OVERBOOKING WITH MULTI-CLASS DEPENDENT WALK OUT COSTS

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ABSTRACT

Hotels may encounter overbooked settings with customers potentially being walked. Walk or re-accommodation costs are not homogeneous across all customer types, with costs potentially different for loyal (branded, direct) versus non/less-loyal (3rd party-intermediated) guests. Owing to cost/satisfaction impacts – hotels may need to proactively determine which guests to re-accommodate. We formulate an overbooking model with class dependent walk out costs for a hotel with two classes of reservations - loyal members with higher walk out costs, and non-members with lower walk out costs, but with each class paying the same room rate. On the morning of the stay-date, customers arrive randomly proportionate to the number of reservations by class with the hotel potentially proactively (i.e. with rooms still available) walking non-members to avoid potentially walking members. By implementing a dynamic walk out model (walk out decisions based on mix of reservations and empty rooms), optimal walkout decisions can be made to minimize total expected walk out costs. We investigate how class dependent no-show rates and walk out costs impact optimal walk out decisions and overbooking levels. We find that changes in the no-show rates for a customer class only impact the overbooking levels of the related class whereas changes in class-specific walk out costs impact all customer class overbooking levels. This thesis offers managerial insight into a proactive/strategic walk out policy for the lodging industry, aiming to achieve optimal overbooking levels.

Biographical Sketch

Shuwen Liang received her Bachelor's degree from Ecole Hoteliere de Lausanne. During her study, she had several half-year internships in Guest Relations, Human Resources, and Sales and Marketing departments. After graduation, She was selected as a revenue management trainee and later became a cluster revenue manager at Marriott. From 2016 to 2020, she took over five hotels, including complex hotels, business hotels, and resorts. She also participated in the preparation of a hotel opening. The fruitful working experience brought her creative, fascinating, and valuable research ideas. Therefore, she decided to come to Cornell and pursue a Research Master's degree in Hotel Administration.

This paper is dedicated to my parents.

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1 Introduction

In the field of Revenue Management (RM) in the hospitality industry, overbooking, as one of the main RM tools, is always a hot topic brought to discuss. Overbooking means accepting more reservations than its capacity. An appropriate overbooking limit can bring optimal revenue and profit for a hotel. However, the hotel might get an unpredictable loss if the overbooking level is in an unreasonably high level. Another factor that has been involved in business performance is the loyalty program. More and more repeated customers choose to join the loyalty program which brings more benefits for both customers and hotels. In this paper, we study the optimal hotel overbooking limits for two types of customers: loyal members and non-members.

Kimes (1989) mentions that RM is a selling strategy dedicated to maximizing revenue with three 'right's: the right type of capacity, the right kind of customer, and the right price. And overbooking directly relates to the capacity mentioned above. Subramanian et al. (1999) developed a dynamic programming model for solving seat allocation problems with consideration of overbooking, cancellation, and no-shows. No-shows are those people booked reservations but they do not show up in the end for the events. There are plenty of published papers focusing on finding optimal overbooking limits. Phumchusri and Maneesophon (2014) developed an optimal overbooking model for one and two-room types by analyzing marginal costs for vacant rooms and walking-out guests. Hotel overbooking is treated as one of the most effective strategies to minimize the loss of revenue with vacant rooms from a no-show, cancellation at the last minute, and early departure (Ivanov, 2015). In this paper, we formulate an overbooking model to calculate the optimal overbooking levels with the help of the marginal analysis and a dynamic programming for the walk out model.

The other hot topic nowadays is the loyalty program. One sample from research indicates that for every 1 percent increase in expenditure on the loyalty program, there are 12 percent increase in operating income and a 3 percent increase in room margin averagely (Jongcheveevat et al., 2018). A loyal customer who is enrolled as a member of the hotel brand tends to have a much higher contribution to hotel revenue than non-loyal customers. This discrepancy reflects in room revenue and other ancillary service revenue such as food and beverage consumption. Therefore, it is meaningful to study further how to optimize the overbooking limits by protecting the member customers in the hotel loyalty program. Several pieces of research have studied optimal overbooking and loyalty programs (Hwang and Wen, 2009; Noone and Lee, 2011; Vaeztehrani et al., 2015). However, there are few studies that make loyal members and non-members the variables in an optimal overbooking model. Not many researchers have realized how the walk out costs for members and non-members can be in the huge difference and how the walk out costs impact the overbooking levels for both members and non-members. This paper is dedicated to deal with optimal overbooking levels for two customer segments – loyalty members and non-members.

The remainder of this paper is organized as follows. First, the background and related literature are reviewed in Section 2. Section 3 describes the walk out model and overbooking models for hotel optimal overbooking levels for two types of customers: loyal members and non-members. Numerical tests followed by the model development are presented in Section 4. Finally, Section 5 provides the conclusion, limitations and extensions of the work.

2 Background and Related Research

In this section, we review some literature on several topics: revenue management, inventory control, and customer loyalty programs. This background knowledge is presented to clearly identify the gaps between the previous literature and this thesis, therefore, to better assist in understanding the models of this paper.

2.1 Revenue Management

Revenue management (RM) is a critical business strategy to optimize revenue performance which has a half-century history and development. Its origin dates to the 1970s when Littlewood (1972) applied a rule to solve a two-fare seat allocation problem and yield revenue in the U.S. airline industry after deregulation. Later, the development of RM in the airline industry is rapid. In the following two decades, loads of studies enriched the content of RM with diverse methodologies in four key areas - forecasting, overbooking, seat inventory control, and pricing (McGill and Van Ryzin, 1999). For instance, Belobaba (1989) formulated a probabilistic decision model to enhance Littlewood's rule on seat inventory control; Lee (1990) combined statistical historical and advanced bookings models to achieve a more accurate forecast; Botimer (1996) optimized revenue through structuring pricing levels for different products; Subramanian et al. (1999) proposed a dynamic programming model allowing overbooking, cancellations, and no-shows to maximize revenue. Moreover, RM can be applied by a single technique but usually appears in a combination of methods. In fact, implementing multiple RM techniques such as pricing and seat allocation together can fulfill the full benefit of yield management (Gallego and Van Ryzin, 1997).

The success of RM practice in the airline industry has fueled the application and development of RM in other industries such as hotels, car rental, and healthcare industries. RM is applied quickly and widely in various industries, contributing to similar characteristics these industries share. More importantly, firms cannot practice RM without these characteristics since they are necessary conditions. These include constrained supply but unconstrained demand, perishable inventory with reservation in advance required, low marginal sales costs but high marginal production costs, fluctuating demand with the property can be segmented (Gallego and Van Ryzin, 1997). In the hotel industry, the fixed capacity matches the available rooms for sale in a hotel. And the different types of customers are the unconstrained segmentable demand. Furthermore, customers need to book in advance to prevent the hotel from being sold out. In addition, the cost of cleaning a room is relatively low while the profit margin is relatively high. In other words, RM is a strategy to optimize revenue by allocating the right type of fixed capacities to the right types of customers at the right time with the right price (Kimes, 1989).

With an overview of RM history and its development in various industries, the focus of this paper will be on overbooking and inventory control among the critical areas of RM research and direction. In addition, the main background of this paper is the hotel industry. Therefore, a literature review of the corresponding RM components such as types of hotel customers will be going through later in this section.

To summarize, we will first review ‘Inventory Control’, ’Dynamic Programming' and 'Overbooking' to illustrate the importance of allocating the right type of fixed capacities at the right time. Moreover, the topic of 'Loyalty Customer' will be reviewed regarding selling to the right types of customers.

2.2 Inventory Control

The early stage of the RM is about inventory management. It connects tightly with the airline industry, where the fixed cost is extraordinarily high and marginal cost is relatively low. Therefore, one direct and effective method to drive revenue and profit is to optimize seat allocation. According to different classifications, the solution for seat allocation problems can be categorized by single-leg based and origin-destination based inventory control; static and dynamic models based on customer arrival patterns. Moreover, inventory management can be applied to various industries such as airlines, car rentals, and hospitality, which share similar properties.

Single-Leg and Origin-Destination Based Inventory Control

Single-leg based inventory control is the most simple inventory management method. The first and most famous inventory control rule, proposed by Littlewood, indicates that the discounted fare should always be sold unless the revenue gained from this part is less than the part expected from future full-fare bookings (Littlewood, 1972). Quantity of distributed topics relating to Littlewood's two-fare classes rule are explored. With continuous research on inventory management in academia, Belobaba created a decision model named Expected Marginal Seat Revenue (EMSRa) (Belobaba, 1987). Unlike Littlewood's two-fare inventory control, the model was designed for multiple nested inventories considering consumer booking patterns. This mechanism was implemented in 1989 and shows a significant revenue increase by automatically optimizing booking limits for nested inventory in airline companies (Belobaba, 1989).

Based on the EMSRa model, optimality conditions are further analyzed under various assumptions, specifically on whether multiple nested fare classes are independent or not (McGill, 1989). A general model for dependent demands was created and proved that Littlewood's rule still offers the optimal seat allocation solution under the assumption that the demands are monotonically dependent between two fare classes. Other than that, a more practical approach was raised to use historical spill costs which the unsatisfied demand need to be re-accommodated. It can be a vital reference for optimal seat allocation instead of prediction on demand distribution. Moreover, the author used a bivariate multiple regression model to explain the impact from the outside. Brumelle and Walczak (2003) also discussed how dependent demand between two fare classes impacts the optimal booking limit for each. The research indicated that the optimal overbooking limit decreases if two fare classes are bivariate normal and positively correlated. This result suggests dynamic policies based on multiple fare classes are worthwhile developing and needed in real practice. Besides, the accuracy of the EMSRa approximation has been tested by Belobaba (1989); Curry (1990); Wollmer (1992); Brumelle and McGill (1993). They found that optimal booking limits did not match that of the EMSRa model even though the revenue penalty of the EMSRa was mostly under 0.5 percent.

Belobaba and Weatherford (1996) upgraded EMSRb model based on EMSRa. The expected marginal revenue for unsold parts is treated in a weighted average price from higher classes. A combined decision rule with customer diversion for multiple fare classes was proved to affect inventory limits significantly. Moreover, the accuracy of the estimate of the sell-up probabilities between fare classes had a significant impact on the combined decision rule.

The opposite of single-leg based inventory control is network inventory control, also known as segment or origin-destination (OD) based inventory control. We consider the travel as A-B and B-C, two individual trips for passengers traveling from destination A to C with one intermediate stop B. In this case, the airline company can have more power to manage all three segments: A-B, B-C, and A-C. Curry (1990) concluded that the expected revenue for the entire OD network is separably convex due to its convexity on each fare class allocation. Similarly, multiple nights in a hotel stay can be treated as OD itineraries in the airline industry. In this paper, we will only discuss the situation of one night instead of multiple nights for a hotel. Therefore, we will not review further on OD based inventory control.

Table 1 and Table 2 present the research on Single-leg based and Origin-destination based inventory control, respectively. The list is summarized chronologically by McGill and Van Ryzin (1999); Chiang et al. (2007), and a book named 'Revenue Management and Pricing Analytics' by Gallego et al. (2019).

Table 1: Single-Leg Based Inventory Control

Year	Reference	Year	Reference
1972	Littlewood	1994	Weatherford
1973	Bhatia and Parekh	1994	Shaykevich
1976	Mayer	1994	Young and Van Slyke
1977	Ladany and Bedi	1995	Bodily and Weatherford
1978	Hersh and Ladany	1995	Robinson
1982	Wang	1996	Belobaba and Weatherford
1982	Buhr	1997	Brumelle and Walczak
1982	Richter	1998	Kleywegt and Papastavrou
1983	Titze and Griesshaber	1998	Li and Oum
1985	Simpson	1998	Li
1986	Alstrup et al.	1998	Van Ryzin and McGill
1986	Kraft et al.	1998	Zhao and Zheng
1986	Pratte	1999	Subramanian et al.
1986	Wollmer	1999	Lautenbacher and Stidham
1985	Gerchak et al.	2000	Ryzin and McGill
1987	Gerchak and Parlar	2002	Gosavi et al.
1989	McGill	2003	Bertsimas and Shioda
1989	Belobaba	2003	Brumelle and Walczak
1989	Pfeifer	2005	Koide and Ishii
1990	Brumelle et al.	2005	Ratliff
1991	Weatherford	2005	Savin et al.
1992	Stone and Diamond	2005	Zhang and Cooper
1992	Sun	2009	Kunnumkal and Topaloglu
1992	Wollmer	2009	Ball and Queyranne
1993	Weatherford, Bodily, and Pfeifer	2010	Diwan
1993	Brumelle and McGill	2018	Ma et al.
1993	Lee and Hersh		

Table 2: Origin-Destination Based Inventory Control

Year	Reference	Year	Reference
1982	D'Sylva	1996	Talluri and van Ryzin
1982	Glover et al.	1997	Garcia-Diaz and Kuyumcu
1983	Wang	1999	Ciancimino et al.
1985	Simpson	1999a,b	Talluri and van Ryzin
1986a,b	Wollmer	2001	Feng and Xiao
1987a,b	Belobaba	2001	Talluri
1988	Dror et al.	2002	De Boer et al.
1988	Smith and Penn	2003	Bertsimas and Popescu
1988	Williamson	2004	Möller et al.
1988	Wysong	2004	El-Haber and El-Taha
1989	Simpson	2004	Pölt
1989	Vinod	2004	Gallego and Phillips
1990	Curry	2005	Lai and Ng
1990	Vinod and Ratliff	2007	Farias and Van Roy
1990	Vinod	2007	Zhang and Adelman
1990	Wong	2008	Liu and van Ryzin
1991	Phillips et al.	2008	van Ryzin and Vulcano
1991	Vinod	2009	Shumsky and Zhang
1992	Williamson	2010	Kunnumkal and Topaloglu
1993	Talluri	2010	Perakis and Roels
1993	Wong et al.	2014	Tong and Topaloglu
1994a,b	Talluri	2015	Vossen and Zhang
1995	Vinod	2016	Kunnumkal and Talluri

Static and Dynamic Inventory Control Model

According to customer arrival patterns, the static and dynamic models are introduced as the second classification for inventory control. To verify whether the model is static or dynamic, we first review the definition of protected seats, protection levels, and the basic concept of two existing approaches to seat inventory control. According to McGill and Van Ryzin (1999), the protection level means the total number of seats being restricted to bookings and protected for that fare class and all higher fare classes in the nested booking systems. Lee and Hersh (1993) compared non-nested seat-allocation approaches and nested booking-limit approaches to seat inventory control. By applying the former approach, revenue might be lost even without full capacity since some high-value reservations might be denied. However, the nested booking-limit approach can easily overcome the enormous drawback.

Based on the properties of the nested booking-limit approach, static models are introduced that allow accepting bookings one time each. Furthermore, the models assumed all bookings with higher values come later than those with lower values. In terms of multiple fare classes, an independence assumption was also required between each. Static models do not adjust the protection levels during the booking process. Static models' treatments for inventory control problems are presented in Littlewood (1972); Bhatia and Parekh (1973); Belobaba (1989); Curry (1990); Wollmer (1992); Brumelle and McGill (1993); Robinson (1995). This literature review for static models is few since this paper will apply a dynamic model for the walk out model.

Different from static models, dynamic models relax some assumptions in static models. Dynamic Programming (DP) models keep revising the booking limits as time goes on, optimizing total expected revenue. In dynamic programming (DP), instead of strict assumptions on customer arrival patterns, the demand for booking requests becomes a stochastic process. Another advantage of DP is allowing multiple bookings to be accepted when the expected marginal seat value is no longer applicable. Ladany and Bedi (1977) and Hersh and Ladany (1978) solved the seat allocation issue with a DP model for a two-leg flight without passenger boarding in the intermediate stop. Gerchak et al. (1985) formulated the backward-recursion DP model to get an optimal solution for a bagel selling discount problem. This research can be treated as equivalent to an approach to evaluate the optimal expected revenues and profits under a multiclass-demand environment by following optimal accept or deny policy decisions.

Afterward, Lee and Hersh (1993) extended DP models for multiple booking classes with and without numerous seat bookings. However, the monotonicity results from Lee and Hersh (1993) are refuted by both Kleywegt and Papastavrou (1998) and Brumelle and Walczak (2003) with counterexamples. The latter showed that the structural properties would break down when multiple seats are treated as one single request to be accepted or denied. On the other hand, subramanian1999airline upgraded Lee and Hersh's single-seat discrete-time DP model considering overbooking, cancellations, and no-shows. According to the numerical tests, the results of the optimal booking policy were not the same compared to the results from the previous DP model without overbooking, cancellations, and no-shows. The booking limits for the rest of the booking periods do not have to be monotonic. Both the book and available capacity left are critical to the optimal booking policy. Moreover, the researchers also extended the DP model with class-dependent cancellation and no-show rates.

Our paper will apply the basic single-seat discrete-time DP model from Subramanian et al. (1999), while the customer types will be separated into two.

Table 3 presents the research on inventory control problems solved by dynamic programming. The list is summarized chronologically by McGill and Van Ryzin (1999); Chiang et al. (2007); Anderson and Xie (2014).

Table 3: Dynamic programming

Year	Reference	Year	Reference
1976	Mayer	1998	Zhao and Zheng
1977	Ladany and Bedi	1999	Lautenbacher and Stidham
1992	Stone and Diamond	1999	Zhao
1992	Sun	1999	Subramanian et al.
1993	Lee and Hersh	2003	Bertsimas and Popescu
1994	Shaykevich	2003	Bertsimas and Shioda
1994	Young and Van Slyke	2003	Brumelle and Walczak
1996	Bertsekas and Tsitsiklis	2004	El-Haber and El-Taha
1997	Brumelle and Walczak	2005	Bertsimas and de Boer
1997	Birge and Louveaux	2005	Savin et al.
1998	Brumelle and Walczak	2007	Powell

Inventory Control Application in Hotel and Other Industries

As mentioned in Section 2.1 Revenue Management, perishable inventory, products sold in advance, fluctuating demand, low marginal sales costs, and high marginal production are the key properties of those industries where yield management exists and is needed (Kimes, 1989). Based on these properties, loads of factors such as booking patterns, overbooking policies, pricing, and segments were studied for optimal solutions in the RM. Later, a new term named perishable asset revenue management or PARM is defined (Weatherford, 1991). Instead of mainly in the airline industry, PARM generalized the optimal trade-off between average revenue and capacity utilization applicable in various industries. Therefore, PARM has become a more appropriate word than yield management to describe properties such as perishability, ability to segment the market, and fixed capacity from all general businesses instead of limiting to airline and hospitality industries. Moreover, he summarized 13 essential elements, including capacity, prices, discount price class, reservation demand, show up of both discount and full-price reservations, decision rules, etc., in the yield management field. This taxonomy helps scholars try not to miss any relevant important element in their research on revenue management models. In the PARM situation, the inventory control is based on an optimal decision rule created under certain assumptions according to the taxonomy of 13 elements (Weatherford et al., 1993). The optimal surface with a possible buildup curve for two price classes represents the advanced static decision rule with an optimal discount sales period.

Regarding hotel inventory control issues, the hotel overbooking problem was studied by Rothstein for the existence of cancellations and no-shows (Rothstein, 1974). An easy-to-apply decision tool was designed to allocate different types of rooms for customers by Ladany (1976). The maximal expected total net profit was generated by data on the book numbers of rooms and the period to arrival date. One year later, Ladany (1977)further developed a model to find the optimal decision rules for single-bed and double-bed allocations with consideration of the complexity of customer arrival patterns which includes overbooking, cancellations, no-shows, and standbys. Moreover, the OD based inventory control in the airline industry and the multiple-night stay in the hotel industry are analogical. A stochastic and dynamic programming model derived optimal heuristic solutions for multiple types of hotel rooms and multiple nights (Bitran and Mondschein, 1995). The literature review on multiple-night stays is limited since we only discuss one-night stays instead of multiple-night stays in the present paper.

The inventory control problem for hotels is studied since the 1970s for decades. It includes customer booking patterns, with the extension on the application of overbookings, numbers of night stays, etc. As we entered the 2000s, a new generation of internet, inventory control is more than focusing only on segments but channels. Online travel agents (OTA) are critical platforms that help on selling unsold inventory for hotels. Moreover, the reservation volume increases on the hotels' own websites when they list themselves on OTA channels (**?**). Another aspect of inventory management is related to the loyalty program. We will give a thorough review of the loyalty program and its impact on inventory control and revenue management in Section 2.3.

Overbooking

Overbooking is one of the most critical tools in inventory management. Hence, we review what overbooking is, why overbooking is important and how it impacts the business performance independently. It is documented that 10-15 percent of flight travelers do not show up without any notice which directly caused around 50 million dollars in revenue loss per year for an airline. And American Airlines saved an estimated 1.4 billion dollars over a three-year period by applying the overbooking strategy (Suzuki, 2006).

The origin of overbooking is due to the existence of uncertainty of customers showing up for an event, such as taking a flight and checking in a hotel room, etc. No-shows, cancellations, change of plans including postpone or early departure are all possible reasons that might cause a vacant seat or room for an event. The vacancy, therefore, brings a revenue loss. In the early stage of overbooking studies, Beckmann (1958) calculated the optimal limits of overselling problems with consideration of gamma distributions of cancellation and no-shows. The author mentioned the reason for studying the overbooking problem is to solve the dilemma of whether the company should stop selling goods when the total capacity is fully booked. The optimal overbooking limits are dedicated to minimizing the revenue loss and optimizing profit since stopping selling might cause revenue loss while overselling puts the company at risk of walking out valid customers. If a customer is walked out, this means that the hotel need to help to re-accommodate the customers with extra compensation. Afterward, a model that gave a negative binomial distribution for total demand is formulated in the form of gamma distribution with Poisson random errors by Lyle (1970).

McGill and Van Ryzin (1999) reviewed overbooking papers which are separated into dynamic and non-dynamic optimization models before 2000. Various complex statistic models in static fashion were studied to obtain optimal overbooking limits in the early stage. Rothstein (1968) developed the first dynamic programming models for overbooking which the results are run and reviewed in airline industries. Similarly, Ladany (1976, 1977); Ladany and Arbel (1991); Liberman and Yechiali (1977, 1978) solved optimal overbooking limits by dynamic models in motel and hotel industries with consideration of the cancellation and booking status based on each time period instead of omitting updated information after any single overbooking decision.

Based on McGill and Van Ryzin (1999); Chiang et al. (2007) continued to review papers relating to overbooking before 2007. Research on overbooking went wider and deeper. Hadjinicola and Panayi (1997) concluded that a hotel could save more costs if the overbooking limit policy is treated integrally instead of distributed to multiple tour operators. Biyalogorsky et al. (1999) mentioned that sellers can increase the overbooking limits when consumers who have high buying power show up. Instead, they can cancel the other consumers with lower buying power with some compensation. Toh and Dekay (2002) discussed thorough details on factors that might be involved in executing an overbooking model for a hotel. This includes early departure and stay over, walk out as well as how these factors connect to customer service level.

Additionally, we reviewed some papers specifically relating to hotel overbooking since 2007. Ivanov (2007) discovered dynamic overbooking limits based on whether the hotel bookings are guaranteed or not. Moreover, Ivanov (2015) explored optimal overbooking limits in another aspect which considers both upgrades and downgrades between three room types. Phumchusri and Maneesophon (2014) proposed optimal overbooking decisions based on the marginal cost involved with walk-out costs and vacant rooms from no-shows. Jongcheveevat et al. (2018) calculated optimal overbooking limits with joint stochastic bookings and show-up requests and room upgrades allowed. Chun and Ovchinnikov (2019)was the first paper to discuss optimal overbooking decisions for different distribution channels which are hotel own channels and OTA channels.

In summary, we conclude that many factors need to be discussed when the overbooking strategy is designed and executed. To develop a dynamic programming model for hotel overbooking, the form of demand distributions, types of cancellation reasons forming upon no-show probabilities, penalties for no-shows, pricing, empty room costs, and walk out costs are all essential to be carefully considered. In this paper, we analyze an optimal overbooking limit by considering of types of customers who are members and non-members respectively. We will offer a further review on the connection between overbooking practice and customer loyalty.

Table 4 presents the research on overbooking problems. The list is summarized chronologically by McGill and Van Ryzin (1999); Chiang et al. (2007), and this paper.

Table 4: Overbooking

Year	Reference	Year	Reference
1958	Beckmann	1989	Alstrup
1960	Kosten	1989	Brumelle and McGill
1962	Taylor	1989	McGill
1964	Deetman	1989	Alstrup
1967	Rothstein and Stone	1993	Chatwin
1968	Rothstein	1995	Dunleavy
1968	Simon	1997	Hadjinicola and Panayi
1969	Falkson	1998	Chatwin
1971a,b	Rothstein	1998	Karaesman and van Ryzin
1972	Andersson	1999	Biyalogorsky et al.
1972	Simon	1999a,b	Chatwin
1972	Vickrey	1999	Coughlan
1974	Etschmaier and Rothstein	1999	Subramanian et al.
1975	Bierman and Thomas	2002	Toh and Dekay
1975	Rothstein	2002	Ringbom and Shy
1975	Shlifer and Vardi	2004	Karaesmen and van Ryzin
1978	Liberman and Techiali	2005	Bertsimas and de Boer
1979	Nagarajan	2007	Ivanov
1983	Ruppenthal and Toh	2014	Phumchusri and Maneesophon
1985	Rothstein	2015	Ivanov
1986	Alstrup et al.	2018	Jongcheveevat et al.
1987a,b	Belobaba	2019	Ye et al.

2.3 Customer Loyalty Program

The origin of loyalty programs (LPs) started in the airline industry for enlarging the market share in the 1980s. Afterward, LPs have been well applied in various industries such as airlines, hotels, car rental firms, book retailers, supermarkets, financial services firms, etc. Empirically, Bain Company claimed that LP makes firms more profitable by reducing service costs and price sensitivity from loyal members while increasing their spending and the power of word of mouth (Dowling and Uncles, 1997). The paper also stated that it cost less for a firm to do business with a current customer who is highly possible to be a repeat customer rather than to spend money on gaining a new customer. The objective of LPs is obvious which is to increase the number of members and their purchase frequency, reduce member attrition, generate a satisfactory return on loyalty program investment, and receive valuable market research data. Berman (2006) summarized both the potential benefits and pitfalls of an LP. An effective LP can lower price sensitivity and enhance the strong attitudes to the brand and the company. Meanwhile, the company can get more data and information from an LP. Therefore, it helps the company knows better about consumer behaviors and take more effective actions to optimize their products and business performance. The author also revealed that LPs became mandatory for companies to secure market saturation most LPs did not have any nontrivial difference.

To well understand the mechanism of LPs, it is necessary to review the definition of loyalty and customer loyalty. The economics of loyalty was presented by Shoemaker and Lewis (1999). In other words, a customer's loyalty value can be calculated by the net profit from a customer to a firm in the lifetime based on the retention rate, spending rate, costs, and discount rate. Shoemaker and

Lewis (1999) summarized some key attributes of customer loyalty from Smith (1998); Shoemaker and Lewis (1999); Utami (2015). Customer loyalty is characterized that customers choosing the firm exclusively and having emotional attachment and repeat purchase behaviors. They always have a strong attachment with the firm even with seldom purchase. The paper also explained the difference between frequency programs and loyalty programs. The latter not only focused on profitability but also on enhancing the brand relationship with loyal customers and their word-of-mouth. Uncles et al. (2003) proposed three conceptualizations of customer loyalty which were attitudinal commitment to the brand, repeat purchase behaviors, and above two perspectives combined with customer's individual characteristics. To get a better understanding of customer loyalty, they studied three aspects: Customer Brand Commitment, Customer Brand Acceptance, and Customer Brand Buying. They believe different types of definitions of loyalty help a firm to launch a more efficient loyalty program based on these three approaches respectively.

The Financial and Non-financial Impact from the Loyalty Program

There is no doubt that LPs have a strategic position within the marketing field which generates a significant impact from non-financial perspectives. Duygun (2015) stated that value perception of loyalty programs can moderate brand loyalty both directly and indirectly via direct rewards. Lentz et al. (2022) mentioned that revenue managers always ignored the overall spending from a loyal customer but focused on the average daily rate instead. The LP was considered mainly for getting repeat business by revenue managers. However, they omitted how important loyalty members' emotional attachment is to the hotel brand. The authors conducted in-depth interviews for content analysis. The content model from Figure 1 well presents how revenue management and hotel LPs are complicatedly connected with each other. The paper concluded that the goal of understanding customers is the bond between RM and LPs.

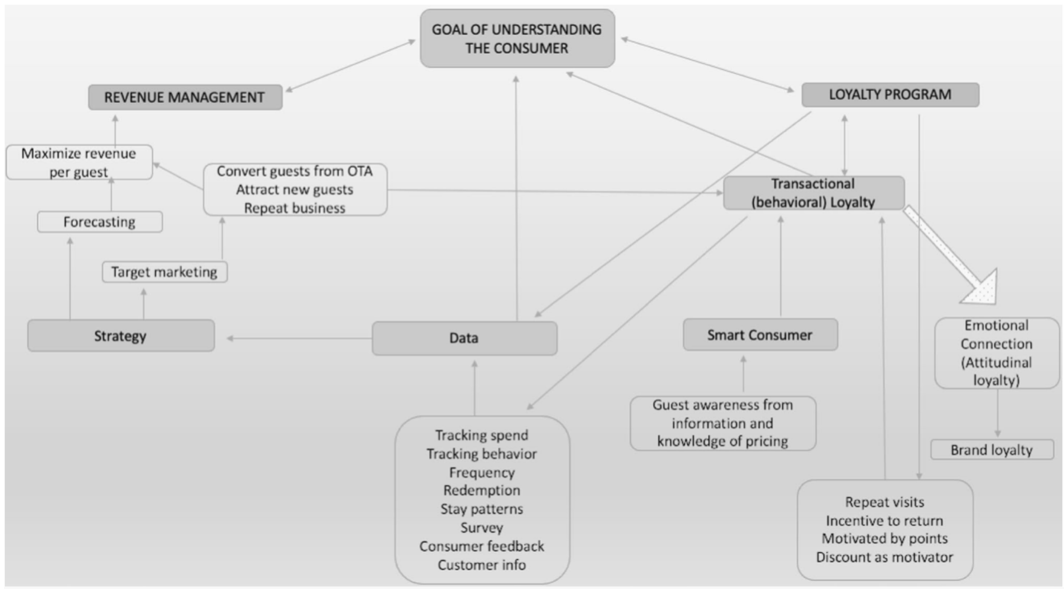


Figure 1: Content Model (Lentz et al., 2021)

However, the function and purpose of LPs are much more than only about marketing impact. Duffy (1998) formulated a consistent framework for structuring the LP. To define LP as a marketing program, it is more precise to treat it as a business strategy that optimizes the share of customers in the market. Meanwhile, points and miles in the LPs are considered as a currency for redemption, which somehow connects to both inventory control and pricing strategy. Noone et al. (2003) discussed the relationship between RM and CRM (Customer Relationship Management) based on the customer lifetime value which was analyzed by Reinartz and Kumar (2002). Evanschitzky et al. (2012) investigated the impact and the difference between customers loyal to a loyalty program and loyal to the company. The result showed that the program has a significant impact on the purchase behaviors of consumers while company loyalty focuses more on attracting consumers to visit a particular provider. Kreis and Mafael (2014) explored the relationship between perceived customer value of customer loyalty programs and motives from economic and socio-psychological perspectives. The result from the scenario-based experimental design indicated that a tailor-made customer LP helps the firm to get higher perceived customer value. Moreover, the benefits of a reward-centered LP are not only customer retention but also gaining a strong attitude to the firm. In other words, an LP with a customized design can bring significant influence from both financial and non-financial sides.

Several pieces of the literature indicated that LPs have a strong financial impact on a firm. Evanschitzky et al. (2012) took one example from the real-life is that co-branded airline customer loyalty cards generated more than 4 billion U.S. dollars in annual revenue for the top seven legacy airlines. Moreover, Singh et al. (2008) mentioned that the customer LP is affirmatively profitable no matter whether a market supported a symmetric or asymmetric equilibrium on pricing for competition. Lee et al. (2014) claimed that though hotel LPs do not support increasing revenue, the programs have a significant financial impact on both hotel's occupancy rate and profitability based on empirical results from 36 hotel brands database. The result indicated that a 1 percent increase in LP investment can bring a 12 percent increase in total operating margin. Chun et al. (2020)also considered loyalty points as a new currency. They redefined the function of LP as a hedging tool to offset some uncertainty in the operating performance.

Overbooking Practice and Customer Loyalty

Knowing the importance and benefits of customer loyalty, we narrow the exploration into the relationship between overbooking and customer loyalty. Several literatures studied how overbooking and customer loyalty interact with each other in various methods.

Hwang and Wen (2009) is the first paper to empirically study the customer reactions to hotel overbooking and how overbooking involves customer relationship management. According to a one-way ANOVA analysis, the paper indicated that gender and membership status are the main two factors that significantly affect the customer's perceived fairness of the hotel overbooking. To make customers have a positive perception of fairness in the overbooking policy, hotels must spend money on compensation for walking out customers. Moreover, the customers perceived fairness to the overbooking and compensation policies has a strong impact on the word of mouth for the hotels. Additionally, the study suggests that hotels should avoid walking out female members for minimal impact on word of mouth for the hotels.

Noone and Lee (2011) found out that the intention of retention from customers who were walked out due to hotel overbooking cannot be easily changed even with overcompensation. Moreover, the customer satisfaction rating was impacted by additional compensation types such as cash-based and voucher-based compensation. The former method led to a more positive result in customer satisfaction with the service failure experience.

Vaeztehrani et al. (2015) proposed a method to optimize revenue on capacity allocation and overbooking decisions with consideration of customer relationship management (CRM). They defined the customer types into occasional customers and loyal customers based on their lifetime values. Then stochastic dynamic programming was formulated to find the optimal solution with consideration of loyal customers who are allowed to receive a guarantee on price discount and room availability. The results indicated that loyalty programs may generate a decrease in net revenue in a short run while may also have a positive increase in expected net revenue up to 3.5 percent in a long run.

In summary, hotel overbooking is one of the service failure experiences for customers who are walked out. This action from a hotel directly affects customer satisfaction, especially to the customers from the loyalty program. Whether the operation of a loyalty program is appropriate can bring a significant impact directly and indirectly on many aspects such as marketing, brand image, and profitability. Therefore, this paper is dedicated to exploring the optimal overbooking limits for different types of customers. This study has managerial implications to minimize both financial and non-financial damages caused by overbooking loyal customers.

3 Model Development

In this section, two models are introduced: the walk out model and the overbooking model with the walk out model integrated. The walk out model is formulated as a dynamic programming model, whereas the overbooking model is formulated with the method of marginal analysis. The walk out model serves at the stay date for achieving the minimal walk out cost in total under the multi-class scenario. Therefore, the walk out model and the overbooking model is tightly connected to solving the optimal overbooking levels for different types of customers. In the following subsections, we show further details on the walk out model first in section 3.1, followed by the overbooking model with the walk out model integrated in section 3.2.

3.1 Walk Out Model

The walk out model is the first part of the entire model development. It offers the decision whether to accept or walk out a non-member customer coming to the front desk at a specific time on the stay date based on the reservation on hands from different types of customers and the current capacity availability. In this section, model notations and assumptions are introduced first, followed by the explanation of the walk out model development.

3.1.1 Walk Out Model Notations and Assumptions

Table 5 presents the notations for the walk out model.

Table 5: Walk Out Model Notations

Notations	
C	total available capacity for a hotel standard room
q_m	no-show probability for member customers
q_n	no-show probability for non-member customers
r	hotel standard room price for both member and non-member customers
w_m	walk out cost of one room night for a member customer
w_n	walk out cost of one room night for a non-member customer
c	available capacity rooms left at the current stage
m	numbers of reserved room nights for member customers left to check-in, random variable
n	numbers of reserved room nights for non-member customers left to check-in, random variable
$Y(m)$	member shows up follows a binomial(m, $1 - q_m$) distribution
$Y(n)$	non-member shows up follows a binomial(n, $1 - q_n$) distribution
$U_c(m, n)$	the minimum walk out cost on the stay-date at stage c with m and n on hands left to check in
$D_c(m, n)$	the walk out out decision to the non-member customer on the stay-date at stage c with m and n on hands left to check in

We build our models based on 13 elements summarized from a taxonomy and research overview of perishable-asset revenue management - Yield management, overbooking, and pricing by Weatherford and Bodily (1992). From Figure 2, A. resources, B. capacity, C. prices, F. reservation demand, H. show up of full-price reservation, and M. Decision Rule are the key elements assist to describe our model clearly.

TABLE I

Comprehensive Taxonomy

Elements	Descriptors
A. RESOURCE	*Discrete / Continuous*
B. CAPACITY	*Fixed / Nonfixed*
C. PRICES	*Predetermined / Set optimally / Set jointly*
D. WILLINGNESS TO PAY	*Buildup / Drawdown*
E. DISCOUNT PRICE CLASSES	*1 / 2 / 3 / . . . /I*
F. RESERVATION DEMAND	*Deterministic / Mixed / Random-independent / Random-correlated*
G. SHOW UP OF DISCOUNT RESERVATION	*Certain / Uncertain without cancellation / Uncertain with cancellation*
H. SHOW UP OF FULL-PRICE RESERVATION	*Certain / Uncertain without cancellation / Uncertain with cancellation*
I. DIVERSION	*No / Yes*
J. DISPLACEMENT	*No / Yes*
K. BUMPING PROCEDURE	*None / Full-price / Discount / FCFS / Auction*
L. ASSET CONTROL MECHANISM	*Distinct / Nested*
M. DECISION RULE	*Simple static / Advanced static / Dynamic*

Figure 2: Taxonomy of Perishable-asset Revenue Management

Based on the key elements mentioned in the Figure 2, we introduce the assumption of the walk out model. For the element B. capacity and C. prices, let hotel total capacity be fixed at C ($C > 0$) and price be predetermined at r ($r > 0$) for the standard room type for two types of customers: member and non-member customers. The m and n ($m, n \geq 0$) represent the reservations on the book for member and non-member customers, where the reservation demand for two types of customers are defined as random-independent.For the element H. show up of full-price reservation, we assume that the no-show probability for members and non-members are q_m and q_n ($q_m, q_n \geq 0$), respectively. Let the walk out cost of one standard room for one night be W_m for a member customer and be W_n for a non-member customer. Additionally, we assume that the walk out costs $W_m > W_n > r$. Last but not least, the decision rule (element M) of the walk out model is dynamic. Using the backward algorithm, the $U_c(m, n)$ is calculated as the total expected walk out costs at c stage. Accordingly, the $D_c(m, n)$ is presented as the walk out decision to follow for the hotel.

After the basic elements are satisfied, we have some extra fundamental assumptions for the walk out model.

- *Assumption* 1 - The hotel stops accepting neither new reservations or walk-ins on the stay-date (i.e. all the reservations have been made into the reservation system prior to the stay-date).
- *Assumption* 2 - The length of stay for all reservations is one. We do not consider multiple length of stay in this model.
- *Assumption* 3 - The walk out cost for each room is only for one customer instead of two or more.
- *Assumption* 4 - On the stay-date, we assume customers arrive randomly proportionate to the number of reservations by class with the hotel potentially proactively (i.e. with rooms still available) walking non-members to avoid potentially walking members.
- *Assumption* 5 - Each stage of the walk out model, at most one walk out decision is made.
- *Assumption* 6 - If the upcoming customer does not show up, no walk out decision need to be made at the current stage.
- *Assumption* 7 - Each customer holding a room reservation is with a no-show probability q_m or q_n based on the customer types. Take members as an example, let $Y(m)$ denote the number of members who show up for the reservation on the stay-date, given that the number of reserved rooms is m before end of the stay-date, so that $m - Y(m)$ is the number of no-shows for members. Because each customer has a probability $1-q_m$ of showing up for the reservation, it is clear that $Y(m)$ has a binomial $(m, 1 - q_m)$ distribution. Same assumption on the no-show probability is set for non-members.

3.1.2 Walk Out Model Development

This section, the main mechanism is introduced for the walk out model. Figure 3 shows the logic of one stage among all stages of the walk out model on the stay-date. The stage should starts from C, the total capacity of the hotel. For each stage, the stage name is addressed at Stage c, which represents the hotel has c available rooms left for the stay-date. We assume that there are m and n reservations on hands to be handled at the Stage c. At each stage, the model first checks whether the hotel still have available rooms (i.e. $c \neq 0$). If the hotel is still available to accommodate customers, then whether total reservations on the books (i.e. $m+n = 0$) is checked. The walk out model immediately terminates the entire process once c or $m + n$ hits 0 (The terminate stage is displayed with * in the Figure 3).

Otherwise, the model proceeds to make the walk out decisions and continue to the next stage which is addressed at Stage $c - 1$. Accordingly, we have total eight possible scenarios at each stage. We first introduce two scenarios where the conditions of termination are hit. Then we explain the rest six scenarios after several complicate conditions are checked.

$Scenario$ 1

In Scenario 1, $c = 0$ represents no available room for the entire hotel on the stay-date. All customers regardless of the customer types (m and n) have to be walked out. Additionally, the current stage is the final stage of the walk out model since no further walk out decision need to be made. The expected walk out cost at the current stage is expressed by the number of the on the books for members and non-members, related no-show rates, and walk out costs for

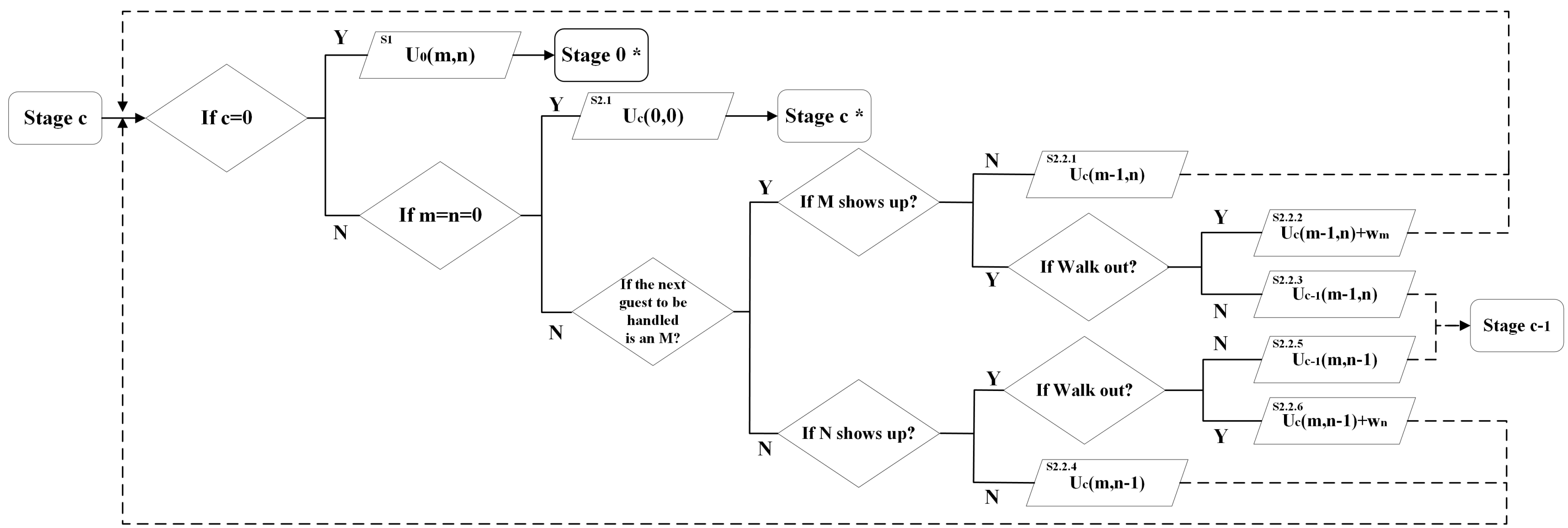


Figure 3: Walk Out Model Flowchart

each type of customer. The expression of the expected walk out costs and the decision show in the $S1$.

- $S1$ - Decision: No decision is needed (the stage is terminated).

$$U_0(m,n) = (1-q_m)*m*W_m + (1-q_n)*n*W_n, \quad (m,n \geq 0, c=0) \tag{1}$$

Scenario 2

In Scenario 2, whether no reservation in the system (i.e. $m = n = 0$) needs to be handled is checked based on $c \neq 0$. If no reservation in the system awaits to be handled, $S2.1$ is realized. Otherwise, the model checks which type of customer is going to be handled in the current stage. Though our model does not require a specific sequence of customers coming, to explain the process of the model better, we assume that each reservation in the system is numbered in order and will come with a certain no-show probability. There are three possible scenarios for each type of customers (i.e. $S2.2.1 - S2.2.3$ for members and $S2.2.4 - S2.2.6$ for non-members).

$S2.1$ represents all reservations on hands have been checked in for the stay-date while hotel still has empty rooms. Therefore, there is no walk out cost generated. The stage is the final stage since no further walk out decision need to be made.

- $S2.1$ - Decision: No decision is needed (the stage is terminated).

$$U_c(0,0) = 0, \quad (m = n = 0, c \neq 0) \tag{2}$$

If the next reservation in the system awaits to be handled is a member reservation, the model first checks whether the customer shows up or not. This member reservation is handled by default if the customer does not show up. Meanwhile, the number of member reservations that await to be handled should decrease by one. At the current stage, the available capacity remains unchanged. The model proceeds to $S2.2.1$ with the updated expression on the expected walk out costs and decisions.

- $S2.2.1$ - Decision: No decision is needed.

$$U_c(m-1,n) \tag{3}$$

However, if the member reservation shows up, the hotel need to make a decision whether to walk out this customer or not based on current available capacity left. If the hotel decides to walk out this member, $S2.2.2$ is realized where the walk out cost for a member is spent. Similar to the 2.2.1, the number of member reservations that await to be handled should decrease by one. At the current stage, the available capacity still remains unchanged. The model proceeds to 2.2.2 with the updated expression on the expected walk out costs and decisions.

- $S2.2.2$ - Decision: The current member is walked out.

$$U_c(m-1,n) + w_m \tag{4}$$

The other possible decision is to accept the current member. The current member will occupy one available room. Therefore, the number of member reservations that await to be handled should decrease by one. Meantime, the number of available capacity should also decrease by one. The model proceeds to 2.2.3 with the updated expression on the expected walk out costs and decisions.

- $S2.2.3$ - Decision: The current member is accepted.

$$U_{c-1}(m-1,n) \tag{5}$$

Similarly, $S2.2.4 - S2.2.6$ are realized for non-members based on different conditions. The non-member reservation is handled by default if the customer does not show up. Meanwhile, the number of non-member reservations that await to be handled should decrease by one. At the current stage, the available capacity remains unchanged. The model proceeds to $S2.2.4$ with the updated expression on the expected walk out costs and decisions.

- $S2.2.4$ - Decision: No decision is needed.

$$U_c(m,n-1) \tag{6}$$

If the non-member reservation shows up, the hotel need to make a decision whether to walk out this customer or not based on current available capacity left. If the hotel decides to accept this non-member, $S2.2.5$ is realized. The current non-member will occupy one available room. Therefore, the number of non-member reservations that await to be handled should decrease by one. Meantime, the non-number of available capacity should also decrease by one. The model proceeds to 2.2.5 with the updated expression on the expected walk out costs and decisions.

- $S2.2.5$ - Decision: The current non-member is accepted.

$$U_{c-1}(m,n-1) \tag{7}$$

The other possible decision is to walk out the non-member. If the hotel decides to walk out this member, $S2.2.6$ is realized where the walk out cost for a non-member is spent. The number of non-member reservations that await to be handled should decrease by one. At the current stage, the available capacity still remains unchanged. The model proceeds to 2.2.6 with the updated expression on the expected walk out costs and decisions.

- $S2.2.6$ - Decision: The current non-member is walked out.

$$U_c(m, n-1) + w_n \tag{8}$$

In summary, there are total eight scenarios for the entire walk out model. Figure 3 thoroughly describes each outcome after the decision with current conditions displayed. In the Figure 3, we can observe that $S1$ and $S2.1$ among all the scenarios represent the termination stages of the walk out model. The expected walk out cost can be calculated based on the given parameters. However, the total expected walk out cost values from scenario $S2.2.1 - S2.2.6$ seems not easy to calculate since all of these stages are not the termination stages. $S2.2.1$, $S2.2.2$, $S2.2.4$ and $S2.2.6$ are the four scenarios where the model will recursively move the Stage c with updated values of m and n. $S2.2.3$ and 2.2.5 are the two scenarios where the model will proceed recursively to Stage $c-1$.

Moreover, the walk out model follows several critical rules to secure the walk out cost are minimized at each stage. *Rule* 1, *Rule* 2, *Rule* 3 and *Rule* 4 represent each logic rule need to be followed behind the decision made. $S2.2.2$, $S2.2.3$, $S2.2.5$ and $S2.2.6$ are the outcomes from those decisions accordingly. We further introduce four logic rules.

- *Rule* 1 - the decision is to walk out the current member if

$$U_c(m-1,n) + w_m < U_{c-1}(m-1,n) \tag{9}$$

- *Rule* 2 - the decision is to accept the current member if

$$U_{c-1}(m-1,n) \le U_c(m-1,n) + w_m \tag{10}$$

- *Rule* 3 - the decision is to accept the current non-member if

$$U_{c-1}(m,n-1) \le U_c(m,n-1) + w_n \tag{11}$$

- *Rule* 4 - the decision is to walk out the current non-member if

$$U_c(m,n-1) + w_n < U_{c-1}(m,n-1) \tag{12}$$

According to the assumptions and sequence of events discussed above, our objective is to minimize the expected walk out costs of operating system on the stay-date for any Stage c where $0 \le c \le C$. The beginning stage starts from $c = C$, representing the total available capacity left to check in is C. For each period, or stage, one customer who reserved his or her reservation comes to the hotel for checking in, at most a accept or walk out decision is made at each stage. A general function is formulated to denote the minimal expected walk out cost of operating the system on the stay-date over stages from c to 0. When $c \neq 0$, the minimal expected walk out cost value functions, U_c, are determined recursively by

$$U_c(m,n) = \begin{cases} 0, & m=n=0 \\ \frac{m}{m+n} * (1-q_m) * \min\{U_c(m-1,n)+w_m, U_{c-1}(m-1,n)\} & \\ +\frac{m}{m+n} * q_m * U_c(m-1,n) & \\ +\frac{n}{m+n} * (1-q_n) * \min\{U_c(m,n-1)+w_n, U_{c-1}(m,n-1)\} & \\ +\frac{n}{m+n} * q_n * U_c(m,n-1), & \text{otherwise} \end{cases} \tag{13}$$

and the minimal expected walk out cost value function at stage 0, U_0, is determined by

$$U_0(m,n) = (1-q_m) * m * w_m + (1-q_n) * n * w_n, \quad m,n \geq 0 \tag{14}$$

Using backwards-recursive algorithm, we show how to write this optimal equation for walk out model. Based on the value calculated via equation 13 and 14, the optimal decision for accepting or walking out a non-member at any stage can be presented in equation 15 and 16.

Similarly, a general function is formulated to denote the optimal walk out decision for non-members over stages from c to 0. When $c \neq 0$, the optimal walk out decision functions, D_c, are

$$D_c(m,n) = \begin{cases} -, & \text{if } m=0 \text{ or } n=0 \\ 1, & \text{if } U_{c-1}(m,n-1) - Uc(m,n-1) \leq w_n * (1-q_n) \text{ and } m,n>0 \\ 0, & \text{otherwise} \end{cases} \tag{15}$$

and the optimal walk out decision function at stage 0, D_0, is determined by

$$D_0(m,n) = 0, \quad m,n \geq 0 \tag{16}$$

where '-' represents 'No decision is needed.'; '1' represents 'The current customer is accepted.' and '0' represents 'The current customer is walked out.'.

In summary, the walk out model formulates the optimal decision on accepting or walking out a customer at any stages on the stay-date. The objective of the walk out model is get the minimal walk out cost for the hotel by following the optimal walk out decisions. This section, we demonstrate an example of the outputs from the walk out model.

Example The capacity of a hotel is 500 rooms ($C = 500$). The current time is late night of the stay-date. The number of available rooms left is 3 (Stage $c = 3$). The hotel room price r is 100 dollars per room per night for all customers. Both the no-show probabilities for members (q_m) and non-members (q_n) are 0.4. The walk out costs for members (w_m) and non-members (w_n) are 300 dollars and 150 dollars respectively. Figure 4 and Figure 5 represents the minimal walk out costs and optimal walk out decisions of the walk out model.

c = 2

m \ n	0	1	2	3	4	5	6
0	0	0	0	32	91	165	247
1	0	0	40	103	178	260	346
2	0	50	121	199	281	364	449
3	65	141	222	306	392	478	565
4	181	266	353	440	528	616	705
5	329	417	506	595	684	773	862
6	494	583	672	762	851	941	1031

c = 3

m \ n	0	1	2	3	4	5	6
0	0	0	0	0	19	62	124
1	0	0	0	23	69	133	208
2	0	0	27	78	146	222	303
3	0	32	89	160	239	321	404
4	39	103	177	256	338	422	508
5	124	203	285	370	455	542	629
6	247	332	419	506	594	682	771

Figure 4: Walk Out Model Output - Minimal Walk Out Costs

c = 2

m \ n	0	1	2	3	4	5	6
0	-	-	-	-	-	-	-
1	-	1	1	0	0	0	0
2	-	0	0	0	0	0	0
3	-	0	0	0	0	0	0
4	-	0	0	0	0	0	0
5	-	0	0	0	0	0	0
6	-	0	0	0	0	0	0

c = 3

m \ n	0	1	2	3	4	5	6
0	-	-	-	-	-	-	-
1	-	1	1	1	1	0	0
2	-	1	1	0	0	0	0
3	-	1	0	0	0	0	0
4	-	0	0	0	0	0	0
5	-	0	0	0	0	0	0
6	-	0	0	0	0	0	0

Figure 5: Walk Out Model Output - Optimal Walk Out Decisions

We demonstrate two cases which the total numbers of the reservations are the same, while the mix of members and non-members are different. It shows how different minimal expected walk out costs are in Figure 4 and how different the optimal walk out decisions are in Figure 5.

Case 1: m=1, n=4 The value of $U_{c=3}(1,4)$ is 69 dollars which is calculated to get the minimal expected walk out cost based on four values from $U_{c=3}(1,3)$, $U_{c=3}(0,4)$,$U_{c=2}(1,3)$ and $U_{c=2}(0,4)$. The value of $D_{c=3}(1,4)$ is 1, which represents the decision is to accept the current non-member.

Case 2: m=4, n=1 The value in $U_{c=3}(4,1)$ is 103 dollars which is calculated to get the minimal expected walk out cost based on four values from $U_{c=3}(4,0)$, $U_{c=3}(3,1)$,$U_{c=2}(4,0)$ and $U_{c=2}(3,1)$. The value of $D_{c=3}(4,1)$ is 0, which represents the decision is to walk out the current non-member. We should proactively walk out the non-member to avoid potentially walking out members in this case.

3.2 Overbooking Model

This section illustrates the integrated overbooking model with the walk-out model. Since the overbooking model is formulated with the marginal analysis approach, we first review the traditional overbooking model with marginal analysis. The traditional overbooking model for one type of customer helps build the foundation for the new overbooking model serving multi-class. The upcoming subsections first go through the traditional overbooking model, followed by the development of the new overbooking model. Last but not least, we conduct the validation for the new overbooking model.

3.2.1 Traditional Overbooking Model via Marginal Analysis

Before introducing the overbooking model with the walk out model, we first review the overbooking model for one type of customer via the traditional marginal analysis. The optimal overbooking level is calculated by comparing the marginal revenue and marginal cost for an incremental customer. The parameters and notations are introduced in the Table 6.

The probability $P(x)$ representing that the $(x-C)^{th}$ incremental customer will be walked out is calculated based on the total capacity C, no-show probability q and numbers of reserved rooms x, where $x > C$ is set to discussed since the scenario we focus is overbooking scenario. The probability is calculated in the cumulative density function from binomial distribution. The $P(x)$ can be presented as the equation 17

$$P(x) = \sum_{i=0}^{x-C-1} \binom{x}{i} * q^i * (1-q)^{n-i}, \quad x > C \tag{17}$$

Table 6: Traditional Marginal Analysis Overbooking Model Notations

Notations	
C	total available capacity for a hotel standard room
q	no-show probability
r	hotel standard room price
w	walk out cost of one room night
x	number of reserved rooms, random variable
$Y(x)$	customer shows up follows a binomial $(x, 1-q)$ distribution
$P(x)$	probability that the $(x-C)^{th}$ incremental customer will be walked out based on x reservations on hands
MC_x	the marginal cost of the $(x-C)^{th}$ incremental customer
MR_x	the marginal revenue of the $(x-C)^{th}$ incremental customer
b	booking limit
OB	overbooking level (OB^*: optimal overbooking level)

The marginal walk out cost (MC_x) is calculated based on equation 17 and the walk out cost w.

$$MC_x = w * P(x) \tag{18}$$

The marginal revenue (MR_x) is calculated based on equation 17 and the hotel standard room price r.

$$MR_x = r * (1 - P(x)) \tag{19}$$

The booking limit is calculated based on marginal walk out cost from equation 18 and marginal revenue from equation 19.

$$b = \max\{x : MC_x \leq MR_x\} \tag{20}$$

Accordingly, the optimal overbooking level is calculated based on the booking limit from equation 20.

$$OB^* = \begin{cases} b - C, & \text{if } b > C \\ 0, & \text{otherwise} \end{cases} \tag{21}$$

3.2.2 New Overbooking Model Development

In Table 7, we first introduce the additional parameters and notations of the new overbooking model with the walk out model we formulated in the section 3.1.

Table 7: Overbooking Model Notations

Notations	
$P_{(m,n)}$	probability that the incremental customer will be walked out based on m and n reservations on hands
MC_m	the marginal cost of an incremental member customer based on m and n reservations on hands
MC_n	the marginal cost of an incremental non-member customer based on m and n reservations on hands
MR	the marginal revenue of an incremental customer based on m and n reservations on hands
b_m	booking limit of member customers
b_n	booking limit of non-member customers
OB_m	overbooking level of member customers (OB_m^*: optimal overbooking level of member customers)
OB_n	overbooking level of non-member customer (OB_n^*: optimal overbooking level of non-member customers)

Compared with the traditional overbooking model via marginal analysis, we add parameters for an additional type of customer. In our model, We assume that there are only two types of customers who are members and non-members. To differentiate these two types of customers, we use m and n in the subscript to represent member and non-members related parameters respectively.

By implementing the walk out model, the new overbooking model is dedicated to achieving optimal overbooking levels for multiple classes (i.e. loyal members and non-members). There are two main challenges in the formulation of the new overbooking model. First, members and non-members who show up following different binomial distributions should be considered when calculating the probability that an incremental customer will be walked out. Second, the walk out model's minimal walk out costs are expected instead of marginal walk out costs. It is necessary to transfer the expected minimal walk out costs to marginal walk out costs since we still apply the marginal analysis technique in the new overbooking model.

To solve the first challenge, the probability that the incremental customer will be walked out based on m and n reservation on hands are calculated. The probabilities vary based on the value of m and n. Therefore, a 2-Dimensional probability table is generated with the help of numpy package and binom from scipy.stats package in python. The table is in the scale of $(m + 1) * (n + 1)$. The Figure 6 is the calculation code for the $P_{(m,n)}$. The output Ptable[m][n] matches the value of $P_{(m,n)}$.

```
Ptable = np.zeros((m + 1, n + 1))
P = 0
for x in range(m + 1):
    for y in range(n + 1):
        if m + n > C:
            for i in range(x + y - C):
                for k in range(x + 1):
                    if k <= i:
                        P += binom.pmf ( k, x, qm ) * binom.pmf ( i-k, y, qn )
                        Ptable[m][n] += P
                        P = 0
```

Figure 6: $P(m, n)$ Calculation Coding in Python

The marginal walk out cost (MC_m) is calculated based on The Figure 6, the member no-show probability q_m, and the expected walk out costs from the walk out model.

$$MC_m = \frac{U_{c=C}(m, n) - U_{c=C}(m-1, n)}{1 - q_m} \tag{22}$$

The marginal walk out cost (MC_n) is calculated based on The Figure 6, the non-member no-show probability q_n, and the expected walk out costs from the walk out model.

$$MC_n = \frac{U_{c=C}(m, n) - U_{c=C}(m, n-1)}{1 - q_n} \tag{23}$$

The marginal revenue (MR) is calculated based on Figure 6 and the hotel standard room price r.

$$MR = r * (1 - P_{(m,n)}) \tag{24}$$

The booking limit of members is calculated based on the marginal walk out cost from equation 22 and marginal revenue from equation 24.

$$b_m = \max\{m : MC_m \leq MR\} \tag{25}$$

The booking limit of non-members is calculated based on the marginal walk out cost from equation 23 and marginal revenue from equation 24.

$$b_n = \max\{n : MC_n \leq MR\} \tag{26}$$

Accordingly, the optimal overbooking level of members is calculated based on the booking limit of members from equation 25.

$$OB_m^* = \begin{cases} b_m - C, & \text{if } b_m > C \\ 0, & \text{otherwise} \end{cases} \tag{27}$$

Similarly, the optimal overbooking level of non-members is calculated based on the booking limit of non-members from equation 26.

$$OB_n^* = \begin{cases} b_n - C, & \text{if } b_n > C \\ 0, & \text{otherwise} \end{cases} \tag{28}$$

In summary, the overbooking model with the walk out model provides the optimal overbooking levels for multiple classes (i.e. loyal members and non-members). The integrated model helps a hotel to optimize profit via specifying the optimal overbooking levels by class and executing proactive walk out decisions during the operational process. This section, we demonstrate an example of the outputs from the overbooking model with the walk out model.

Example The capacity of a hotel is 5 rooms ($C = 5$). The hotel room price r is 100 dollars per room per night for both members and non-members. Both the no-show probabilities for members (q_m) and non-members (q_n) are 0.4. The walk out costs for non-members (w_n) is 150 dollars. Two different values are assigned to the walk out costs for members (w_m) in two cases discussed in this section. The optimal overbooking levels for members and non-members are provided.

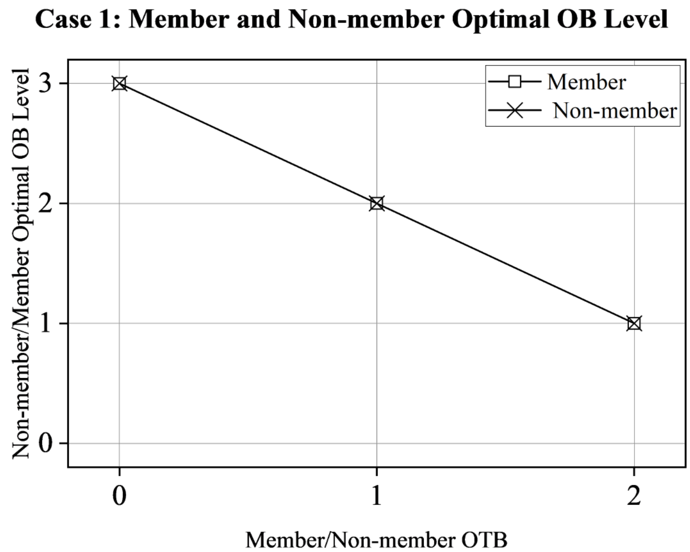


Figure 7: Walk Out Model Output - Minimal Walk Out Costs

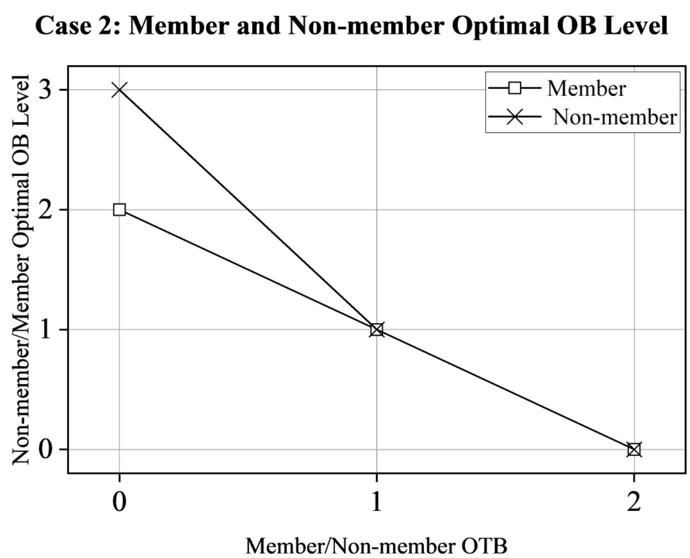


Figure 8: Walk Out Model Output - Optimal Walk Out Decisions

We demonstrate two cases with different walk out costs for members. The Case 1 is with same walk out cost for members and non-members. Therefore, the optimal overbooking levels for both types of customers should be at the same level. For the Case 2, we show the discrepancy between w_m and w_n. And the optimal overbooking levels for each type of customers are displayed respectively.

Case 1: $w_m = 150$**,** $w_n = 150$. From the Figure 7, the optimal overbooking level for members and non-members are at the same level. Both OB_m^* and OB_n^* are 3, 2 and 1 when the on the book of the other type of customers are 0, 1 and 2.

Case 2: $w_m = 300$**,** $w_n = 150$. From the Figure 8, the optimal overbooking level for members and non-members are at the different levels. OB_m^* are 2, 1 and 0 when the on the book of non-members are 0, 1 and 2. While, OB_n^* are 3, 1 and 0 when the on the book of members are 0, 1 and 2.

3.2.3 Validation for the New Overbooking Model

This section is to validate the new overbooking model with the walk out model. To verify the accuracy of the model, we compare the results from the new overbooking model integrated with the walk out model with the results from the traditional overbooking model which uses the marginal analysis technique.

For the traditional overbooking model with marginal analysis, We test some cases with capacity C at 20. The hotel standard room price r is 100 dollars. The walk out cost w is 150 dollars. The no-show probability q is tested at the level of 0.05, 0.1, 0.15, 0.2, 0.25, 0.3 and 0.35. Figure 9 shows the optimal overbooking levels from the traditional overbooking model with different no-show rates.

As for the overbooking model with the walk out model, we set the walk out cost for members w_m and the walk out cost for non-members w_n at the same

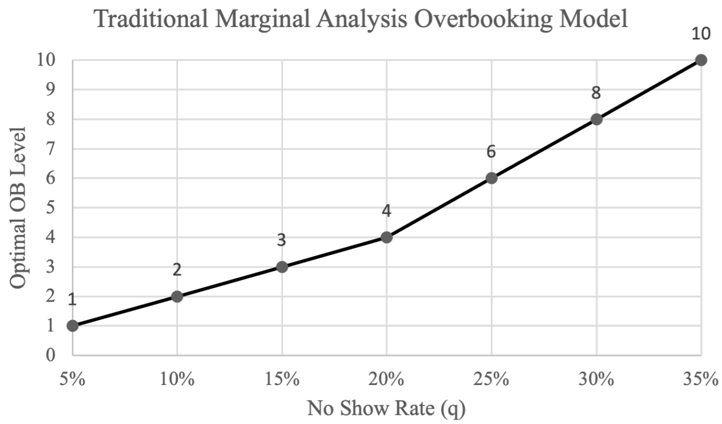


Figure 9: Traditional Marginal Analysis Overbooking Model - Standard

level, which is 150 dollars. Similarly, we set the no-show probabilities for members and non-members at the same levels, which the tests are run at the level of 0.05, 0.1, 0.15, 0.2, 0.25, 0.3 and 0.35. Figure 10 shows the optimal overbooking levels from the overbooking model with the walk out model under the different scenarios of the no-show probabilities.

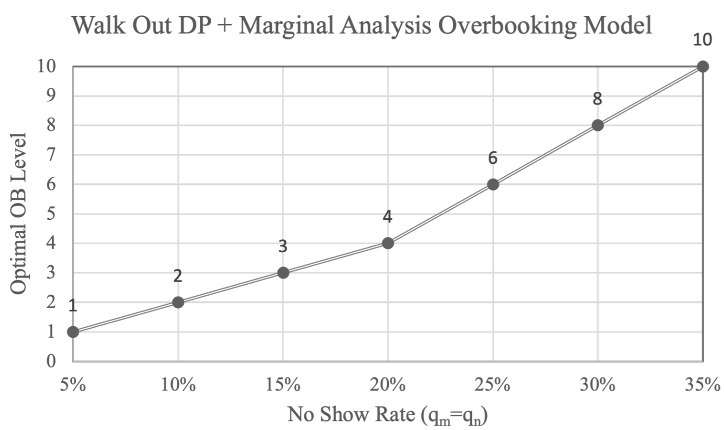


Figure 10: Overbooking Model with the Walk Out Model - Validation

The results shown from Figure9 and Figure10 are the same. Therefore, the preliminary validation is confirmed for the overbooking model with the walk out model.

4 Numerical Analysis

In this section, we run numerical tests for both the walk out model and the overbooking model. For the walk out model, we first explore how the total optimal walk out cost changes by different proportions of member and non-member reservations. Then we show how the model proactively walks the non-member customers. Moreover, we explore how the changes in the no-show rates and walk out costs impact the walk out decisions respectively. For the overbooking model, we show the optimal overbooking levels for two types of customers under several scenarios. Meanwhile, the results indicate how the no-show rates and walk out costs impact the optimal overbooking levels respectively. Last but not least, we discuss the computation complexity for both models.

4.1 Walk Out Model Numerical Tests

Optimal Walk Out Costs

The optimal walk out cost is one of the outputs of the walk out model. In this section, we explore how the total optimal walk out cost changes by the proportion of member and non-member reservations. We set two cases with different walk out costs for members at 300 and 600 dollars respectively. The other parameters are set at the same level for both cases, where the capacity is 15 and the price is 100 dollars. Both no-show rates for members and non-members are 0.15. The walk out cost for non-member is fixed at 150 dollars.

Case 1: $C = 15$**,** $r = 100$**,** $q_m = q_n = 0.15$**,** $w_m = 300$**,** $w_n = 150$

Case 2: $C = 15$**,** $r = 100$**,** $q_m = q_n = 0.15$**,** $w_m = 600$**,** $w_n = 150$

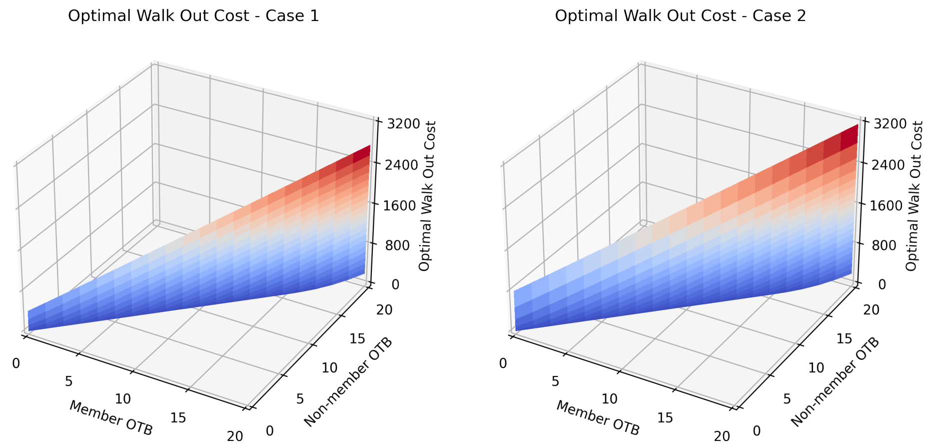


Figure 11: Optimal Walk Out Costs Surface Plots

The figure 11 shows two 3D surface plots with different levels of walk out costs for members (i.e. 300 dollars for Case 1 and 600 dollars for Case 2). The plots display the total optimal walk out costs with reservations up to 20 for each customer type. We observe that the optimal walk out cost increases monotonously when reservations increase, no matter members or non-members. Furthermore, more member reservations bring larger optimal walk out costs than the same quantity of non-member reservations. Additionally, we find that higher walk out cost for members increase the total optimal walk out costs by comparing Case 1 with Case 2.

Proactive Walk Out Decisions

The walk out model is dedicated to walking out the customers with lower walk out costs to avoid walking the customers with higher walk out costs. In this section, we run three numerical tests with different capacities to see how the walk out model proactively walks the customers as an optimal decision. For each case, we show the situation that the current stage is with full capacity available left (i.e. the beginning of the stay-date). The plotting indicates the maximum number of non-members being accepted with certain members on the book. In other words, exceeding the maximum number indicates that the non-member customer has to be walked out.

Case 1: $C = 5$**,** $r = 100$**,** $q_m = q_n = 0.3$**,** $w_m = 300$**,** $w_n = 150$

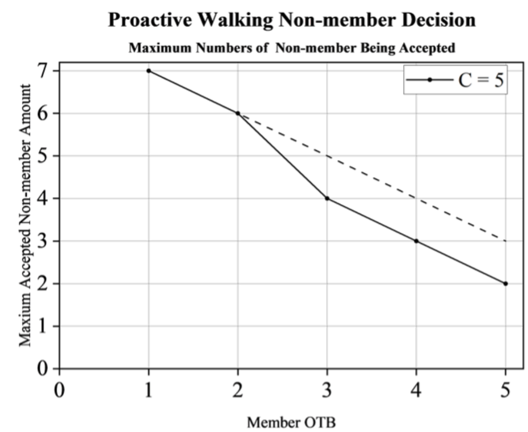


Figure 12: Proactive Walking Non-member Decision (1)

Case 2: $C = 10$**,** $r = 100$**,** $q_m = q_n = 0.3$**,** $w_m = 300$**,** $w_n = 150$

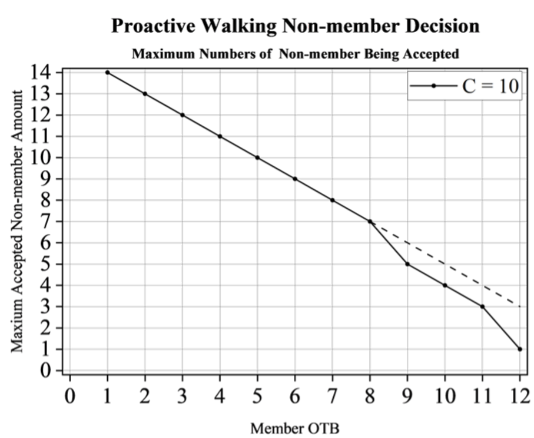


Figure 13: Proactive Walking Non-member Decision (2)

Case 3: $C = 20$**,** $r = 100$**,** $q_m = q_n = 0.3$**,** $w_m = 300$**,** $w_n = 150$

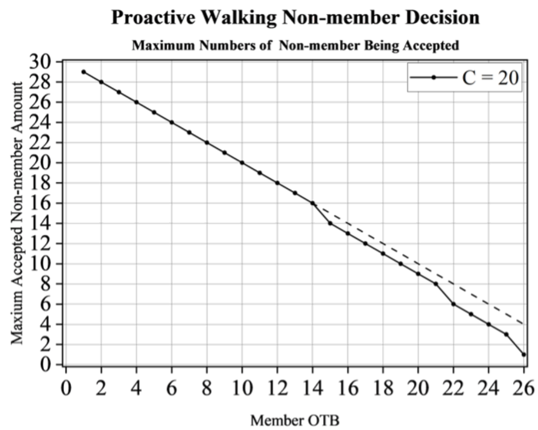


Figure 14: Proactive Walking Non-member Decision (3)

The Figure 12,13 and 14, are examples of capacity at 5,10 and 20. The parameters are set with both no-show rates fixed at the same level, while the walk out costs for members and non-members are 300 and 150 dollars, respectively. Since there are discrepancies in walk out costs between members and non-members, the walk out model has its proactive walking mechanism which walks the non-members due to their lower walk out costs. The solid lines with dots in the figures show the maximum amount of non-members being accepted based on members on the book at stage$c = 5$, $c = 10$, and $c = 20$ respectively. Each figure has another dashed line, which is always above the solid line when members on the book increase. This indicates that the maximum amount of non-members being accepted can increase to that level if both the walk out costs for members and non-members are the same. However, it is the proactive walking mechanism that exceeding non-members (i.e. above the solid line with dots) have to be walked out proactively.

Walk Out Decision Changes by No-show Rates

In this section, we study how the no-show rate impacts the optimal walk out decisions. We test three cases with different member no-show rates at 0.2, 0.3 and 0.4. The other parameters are set as capacity C at 5, hotel standard room price r is 100 dollars, non-member no-show rate is fixed at 0.2, both member and non-member walk out costs w_m and w_n are 150 dollars. The test results from

$q_m = 0.2$

$q_m = 0.3$

$q_m = 0.4$

Parameters
$C = 5$
$r = 100$
$q_m = 0.2 / 0.3 / 0.4$
$q_n = 0.2$
$w_m = 150$
$w_n = 150$

c = 5

m \ n	0	1	2	3	4	5	6	7	8	9
0	-	-	-	-	-	-	-	-	-	-
1	-	1	1	1	1	1	1	1	1	0
2	-	1	1	1	1	1	1	1	0	0
3	-	1	1	1	1	1	1	0	0	0
4	-	1	1	1	1	1	0	0	0	0
5	-	1	1	1	1	0	0	0	0	0
6	-	1	1	1	0	0	0	0	0	0
7	-	1	1	0	0	0	0	0	0	0
8	-	1	0	0	0	0	0	0	0	0
9	-	0	0	0	0	0	0	0	0	0

Figure 15: Walk Out Decision Changes by No-show Rates

Figure 15 indicate that the number of non-members to be accepted increases by the increase of the member no-show rate. The light grey cells represent the acceptance of the non-members when $q_m = 0.4$. Under this condition, we can observe that the non-member customer will be accepted when $m = 8, n = 1$ and $m = 1, n = 8$. However, as the member no-show rate decreases, (i.e. the dark grey area represents non-member acceptance when $q_m = 0.2$), the non-member can be accepted when $m = 1, n = 7$ but cannot be accepted when $m = 7, n = 1$. It requires the hotel proactively to walk the non-members to secure available rooms for those members with the small no-show rate.

Walk Out Decision Changes by Walk Out Costs

In this section, we study how the walk out cost impacts the optimal walk out decisions. We test two cases with different member walk out costs at 150 and 300 dollars. The other parameters are set as capacity C at 5, hotel standard room price r is 100 dollars, both member and non-member no-show rates are fixed at 0.4, the non-member walk out cost w_n is fixed at 150 dollars.

$w_m = 300$

$w_m = 150$

Parameters
$C = 5$
$r = 100$
$q_m = 0.4$
$q_n = 0.4$
$w_m = 150 / 300$
$w_n = 150$

c = 5

m \ n	0	1	2	3	4	5	6	7	8	9
0	-	-	-	-	-	-	-	-	-	-
1	-	1	1	1	1	1	1	1	1	0
2	-	1	1	1	1	1	1	1	0	0
3	-	1	1	1	1	1	1	0	0	0
4	-	1	1	1	1	1	0	0	0	0
5	-	1	1	1	1	0	0	0	0	0
6	-	1	1	1	0	0	0	0	0	0
7	-	1	1	0	0	0	0	0	0	0
8	-	1	0	0	0	0	0	0	0	0
9	-	0	0	0	0	0	0	0	0	0

Figure 16: Walk Out Decision Changes by Walk Out Costs

The test results from Figure 16 indicate that the number of non-members to be accepted decreases when the member walk out cost increases. The grey cells represent the acceptance of the non-members when $w_m = 150$. The non-member customer will be accepted when $m = 8, n = 1$ and $m = 1, n = 8$. However, as the member walk out cost increases to 300 (i.e. the dark grey area), the non-member can be accepted when $m = 1, n = 7$ but cannot be accepted when $m = 7, n = 1$. It requires the hotel proactively to walk the non-members to avoid members being walked out with higher walk out cost.

4.2 Overbooking Model Numerical Tests

Optimal Overbooking Level Changes by No-show Rates

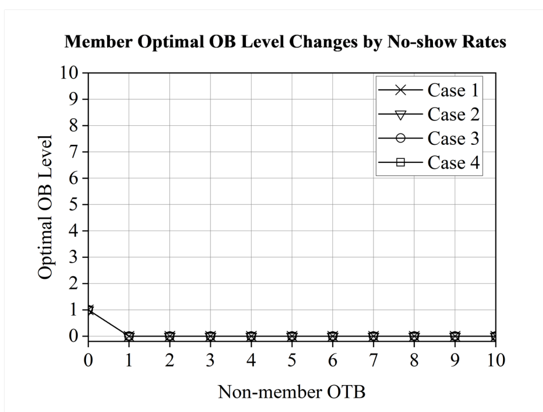
In this section, we study how the no-show rate impacts the optimal overbooking (OB) levels for members and non-members. We test 4 cases with adjustment only on non-member no-show rate. The walk out cost set up for member and non-member is at the same level.

Case 1: $C = 20$, $r = 100$, $w_m = w_n = 150$, $q_m = 0.05$, $q_n = 0.05$

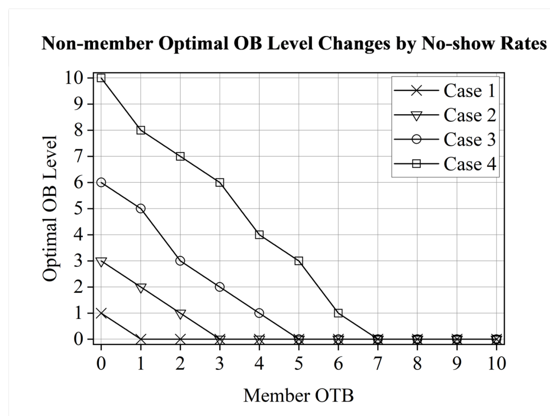
Case 2: $C = 20$, $r = 100$, $w_m = w_n = 150$, $q_m = 0.05$, $q_n = 0.15$

Case 3: $C = 20$, $r = 100$, $w_m = w_n = 150$, $q_m = 0.05$, $q_n = 0.25$

Case 4: $C = 20$, $r = 100$, $w_m = w_n = 150$, $q_m = 0.05$, $q_n = 0.35$



(a) Member Optimal OB Level Changes



(b) Non-member Optimal OB Level Changes

Figure 17: Optimal Overbooking Level Changes by No-show Rates

From the Figure 17, We can observe that the non-member no-show rate only impacts the non-member optimal overbooking level. From the Figure 17(a), it indicates the member optimal overbooking levels always maintain at the same level for all four cases.From the Figure 17(b), it shows the non-member optimal overbooking level increases by the increase of non-member no-show rate.

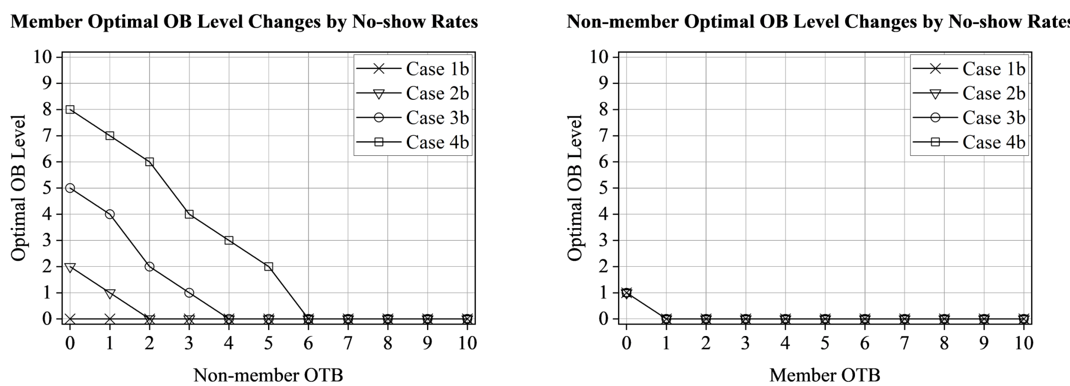
It is worth mentioning that the walk out cost set at same level for both members and non-members in above four cases. However, whether the no-show rate can impact the other type of customer's optimal overbooking level need to be checked when the walk out costs for members and non-members are set in a different level. We test additional four cases. This time, we set member walk out cost at 300 dollars while 150 dollars for non-members. Moreover, we make adjustments on q_m for the current four cases instead of q_n.

Case 1b: $C = 20$, $r = 100$, $w_m = 300$, $w_n = 150$, $q_m = 0.05$, $q_n = 0.05$

Case 2b: $C = 20$, $r = 100$, $w_m = 300$, $w_n = 150$, $q_m = 0.15$, $q_n = 0.05$

Case 3b: $C = 20$, $r = 100$, $w_m = 300$, $w_n = 150$, $q_m = 0.25$, $q_n = 0.05$

Case 4b: $C = 20$, $r = 100$, $w_m = 300$, $w_n = 150$, $q_m = 0.35$, $q_n = 0.05$



(a) Member Optimal OB Level Changes (b) Non-member Optimal OB Level Changes

Figure 18: Optimal Overbooking Level Changes by No-show Rates (b)

From the Figure 18, we confirm the speculation that the no-show rate only impact its related type of customer's optimal overbooking level. In the Figure 18(a), it shows that the member optimal overbooking level increases when the member no-show rate increases. However, the optimal overbooking level for non-member shown in the Figure 18(b) remains at the same level for all current four cases.

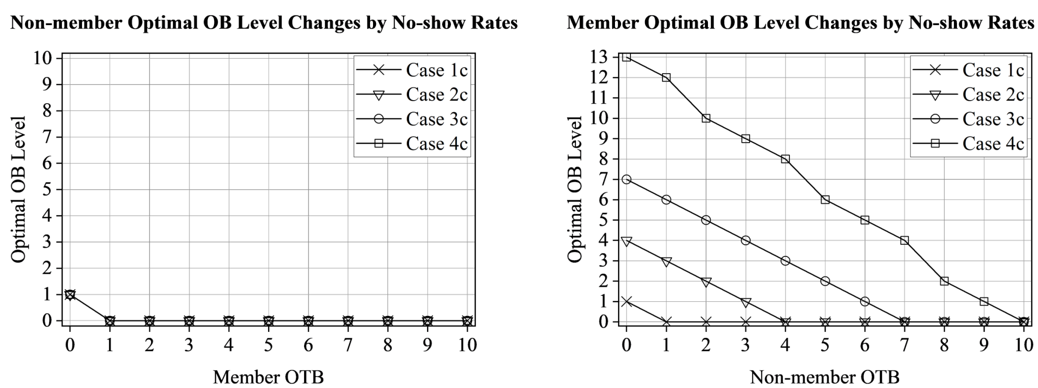
We then change the capacity to double check whether the result is validated. Based on previous cases, the capacity is adjusted to 30 instead of 20. The other parameters remain the same as the parameters from **Case 1b, Case 2b, Case 3b and Case 4b**.

Case 1c: $C = 30$, $r = 100$, $w_m = 300$, $w_n = 150$, $q_m = 0.05$, $q_n = 0.05$

Case 2c: $C = 30$, $r = 100$, $w_m = 300$, $w_n = 150$, $q_m = 0.15$, $q_n = 0.05$

Case 3c: $C = 30$, $r = 100$, $w_m = 300$, $w_n = 150$, $q_m = 0.25$, $q_n = 0.05$

Case 4c: $C = 30$, $r = 100$, $w_m = 300$, $w_n = 150$, $q_m = 0.35$, $q_n = 0.05$



(a) Member Optimal OB Level Changes (b) Non-member Optimal OB Level Changes

Figure 19: Optimal Overbooking Level Changes by No-show Rates (c)

From the Figure 19, we observe that the trend is same as cases with capacity at 20. The member optimal overbooking levels increase when the member no-show rate increases. However, changes in the member no-show rate does not impact the non-member optimal overbooking levels.

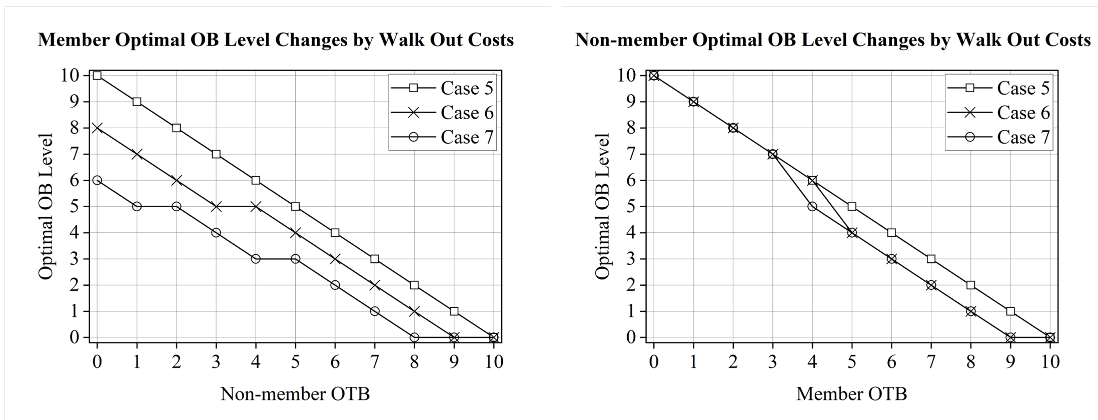
Optimal Overbooking Level Changes by Walk Out Costs

In this section, we check how the walk out cost impacts the optimal overbooking levels for members and non-members. We test cases with only adjusting the walk out cost for members.

Case 5: $C = 20$, $r = 100$, $w_m = 150$, $w_n = 150$, $q_m = q_n = 0.35$

Case 6: $C = 20$, $r = 100$, $w_m = 300$, $w_n = 150$, $q_m = q_n = 0.35$

Case 7: $C = 20$, $r = 100$, $w_m = 900$, $w_n = 150$, $q_m = q_n = 0.35$



(a) Member Optimal OB Level Changes (b) Non-member Optimal OB Level Changes

Figure 20: Optimal Overbooking Level Changes by Walk Out Costs

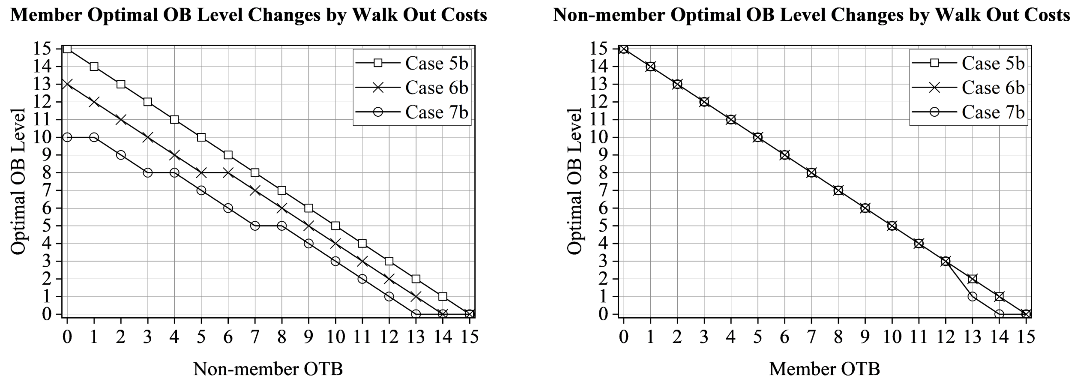
From the Figure 20, we can observe that the change in walk out cost for members will not only impact the member optimal overbooking levels as shown from the Figure 20(a) but also impact the non-member optimal overbooking levels as shown from the Figure 20(b). The increase of the walk out cost for members causes the decrease of both optimal overbooking levels for members and non-members.

We also test cases with bigger capacity to validate the result. Based on previous cases, the capacity is adjusted to 30 instead of 20. The other parameters remain the same as the parameters from **Case 5, Case 6, Case 7**.

Case 5b: $C = 30$, $r = 100$, $w_m = 150$, $w_n = 150$, $q_m = q_n = 0.35$

Case 6b: $C = 30$, $r = 100$, $w_m = 300$, $w_n = 150$, $q_m = q_n = 0.35$

Case 7b: $C = 30$, $r = 100$, $w_m = 900$, $w_n = 150$, $q_m = q_n = 0.35$



(a) Member Optimal OB Level Changes (b) Non-member Optimal OB Level Changes

Figure 21: Optimal Overbooking Level Changes by Walk Out Costs (b)

Similarly, we find that the optimal overbooking levels decrease when the walk out cost for members increases. This set of cases validate the changes in the walk out cost for one type of customers impact optimal overbooking levels for both type of customers.

4.3 Computational Complexity

In this section, we present the computational complexity for both the walk out model and the overbooking model. We also discuss how numbers of customer types and capacity impact the computation time respectively. Additionally, the computation time means how long it takes in the technique of python.

Walk Out Model

The methodology of the walk out model is dynamic programming. The computational complexity of solving the dynamic programming problem is impacted by the numbers of stages and dimensions.

The number of stages depends on the total capacity of a hotel. We run three numerical tests with capacity at 20, 50 and 150 respectively. Both no-show rates for members and non-members are set at 0.15 for three cases. The price is 100 dollars and both walk out costs for members and non-members are 150 dollars. The result of computation times for three cases are less 1 second, 3 seconds and 134 seconds, respectively. It is noticeable that the computation time does not increase significantly when the capacity increases from 20 to 50, however when capacity constantly increases to three times of 50 (i.e. 150), the computation time is much longer than the computation time with capacity at 50.

In terms of dimensions, two customer types (i.e. members and non-members) in our walk out model makes it a two dimensional model. The dimension increases to three if we add one more customer type. The walk out model becomes more complicated with computation time increase dramatically.

Overbooking Model

For the overbooking model, the computational complexity is impacted by the walk out model and the probability table for calculating marginal costs and marginal revenue.

Since the overbooking model is integrated with the walk out model, we run the same cases we discussed previously to see how the walk out model complexity affects the overbooking model. The result of computation times for capacity at 20, 50, and 150 are 10 seconds, 278 seconds, and approximately 7 hours respectively, for the overbooking model. It is obvious that the more stages (i.e. capacity) in the walk out model are, the much longer time it takes in the overbooking model. Furthermore, with more dimensions of the walk out model, the overbooking model's computation time also changes significantly.

On the other hand, the probability table also impacts the computation times. It calculates the probability of an incremental customer being walked. If the customer types increase to three or more, more binomial distributions are involved in the calculation. In python, we have to increase the loop layers for adding more types of customers, which costs a longer computation time.

5 Discussions and Limitations

We connect overbooking model with the walk out model to analyze how to obtain optimal overbooking levels for member and non-member customers respectively. The models assist hotel to generate an optimal business performance if the proportion of overbooked member and non-member customers is carefully calculated. We use a dynamic programming for the walk out model. The analysis allows the walk out model to offer the optimal walk out decisions based on the number of available capacity left and the numbers of reservations on hands for members and non-members.

The numerical analysis illustrates that the changes in one specific class no-show rates can only impact the optimal overbooking levels for the related class. However, the changes in the walk out costs impact all customer class optimal overbooking levels. With the model implementation, it offers specific guidelines for both hotel revenue managers and operations managers to minimize the walk out costs and optimize rooms profit, especially to those hotels with hug mix of loyalty members. Our analysis leads to several managerial implications that (1) revenue managers can apply the optimal overbooking policy which is specified by class; (2) operation managers can proactively walk the non-member customers by following the walk out policy to avoid potentially walking the members; (3) hotels can get a balance between optimizing room revenue and valuing member customers with higher customer satisfaction.

However, there are several limitations and extensions to this paper. The model can be investigated for two or more room types instead of one. This can extend the study with consideration of room upgrade and downgrade. Moreover, the room price can be different for different types of customers. The distribution channels should be considered since the commission rate is also a factor impacting the cost and profit. In summary, this paper is the first step of proactively walking guests with simple set up at single room type and homogeneous price, the next step can be a complexity improvement of the overbooking model with the walk out model.

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