξ_{n-1} in each scenario, $V_n(y_{n-1})$ is convex in $y_{n-1} \in [x_{n-1}, x_{n-1} + w_{n-1}]$, we need to show that V_n^1, V_n^2, V_n^3, V_n^4 are convex; we also need to show that in the kinky point of $V_n(y_{n-1})$, $\frac{dV_n^1(y_{n-1})}{dy_{n-1}} \le \frac{dV_n^2(y_{n-1})}{dy_{n-1}} \le \frac{dV_n^3(y_{n-1})}{dy_{n-1}} \le \frac{dV_n^4(y_{n-1})}{dy_{n-1}}$. Obviously, $\frac{dV_n^1(y_{n-1})}{dy_{n-1}} = \frac{dV_n^2(y_{n-1})}{dy_{n-1}} = -c_n(r)$, and V_n^1, V_n^2 are both convex. For V_n^3, V_n^4, we have $EV_{n+1}(y_{n-1} - \xi_{n-1} - \xi_n, w_{n-1} + x_{n-1} - y_{n-1})$ to be convex in $y_{n-1} \in [x_{n-1}, x_{n-1} + w_{n-1}]$. This is because when redefining new random shock as $(\xi_{n-1} + \xi_n)$, we can use the assumption that $E[V_{n+1}(y_n \vee x_n - \xi_n, w_n - (y_n - x_n)^+]$ is convex in $y_n \in [x_n, x_n + w_n]$ given any (x_n, w_n). and this assumption holds in the last period regardless of the distribution of random shock ξ_n (and the induction backward will also not depend on the distribution of ξ_{n-1}). Hence, V_n^3 and V_n^4 are convex in y_{n-1}. For V_n^3, since $V_n^3(y_{n-1}) = G_n(x_n)$, i.e., the optimal cash decision is to keep the original case, we must have $\frac{G_n(y_n)}{dy_n} \ge 0$ for $y_n \in [x_n, x_n + w_n]$. To compare the difference in $\frac{dV_n^3(y_{n-1})}{dy_{n-1}}$ and $\frac{G_n(y_n)}{dy_n}$, we have $\frac{dV_n^3(y_{n-1})}{dy_{n-1}} = -c_n(r) + \frac{G_n(y_n)}{dy_n}$, so $\frac{dV_n^3(y_{n-1})}{dy_{n-1}} \ge -c_n(r) = \frac{dV_n^2(y_{n-1})}{dy_{n-1}}$. Finally, we need to compare $\frac{dV_n^3(y_{n-1})}{dy_{n-1}}$ and $\frac{dV_n^4(y_{n-1})}{dy_{n-1}}$, which is equivalent as comparing $\frac{dL(y_{n-1}-\xi_{n-1},\xi_n)}{dy_{n-1}}$ and $-r_C$. Since $\frac{dL(y_{n-1}-\xi_{n-1},\xi_n)}{dy_{n-1}} = -r_S + r_S \cdot F(y_{n-1} - \xi_{n-1})$ and $\xi_{n-1} > y_{n-1} - \overline{y}$ for V_n^3, we have $\frac{dL(y_{n-1}-\xi_{n-1},\xi_n)}{dy_{n-1}} < -r_S + r_S \cdot F(\overline{y}) = -r_C$. Thus, $\frac{dV_n^3(y_{n-1})}{dy_{n-1}} \le \frac{dV_n^4(y_{n-1})}{dy_{n-1}}$ is also proved.

Now we have for each ξ_{n-1} in each scenario, $V_n(y_{n-1} - \xi_{n-1}, x_{n-1} + w_{n-1} - y_{n-1})$ is convex in $y_{n-1} \in [x_{n-1}, x_{n-1} + w_{n-1}]$ given any pair of (x_{n-1}, w_{n-1}). Thus $EV_n(y_{n-1} - \xi_{n-1}, x_{n-1} + w_{n-1} - y_{n-1})$ is convex in $y_{n-1} \in [x_{n-1}, x_{n-1} + w_{n-1}]$ given any pair of (x_{n-1}, w_{n-1}), induction is completed. So we have $G_n(y_n)$ to be convex in y_n for any general n, and the optimal cash policy in each period is a modified (L_n, U_n) policy.

To further prove that (L_n, U_n) **does not depend on** w_n**.** It is obvious that, $U_n = U$ does not depend on w_n. Now we need to show that U_n does not depend on w_n as well. For the "L" bound, it is determined by the FOC of the following (for $y_n > x_n$):

$$\begin{aligned}
\frac{dG_n(y_n)}{dy_n} =& c_n(r) - r_S \cdot Pr(y_n < \xi_n) + \delta \cdot \frac{dEV_{n+1}(y_n - \xi_n, x_n + w_n - y_n)}{dy_n} \\
=& c_n(r) - r_S + r_S \cdot F(y_n) + \delta \cdot \frac{dEV_{n+1}(y_n - \xi_n, x_n + w_n - y_n)}{dy_n}
\end{aligned} \tag{B.6}$$

Starting from (B.5), we can look at the the derivative of $EV_{n+1}(y_n - \xi_n, x_n + w_n - y_n)$ with respect to y_n for $y_n > x_n$. We have

$$\begin{aligned}
&\frac{dEV_{n+1}(y_n - \xi_n, x_n + w_n - y_n)}{dy_n} \qquad \text{(B.7)} \\
=& \int_{x_n + w_n - \underline{y}^*_{n+1}}^{\infty} [-c_{n+1}(r)] \cdot f(\xi_n) d\xi_n + \int_{y_n - \underline{y}^*_{n+1}}^{x_n + w_n - \underline{y}^*_{n+1}} [-c_{n+1}(r)] \cdot f(\xi_n) d\xi_n \\
&+ \int_{y_n - \bar{y}}^{y_n - \underline{y}^*_{n+1}} \left[\frac{dL(y_n - \xi_n, \xi_{n+1})}{dy_n} + \delta \cdot \frac{dEV_{n+2}(y_n - \xi_n - \xi_{n+1}, w_n + x_n - y_n)}{dy_n} \right] \cdot f(\xi_n) d\xi_n \\
&+ \int_{-\infty}^{y_n - \bar{y}} \left[-r_C + \delta \cdot \frac{dEV_{n+2}(y_n - \xi_n - \xi_{n+1}, w_n + x_n - y_n)}{dy_n} \right] \cdot f(\xi_n) d\xi_n
\end{aligned}$$

Similarly, for any random variable $\tilde{\xi}$, we have

$$\begin{aligned}
&\frac{dEV_{n+1}(y_n-\tilde{\xi}, x_n+w_n-y_n)}{dy_n} \qquad \text{(B.8)}\\
&= \int_{x_n+w_n-\underline{y}^*_{n+1}}^{\infty} [-c_{n+1}(r)]\cdot f(\tilde{\xi})d\tilde{\xi} + \int_{y_n-\underline{y}^*_{n+1}}^{x_n+w_n-\underline{y}^*_{n+1}} [-c_{n+1}(r)]\cdot f(\tilde{\xi})d\tilde{\xi}\\
&\quad + \int_{y_n-\overline{y}}^{y_n-\underline{y}^*_{n+1}} \left[\frac{dL(y_n-\tilde{\xi},\xi_{n+1})}{dy_n} + \delta\cdot\frac{dEV_{n+2}(y_n-\tilde{\xi}-\xi_{n+1}, w_n+x_n-y_n)}{dy_n}\right]\cdot f(\tilde{\xi})d\tilde{\xi}\\
&\quad + \int_{-\infty}^{y_n-\overline{y}} \left[-r_C + \delta\cdot\frac{dEV_{n+2}(y_n-\tilde{\xi}-\xi_{n+1}, w_n+x_n-y_n)}{dy_n}\right]\cdot f(\tilde{\xi})d\tilde{\xi}
\end{aligned}$$

We know from (B.2) that $\frac{dEV_2(y_1-\tilde{\xi},x_1+w_1-y_1)}{dy_1} = -r + r_S\cdot\int_{\underline{y}_2}^{\overline{y}}\tilde{F}(y_1-t)\cdot f(t)dt$ ($y_1 > x_1$) does not depend on w_1 for any random variable $\tilde{\xi}$. Now we do backward induction in (B.8). Assume $\frac{dEV_{n+2}(y_n-\tilde{\xi},w_n+x_n-y_n)}{dy_n}$ ($y_n > x_n$) does not depend on w_n for any random variable $\tilde{\xi}$, then $\frac{dEV_{n+2}(y_n-\tilde{\xi}-\xi_{n+1},w_n+x_n-y_n)}{dy_n}$ does not depend on w_n for each value of $\tilde{\xi}\in(-\infty, y_n-\overline{y}]$, where $\xi_{n+1}\sim F(\cdot)$. Hence, $\frac{dEV_{n+1}(y_n-\tilde{\xi},x_n+w_n-y_n)}{dy_n}$ ($y_n > x_n$) does not depend on w_n for any random variable $\tilde{\xi}$. Specifically, $\frac{dEV_{n+1}(y_n-\xi_n,x_n+w_n-y_n)}{dy_n}$ ($y_n > x_n$) does not depend on w_n. Because L_n is determined by the FOC of (B.6) ($y_n > x_n$), we then conclude that L_n does not depend on w_n. Thus, the optimal cash policy is a modified (L_n, U_n) policy. Proof is done.

B.3 Proof of Proposition 3

We want to show how the modified (L_n, U_n) policy changes over time, i.e., how the "L" (borrow-up-to) bound and the "U" (invest-down-to) bound changes with n. In the proof of two-period problem, we have shown that the "U" bound is $\overline{y} = F^{-1}(\frac{r_S-r_C}{r_S})$ and it does not change with n. For the "L" bound, it is determined by (B.6). In the two period problem, we have shown $L_1 \geq F^{-1}(\frac{r_S-c_1(r)+\delta\cdot r_C}{r_S})$ and $L_2 = F^{-1}(\frac{r_S-c_2(r)}{r_S})$. One sufficient condition for $L_1 \geq L_2$ is $-c_1(r)+\delta\cdot r_C \geq c_2(r)$, which translates into $r_C \geq \frac{1}{1+(r+1/r)}$ or $r \leq \frac{(1-r_C)-\sqrt{1-2r_C-3r_C^2}}{2r_C}$. For the existence of $\frac{(1-r_C)-\sqrt{1-2r_C-3r_C^2}}{2r_C}$, we need $r_C < \frac{1}{3}$ and it is generally true since r_C is the market risk free rate.

We want to show that $\frac{dG_{n-1}(y)}{dy} \leq \frac{dG_n(y)}{dy}$ holds for all n while holding the initial state (x,w) to be same for period $n-1$ and n under the condition of $r \leq \frac{(1-r_C)-\sqrt{1-2r_C-3r_C^2}}{2r_C}$. One sufficient condition for $\frac{dG_{n-1}(y)}{dy} \leq \frac{dG_n(y)}{dy}$ to be true is $\frac{dEV_n(y)}{dy} \leq \frac{dEV_{n+1}(y)}{dy} - \delta^{-1} \cdot (c_{n-1}(r) - c_n(r))$. We have this to be true in the two-period problem as specified by the condition. The resulting "L" bound in the two-period problem is $L_1 \geq L_2$ ($\underline{y}_1^* \leq \underline{y}_2$ in the two-period problem proof). So we assume that $\frac{dEV_{n+1}(y)}{dy} \leq \frac{dEV_{n+2}(y)}{dy} - \delta^{-1} \cdot (c_n(r) - c_{n+1}(r))$ is true and thus $L_n \geq L_{n+1}$ (i.e., $\underline{y}_n^* \leq \underline{y}_{n+1}^*$), we want to induce backward to see if $\frac{dEV_n(y)}{dy} \leq \frac{dEV_{n+1}(y)}{dy} - \delta^{-1} \cdot (c_{n-1}(r) - c_n(r))$ holds. Starting from (B.7), we look at the the derivative of $EV_n(y_{n-1}-\xi_{n-1}, x_{n-1}+w_{n-1}-y_{n-1})$ with respect to y_{n-1}. We have

$$
\begin{aligned}
&\frac{dEV_n(y_{n-1}-\xi_{n-1}, x_{n-1}+w_{n-1}-y_{n-1})}{dy_{n-1}} \\
=& \int_{x_{n-1}+w_{n-1}-L_n}^{\infty} [-c_n(r)] \cdot f(\xi_{n-1}) d\xi_{n-1} + \int_{y_{n-1}-L_n}^{x_{n-1}+w_{n-1}-L_n} [-c_n(r)] \cdot f(\xi_{n-1}) d\xi_{n-1} \\
&+ \int_{y_{n-1}-U}^{y_{n-1}-L_n} \left[\frac{dL(y_{n-1}-\xi_{n-1}, \xi_n)}{dy_{n-1}} + \delta \cdot \frac{dEV_{n+1}(y_{n-1}-\xi_{n-1}-\xi_n, w_{n-1}+x_{n-1}-y_{n-1})}{dy_{n-1}} \right] \cdot f(\xi_{n-1}) d\xi_{n-1} \\
&+ \int_{-\infty}^{y_{n-1}-U} \left[-r_C + \delta \cdot \frac{dEV_{n+1}(y_{n-1}-\xi_{n-1}-\xi_n, w_{n-1}+x_{n-1}-y_{n-1})}{dy_{n-1}} \right] \cdot f(\xi_{n-1}) d\xi_{n-1} \\
\leq& \int_{x_{n-1}+w_{n-1}-L_{n+1}}^{\infty} [-c_n(r)] \cdot f(\xi_{n-1}) d\xi_{n-1} + \int_{y_{n-1}-L_{n+1}}^{x_{n-1}+w_{n-1}-L_{n+1}} [-c_n(r)] \cdot f(\xi_{n-1}) d\xi_{n-1} \\
&+ \int_{y_{n-1}-U}^{y_{n-1}-L_{n+1}} \left[\frac{dL(y_{n-1}-\xi_{n-1}, \xi_n)}{dy_{n-1}} + \delta \cdot \frac{dEV_{n+1}(y_{n-1}-\xi_{n-1}-\xi_n, w_{n-1}+x_{n-1}-y_{n-1})}{dy_{n-1}} \right] \cdot f(\xi_{n-1}) d\xi_{n-1} \\
&+ \int_{-\infty}^{y_{n-1}-U} \left[-r_C + \delta \cdot \frac{dEV_{n+1}(y_{n-1}-\xi_{n-1}-\xi_n, w_{n-1}+x_{n-1}-y_{n-1})}{dy_{n-1}} \right] \cdot f(\xi_{n-1}) d\xi_{n-1} \\
\leq& \int_{x_{n-1}+w_{n-1}-L_{n+1}}^{\infty} [-c_{n+1}(r) - (c_n(r) - c_{n+1}(r))] \cdot f(\xi_{n-1}) d\xi_{n-1} + \int_{y_{n-1}-L_{n+1}}^{x_{n-1}+w_{n-1}-L_{n+1}} [-c_{n+1}(r) - (c_n(r) - c_{n+1}(r))] \cdot f(\xi_{n-1}) d\xi_{n-1} \\
&+ \int_{y_{n-1}-U}^{y_{n-1}-L_{n+1}} \left[\frac{dL(y_{n-1}-\xi_{n-1}, \xi_n)}{dy_{n-1}} + \delta \cdot \left(\frac{dEV_{n+2}(y_{n-1}-\xi_{n-1}-\xi_n, w_{n-1}+x_{n-1}-y_{n-1})}{dy_{n-1}} - \delta^{-1} \cdot (c_n(r) - c_{n+1}(r)) \right) \right] \cdot f(\xi_{n-1}) d\xi_{n-1} \\
&+ \int_{-\infty}^{y_{n-1}-U} \left[-r_C + \delta \cdot \left(\frac{dEV_{n+2}(y_{n-1}-\xi_{n-1}-\xi_n, w_{n-1}+x_{n-1}-y_{n-1})}{dy_{n-1}} - \delta^{-1} \cdot (c_n(r) - c_{n+1}(r)) \right) \right] \cdot f(\xi_{n-1}) d\xi_{n-1} \\
=& \frac{dEV_{n+1}(y_{n-1}-\xi_{n-1}, x_{n-1}+w_{n-1}-y_{n-1})}{dy_{n-1}} - (c_n(r) - c_{n+1}(r)) \\
\leq& \frac{dEV_{n+1}(y_{n-1}-\xi_{n-1}, x_{n-1}+w_{n-1}-y_{n-1})}{dy_{n-1}} - \delta^{-1} \cdot (c_{n-1}(r) - c_n(r)).
\end{aligned}
$$

The first inequality follows the fact that $\frac{dL(y_{n-1}-\xi_{n-1},\xi_n)}{dy_{n-1}} + \delta \cdot \frac{dEV_{n+1}(y_{n-1}-\xi_{n-1}-\xi_n, w_{n-1}+x_{n-1}-y_{n-1})}{dy_{n-1}} \geq -c_n(r)$ and $L_n \geq L_{n+1}$. The second inequality follows the inductiona assumption.

The last inequality holds because $c_n(r) - c_{n+1}(r) \geq \delta^{-1} \cdot (c_{n-1}(r) - c_n(r))$. So we have proved that $L_n \geq L_{n+1}$ ($\underline{y}_n^* \leq \underline{y}_{n+1}^*$) for all n, meaning that the borrow-up-to bound decreases with time given $r \leq \frac{(1-r_C)-\sqrt{1-2r_C-3r_C^2}}{2r_C}$.

Further, $EV_n(y_{n-1} - \xi_{n-1}, x_{n-1} + w_{n-1} - y_{n-1})$ can be decomposed into two parts:

$$
\begin{aligned}
&EV_n(y_{n-1} - \xi_{n-1}, x_{n-1} + w_{n-1} - y_{n-1}) \\
=&\left\{ \int_{x_{n-1}+w_{n-1}-\underline{y}_n^*}^{\infty} [c_n(r) \cdot (x_{n-1} + w_{n-1} - y_{n-1}) + L(x_{n-1} + w_{n-1} - \xi_{n-1}, \xi_n)] \cdot f(\xi_{n-1}) d\xi_{n-1} \right. \\
&+ \int_{y_{n-1}-\underline{y}_n^*}^{x_{n-1}+w_{n-1}-\underline{y}_n^*} \left[c_n(r) \cdot (\underline{y}_n^* - y_{n-1} + \xi_{n-1}) + L(\underline{y}_n^*, \xi_n)\right] \cdot f(\xi_{n-1}) d\xi_{n-1} \\
&+ \int_{y_{n-1}-\overline{y}}^{y_{n-1}-\underline{y}_n^*} [L(y_{n-1} - \xi_{n-1}, \xi_n)] \cdot f(\xi_{n-1}) d\xi_{n-1} \\
&+ \left. \int_{-\infty}^{y_{n-1}-\overline{y}} [-r_C \cdot (y_{n-1} - \overline{y} - \xi_{n-1}) + L(\overline{y}, \xi_n)] \cdot f(\xi_{n-1}) d\xi_{n-1} \right\} \\
&+ \delta \cdot \left\{ \int_{x_{n-1}+w_{n-1}-\underline{y}_n^*}^{\infty} [EV_{n+1}(x_{n-1} + w_{n-1} - \xi_{n-1} - \xi_n, 0)] \cdot f(\xi_{n-1}) d\xi_{n-1} \right. \\
&+ \int_{y_{n-1}-\underline{y}_n^*}^{x_{n-1}+w_{n-1}-\underline{y}_n^*} \left[EV_{n+1}(\underline{y}_n^* - \xi_n, w_{n-1} - \underline{y}_n^* + x_{n-1} - \xi_{n-1})\right] \cdot f(\xi_{n-1}) d\xi_{n-1} \\
&+ \int_{y_{n-1}-\overline{y}}^{y_{n-1}-\underline{y}_n^*} [EV_{n+1}(y_{n-1} - \xi_{n-1} - \xi_n, w_{n-1} + x_{n-1} - y_{n-1})] \cdot f(\xi_{n-1}) d\xi_{n-1} \\
&+ \left. \int_{-\infty}^{y_{n-1}-\overline{y}} [EV_{n+1}(y_{n-1} - \xi_{n-1} - \xi_n, w_{n-1} + x_{n-1} - y_{n-1})] \cdot f(\xi_{n-1}) d\xi_{n-1} \right\}
\end{aligned}
$$

Let

$$
\begin{aligned}
&E\hat{V}_n(y_{n-1}) \\
=&\int_{x_{n-1}+w_{n-1}-\underline{y}_n^*}^{\infty} [c_n(r) \cdot (x_{n-1} + w_{n-1} - y_{n-1}) + L(x_{n-1} + w_{n-1} - \xi_{n-1}, \xi_n)] \cdot f(\xi_{n-1}) d\xi_{n-1} \\
&+ \int_{y_{n-1}-\underline{y}_n^*}^{x_{n-1}+w_{n-1}-\underline{y}_n^*} \left[c_n(r) \cdot (\underline{y}_n^* - y_{n-1} + \xi_{n-1}) + L(\underline{y}_n^*, \xi_n)\right] \cdot f(\xi_{n-1}) d\xi_{n-1} \\
&+ \int_{y_{n-1}-\overline{y}}^{y_{n-1}-\underline{y}_n^*} [L(y_{n-1} - \xi_{n-1}, \xi_n)] \cdot f(\xi_{n-1}) d\xi_{n-1} \\
&+ \int_{-\infty}^{y_{n-1}-\overline{y}} [-r_C \cdot (y_{n-1} - \overline{y} - \xi_{n-1}) + L(\overline{y}, \xi_n)] \cdot f(\xi_{n-1}) d\xi_{n-1}
\end{aligned}
$$

and

$$
\begin{aligned}
& E\tilde{V}_n(y_{n-1}) \\
= & \int_{x_{n-1}+w_{n-1}-\underline{y}_n^*}^{\infty} [EV_{n+1}(x_{n-1}+w_{n-1}-\xi_{n-1}-\xi_n, 0)] \cdot f(\xi_{n-1}) d\xi_{n-1} \\
& + \int_{y_{n-1}-\underline{y}_n^*}^{x_{n-1}+w_{n-1}-\underline{y}_n^*} \left[EV_{n+1}(\underline{y}_n^* - \xi_n, w_{n-1} - \underline{y}_n^* + x_{n-1} - \xi_{n-1})\right] \cdot f(\xi_{n-1}) d\xi_{n-1} \\
& + \int_{y_{n-1}-\overline{y}}^{y_{n-1}-\underline{y}_n^*} [EV_{n+1}(y_{n-1}-\xi_{n-1}-\xi_n, w_{n-1}+x_{n-1}-y_{n-1})] \cdot f(\xi_{n-1}) d\xi_{n-1} \\
& + \int_{-\infty}^{y_{n-1}-\overline{y}} [EV_{n+1}(y_{n-1}-\xi_{n-1}-\xi_n, w_{n-1}+x_{n-1}-y_{n-1})] \cdot f(\xi_{n-1}) d\xi_{n-1}
\end{aligned}
$$

To prove that $L_n \geq L_n^m$.

We already have $EV_n(y_{n-1}-\xi_{n-1}, x_{n-1}+w_{n-1}-y_{n-1}) = E\hat{V}_n(y_{n-1}) + \delta \cdot E\tilde{V}_n(y_{n-1})$.

We have $\frac{dE\hat{V}_n(y_{n-1})}{dy_{n-1}} = -c_n(r) + r_S \cdot \int_{\underline{y}_n^*}^{\overline{y}} F(y_{n-1}-t) \cdot f(t) dt$ using the same logic in two-period problem and

$$
\begin{aligned}
\frac{dE\hat{V}_n(y_{n-1})}{dy_{n-1}} &= -c_n(r) + r_S \cdot \int_{\underline{y}_n^*}^{\overline{y}} F(y_{n-1}-t) \cdot f(t) dt \\
&\leq -c_n(r) + r_S \cdot \int_{\underline{y}_n}^{\overline{y}} F(y_{n-1}-t) \cdot f(t) dt \\
&\leq -c_n(r) + r_S \cdot \left[F(\overline{y}) - F(\underline{y}_n)\right] \\
&\leq -r_C < 0
\end{aligned}
$$

Here $\underline{y}_n$ is the myopic bound L_n^m, and we have $\underline{y}_n^* \geq \underline{y}_n$ by induction for the first step to hold.

In addition, we have $\frac{dE\tilde{V}_n(y_{n-1})}{dy_{n-1}} \leq 0$, because the expression of $\frac{dE\tilde{V}_n(y_{n-1})}{dy_{n-1}}$ is as the following:

$$
\begin{aligned}
&\frac{dE\tilde{V}_n(y_{n-1})}{dy_{n-1}} \\
=& -[EV_{n+1}(\underline{y}_n^* - \xi_n, x_{n-1} + w_{n-1} - y_{n-1})]f(y_{n-1} - \underline{y}_n^*) \\
&+ [EV_{n+1}(\underline{y}_n^* - \xi_n, x_{n-1} + w_{n-1} - y_{n-1})]f(y_{n-1} - \underline{y}_n^*) \\
&+ \int_{-\infty}^{y_{n-1}-\underline{y}_n^*} \left[\frac{dEV_{n+1}(y_{n-1} - \xi_{n-1} - \xi_n, x_{n-1} + w_{n-1} - y_{n-1})}{dy_{n-1}}\right] \cdot f(\xi_{n-1}) d\xi_{n-1} \\
=& \int_{-\infty}^{y_{n-1}-\underline{y}_n^*} \left[\frac{dEV_{n+1}(y_{n-1} - \xi_{n-1} - \xi_n, x_{n-1} + w_{n-1} - y_{n-1})}{dy_{n-1}}\right] \cdot f(\xi_{n-1}) d\xi_{n-1} \\
=& \int_{\underline{y}_n^*}^{\infty} \left[\frac{dEV_{n+1}(y_{n-1} - \xi_{n-1} - \xi_n, x_{n-1} + w_{n-1} - y_{n-1})}{dy_{n-1}}\Big|_{\xi_{n-1}=y_{n-1}-t}\right] \cdot f(y_{n-1} - t) dt
\end{aligned}
$$

The last step is arrived by using $t = y_{n-1} - \xi_{n-1}$. Because $\underline{y}_n^* \geq F^{-1}(\frac{r_S - c_n(r)}{r_S}) = L_n^m$ (true for last period and can be induced backward), $\frac{dE\hat{V}_n(y_{n-1})}{dy_{n-1}} \leq -r_C$. So we have $\frac{dE\hat{V}_n(y_{n-1})}{dy_{n-1}} + \frac{dE\tilde{V}_n(y_{n-1})}{dy_{n-1}} \leq 0$, resulting in $\underline{y}_{n-1}^* \geq F^{-1}(\frac{r_S - c_{n-1}(r) + \delta \cdot r_C}{r_S}) \geq F^{-1}(\frac{r_S - c_{n-1}(r)}{r_S})$. (Induction completed, similar lower bound of $\underline{y}_n^*$ achieved as the two-period problem.)

Above all, we have proved that $L_n \leq L_{n+1}$ and $L_n \geq F^{-1}(\frac{r_S - c_n(r)}{r_S})$. Also, from the expression of $\frac{dE\hat{V}_n(y_{n-1})}{dy_{n-1}}$ and $\frac{dE\tilde{V}_n(y_{n-1})}{dy_{n-1}}$, we can see they are not related with $\{w_n\}$, thus the "L" bound is not impacted by $\{w_n\}$ and the current capacity w_n which modifies the (L_n, U_n) policy is determined by the current outstanding account receivables.

B.4 Proof of Proposition 4

Define

$$G_n(y) = -r_C \cdot (x_n - y) + L(y, \xi_n) \tag{B.9}$$

$$H_n(y) = r_S \cdot (y - x_n) + L(y, \xi_n) \tag{B.10}$$

Both G_n and H_n are convex in y. The derivative of $G_n(y)$ and $H_n(y)$ with respect to y is as below:

$$
\begin{aligned}
G_n'(y) &= r_C - r_S \cdot E\delta(\xi_n - y) \\
&= r_C - r_S \cdot Pr(\xi_n > y) \\
&= r_C - r_S + r_S \cdot Pr(\xi_n < y)
\end{aligned}
$$

$$H_n'(y) = r_S \cdot Pr(\xi_n < y) > 0$$

So $G_n(y)$ is minimized at $y^* = F^{-1}(\frac{r_S - r_C}{r_S})$, i.e., $U^0 = F^{-1}(\frac{r_S - r_C}{r_S})$. Proof is done.

B.5 Proof of Proposition 5

The U_n^m bound is the same as U^0 for all n following the same logic in the no RF case. For the L_n^m, consider the cost function in the current period $K_n(y)$ when $x_n \leq y \leq x_n + w_n$:

$$K_n(y) = c_n(r) \cdot (y - x_n) + L(y, \xi_n) \tag{B.11}$$

The derivative of $K_n(y)$ with respect to y is

$$
\begin{aligned}
K_n'(y) &= c_n(r) - r_S \cdot E\delta(\xi_n - y) \\
&= c_n(r) - r_S + r_S \cdot Pr(\xi_n < y)
\end{aligned}
$$

So $K_n(y)$ is minimized at $y^* = F^{-1}(\frac{r_S - c_n(r)}{r_S})$ if $r_S > c_n(r)$ and $y^* = -\infty$ otherwise. Proof is done.

B.6 Proof of Corollary 1

See Appendix B.2.1.

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