Characteristic Crack Dimension of Saturated Drying Soils: Theory and Applications

V.Y. Chertkov
Faculty of Agricultural Engineering, Technion, Haifa 32000, Israel
E-mail: agvictor@tx.technion.ac.il

Abstract. *Shrinkage cracks in swelling soils determine their transport properties. An available model of shrinkage cracks in a brittle medium, having constant relevant properties and desiccating in conditions of shock drying, is used as a basis for the description of crack growth in water saturated soils undergoing desiccation and shrinkage. A key point is the existence of a minimum crack capable of developing at shrinkage. The concept of the minimum quasi-brittle crack capable of developing at shrinkage leads to two possible types of shrinkage cracks in clay soils. The major objective of this work is a synthesis of existing information relating to interconnections between crack networks, aggregates, and physical properties of drying and shrinking, but staying saturated clay soils.*
Keywords. *Shrinkage, Characteristic crack dimension, Saturated drying soils, Theory, Applications.*

Introduction

The network of shrinkage cracks brings in an essential contribution to the hydraulic conductivity of a clay soil. In the broad range of water content beginning from the saturated state this contribution is decisive compared to that of the soil matrix (Chertkov and Ravina, 2001). For this reason the geometrical characteristics of crack networks in swelling soils are of great interest. A number of such characteristics were considered earlier (Chertkov, 1995, 2000; Chertkov and Ravina, 1998, 2001). It follows from general physical considerations that the different geometrical characteristics of both the interaggregate and the interblock crack networks of a given soil flow out of the united basis, namely from its elastic, strength, diffusivity and shrinkage properties. Therefore, one can assume that these crack network characteristics should be interconnected. The demonstration of these interconnections is the objective of this work. Salganik and Chertkov (1969) considered theoretically and Chertkov (2002) validated by available data on a clay soil, the existence of a characteristic crack dimension, l_* corresponding to the minimum crack capable of developing at shrinkage and to be expressed through the above physical properties of the soil. In the following the model to be discussed in Chertkov (2002) will be referred to as the basic one. Actually, we are going to show a relation between l_* and:

- interaggregate cracks, using the basic model and results from Chertkov and Ravina (2001);
- interblock cracks, using the basic model and results from Chertkov and Ravina (1998) and Chertkov (2000);
- aggregate-size distribution, using the basic model and results from Chertkov (1995).

We will thereby show that the characteristics of interaggregate and interblock cracks, and aggregate-size distribution of the soil are mutually connected and connected (through l_*) with its physical properties as indicated above. First, for the reader's convenience we briefly summarize the major concepts and results of the basic model that will be needed.

The Brief Review of the Basic Model

Theory

Shrinkage cracks are observable even in saturated clay soils under desiccation in restrained conditions (Konrad and Ayad, 1997a, b; Ayad et al., 1997). A model of the development of an isolated shrinkage crack under the action of shock drying was earlier considered in the frame of linear elastic fracture mechanics (Salganik and Chertkov, 1969). A body with the initial (gravimetric) water content, $\theta = \theta_o$ (Fig.1) was considered to be suddenly placed in a medium with (gravimetric) water content $\theta = \theta_1 < \theta_o$. The simplest condition of water exchange on the boundary of the body, $\theta = \theta_1$, was assumed (Fig.1). It was supposed that there exists a crack of length l going into the depth of the body along the z-axis perpendicular to its boundary (the y-axis, Fig.1). The maximum shrinkage stress σ_* (reached on the boundary at z=0, Fig.1) was determined to be

$$\sigma_* = \frac{E\alpha}{3(1-\nu)}(\theta_o - \theta_1), \tag{1}$$

where E is Young's modulus; ν is Poisson's ratio; α is the shrinkage coefficient. The characteristic dimension l_* (Fig.1) to be defined as

$$l_* = \frac{1}{\pi}\left(\frac{K_{Ic}}{\sigma_*}\right)^2 \tag{2}$$

(K_{Ic} is the critical stress-intensity factor or fracture toughness of the soil) represents Griffith's formula and is the length of the crack for which the body would fracture if it were loaded by the uniformly distributed stress σ_*. A crack with $l < l_*$ will not develop. When the length of the initial crack $l > l_*$ and is close to l_* after a definite delay time the crack with a jump goes over into a new moving equilibrium state and further will increase continuously and with time goes into the regime of propagation with a constant velocity. The initial depth (l_o) of a quasi-brittle crack developing by jump is in the range $l_* < l_o < l_\tau \cong 1.43 l_*$. The average initial depth of cracks capable of developing by jump is found to be

$$l_{av} = 1.22 l_* . \tag{3}$$

The average delay time, T_{av} and average crack depth after jump, L_{av} are determined by

$$T_{av} = 11.10(l_*^2 / D), \qquad L_{av} \cong 5.72 l_* \tag{4}$$

(D is the hydraulic diffusivity).

The development of cracks reaching the maximum depth z_m passes, on the average, stages of delay (or forming of an initial crack) ($l_* < l \le l_\tau$), crack jump ($l_\tau < l <\cong L_{av}$), quasi-stable growth ($L_{av} \cong< l < z_*$, where $z_* \cong\le z_m$), and quick slowing-down and stop ($z_* < l \le z_m$).

E, ν, K_{Ic}, *D*, and α can vary in dependence on water content (at given θ_o and θ_1). However, at saturated state ($\theta > \theta_s$ - shrinkage limit) the fracture toughness of the studied clays (Haberfield and Johnston, 1990; Murdoch, 1993; Harison et al., 1994)

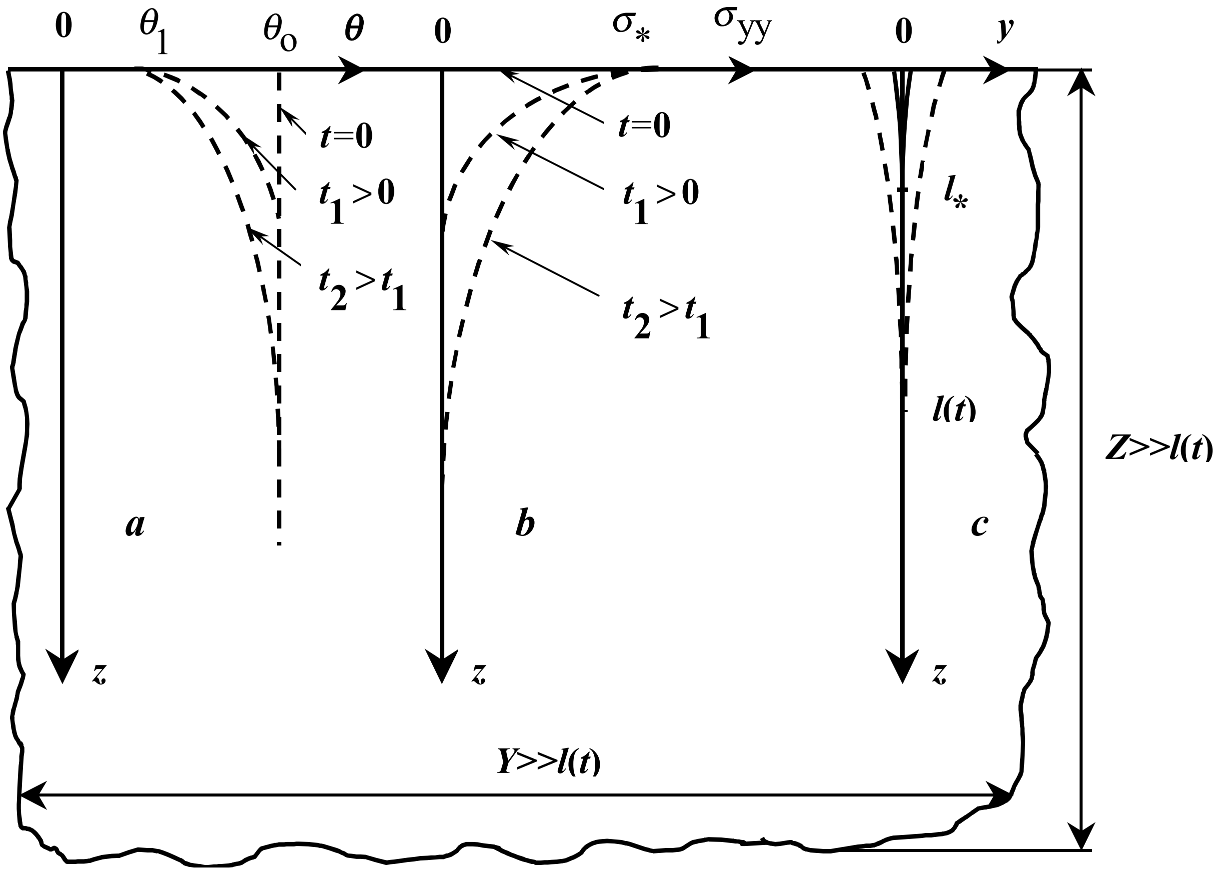

Figure 1. Sketch of the growth of a small quasi-brittle crack at the surface of a large body as it dries and shrinks at initial conditions corresponding to ‘shock drying’. (a) Water content profiles. (b) Shrinkage stresses. (c) Crack extension. *Y* and *Z* are the horizontal and vertical dimensions, respectively, of an area that one crack takes over;

t=0, t_1, and t_2 are consecutive time instants; σ_{yy} is tensile stresses on the line of a vertical crack; $l(t)$ is a current crack depth.

changes with water content very little, if at all. The same is true with regard to E and ν. The soil water diffusivity of disaggregated clays, D is nearly constant with soil water content (Kutilek et al., 1985). The shrinkage coefficient α is defined as $\alpha = (1/V)(dV/d\theta)$ where V is the specific volume of a clay soil (per unit mass of oven-dried soil (m^3/kg)). The derivative is at a constant pressure. For soils with high clay content the dependence between V and θ is linear (McGarry and Malafant, 1987) and $\alpha = (1/\rho_w)/(1/\rho_s + \theta/\rho_w)$ where ρ_s is the clay particle (or solid phase) density, and ρ_w is the water density. The α value is considered as constant in a range $\theta'' < \theta < \theta'$ if the absolute value of a relative deflection of the average, $\bar{\alpha} \equiv (\alpha' + \alpha'')/2$ of corresponding α' and α'' does not surpass 0.1, i.e. $(\alpha'' - \alpha')/(\alpha' + \alpha'') < 0.1$.

The fact that the actual water content at the soil surface $\theta_1 = \theta(0,t)$ decreases means the existence of small water content differences between the soil surface and a certain superficial layer. To estimate these differences one can formally consider a small interval $\Delta\theta \equiv \theta_o - \theta_1 << (\theta' - \theta'')$ as a constant small difference between a higher (decreasing) water content within the superficial layer, θ_o and a lower (decreasing) water content at the soil surface, θ_1. The difference $\Delta\theta$ should not surpass $3\delta\theta$ where $\delta\theta$ is a statistical (thermodynamic) fluctuation of the θ value (Landau and Lifshitz, 1980). Statistical fluctuations $\delta\theta$ limit the accuracy of θ value measurements. Absolute error of θ values from Konrad and Ayad (1997b), Ayad et al. (1997), Konrad and Seto (1994) is ~0.001. Therefore, in practical estimation the value $\delta\theta = 0.001$ is used.

Validation

Works of Konrad and Ayad (1997b), Ayad et al. (1997), Konrad and Seto (1994), and Virely (1987) were dedicated to studying the different aspects of the cracking of an intact Saint-Alban sensitive marine clay (80 km west of Quebec City in the Saint Lawrence Valley, Canada) undergoing desiccation and shrinkage in saturated state under restrained conditions. Two sets of data: E=4MPa, $\nu = 0.3$, $K_{Ic} = 1.35\mathrm{kPa}\cdot\mathrm{m}^{1/2}$ and E=6MPa, $\nu = 0.5$, $K_{Ic} = 1.6\mathrm{kPa}\cdot\mathrm{m}^{1/2}$ (Ayad et al., 1997) give the model prediction of $l_* = 0.0200 \pm 0.0104\mathrm{m}$. One can conduct a number of comparisons between the model prediction and the data (Konrad and Ayad, 1997b; Ayad et al., 1997; Konrad and Seto, 1994; Virely, 1987). It can be shown that: a) the observable values of the crack length before and after jump, $l_o = 0.03 - 0.04\mathrm{m}$ and $l_j = 0.08 - 0.11\mathrm{m}$, respectively, (Ayad et al., 1997) are in agreement with Eqs.(3) and (4) and the l_* estimate; b) the model inequality $l_* < l_o < l_\tau$ is fulfilled with $l_o = 0.03 - 0.04\mathrm{m}$ (Ayad et al., 1997), the l_* estimate, and $l_\tau \cong 1.43 l_*$; c) values of $l_0 = 0.03 - 0.04\text{m}$ (Ayad et al., 1997) and the l_* estimate confirm Eq.(3) for the average initial depth of cracks before jump; d) crack length before and after jump, $l_0 = 0.03 - 0.04\text{m}$ and $l_j = 0.08 - 0.11\text{m}$, respectively, (Ayad et al., 1997) and the observable time before jump, T_j=17hours (Ayad et al., 1997) are in agreement with the l_* estimate and Eq.(4).

Crack and Aggregate Characteristics to Be Considered in This Work

Characteristics of Interaggregate Cracks

Considering the hydraulic conductivity of interaggregate capillary cracks in swelling soils Chertkov and Ravina (2001) introduced a number of values that are inversely proportional with respect to the mean spacing between the cracks or mean dimension of soil aggregates, *d*. Those include the total specific length, L_t of traces of connected vertical interaggregate cracks at a horizontal cross-section (per unit area), the specific length, $L(R)$ of crack traces of width < R at a horizontal cross-section, the crack volume distribution, $V_{cr}(R)$ with respect to crack width, the total specific crack volume, $V_{cr}(R_m)$ per unit volume of soil (R_m is the maximum capillary crack width), and the vertical hydraulic conductivity, $K_{cr}(R_w)$ of the network of vertical interaggregate capillary cracks in a clay soil (R_w is the maximum width of water-filled capillary cracks).

Characteristics of Aggregate-Size Distribution

Considering two- and three-dimensional fragmentation of clay soils by a shrinkage crack network Chertkov (1995) introduced a fragment (aggregate)-size distribution based on the concept of multiple crack formation. It is important for us here that this distribution also contains as a scaling factor the mean dimension of soil fragments (aggregates), *d*. That is, the distribution depends on the aggregate dimension, *x* through an *x*/*d* ratio.

Characteristics of Interblock Cracks

Modeling the crack network of swelling clay soils Chertkov and Ravina (1998) introduced the thickness, z_0 of the intensive-cracking layer at the quasi-steady state and the maximum crack-depth, z_m (Fig.2). These characteristics were shown to be interconnected as

$$z_0 \cong 0.1\, z_m . \quad (5)$$

Below we also consider the mean spacing, S_0 between initial cracks at the soil surface. This crack characteristic is connected with z_0 (Chertkov, 2000) as

$$z_0 \cong S_0 / 2 . \quad (6)$$

Results and Discussion

Characteristics of Interaggregate Cracks and Aggregate-Size Distribution

The major concept (of the basic model) of the minimum quasi-brittle crack capable of developing at shrinkage, enables one to assume that in saturated clay soils there are cracks of depth $l < l_*$ incapable of developing in dimension in desiccation, and cracks reaching dimension $l > l_*$, that are initial in developing large shrinkage cracks. Observations show that the network of shrinkage cracks actually consists of larger seasonal macrocracks (Zein el Abedine and Robinson, 1971; Dasog et al.,1988; Morris et al., 1992) and smaller quasi-steady interaggregate microcracks (Guidi et al., 1978; Ringrose-Voase and Nys, 1990; Scott et al., 1988; Velde et al., 1996). That is, the above assumption is justified, and we can consider the l_* dimension to be the maximum spacing between interaggregate cracks or as the maximum aggregate dimension.

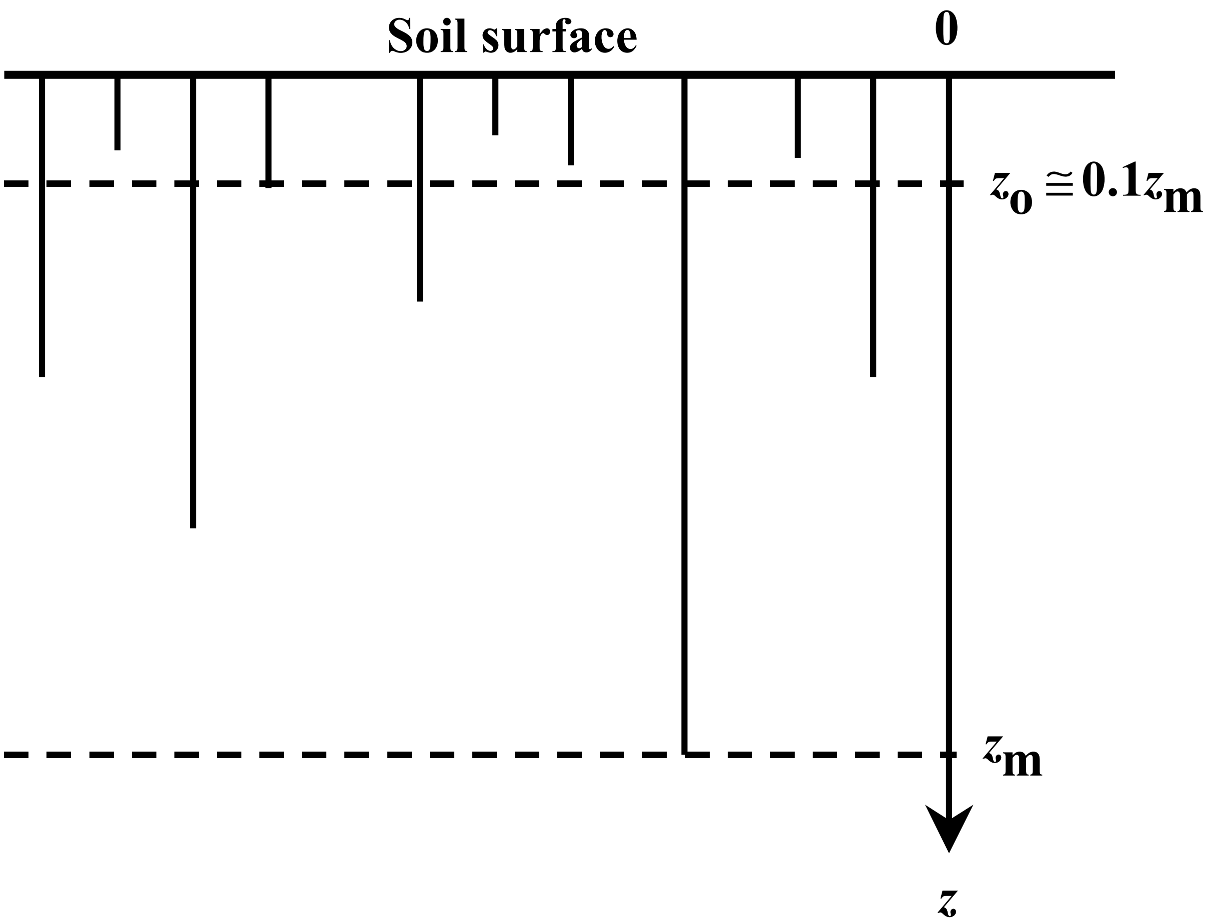

Figure 2. Sketch of the quasi-steady state of a crack system in a swelling soil. z_o is the lower boundary of the intensive-cracking layer; z_m is the maximum crack depth; the vertical lines are the traces of the shrinkage cracks shown in cross-section.

In the two-dimensional case the mean (d) and maximum (l_*) dimensions of (macro)aggregates are connected (Chertkov, 1995) as

$$d = l_* / 3 . \quad (7)$$

In this case the estimates of d=0.0066m (Chertkov, 1995) and $l_* = 0.0200 \pm 0.0104$m (Chertkov, 2002) are in agreement with Eq.(7). In the three-dimensional case Eq.(7) transforms to (Chertkov, 1995)

$$d = l_* / 4 . \quad (8)$$

Equations (7) and (8) connect the indicated characteristics of interaggregate cracks and aggregate-size distribution with the characteristic crack dimension, l_* of swelling clay soils.

Characteristics of Interblock Cracks

Let us estimate relations between S_o, z_o, and z_m crack characteristics and the characteristic crack dimension l_* (from the basic model). The mean dimension of (surface) cracks developing by the merging of smaller initial cracks of dimension l_* is $(K_* + 1)l_*$ (Chertkov and Ravina, 1998) where K_* is the critical value of the ratio of the mean linear dimension of an area that one initial (surface) crack takes over to the mean crack dimension proper (for soils $K_* \cong 5$ (Zhurkov et al., 1981)). In connecting these enlarged surface cracks and forming the network their mean dimension $(K_* + 1)l_*$ should coincide with the mean spacing, S_o between initial cracks of dimension l_*, i.e.,

$$S_o \cong (K_* + 1)l_* \cong 6l_* . \quad (9)$$

The data for Saint-Alban clay, S_o=0.20-0.24m (Konrad and Ayad, 1997b) do not contradict this relation at the above estimate of $l_* = 0.0200 \pm 0.0104$m . Note that according to Eq.(4) and the S_o estimate (Eq.(9)), the mean depth of primary cracks after jump (L_{av}) and their mean spacing at the surface (S_o) are of the same order of magnitude, i.e., $L_{av} \cong S_o$. This equality is in agreement with the initial condition of an intensive-cracking layer (Chertkov, 2000).

Using Eqs.(6) and (9) one can get an estimate of z_o as

$$z_o \cong 3l_* . \quad (10)$$

Finally, accounting for Eqs.(5) and (10) we obtain an estimate for the maximum crack depth, z_m as

$$z_m \cong 30l_* . \quad (11)$$

Thus, in the saturated drying clay soil the interaggregate-crack characteristics (L_t, $L(R)$, $V_{cr}(R)$, $V_{cr}(R_m)$, and $K_{cr}(R_w)$), fragment (aggregate)-size distribution, and interblock-crack characteristics (S_o, z_o, and z_m) are interconnected through the characteristic crack dimension, l_*. Through l_* and Eqs.(1) and (2) they also depend on the physical properties of the soil.

Conclusion

Results of the work demonstrate the connections between the minimum dimension of a quasi-brittle crack capable of developing at shrinkage and other characteristic dimensions of a crack network in a swelling clay soil. These interconnections create the prerequisites for a better understanding of cracking and hydraulic processes in clay soils as well as for the simplification of relevant prediction.

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