Design Autonomous Vehicle Behaviors in Heterogeneous Traffic Flow

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by
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16. Abstract

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Design Autonomous Vehicle Behaviors in Heterogeneous Traffic Flow

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While much attention was paid to the interactions of human-driven and automated vehicles at the microscopic level in recent years, the understanding of the macroscopic properties of mixed autonomy traffic flow still remains limited. In this report, we present an equilibrium model of traffic flow with mixed autonomy based on the theory of two-player games. We consider self-interested traffic agents (i.e. human-driven and automated vehicles) endowed with different speed functions and interacting with each other simultaneously in both longitudinal and lateral dimensions. We propose a two-player game model to encapsulate their interactions and characterize the equilibria the agents may reach. We show that the model admits two types of Nash equilibria, one of which is always Pareto efficient. Based on this equilibrium structure, we propose a speed policy that guarantees the realized equilibria are Pareto efficient in all traffic regimes. We present two examples to illustrate the applications of this model. In one example, we construct flux functions for mixed autonomy traffic based on behavior characteristics of agents. In the other example, we consider a lane policy and show that mixed autonomy traffic may exhibit counterintuitive behaviors even though all the agents are rational. In addition, we present empirical evidence concerning the assumptions made in the model.

Introduction

The advent of automated vehicles (AVs) can create complications for traffic control and operations, because characters of AVs and human-driven vehicles (HVs) may be much different and need to be addressed. Their differences range from basic ones such as reaction time, sight or sensing distance, and stopping distance, to more sophisticated ones, such as driving style, capability to learn and drive cooperatively with peers, and the possibility to develop crowd intelligence and self-organize. To resolve the complications and explore the full potential of automated driving technologies, it is imperative to have a new theory of traffic flow that can capture the characters of heterogeneous traffic agents (namely vehicles and/or drivers; referred to as "agents" hereafter), describe how they interact, and ultimately, explain and forecast behaviors of mixed autonomy traffic flow. It is particularly interesting to have an analytical understanding into macroscopic behaviors of mixed autonomy traffic and how these behaviors are shaped by agent characters and interactions.

Characterizing the equilibrium relationships, either empirically or analytically, is usually the first step towards understanding traffic flow behaviors. The equilibrium relationships encapsulate key information about traffic flow (such as capacity and wave speeds) and constitute the foundation of first-order traffic flow models. In this report, we present a new approach to model the equilibriaof mixed autonomy traffic. We are interested in what equilibria can be attained by agents in mixed autonomy traffic flow when they are endowed with different behavior characters and interact with each other simultaneously. Our approach is intended to be behavior-based and constructive, as opposed to heuristic and descriptive. Our quest is related two lines of research.

Along the first line, dating back to two decades ago, researchers have started developing models of mixed human-driven traffic consisting of cars and trucks. One group of models extends the classical LWR model by specifying flux functions for the mixed flow without

explicitly considering underlying agent behaviors or lane settings. We call such models "descriptive mixed flow model". Some representative models in this group include Wong and Wong (2002) and van Lint et al. (2008). The similar idea was also used to model mixed autonomy traffic, see e.g., Levin and Boyles (2016). The advantage of the descriptive approach mainly lies in the simplicity of flux functions,which can usually be analytically defined and offer much convenience in solving a dynamic model. Nonetheless, since these equilibrium relations are descriptive in nature and do not explicitly account for how agent interact, they fall short when complex agent behaviors or control strategies need tobe considered.

In contrast, another group of models explicitly considers agent interactions and lane settingsand constructs the equilibrium relations of traffic flow from behavior rules. We call these models "behavioral mixed flow model". An early work along this line is Daganzo (1997), which considered two classes of vehicles that are endowed with the same triangular fundamental diagram and only differ in their priority to access the special lane on a two-lane road. One major contribution of Daganzo (1997) is the introduction of the Wardrop's User Equilibrium (UE) principle to describethe behavior of the prioritized vehicles. Based on this behavior rule, the equilibrium flow-density relationships (i.e. flux functions) for the mixed traffic were derived. In this model, the mixed traffic will reach either the so-called 1-pipe or 2-pipe equilibrium. Several later works pursued the similar basic idea, i.e., analyzing or deriving equilibria of mixed traffic from agent behaviors, but considered more general settings and further extended this approach. Daganzo (2002a,b) considered the interactions of two groups of agents (called "slugs" and "rabbits") with more sophisticated behaviors on multilane freeways and provided explanations to a few phenomena not well explained by the single-class theory. Nonetheless, the equilibria of mixed traffic are not explicitly derived. Logghe and Immers (2008) considered mixed traffic of cars and trucks, which are endowed with different triangular fundamental diagrams. A division factor, which represents the lateral space division between the two classes of vehicles, was introduced to model the interaction of the two classes of vehicles. This division factor is heuristically determined by prescribing that either 1-pipe or 2-pipe equilibrium will be attained. Qian et al. (2017) adopted a similar modeling framework, but defined the division factor from a new angle: the inter-class interactions are interpreted as frictions between the two traffic streams. Similar to Logghe and Immers (2008), the division factor is still heuristically defined so that the mixed traffic attains 1-pipe and 2-pipe equilibria in a prescribedway. Jin and Wada (2018) revisited the model of Daganzo (1997) and discussed discretization and approximate solvers to the problem.

Along another line, research was conducted concerning the macroscopic equilibria of mixed autonomy traffic. An important character of AVs is that they can change driving modes based onthe type of leading vehicle. Such a property is the key consideration in a few models. For example, Chen et al. (2017) discussed capacity of steady-state mixed autonomous flow on multilane freeway under different combination of static lane access policies and how agent characters come into play. Ghiasi et al. (2017) derived the capacity of mixed autonomy traffic in a similar context, assuming the spatial distribution of mixed flow follows a Markov process, thus incorporating the randomness in the mixed autonomy traffic. Both works are focused on traffic capacity and did not consider equilibria reached by agents in general situations. Concerning equilibria of mixed autonomy traffic in all regimes, Qin et al. (2021) derived a fundamental diagram of mixed autonomy traffic by incorporating the type-dependency of AVs. Nonetheless,

their model implicitly assumes that AVs and HVs always have the same speed, which is unlikely in multilane settings when traffic is not heavy. We call this a "single-lane" approach as only the longitudinal interactions (i.e. car-following) are considered. This approach is limited in that it ignores lateral interactions of agents and potential lane policies. Huang et al. (2019) considered the micro-macroscopic connection and speed control in mixed autonomy traffic using a mean-field game approach. Though their model is a dynamic one, it suggests the equilibria that mixed autonomy traffic may attain. In the limiting case, their mean-field game model reduces to the classical LWR model. However, the focus of their model is also limited to the longitudinal interactions in traffic.

We draw inspiration from both lines of works. On one hand, the first line of research illuminates ways to bridge agent behaviors and macroscopic traffic flow properties, especially when lateral interactions are considered, and it was shown that such connections are valuable in understanding and controlling mixed traffic. On the other hand, the second line of research pinpoints an important character of mixed autonomy traffic, i.e., the dependency of headways on leading vehicle types, and also provides great insights into the capacity of mixed autonomy traffic and how longitudinal interactions shape mixed autonomy traffic behaviors.

In this report, we propose a new game theoretic approach to model the macroscopic equilibria of mixed autonomy traffic flow, considering simultaneous longitudinal and lateral interactions of self-interested agents. We are interested in the equilibria that heterogeneous agents can reach in all traffic regimes, and ultimately, how agent characters and their interactions determine the macro-scopic equilibrium properties of mixed autonomy traffic. We consider a more generalized setting compared to existing behavior mixed flow models. Also, unlike the existing approaches where the behavior rules usually presume a certain macroscopic equilibrium structure (i.e., when 1-pipe and 2-pipe equilibriums are attained), our approach is intrinsically bottom-up and the equilibria are fully determined from agent characteristics. This makes our approach more coherent and flexible to embrace a wide range of agent behaviors and lane policies. Our model also deepens the under-standing of mixed autonomy traffic flow: we show that under mild conditions, mixed autonomy traffic can reach two types of equilibria, and one of them is always Pareto efficient. This finding sheds light on the self-organization of mixed autonomy traffic.

The report is organized as follows. Our model consists of two parts, which are respectively presented in Section 2 and 3. In Section 2, we first define 1-pipe speed of mixed autonomy traffic, where agents are endowed with general speed functions and have only longitudinal interactions with each other. In Section 3, we formulate a two-player game to capture agent interactions in

multilane settings and analyze the equilibria of this game. We also describe a speed policy that guarantees the realized equilibrium is Pareto efficient. In Section 4, we show two applications of the model and illustrate a counterintuitive phenomenon in mixed autonomy traffic. In section 5, we empirically verify the assumption of the scaling parameter that seizes the dependency of speed on vehicle types and extends to the mixed autonomy traffic. We also examined the existence of 1-pipe speed in mixed traffic. We conclude the paper with remarks on the key findings and future works in Section 6.

1-pipe speed

In this section we consider a simple scenario, i.e. mixed traffic traveling on a single lane. We define the equilibrium traffic speed in this scenario and call it "1-pipe speed". As will be shown shortly, the 1-pipe speed may or may not be the equilibrium speed of mixed traffic in multilane settings, but it is a key component to construct equilibrium traffic speeds in more complicated scenarios.

We start from the simplest case, assuming that there are two classes of agents and each class of agents is endowed with a nominal speed function, respectively denoted as $u_1(\rho)$ and $u_2(\rho)$. This assumption means the speed of an agent is completely determined from the density or spacing. We also require that the inverse of a nominal speed function is unique. When a speed function $u(\cdot)$ is not strictly decreasing, we define its inverse as $u^{-1}(y) = \sup_\rho\{\rho : u(\rho) = y\}$.

Now we consider a single-lane circular road of length L, with n_1 agents in class 1 and n_2 agents in class 2. When this system settles to an equilibrium, all agents move at the same speed, which is the equilibrium speed to solve, denoted as u^*. Since nominal speed functions are strictly decreasing, the spacings for agents in the same class must be identical; otherwise, their speeds are not the same, meaning the equilibrium isn't reached yet. Therefore, the equilibrium spacings of the agents, denoted as s_1^* and s_2^*, satisfies,

$$L = n_1 s_1^* + n_2 s_2^* = \frac{n_1}{\rho_1^*} + \frac{n_2}{\rho_2^*} = \frac{n_1}{u_1^{-1}(u^*)} + \frac{n_2}{u_2^{-1}(u^*)} \tag{1}$$

Note that in (1), the ρ_1^* and ρ_2^* are interpreted as perceived densities for the two classes of agents, which are not equal to their averaged densities $\rho_1 = n_1/L$ and $\rho_2 = n_2/L$. The variables L, n_1 and n_2 can be cancelled from the equation by rewriting (1) as,

$$\frac{\rho_1}{u_1^{-1}(u^*)} + \frac{\rho_2}{u_2^{-1}(u^*)} = 1 \tag{2}$$

The solution of this equation defines an equilibrium speed $u^*(\rho_1, \rho_2)$ for mixed traffic.

Definition 1 (Equilibrium speed of single-lane mixed traffic flow). *The equilibrium speed $u^*(\rho_1, \rho_2)$ of mixed traffic flow on a single lane is given by (2).*

One may note that in this definition, the ordering of agents does not influence the equilibrium speed. This is because the nominal speed functions of agents do not depend on the type of its leading agent. Extending (2) to account for the type-dependency is straightforward. We introduce four nominal speed functions in this case, denoted as $u_{ij}(\rho)$ ($i, j = 1,2$). Here $u_{ij}(\rho)$ represents

the speed of a class i agent when it follows with class j agents, where the spacing between them is s and density is $\rho = 1/s$.

Then we have,

$$L = \sum_{i=1}^{2} \sum_{j=1}^{2} n_{ij} \, s_{ij}^{*} = \sum_{i=1}^{2} \sum_{j=1}^{2} \frac{n_{ij}}{\rho_{ij}^{*}} = \sum_{i=1}^{2} \sum_{j=1}^{2} \frac{n_{ij}}{u_{ij}^{-1}(u^{*})} \tag{3}$$

which leads to a new governing equation of u^{*},

$$\sum_{i=1}^{2} \sum_{j=1}^{2} \frac{\rho_{ij}}{u_{ij}^{-1}(u^{*})} = 1 \tag{4}$$

where ρ_{ij} is the density of agents in class i that follows a class j agent.

The equation (4) tells how the way of mixing (i.e. ordering when traffic travels on a single lane) influences mixed traffic behaviors, which is a peculiar property stemmed from the dependency of speed not only on spacing, but also on vehicle types in the leading-following pair. The vector $(\rho_{11}, \rho_{12}, \rho_{21}, \rho_{22})$ follows a joint probabilistic distribution, under the only constraints $\sum_{j} \rho_{ij} = \rho_{i}, (i = 1,2)$. The exact form of this distribution may be derived by considering all possible ways of permutations which allocate $n_{1} + n_{2}$ agents to $n_{1} + n_{2}$ "slots", but a simpler way to approximate this distribution and obtain the expectation of random vector is as follows. The approximation assumes n_{1} and n_{2} are large numbers. Under this assumption, a slot is filled with agent 1 with probability p[1], and filled with agent 2 with probability q, where $p = n_{1}/(n_{1} + n_{2})$, and $q = 1 - p = n_{2}/(n_{1} + n_{2})$. In addition, allocation of agents to consecutive slots are independent. Then it is straightforward to obtain the probabilities $p_{ij} = P(\text{agent } i \text{ follows agent } j)$ (we omit the details here), and finally the expectation of $(\rho_{11}, \rho_{12}, \rho_{21}, \rho_{22})$ as $E(\rho_{ij}) = \rho_{i} p_{ij} / \sum_{j} p_{ij}$, which can be succinctly expressed as $E(\rho_{ij}) = \rho_{i}\rho_{j}/(\rho_{i} + \rho_{j})$.

The expected values are now plugged back into (4) to cancel out all ρ_{ij} terms. Then we have a new governing equation for u^{*} as follows,

$$\frac{1}{\rho_{tot}} \sum_{i=1}^{2} \sum_{j=1}^{2} \frac{\rho_{i}\rho_{j}}{u_{ij}^{-1}(u^{*})} = \frac{1}{\rho_{tot}} \sum_{i=1}^{2} \rho_{i} \left(\sum_{j=1}^{2} \frac{\rho_{j}}{u_{ij}^{-1}(u^{*})} \right) = 1 \tag{5}$$

The unknown u^{*} solved from this equation represents the expected mixed flow speed over the possible ways of mixing the two classes of agents randomly on a single-lane road.

For analytical tractability (it turns out this is also a close approximation to empirical data, see 5), we look into a special case of the equilibrium speed u^{*} in (5), when it can be analytically solved. In this special case, we assume all speed functions have the form,

$$u_{ij}(\rho) = u(\rho/a_{ij}), \ i,j = 1,2 \tag{6}$$

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[1] We abuse the notation a bit because the context is clear. In the next section, p_{1} and p_{2} denote players' bids for road share.

where a_{ij} is a scaling parameter capturing the dependency of speed on vehicle types (see Definition Definition 2 below), and $u(\cdot)$ is a reference speed-density relation. Here we introduce a character of agents called type sensitivity.

Definition 2 (Type sensitivity). *We call the class i of agents type sensitive, if $a_{ij} \neq a_{ij'}$, for $j \neq j'$. Otherwise, the class is type insensitive.*

The larger value of a_{ij} corresponds to smaller headways at the same speed. Therefore, in general, AVs should have larger values of a_{ij} than HVs. With this simplification, we have $u_{ij}^{-1}(u^*) = a_{ij}u^{-1}(u^*)$ and the equilibrium speed u^* with the scaling assumption can be solved as,

$$u^*(\rho_1, \rho_2) = u\left(\frac{1}{\rho_{tot}}\sum_{i=1}^{2}\sum_{j=1}^{2}\frac{\rho_i\rho_j}{a_{ij}}\right) \tag{7}$$

It is straightforward to check that (7) reduces to $u^*(\rho_1, \rho_2) = u(\rho_{tot})$ when $a_{ij} = 1$ for all i, j. This means when all agents have the identical speed functions, the mixed flow equilibrium speed is only dependent on the total density and does not depend on the composition of traffic flow. In this case, the speed function of mixed flow degenerates to single-class speed function. This is necessary for (7) to be well-posed. We may interpret the term $\frac{1}{\rho_{tot}}\sum_{i=1}^{2}\sum_{j=1}^{2}\frac{\rho_i\rho_j}{a_{ij}}$ in (7) as *effective density* of mixed traffic flow.

Equilibria on multilane road

On roads with multiple lanes, agents will have lateral interactions, such as changing lanes and overtaking other agents. Such interactions are conceivably more complicated than the longitudinal one we discussed, since the latter usually only involves agent decisions that is one-directional, i.e. an agent can determine its speed from its spacing with one or more leading vehicles, but the inverse is not true. In contrast, on the lateral dimension, the decisions of agents are mutually dependent. That is, one agent's decision to choose lane depends on the decisions of other agents; meanwhile, its own decision can impact other agents as well. The equilibria thus formed are conceivably different (and more complicated) than the 1-pipe equilibrium we considered above. It is the purpose of this section to depict the relation of individual agent behaviors with the collective equilibria.

Two-player game model

We formulate the following two-player game to capture the interactions in mixed autonomy traffic. In this game, each player places a bid (respectively p_1 and p_2) for the lateral space, and the payoffs to the players are determined as,

$$U_i(\rho_1, \rho_2, p_1, p_2) = \begin{cases} u_i(\rho/p_i) & \text{if } p_1 + p_2 \leq 1 \\ u_{1-pipe}(\rho_1, \rho_2) & \text{if } p_1 + p_2 > 1 \end{cases}, \quad i = 1,2 \tag{8}$$

Here (ρ_1, ρ_2) is the system state, and the bids, which take value in $[0,1]$, constitutes strategies of the players. With (8), we have the complete strategic form of a two-player game, and we are ready to analyze its equilibrium properties.

Nash equilibria of the game

To derive the Nash equilibria of the game, we first introduce a concept called "minimum road share".

Definition 3 (Minimum road share). *For agents of class i, we define minimal road share p_i^* as follows*

$$p_i^*(\rho_1, \rho_2) = \inf\{p: 0 \leq p \leq 1, u_i(\rho_i/p) \geq u_{1-pipe}(\rho_1, \rho_2)\} \tag{9}$$

The minimum road share is the minimal lateral share of the road (i.e. percentage of lanes) that a class of agents need to maintain the same speed as when they share all the lanes with the other class of agents. In another word, the class i of agents will maintain the same speed in two scenarios: 1) when they share all lanes with the other class of agents with density ρ_j; and 2) they use p_i^* of total number of lanes exclusively. In essence, this equivalency reflects the trade-offs when inter-class interactions exist. We also note that this definition is independent of the specific definitions of $u_i(\cdot)$ and $u_{1-pipe}(\cdot,\cdot)$, as long as they all make physical senses.

We give a simple example here to illustrate the concept of minimum road share. Consider identical agents, i.e. agents with the same nominal speed function and $a_{ij} = 1$ for $i, j = 1,2$, following the specification in (6). Then we have the equilibrium speed $u^*(\rho_1, \rho_2) = u(\rho_1 + \rho_2)$. In this case, $p_1^* = \rho_1/(\rho_1 + \rho_2)$ and $p_2^* = \rho_2/(\rho_1 + \rho_2)$, and $p_1^* + p_2^* = 1$ for all values of (ρ_1, ρ_2). This is a degenerate case of the 1-pipe equilibrium.

A key property of minimum road share is stated in Lemma 1, which tells that based on the value of $p_1^* + p_2^*$, one can determine whether or not all agents can better off in a fully separate configuration compared to the fully mixed configuration.

An important usage of the minimum road share is to characterize when fully mixed traffic

Lemma 1 (1-pipe characterization). *There exists $p \in (0,1)$ such that $u_1(\rho_1/p) \geq u_{1-pipe}(\rho_1, \rho_2)$ and $u_2(\rho_2/(1-p)) \geq u_{1-pipe}(\rho_1, \rho_2)$ if and only if $p_1^* + p_2^* \leq 1$.*

Proof. The "if" part. When $p_1^* + p_2^* < 1$, let $p = p_1^* + (1 - (p_1^* + p_2^*))/2$. It is easy to see $p_1^* < p < 1$ and $p_2^* < 1 - p < 1$. By the definition of minimum road share and monotonicity of $u_1(\cdot)$ and $u_2(\cdot)$, we have $u_1(\rho_1/p) \geq u_2(\rho_1/p_1^*) = u_{1-pipe}(\rho_1, \rho_2)$. Similarly, $u_2(\rho_2/(1-p)) \geq u_{1-pipe}(\rho_1, \rho_2)$.

The "only if" part. When such p exists, by the definition of minimum road share, $p_1^* \leq p$ and $p_2^* \leq 1 - p$. Therefore, $p_1^* + p_2^* \leq 1$. $\square$

Now we derive the Nash equilibria of the game in below.

Theorem 1 (Nash equilibria of the game). *The game has the following two types of equilibria:*

(1) 1-pipe equilibrium: Pair (p_1, p_2) satisfying $p_1 > 1 - p_2^*$, $p_2 > 1 - p_1^*$ and $p_1 + p_2 > 1$. In this case, the payoffs to both players are identical, which is $u_{1-pipe}(\rho_1, \rho_2)$;

(2) 2-pipe equilibrium: Pair (p_1, p_2) satisfying $p_1^* < p_1 \leq 1 - p_2^*$ and $p_1 + p_2 = 1$. In this case, the payoffs to the players are respectively $u_1(\rho_1/p_1)$ and $u_2(\rho_2/p_2)$.

Proof. We can verify as follows:

Case (1): Consider a strategy pair (p_1, p_2) satisfying $p_1 + p_2 > 1$ and $p_2 > 1 - p_1^*$. In this case, payoffs to the players are,

$$u_1 = u_2 = u_{1-pipe}(\rho_1, \rho_2) \tag{10}$$

Now suppose the player 1 can better off in another strategy profile (p_1', p_2). Then first, there must be $p_1' + p_2 < 1$; otherwise by the definition of the game, the payoffs to the players remain the same. Given this, the payoffs to the players respectively become $u_1(\rho_1/p_1')$ and $u_2(\rho_2/p_2)$. That the player 1 will better off means $u_1(\rho_1/p_1') > u_{1-pipe}(\rho_1, \rho_2)$, which implies $p_1' > p_1^*$, by the definition of minimum road share. So now there is $p_1' + p_2 \geq p_1^* + (1 - p_1^*) = 1$, which conflicts with $p_1' + p_2 < 1$ as shown at the beginning. This means the player 1 cannot improve its payoff with a new strategy p_1' as supposed. By symmetry, the similar holds for player 2. Therefore, we proved any pair (p_1, p_2) satisfying $p_1 + p_2 > 1$ and $p_1 \in (1 - p_2^*, 1]$ and $p_2 \in (1 - p_1^*, 1]$ is a Nash equilibrium.

Case (2): Consider a strategy pair (p_1, p_2) satisfying $p_1 + p_2 = 1$ and $p_1^* < p_1 \leq 1 - p_2^*$ and $p_2 \in [p_2^*, 1 - p_1^*]$. In this case, the payoffs to the players are,

$$u_1 = u_1(\rho_1/p_1), \ u_2 = u_2(\rho_2/p_2) \tag{11}$$

Now suppose the player 1 can better off in another strategy pair (p_1', p_2). Then there must be $p_1' > p_1$; otherwise, because $p_1' + p_2 < p_1 + p_2 = 1$, the new payoff is $u_1(\rho_1/p_1')$ and it is strictly less than the original payoff $u_1(\rho_1/p_1)$. So we have $p_1' + p_2 > 1$, and therefore the payoff to player 1 becomes $u_{1-pipe}(\rho_1, \rho_2)$. The new payoff is better means $u_{1-pipe}(\rho_1, \rho_2) > u_1(\rho_1/p_1')$, which implies $p_1' < p_1^*$. Meanwhile, we know $p_1 \geq p_1^*$, so $p_1 > p_1'$, which conflicts with $p_1' > p_1$ that we show at the beginning. This means player 1 cannot better off with the new strategy p_1'. By symmetry, the similar holds for player 2. We thus proved a pair (p_1, p_2) satisfying $p_1 + p_2 = 1$ and $p_1 \in [p_1^*, 1 - p_2^*]$ and $p_2 \in [p_2^*, 1 - p_1^*]$ is a Nash equilibrium. $\square$

Based on the theorem, we further characterize regimes in which the different equilibria are attainable.

Corollary 1 (Attainability of the equilibria). *1-pipe equilibrium can be attained at any density $\rho_1, \rho_2 > 0$; 2-pipe equilibria can be attained if and only if $p_1^* + p_2^* \leq 1$.*

Proof. Given $\rho_1, \rho_2 > 0$, we have p_1^* and p_2^* both larger than zero. Then by Theorem 1, (1,1) is always an strategy that leads to the 1-pipe equilibrium.

Regarding 2-pipe equilibria, for corresponding strategies to exist, the condition in Theorem 1 requires $p_1^* < 1 - p_2^*$, i.e. $p_1^* + p_2^* \leq 1$. On the other hand, if $p_1^* + p_2^* \leq 1$, we can let. $p_1 = p_1^* + \frac{1-(p_1^*+p_2^*)}{2}$ and $p_2 = p_2^* + \frac{1-(p_1^*+p_2^*)}{2}$. It can be verified that (p_1, p_2) leads to a 2-pipe equilibrium. $\square$

Pareto efficiency of the equilibria

Theorem 1 and Corollary 1 together suggest that the payoffs to the players are not unique when $p_1^* + p_2^* \leq 1$, i.e. which roughly says that the traffic is light. In this case, the players may or may

not attain the same speed, depending on whether they reach the 1-pipe or 2-pipe equilibrium. Nonetheless, when $p_1^* + p_2^* > 1$, i.e. when traffic is heavy, the only equilibrium can be reached is 1-pipe equilibrium, and in this case, the players' speed must be identical.

When equilibria are not unique, it is natural to ask which equilibrium is more desirable from a system perspective. The Pareto efficiency is a useful notion towards analyzing such problems. In plain language, an equilibrium is Pareto efficient if and only if there is no other outcomes that make all the players better off. The major result we derived is that 2-pipe equilibrium is always Pareto efficient, while 1-pipe equilibrium is Pareto efficient only when $p_1^* + p_2^* > 1$. This means there always exists at least one Pareto efficient equilibrium for any (ρ_1, ρ_2).

Theorem 2 (Pareto efficiency of the equilibria). *We have: (1) The 1-pipe equilibria are Pareto efficient when $p_1^* + p_2^* > 1$; (2) The 2-pipe equilibria are Pareto efficient when $p_1^* + p_2^* \leq 1$.*

Proof. When $p_1^* + p_2^* > 1$, by Theorem 1, 1-pipe equilibria are the only equilibrium and the payoffs are always equal to $u_{1-pipe}(\rho_1, \rho_2)$. Therefore, the 1-pipe equilibria are Pareto efficient, i.e. (1) is proved.

When $p_1^* + p_2^* \leq 1$, we first consider an equilibrium strategy (p_1, p_2), with payoffs $u_1(\rho_1/p_1)$ and $u_2(\rho_2/p_2)$. If another 2-pipe equilibrium strategy (p_1', p_2') improves this strategy for both players, we have $p_1' \geq p_1$ and $p_2' \geq p_2$ due to the monotonicity of speed functions, and here at least one inequality is strict. This leads to $p_1' + p_2' > p_1 + p_2 = 1$. This conflicts with the definition of a 2-pipe equilibrium.

Now suppose a 1-pipe equilibrium improves (p_1, p_2) for both players. This leads to $u_1(\rho_1/p_1) \leq u_{1-pipe}(\rho_1, \rho_2)$ and $u_2(\rho_1/p_1) \leq u_{1-pipe}(\rho_1, \rho_2)$. Therefore, by the definition of minimum road share, $p_1^* \geq p_1$ and $p_2^* \geq p_2$. As a result, $p_1^* + p_2^* \geq p_1 + p_2 = 1$. This conflicts with the assumed initial condition. So (2) is proved. □

The theorem suggests that when $p_1^* + p_2^* > 1$, no lane policies that split the agents to different lanes will improve the speeds of both classes, compared to the 1-pipe equilibrium reached. When $p_1^* + p_2^* \leq 1$, on the other hand, there are infinite many 2-pipe equilibria, where no equilibrium is uniformly better than the others.

We note that the existence and structure of sets $\{(\rho_1, \rho_2): p_1^* + p_2^* > 1\}$ and $\{(\rho_1, \rho_2): p_1^* + p_2^* < 1\}$ depend on how the 1-pipe speed function is defined. We show two extreme situations. In the first example, we let $u_{1-pipe}(\rho_1, \rho_2) = \max\{u_1(\rho_1 + \rho_2), u_2(\rho_1 + \rho_2)\}$. This depicts a scenario that when agents are mixed, they both adopt the faster speed function. In this case, it can be shown that there are always $p_1^* \geq \rho_1/(\rho_1 + \rho_2)$ and $p_2^* \geq \rho_2/(\rho_1 + \rho_2)$. Therefore, $p_1^* + p_2^* \geq 1$ for all (ρ_1, ρ_2), implying that the 1-pipe equilibrium is always Pareto efficient, and all the agents are better off when mixing with each other. Similarly, when we let $u_{1-pipe}(\rho_1, \rho_2) = \min\{u_1(\rho_1 + \rho_2), u_2(\rho_1 + \rho_2)\}$, then there is always $p_1^* + p_2^* \leq 1$, and in this case, all the agents are always better off in 2-pipe equilibria, compared to adopting the 1-pipe equilibrium.

Equilibrium speed policy

We show above that 2-pipe equilibria are not unique and each equilibrium corresponds to a different pair of payoffs (see Theorem 1). In contrast, the payoffs corresponding to the 1-pipe

equilibrium are unique. In below we discuss behavior scenarios that leads to a unique 2-pipe equilibrium, and thus unique equilibrium relationships between density and speed.

In our model, $p_1^* + p_2^*$ is the minimum of total lateral space needed so that both players are better off compared to the 1-pipe equilibrium. When attaining 2-pipe equilibria, players take their own lateral space, and they collectively improve the system from the fully mixed (i.e. 1-pipe) state. In this perspective, we may interpret the attaining of 2-pipe equilibria as a form of player cooperation (note though, the players are still self-interested, and the collaboration is possible because of the Pareto efficiency of the 2-pipe equilibrium). Therefore, we can interpret $1 - p_1^* - p_2^*$ as the surplus of road share from the player's cooperation, and the problem of determining a unique 2-pipe equilibrium is reduced to splitting the surplus between the two players.

We thus define a general class of speed policies, which we call "surplus split policies", as follows,

$$\begin{cases} p_1 = p_1^* + \lambda(\rho_1, \rho_2)s \\ p_2 = p_2^* + \left(1 - \lambda(\rho_1, \rho_2)\right)s \end{cases} \tag{12}$$

where $s := 1 - p_1^* - p_2^*$ is the road share surplus we just mentioned, and $\lambda \in [0,1]$ is a mapping from (ρ_1, ρ_2) to $[0,1]$, which governs how the road share is divided. Note the road share surplus is fully determined when the densities and nominal speed functions of the players are known.

As some examples, when $\lambda = 1/2$ (it is independent of (ρ_1, ρ_2) in this case), the road share surplus is divided equally between the players; when $\lambda = 1$, the player 1 takes all the road share surplus. When sophisticated behavior rules are considered, λ may necessarily involve implicit definitions. For instance, when the players intend to have speeds that are as close as possible, then λ can be defined as,

$$\lambda(\rho_1, \rho_2) = \arg\min_{\lambda \in [0,1]} ||u_1\left(\frac{\rho_1}{p_1}\right) - u_2\left(\frac{\rho_2}{p_2}\right)|| \tag{13}$$

The point here is that by defining the function λ differently, the model (12) can capture a wide range of agent interactions, and albeit the difference, we have the following important property, which guarantees the lower bound of speeds for both players.

Proposition 1. *Under any surplus split policy, both players' speeds are always lower bounded by $u_{1-pipe}(\rho_1, \rho_2)$.*

Proof. Given any surplus split policy, there are always $p_1^* \leq p_1 \leq 1 - p_2^*$ and $p_2^* \leq p_2 \leq 1 - p_1^*$, so this proposition holds as a corollary of Theorem 2. □

Last, we go back to the concept of type-sensitivity and consider its role in a speed policy. Recall that in our model this character differentiates the automated vehicles from human-driven vehicles. We have that when all agents are type-insensitive, then there is no 2-pipe equilibrium that make all the agents strictly better off compared to $u_{1-pipe}(\rho_1, \rho_2)$, i.e. in this case the lower bound $u_{1-pipe}(\rho_1, \rho_2)$ is tight. Actually, the 2-pipe equilibria degenerate into 1-pipe equilibria in this case. This result implies that in mixed traffic consisting of type-insensitive agents, one cannot improve the speeds of both classes of agents by lane policies that separating them into

different lanes. In another word, type-sensitivity is necessary for such policies to work. Theorem 3 below states this result formally. We omit the proof here for brevity.

Theorem 3 (Mixed flow of type-insensitive agents). *Consider mixed traffic consists of two classes of type-insensitive agents with endowed speed function $u_1(\cdot)$ and $u_2(\cdot)$ respectively, then the 2-pipe equilibria degenerate into 1-pipe equilibrium, and the only equilibrium speed for both classes of agents is $u_{1-pipe}(\rho_1, \rho_2)$.*

Example

The example presented here is a direct application of the model to construct flux of mixed traffic from agent characters. We compare two speed policies described in the last section: the base one is the 1-pipe equilibrium, and the other one is the surplus split policy with $\lambda = 0.5$. We assume both classes of agents are endowed with the Greenshields function as its nominal speed function, where the free flow speed is 60 mph (miles per hour) and jam density 200 vpm (vehicles per mile), i.e. the nominal speed function reads $u_{HV}(\rho) = 60 - \frac{60}{200}\rho$ for HVs and $u_{AV}(\rho) = \min\left(60, 60 - 60\frac{50-\rho}{50-200}\right)$ for AVs. Interactions of the agents are governed by the four scaling parameters a_{ij}, whose values are set to be $(a_{11}, a_{12}, a_{21}, a_{22}) = (1,1,1.2,1.3)$. This setting means HVs are type-insensitive, and AVs take less spacing when following an HV or another AV.

The equilibrium speeds and flows of the base scenario is presented in Figure 1, and those for the surplus split policy is presented in Figure 2. We can see that though the behavior rules are different, the qualitative patterns of macroscopic equilibria formed in these two scenarios are similar: the speed of each class decreases as the density of either class increases (the bottom rows in the two figures), and the flow-density relations in all the cases appear concave.

Again, recall that in the two scenarios, the agent characters (i.e. the speed functions they are endowed with) are exactly the same, and this improvement is achieved simply because the better Nash equilibrium is attained.
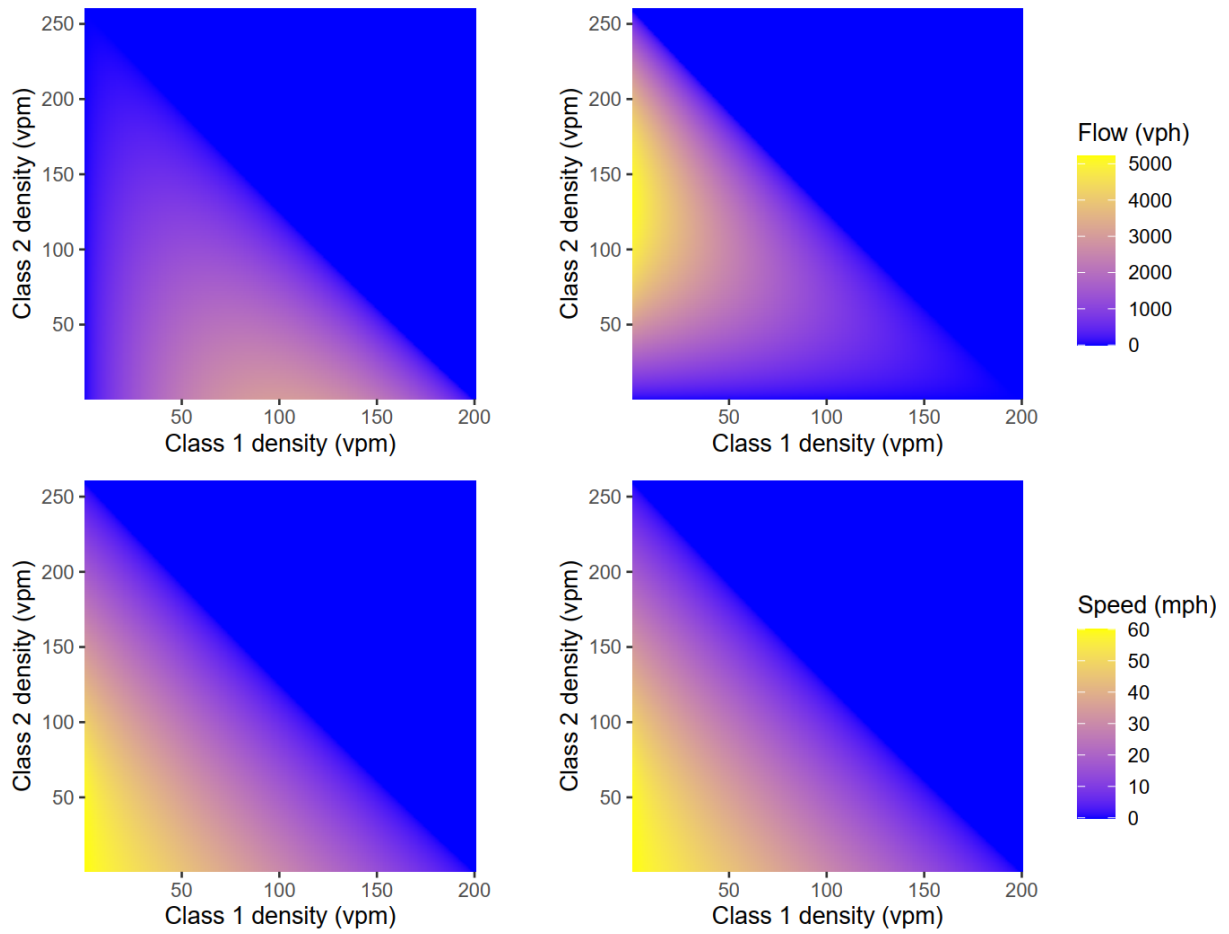
Figure 1: Base scenario: 1-pipe equilibrium (upper left: HV flow; upper right: AV flow; bottom left: HV speed; bottom right: AV speed).
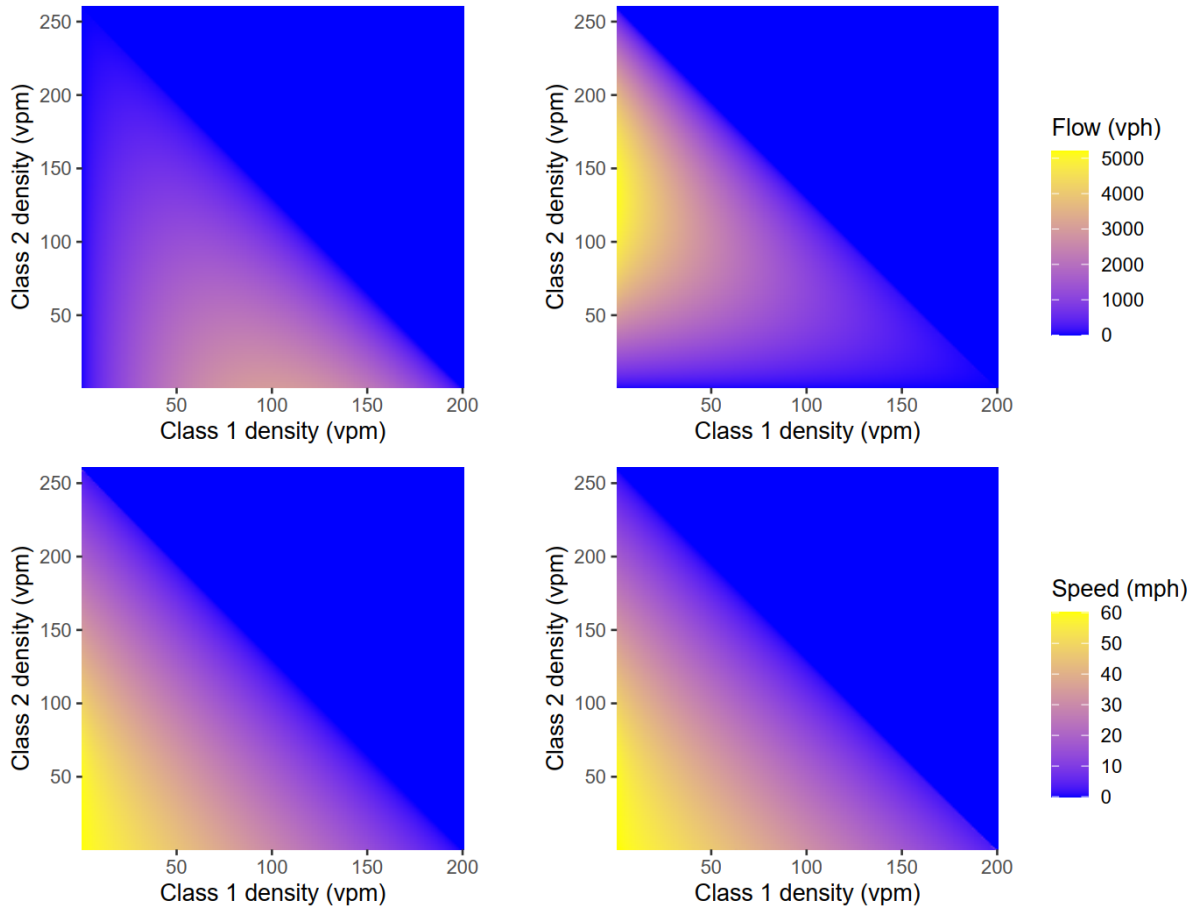
Figure 2: Alternative scenario: equilibrium under surplus split policy, λ = 0.5 (upper left: HV flow; upper right: AV flow; bottom left: HV speed; bottom right: AV speed).

The major difference of the two scenarios is the equilibrium speeds the agents can attain. We show the speed differences of the two classes of agents in the two scenarios in Figure 3. Both classes are better off in the second scenario, which verifies the Pareto efficiency property described in Proposition 1. In general, compared to the base case, one class receives the most significant speed improvement when its density is relatively low, and the density of the other class is relatively high. For HVs, the average and maximum of speed improvement are respectively 0.28 and 12.73 mph. For AVs, these values are respectively 0.54 and 10.43 mph.
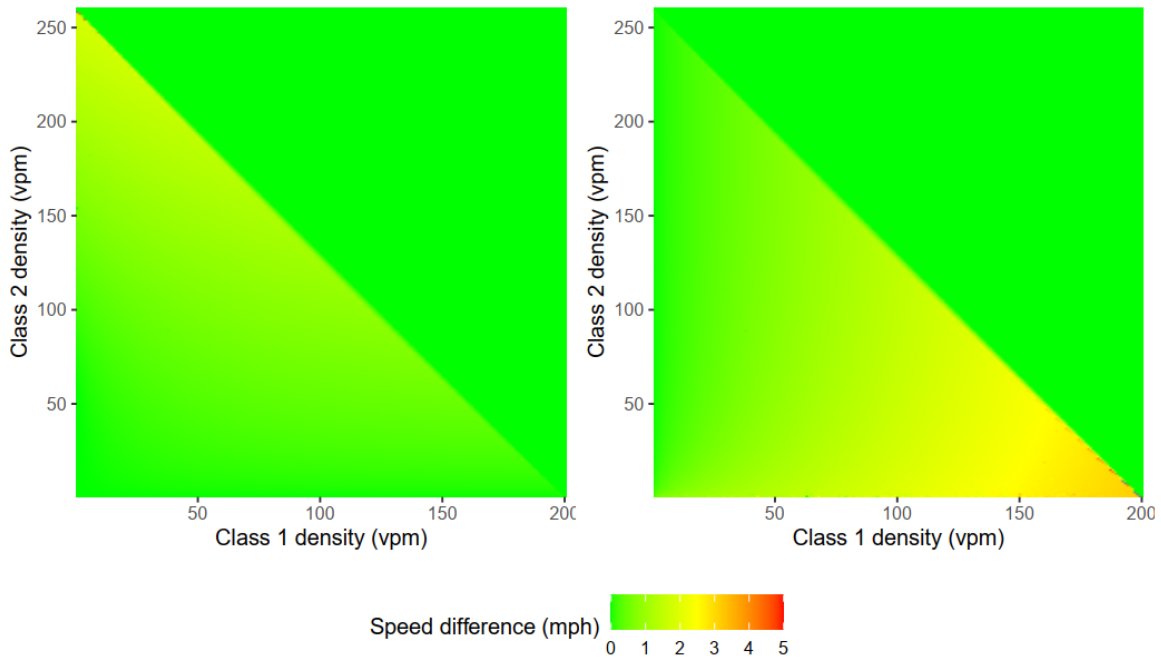
Figure 3: Speed improvements in the surplus split policy (left: human-driven vehicles; right: automated vehicles).
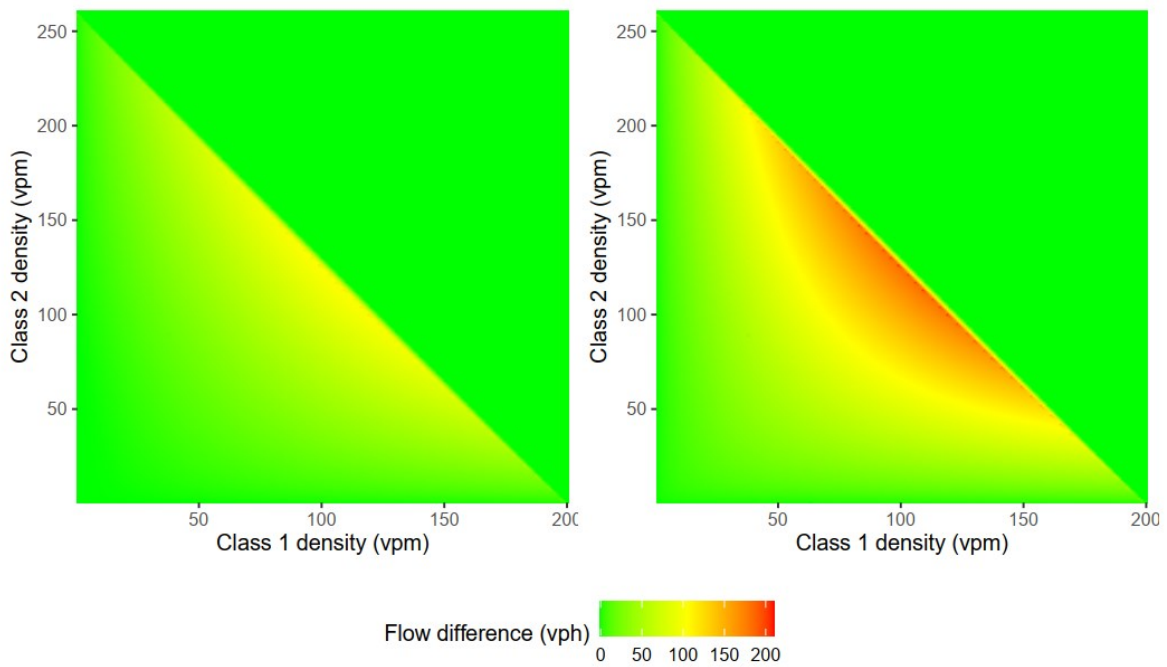

Figure 4: Flow improvements in the surplus split policy (left: human-driven vehicles; right: automated vehicles).

We also compare the difference of flows (throughputs) in the two scenarios, which is shown in Figure 4. Consistent with the case of speed, we also see flow improvements for both classes of agents in the second scenario. Nonetheless, the most significant changes of flow are seen when the densities of both classes of agents are intermediate. In addition, compared to HVs, the AVs experience more significant flow improvement in the new policy. The largest throughput improvements for HVs and AVs are respectively 120 and 208 vph.

Empirical evidence

The proceeding secitons of this report are focused on analytical modeling and numerical examples, where we made assumptions on agent behaviors (i.e., the speed functions they are endowed with) and hypothesized that self-interested agents would reach an equilibrium. It is natural to ask empirically whether these assumptions can be verified. In this section, we provide initial empirical evidence in this regard. We use the NGSIM data and verify that: (1) the scaling property postulated in (6) is a good approximation to real-world data; and (2) 1-pipe equilibrium will be attained in high density, aligned with our model predictions.

Scaling property of speed-density relationships

Data preparation

The Next Generation Simulation (NGSIM) Vehicle Trajectories and Supporting Data () from I80 and US101 is used for the study. The car and truck classes are chosen to study the mixed flow. Some criteria are set to retrieve the equilibrium condition: (1) Lanes 2, 3, and 4 are target lanes; (3) The maximum density is 400 vpm; (4) The maximum acceleration and deceleration rates are $1m/s^2$; (5) The following duration lasts for no less than 60 seconds; (6) A 10 second time window is set at the beginning and end of each following process; (7) No lane changes occur during the following process.

In the equilibrium data, the length of the studied freeway section is 544 meters for I80 and 681 meters for US101. The records have a duration of 93 minutes at I-80 and 46 minutes at US-101.

Verification method

Through explorations, we found the speed-density relation possesses an inverted sigmoid shape, which makes the logistic model a good fit. A five-parameter logistic speed-density model has the form of

$$u(\rho,\theta) = u_b + \frac{u_f - u_b}{\left(1+\exp\left(\frac{\rho-\rho_t}{\theta_1}\right)\right)^{\theta_2}} \tag{14}$$

The parameter u_f is the free-flow speed, u_b is the average speed during the trip, ρ_t marks the point at which the speed-density curve switches from free-flow to congested flow, and θ_1 and θ_2 are two parameters that determines the shape of the curve (). We use the logistic speed-density function $u(\rho,\theta)$ (where $\theta = (\theta_1, \theta_2)$) as the base speed-density relation. Then the logistic model for agent i following agent j can be written as

$$u_{ij}(\rho, \theta) = u(\rho/a_{ij}, \theta) \tag{15}$$

Our purpose is to check whether there exists a set of parameters $\{a_{ij}\}$ such that the fitted model is a close approximation to the empirical relations. Here i, j refer to cars and trucks.

Scaling parameters to capture truck-car interactions

We first calibrated logistic speed-density models from the data. The problem is reduced to minimization under L_1 norm with equality constraints. We consider cars and trucks as two independent classes, and there are four corresponding following relations. The parameters of the four models are presented in Table 1.

Table 1: Optimized parameters for the logistic speed-density model

Location	Vehicle Class	v_b (mph)	v_f (mph)	k_t (vpm)	θ_1	θ_2	MAE (mph)
I-80	Car-Car	6.7980	71.4365	20.4015	3.0098	0.0844	4.1007
I-80	Car-Truck	6.6600	67.0689	24.7778	3.1775	0.2756	4.6513
US-101	Car-Car	13.5693	77.6261	27.5809	5.8921	0.2699	6.1854
US-101	Car-Truck	12.8181	72.9964	21.9073	0.4808	0.0517	5.3907

Vehicle class car-car means a car following a car; car-truck means a car following a truck.

Then we proceed to assuming the scaling property holds and estimating scaling parameters a_{ij}. We denote cars as class 1 agents and trucks as class 2 agents, and attempt to estimate a_{12}. The initial value of a_{12} is set to be 1. To reduce the complexity of solution, we further set the free-flow speed to be 65 mph and the jam density to be 200 vpm. The estimated values for a_{12} are presented in Table 2. Based on these parameters, we plot the three fitted logistic models, namely $u_{11}(\cdot)$ (base model), $u_{12}(\cdot)$ (non-scaled fit), and $u(\cdot/a_{12})$ (scaled fit), in Figure 5. From the figure, we see that the scaled models well approximate the non-scaled models in both cases (I-80 and US-101) and both are different from the base model. This indicates the scaling property is a reasonable approximation to the empirical observations.

Table 2: Scaling parameter a_{12} (class 1 agents following class 2 agents) estimated from data

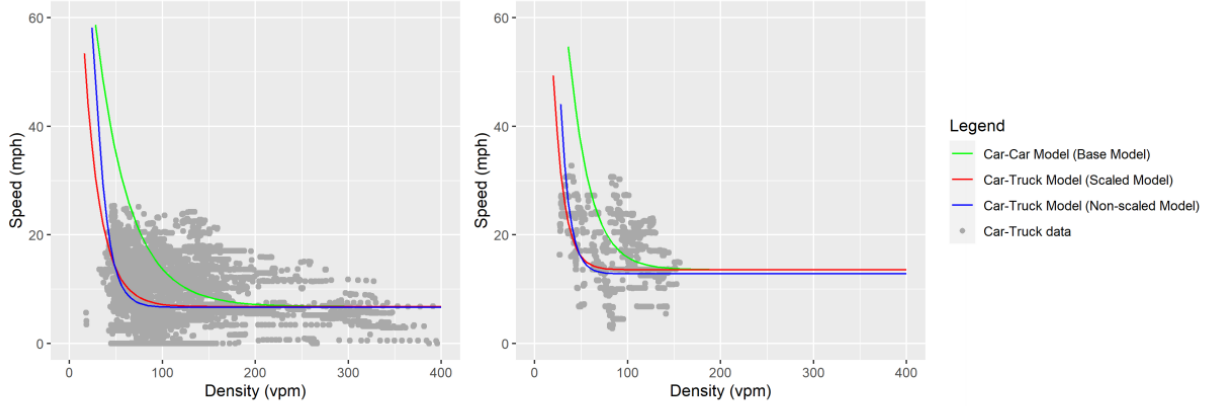| Location	Scaling Parameter	MAE (mph)
I-80	1.9998	4.5586
US-101	1.9793	4.9082

Figure 5: Base, scaled and non-scaled logistic models at I80 (left) and US101 (right)

Scaling parameters to capture HV-AV interactions

The scaling property was meant to capture the HV-AV interactions. Though values of the scaling parameters cannot be directly estimated from the NGSIM data (where all the vehicles are human-driven), we can estimate their values based on the relation between headway and flow, namely, $h = \frac{1}{q} = \frac{1}{\rho \cdot u(\rho)}$ along with other simplifications as follows. We define class 1 agent following class 1 agent as the base case. Assume that the base speed-density relation is linear, $u(\rho) = u_f\left(1 - \frac{\rho}{\rho_j}\right)$, where ρ_j is the jam density. Then its optimal density is $\rho^* = \frac{\rho_j}{2}$, and therefore $\rho_{ij}^* = \frac{a_{ij}\rho_j}{2}$ for the relation of class i agent following class j agent. Also, since $h_{11}^* = \frac{1}{\rho^* u(\rho^*)} = \frac{4}{\rho_j u_f}$, we have $h_{ij}^* = \frac{1}{(a_{ij}\rho_j/2)\left(u_f\left(1-(a_{ij}\rho_j/2)(a_{ij}\rho_j)\right)\right)} = \frac{4}{a_{ij}\rho_j u_f} = \frac{1}{a_{ij}}h_{11}^*$. Hence, the scaling parameter for the pair (i, j) is determined as,

$$a_{ij} = \frac{h_{11}^*}{h_{ij}^*}, \ i, j = 1,2 \tag{16}$$

For the mixed autonomy traffic, we denote HVs as class 1 agents and AVs as class 2 agents. Based on (16), we were able to calculate the scaling parameters based on existing assumptions or simulation results regarding the optimal headways of AVs in literature. The results are summarized in Table 3. The advantage of using a scaling parameter lies in that it is independent of specific assumptions on the shape of the speed-density curves and provides a unified ground for the comparison of different models. This can simplify the equilibrium modeling and analysis substantially.

Table 3: Headway assumptions and scaling parameter estimation based on existing literature

Studies	Headway (sec)				Scaling Parameter	
	HV-HV	HV-AV	AV-AV	AV-HV	HV-AV	AV-HV
Ghiasi et al. 2007	1.50	1.50	0.85	1.10	1.00	0.77
Mohajerpoor and Ramezani 2019	1.80	1.80	0.90	1.20	1.00	0.75
Ma et al. 2020	—	1.50	0.84	1.50	—	0.56
Kerner 2016	—	—	1.10	1.10	—	1.00
Zhou and Zhu 2020	2.00	1.50	0.60	1.50	1.33	0.40
Hussain et al. 2016	1.80	—	0.45	1.20	—	0.38
Qin and Wang 2019	1.50	1.10	0.60	1.10	1.36	0.55
Shladover et al. 2012	1.64	1.10	0.60	1.10	1.49	0.55

Existence of 1-pipe equilibrium

Next, we verify the existence of the 1-pipe equilibrium from empirical data, which is an important qualitative prediction from our model. We verify this by examining class 1 agent speeds and class 2 agent speeds at different density levels using box plots, and the result is shown in Figure 6. From this figure, we can see that the density level where 1-pipe equilibrium starts to occur is 100-120 vpm, which is close to the critical density that is approximately 90 vpm. This result thus indicates that the assumption holds well, at least in a probabilistic sense. Another interesting observation is that, even though the median speeds of the two classes of agents tend to be zero when the density is high, the variations of truck speeds remain relatively high at larger densities. In contrast, the speeds of cars synchronize better. This could potentially result from the better capability of cars to negotiate spaces and thus equilibrate faster collectively, but further evidence is needed to prove or disprove this postulation, which is left to future studies.
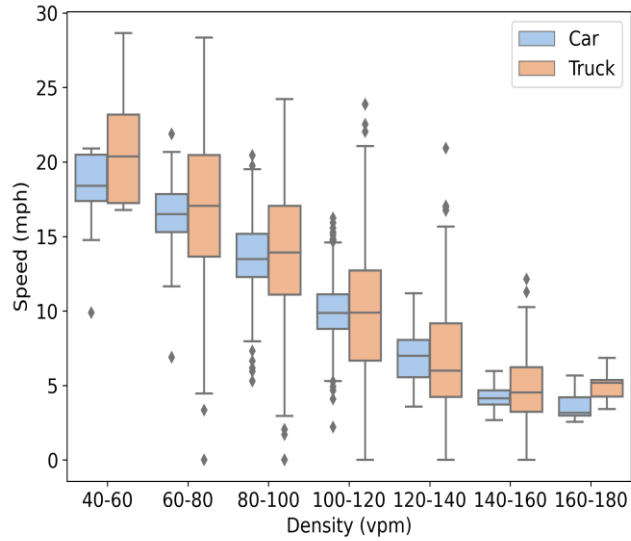
(a) Box plot of class-specific speed at different densities
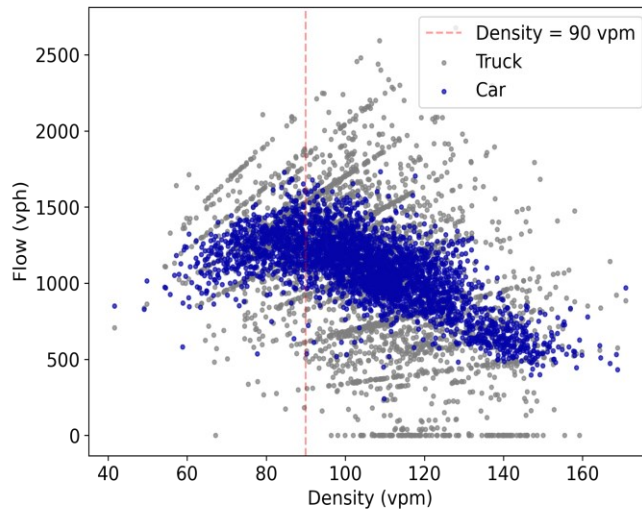
(b) Class-specific flow-density relations

Figure 6: Synchronization of class-specific speeds after reaching a critical threshold

Conclusion

We presented an equilibrium model of mixed autonomy traffic flow based on game theory. Human-driven and automated vehicles are modeled as self-interested agents endowed with different speed functions. Their simultaneous longitudinal and lateral interactions are modeled as

a two-player game. Through this model, we examine the equilibria of mixed autonomy traffic and bridge macroscopic equilibrium properties of traffic flow with behavior characters of agents.

The game theoretic approach presented in this paper is new and advances the existing behavior approach of modeling mixed traffic. Thanks to the power of game theory, in our new approach, equilibria of mixed autonomous traffic can be fully determined from agent characteristics, without presuming a macroscopic equilibrium structure or resorting to heuristic behavior rules. As such, this approach is more behaviorally sound and coherent, making it capable to handle more sophisticated agent behaviors and lane settings.

Our model also brings a new understanding of how mixed autonomy traffic may behave. We found that the agents in mixed autonomy traffic can in general reach two types of Nash equilibria, which may or may not co-exist, depending on traffic regimes. This contradicts the existing equilibrium theories of mixed flow, which presume a unique and well-defined equilibrium relationship. We also show that there always exists at least one Pareto efficient equilibrium for every system state. This implies even when all the agents are self-interested, it is still possible for them to self-organize into a more efficient flow pattern. Based on the equilibrium structure, we propose a speed policy that define an agreement between agents to split the road share surplus. This policy guarantees the attainment of Pareto efficient equilibria.

We provided two examples. In the first example, we compared the equilibrium speeds and fluxes of mixed autonomous traffic in two behavior scenarios, which verified that the Pareto efficient equilibrium improves the 1-pipe equilibrium in all traffic regimes. In the second example, we introduced an AV-exclusive lane policy and investigated the new traffic equilibria. We found discontinuities in the resulted macroscopic equilibrium relations. This suggests that the behaviors of mixed autonomy traffic may not always make common sense, even all the agents are rational. The intriguing micro-macroscopic connection in mixed autonomy traffic should be handled with care when designing AV behaviors.

Empirically, the scaling property of the equilibrium relation is verified. The property simplifies the specification of the flux function since only a nominal speed-density function and associated scaling parameters need to be estimated. If the scaling property holds for AVs, then we can derive the flux function of AVs from the flux function of HVs, which is connected by scaling parameters that are estimable from headway observations or assumptions. In addition, the 1-pipe equilibrium is observable when the system density is high, while the mixed traffic can still behave differently in a stochastic sense.

Our model depicts the connection between macroscopic equilibria of mixed autonomy traffic and agent characteristics. This connection can serve as a basis for designing AV behaviors in mixed autonomy environments. In the future, it is desirable to consider more sophisticated agent behaviors and their macroscopic effects.

References

Danjue Chen, Soyoung Ahn, Madhav Chitturi, and David A Noyce. Towards vehicle automation: Roadway capacity formulation for traffic mixed with regular and automated vehicles. *Transportation research part B: methodological*, 100:196–221, 2017.

Carlos F Daganzo. A continuum theory of traffic dynamics for freeways with special lanes. *Trans- portation Research Part B: Methodological*, 31(2):83–102, 1997.

Carlos F Daganzo. A behavioral theory of multi-lane traffic flow. part i: Long homogeneous freeway sections. *Transportation Research Part B: Methodological*, 36(2):131–158, 2002a.

Carlos F Daganzo. A behavioral theory of multi-lane traffic flow. part ii: Merges and the onset of congestion. *Transportation Research Part B: Methodological*, 36(2):159–169, 2002b.

Amir Ghiasi, Omar Hussain, Zhen Sean Qian, and Xiaopeng Li. A mixed traffic capacity anal- ysis and lane management model for connected automated vehicles: A markov chain method. *Transportation Research Part B: Methodological*, 106:266–292, 2007.

Amir Ghiasi, Omar Hussain, Zhen Sean Qian, and Xiaopeng Li. A mixed traffic capacity anal- ysis and lane management model for connected automated vehicles: A markov chain method. *Transportation Research Part B: Methodological*, 106:266–292, 2017.

Kuang Huang, Xuan Di, Qiang Du, and Xi Chen. A game-theoretic framework for autonomous vehicles velocity control: Bridging microscopic differential games and macroscopic mean field games. *arXiv preprint arXiv:1903.06053*, 2019.

Omar Hussain, Amir Ghiasi, and Xiaopeng Li. Freeway lane management approach in mixed traffic environment with connected autonomous vehicles. *arXiv preprint*, arXiv:1609.02946, 2016.

Wen-Long Jin and Kentaro Wada. A new cell transmission model with priority vehicles and special lanes. *Transportation research procedia*, 34:28–35, 2018.

Boris S. Kerner. Failure of classical traffic flow theories: stochastic highway capacity and automatic driving. *Physica A: Statistical Mechanics and its Applications*, 450:700–747, 2016.

Vijay Gopal Kovvali, Vassili Alexiadis, and Lin Zhang PE. Video-based vehicle trajectory data collection. Technical report, 2007.

Michael W Levin and Stephen D Boyles. A multiclass cell transmission model for shared human andautonomous vehicle roads. *Transportation Research Part C: Emerging Technologies*, 62:103–116, 2016.

S Logghe and Lambertus H Immers. Multi-class kinematic wave theory of traffic flow. *Transportation Research Part B: Methodological*, 42(6):523–541, 2008.

Jiaqi Ma, Edward Leslie, Amir Ghiasi, Zhitong Huang, and Yi Guo. Empirical analysis of a freeway bundled connected-and-automated vehicle application using experimental data. *Journal of Transportation Engineering, Part A: Systems*, 146(6):04020034, 2020.

Reza Mohajerpoor and Mohsen Ramezani. Mixed flow of autonomous and human-driven vehicles: Analytical headway modeling and optimal lane management. *Transportation research part C: emerging technologies*, 109:194–210, 2019.

Zhen Sean Qian, Jia Li, Xiaopeng Li, Michael Zhang, and Haizhong Wang. Modeling heterogeneous traffic flow: A pragmatic approach. *Transportation Research Part B: Methodological*, 99:183–204, 2017.

Yanyan Qin and Hao Wang. Cell transmission model for mixed traffic flow with connected and autonomous vehicles. *Journal of Transportation Engineering, Part A: Systems*, 145(5):04019014, 2019.

Yanyan Qin, Hao Wang, and Daiheng Ni. Lighthill-whitham-richards model for traffic flow mixed with cooperative adaptive cruise control vehicles. *Transportation Science*, 55(4):883–907, 2021.

Steven E. Shladover, Dongyan Su, and Xiao-Yun Lu. Impacts of cooperative adaptive cruise controlon freeway traffic flow. *Transportation Research Record*, 2324(1):63–70, 2012.

J WC van Lint, Serge P Hoogendoorn, and Marco Schreuder. Fastlane: New multiclass first-ordertraffic flow model. *Transportation Research Record: Journal of the Transportation Research Board*, 2088(1):177–187, 2008.

Haizhong Wang, Jia Li, Qian-Yong Chen, and Daiheng Ni. Logistic modeling of the equilibrium speed–density relationship. *Transportation research part A: policy and practice*, 45(6):554–566,2011.

G.C.K. Wong and S.C. Wong. A multi-class traffic flow model–an extension of LWR model withheterogeneous drivers. *Transportation Research Part A*, 36(9):827–841, 2002.

Jiazu Zhou and Feng Zhu. Modeling the fundamental diagram of mixed human-driven and connected automated vehicles. *Transportation research part C: emerging technologies*, 115:102614, 2020.