Tackling the Investment Decision as a “Newsvendor Problem”

by GARY M. THOMPSON

The newsvendor model is a classic approach to determining how to set appropriate inventory levels for products whose value is perishable. While the newsvendor analysis does not yield the richness of data found in a simulation-based model, newsvendor analysis provides a relatively simple way to determine inventory levels.

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In their article, Professors Dittman and Hesford present a simulation-based optimization approach for investment decisions. The case they look at—deciding on the number of rooms to convert to allergy free—is interesting and relevant, and their methodology is sound. I firmly believe in the usefulness of simulation as a modeling tool, as I have written previously

(Thompson and Verma 2003). In this note, I would like to present a simpler approach to the problem, which, though lacking the richness of simulation, is more amenable to a “back of the envelope” calculation. As I will show, it yields the identical decisions in this setting.

The approach I illustrate here is to model the decisions of price and room quantity as variants on the classic “newsvendor” inventory problem from the field of operations management. I will first examine the decision of the room price; once the price is set, I will consider the ideal number of rooms to convert.

In the newsvendor problem, which is also known as a “single period inventory model,” a newsvendor is faced with the task of ordering newspapers. Newspapers unsold at the end of the day have limited value. The assumption is that the newsvendor cannot

restock newspapers during the day and so must make the best decision to maximize revenue, given demand uncertainty. An additional newspaper should be ordered as long as the expected marginal profit from selling that newspaper exceeds the expected marginal cost of not selling it. Formally,

$$\begin{array}{l} \text{Order paper } j \text{ as long as} \\ p \times MP > (1 - p) \times MC, \end{array} \quad (1)$$

where

- p = the probability that paper j will be sold at full price,
- MP = marginal profit of selling paper j at full price,
= $VFR - VC$ (where VFR = variable full revenue, per item and VC = variable cost, per item), and
- MC = marginal cost of not selling paper j at the full price,
= $VC - VSR$ (where VSR = variable salvage revenue, per item, and VC is as defined above).

Simplifying equation (1) yields a decision rule for ordering paper j based on the probability that it sells at full price:

$$p > MC/(MP + MC). \quad (2)$$

In the newsvendor inventory-management problem, equation (2) is also known as a critical fractile.

In the setting described by Dittman and Hesford, one must modify equation (2) to address the issue of occupancy. The probability that an item—in this case an allergy-free room—sells at full price can be thought of as the product of the probability that the room sells at full price, assuming full occupancy times the occupancy proportion. Defining o as the occupancy proportion and p' as the probability that an item sells (assuming full occupancy), we get

$$p' \times o > MC/(MP + MC), \quad (3)$$

which can be rewritten as

$$p' > MC/[o \times (MP + MC)]. \quad (4)$$

The effect of considering occupancy, then, is to raise the critical fractile.

Examining first the room-pricing decision, one can construct a table of critical fractiles, as shown in Exhibit 1. Values in Exhibit 1 are interpreted as follows. Considering a pricing premium of \$5 per room, for example, a room would only be converted if the probability that it could be sold at that price was higher than .75. From the survey reported by Dittman and Hesford (in their Exhibit 2), only 59 percent of customers would be willing to pay a \$5 (or more) premium. Of the different possible price premiums, only \$10 meets the criterion of customer willingness to pay exceeding the critical fractile; the best decision, then, is to price the allergy-free rooms at a \$10 premium.

Having determined the price to charge, one can then examine the decision of the number of rooms to convert. In conducting this analysis, I will look for the number of rooms to convert, such that the probability of the last-converted room being sold exceeds the appropriate critical fractile. To perform this analysis, use the data in Exhibit 4 from Dittman and Hesford's article to calculate the values shown here in Exhibit 2.

Since the data in Exhibit 2 include the times when the property is not occupied, we need to calculate a critical fractile that does not include the effect of occupancy. For the \$10 room premium, the occupancy-free critical fractile, p , is given by $p > 2.77/(7.23 + 2.77) = 0.277$. As such, the ideal number of rooms to convert is given by the largest number of rooms for which probability of needing that number of rooms exceeds 0.277. This is 54.15 rooms, which we round to 54 rooms—the same solution yielded by the simulation optimization performed by Dittman and Hesford.

Exhibit 1:

Critical Fractile Values, by Room Price Premium

<i>Room Price Premium</i>	<i>MC^a</i>	<i>MP</i>	<i>O</i>	<i>Critical Fractile</i>	<i>Probability of a Customer Willing to Pay Premium or Higher</i>
\$0	\$2.77	(\$2.77)	.74	Undefined	1.00
\$5	\$2.77	\$2.23	.74	.750	.59
\$10	\$2.77	\$7.23	.74	.375	.38
\$15	\$2.77	\$12.23	.74	.250	.16
\$20	\$2.77	\$17.23	.74	.187	.10
\$25	\$2.77	\$22.23	.74	.150	.04

Note: MC = marginal cost; MP = marginal profit; O = occupancy proportion.

a. Ignoring the time value of money (for simplicity), the marginal cost, *MC*, is simply the cost per the investment cycle, or $\$1,500 + 7 \times 75$, which works out to \$2.77 per room-night over the 730 days in the two-year horizon.

Exhibit 2:

Calculations to Identify the Decision regarding the Number of Rooms to Convert

<i>Daily Occupancy (%)</i>	<i>Probability of Occupancy</i>	<i>Rooms to Convert (j)^a</i>	<i>Probability of Needing j or More Converted Rooms</i>
0	.0630	0.00	1.0000
15	.0027	8.55	.9370
20	.0082	11.40	.9343
25	.0082	14.25	.9261
30	.0110	17.10	.9179
35	.0137	19.95	.9069
40	.0329	22.80	.8932
45	.0082	25.65	.8603
50	.0438	28.50	.8521
55	.0438	31.35	.8083
60	.0630	34.20	.7645
65	.0301	37.05	.7015
70	.0384	39.90	.6714
75	.0521	42.75	.6330
80	.0685	45.60	.5809
85	.0850	48.45	.5124
90	.0849	51.30	.4274
95	.0877	54.15	.3425
100	.2548	57.00	.2548

a. $j = 150 \text{ rooms} \times .38 (= \text{proportion of customers willing to pay the \$10 price premium}) \times \text{daily occupancy}/100$.

Limitations of the Newsvendor Approach

Applying a newsvendor approach has three limitations relative to using simulation.

First, while the newsvendor approach considers variability, it yields a single (marginal) profitability value. It cannot, as does simulation, provide information on the variability that one might expect in the profitability

associated with different solutions. Second, it is more difficult to incorporate the time value of money. For short-horizon decisions, such as the one examined here, the effects of the time-value of money are less pronounced, however. A third limitation is that one needs to perform additional calculations to determine the total expected payoff of the recommended solution, in contrast to simulation optimization approach, which yielded such information directly. The reason for this difference is that a newsvendor problem bases a decision on marginal values, while the simulation optimization based a decision on total profitability. Nonetheless, the expected value of a decision can be calculated as follows:

Sum, across hotel occupancies, the occupancy probability $\times$ [$MP \times$ smaller of (demand for converted rooms, rooms converted) $- MC \times$ maximum of (rooms converted $-$ demand for converted rooms, 0)],

where

demand for converted rooms =
total rooms $\times$ occupancy proportion $\times$
room conversion rate,
room conversion rate is given by the
probability that customers are willing to
pay the specified premium or higher for the
converted room, and
 MR and MC are as defined earlier.

Applying this calculation to the \$10 price premium and 54 converted rooms yields a total expected value of \$264.40 per night, or \$193,013 over the two-year decision horizon. This value is within 0.6 percent of the value yielded by the simulation optimization performed by Dittman and Hesford.

Additional Considerations When Using the Newsvendor Approach

There are two special considerations when using a newsvendor model for investment

decisions. The first is how to handle a situation where more than one price premium had a valid critical fractile (recall that in this case, only the \$10 price premium had a critical fractile that was met). In a situation with multiple valid premiums, one would determine the best number of rooms to convert at each of the valid premiums and then select the overall choice based on which combination of price premium and rooms converted had the greatest overall expected value (calculated as described above).

A second consideration is how to handle situations where none of the price premiums had a valid critical fractile. In this case, the best decision is to convert no rooms.

Conclusions

A newsvendor model—a classic inventory model from the field of operations management—offers a simpler approach to investment decisions than the simulation optimization approach presented by Dittman and Hesford. While the simulation optimization yields information that cannot be derived from the newsvendor approach, applying two related newsvendor models—the first to set the price premium for the converted rooms and the second to set the number of rooms to be converted—yielded the same answers as the simulation optimization. While this may not always be true, it is always good to tailor a solution to a problem. If one is looking for a quick and reasonably accurate decision model, the newsvendor model belongs in one's model "tool-kit."

References

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