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COMPLETE SETS OF SUM-OF-SQUARES ORTHOGONAL F-SQUARES OF ORDER N

by

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ABSTRACT

Several new concepts and definitions, a method of constructing a complete set of sum-of-squares orthogonal F-squares, and a method of completing a set of combinatorial F-squares with sum-of-squarew orthogonal F-squares are presented. Examples for n = 6, 8, 10, 12, 14, 15, and 18 are used to present the methods and definitions.

Key words: Semi-F-square; Proportional F-square; Combinatorial orthogonality.

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INTRODUCTION

In this paper, several new ideas are presented. In particular, a definition of sum-of-squares orthogonality, a definition of a semi-F-square, a method of constructing complete sets of F-squares, and a method for completing a combinatorially orthogonal set of F-squares with sum-of-squares orthogonal F-squares, SOSOFs. SOSOFs are presented for n = 6, 10, 14, 15, and 18. F-squares obtained by the method of construction for n = 8 and 12 are not a complete set of SOSOFs but SOSOFsare available for these numbers. SOSOFs for $n = 2^k p$, p a prime number, and n = pq, p and q prime numbers, may be obtained by the method of construction. In fact, the method works for any value of n. Use is made of the combinatorial orthogonal properties of factorials and the calculus for prime power factorials.

Combinatorial orthogonality is attained when the frequency of pairs of symbols in two F-squares is equal, e.g. as in orthogonal arrays. Sum-of-squares orthogonality is achieved when the sum of squares of the F-squares formed from an interaction of factors is the same as the sum of squares for the interaction. For example, if F1 and F2 are formed from the two-factor interaction B × D, the sum of the sums of squares for F1 and F2 is the same as for the B × D interaction.

A regular n × n F-square with p symbols has each symbol occurring n/p times in each row and n/p times in each column and is denoted as F(n, n/p). A semi-F-square has the p symbols occurring equally frequently in the square and occurring n/p times in rows (columns) but not in columns (rows). A proportional F-square has the p symbols occurring in each row and in each column of the square but with unequal frequencies.

METHOD OF CONSTRUCTION

Consider a n-row by n-column lattice. For n = pq, number the rows with the two symbols of the p-level factor A by q-level factor B. Thus, 1 = 00, 2 = 01, ..., n = (p - 1)(q - 1). Likewise number the columns with the symbols of the p-level factor C and the q-level factor D. This makes a four factor, p × q × p × q factorial arrangement of the four factors A, B, C, and D. The main and interaction effects of the factorial are combinatorially and sum of squares orthogonal. Type 1 and Type III sums of squares for the main effects and interactions are identical, indicating no confounding of effects. A Type I analysis of variance, ANOVA, sum of squares is a nested one in that the effects of all items above a particular one are eliminated and all those below are ignored. A type III ANOVA contains sums of squares with all other effects eliminated from a particular one. Such sums of squares may be obtained from a number of software packages. The SAS software package was used to construct the F-squares and corresponding sums of squares.

The row by column interaction is partitioned into the interactions of the four factors. All interactions except A × B and C × D are to be partitioned into F-squares. The calculus for prime numbers modulo the larger of p and q is used to determine the symbol number in the F-square. For example, consider the combination 0212 for the four factorial factorial. From the A × B × C × D interaction for p = 2 and q = 3, the F-square formed for this combination has the symbol F = A + 2*B + C + 2*D = 0 + 2(2) + 1 + 2(2) = 9 modulus 3 = 0. For this combination and the B × D interaction, F = B +D = 2 + 2 = 4 modulus 3 = 1. Continuing this process for the n^2 combinations, an F-square is formed with q = 3 symbols. This method is illustrated in the following examples.

N = 6

The rows of and columns of a 6-row by 6-column arrangement are considered to be a four factor factorial with factors A, 2-levels, and B, 3-levels for rows and C, 2-levels, and D, 3-levels, for columns (See Table 1.). The row by column interaction is partitioned into the interactions of the four factors as described in Table 2 . The 13 F-squares are constructed as follows:

F1 = A + C	F6 = B + 2*D	F11 = A + B + 2*D
F2 = A + D	F7 = B + C + D	F12 = A + B + C + D
F3 = A + C + D	F8 = B + C + 2*D	F13 = A + B + C + 2*D
F4 = B + C	F9 = A + B + C	
F5 = B + D	F10 = A + B + D	

F1 is taken modulus two and the remaining F-squares are taken modulus three. The F(6, 3) and the 12 F(6, 2) squares are presented in Table 1 and are given serially from F1 to F13. These were constructed using the SAS code in Appendix A. Squares F2 and F3 are semi-F-squares in that the three symbols 0, 1, and 2 occur equally frequent in the square, appear equally frequent in rows, but not in columns. Squares F4 and F9 are also semi-F-squares where the three symbols occur equally frequent in columns but not in rows. F1, F5, F6, F7, F8, F10, F11, F12, and F13 are regular F-squares. This set is a complete sum-of-squares orthogonal set as the sum of squares for F-squares obtained from an interaction adds to that for the interaction. For example, the sum of the sums of squares for F5 and F6 is equal to that for the B × D interaction. The correspondence between factorial effects and F-squares is given in Table 2.

Pairwise comparisons of the 13 F-squares were made to determine which ones are also combinatorially orthogonal. The results are for O = orthogonal , N = non-orthogonal, and P = proportionally orthogonal.:

	F2	F3	F4	F5	F6	F7	F8	F9	F10	F11	F12	F13
F1	O	O	O	O	O	O	O	O	O	O	O	O
F2	-	N	O	O	O	O	O	O	O	O	O	O
F3	-	-	O	O	O	O	O	O	O	O	O	O
F4	-	-	-	O	O	O	O	N	O	O	O	O
F5	-	-	-	-	O	N	O	O	N	O	P	O
F6	-	-	-	-	-	O	P	O	O	N	O	P
F7	-	-	-	-	-	-	O	O	P	O	N	O
F8	-	-	-	-	-	-	-	O	O	P	O	N
F9	-	-	-	-	-	-	-	-	O	O	O	O
F10	-	-	-	-	-	-	-	-	-	O	N	O
F11	-	-	-	-	-	-	-	-	-	-	O	N
F12	-	-	-	-	-	-	-	-	-	-	-	O

Of the regular F-squares, many pairs of combinatorially squares are possible but triplets and higher sets are not. Since the factorial interactions are combinatorially and sum-of-squares orthogonal, it would appear that more of the F-squares should be. The present method of constructing F-squares is not sufficient for this.

Table 1. Thirteen, F1 to F13, sum-of-squares orthogonal F-squares for n = 6 = 2 × 3.

Row Number	Column number 00	01	02	10	11	12
00	0000000	0110121	0220212	1011001	1121122	1201210
	000000	201212	102121	110011	011220	212102
01	0001111	0111202	0221020	1012112	1122200	1202021
	111111	012020	210202	221122	122001	020210
02	0002222	0112010	0222101	1010220	1120011	1200102
	222222	120101	021010	002200	200112	101021
10	1110000	1220121	1000212	0121001	0201122	0011210
	011111	212020	110202	121122	022001	220210
11	1111111	1221202	1001020	0122112	0202200	0012021
	122222	020101	221010	202200	100112	001021
12	1112222	1222010	1002101	012220	0200011	2010102
	200000	101212	002121	010011	211220	112102

Table 2. Sums of squares for a 2×3×2×3 factorial and for the 13 F-squares.

Source of variation	Degrees of freedom	Sum of squares	F-square number	Sum of squares
Total	36	85,312		
Mean	1	49,580.44		
Row	5	2,375.22		
Column	5	26,196.22		
A×C	1	729.00	F1=A+C	729.00
A×D	2	522.17	F2=A+D	522.17
A×C×D	2	624.50	F3=A+C+D	624.50
B×C	2	360.22	F4=B+C	360.22
B×D	4	843.11	F5=B+D	707.72
			F6=B+2D	135.39
B×C×D	4	805.44	F7=B+C+D	338.39
			F8=B+C+2D	467.06
A×B×C	2	612.67	F9=A+B+C	612.67
A×B×D	4	763.67	F10=A+B+D	487.50
			F11=A+B+2D	276.17
A×B×C×D	4	1,899.33	F12=A+B+C+D	1,356.17
			F13=A+B+C+2D	543.17
Residual	0	0	Residual	0

N = 8

For the 8 × 8 square, let the row numbers be represented as a 2 × 4 factorial for factors A and B. Let the column numbers be represented as a 2 × 4 factorial for the factors C and D. Rather than using the compact form of Table 1, a table from the computer output was used. The row by column interaction is partitioned into the nine interactions of these two sets of effects. The sums of squares for these interactions are as illustrated below. The 17 F-squares were constructed as follows:

F 1 = A + C	F7 = B + 3*D	F13 = A + B + 2*D
F2 = A + D	F8 = B + C + D	F14 = A + B + 3*D
F3 = A + C + D	F9 = B + C + 2*D	F15 = A + B + C + D
F4 = B + C	F10 = B + C + 3*D	F16 = A + B + C + 2*D
F5 = B + D	F11 = A + B + C	F17 = A + B + C + 3*D
F6 = B + 2*D	F12 = A + B + D	

Since the sums of squares corresponding to the F-squares formed from the interactions do not add up to the sums of squares for that interaction in several cases, the 17 F-squares presented below do not form a complete set of sum-of-squares orthogonal F-squares. A code for constructing this set is given in Appendix B. Seven of the degrees of freedom are not accounted for by this set but have been relegated to a residual (error) sum of squares.

Of the 17 F-squares, six are semi-F-squares and 11 are regular F-squares. The semi-F-squares are F2, F3, F4, F6, F11, and F13. The remainding are regular F-squares. If a six factor 2-level factorial has been considered, 49 F(8, 4) squares would have been obtained. These squares would be combinatorially and sum-of-squares orthogonal (See Schwager *et al.*, 1984). Using the marks of the field instead of the simple calculus for prime numbers, a less complete sum-of-squares orthogonal set was obtained. These squares are available.

The 17 F-squares constructed are:

Obs	y	ROW	COL	A	B	C	D	F1	F2	F3	F4	F5	F6	F7	F8	F9	F10	F11	F12	F13	F14	F15	F16	F17
1	92	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	66	1	2	0	0	0	1	0	1	1	0	1	2	3	1	2	3	0	1	2	3	1	2	3
3	19	1	3	0	0	0	2	0	2	2	0	2	0	2	2	0	2	0	2	0	2	2	0	2
4	19	1	4	0	0	0	3	0	3	3	0	3	2	1	3	2	1	0	3	2	1	3	0	1
5	29	1	5	0	0	1	0	1	0	1	1	0	0	0	1	1	1	1	0	0	0	1	1	1
6	16	1	6	0	0	1	1	1	1	2	1	1	2	3	2	3	0	1	1	2	3	2	3	0
7	25	1	7	0	0	1	2	1	2	3	1	2	0	2	3	1	3	1	2	0	2	3	0	3
8	25	1	8	0	0	1	3	1	3	0	1	3	2	1	0	3	2	1	3	2	1	0	0	2
9	60	2	1	0	1	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
10	46	2	2	0	1	0	1	0	1	1	1	2	3	0	2	3	0	1	2	3	0	2	3	0
11	35	2	3	0	1	0	2	0	2	2	1	3	1	3	3	1	3	1	3	1	3	3	0	3
12	35	2	4	0	1	0	3	0	3	3	1	1	3	2	0	3	2	1	0	3	2	0	0	2
13	10	2	5	0	1	1	0	1	0	1	2	1	1	1	2	2	2	2	1	1	1	2	2	2
14	11	2	6	0	1	1	1	1	1	2	2	2	3	0	3	0	1	2	2	3	0	3	0	1
15	5	2	7	0	1	1	2	1	2	3	2	3	1	3	0	2	0	2	3	1	3	0	0	0
16	46	2	8	0	1	1	3	1	3	0	2	1	3	2	1	0	3	2	0	3	2	1	0	3
17	46	3	1	0	2	0	0	0	0	0	2	2	2	2	2	2	2	2	2	2	2	2	2	2
18	81	3	2	0	2	0	1	0	1	1	2	3	0	1	3	0	1	2	3	0	1	3	0	1
19	17	3	3	0	2	0	2	0	2	2	2	1	2	0	0	2	0	2	0	2	0	0	0	0
20	22	3	4	0	2	0	3	0	3	3	2	2	0	3	1	0	3	2	1	0	3	1	0	3
21	22	3	5	0	2	1	0	1	0	1	3	2	2	2	3	3	3	3	2	2	2	3	3	3
22	16	3	6	0	2	1	1	1	1	2	3	3	0	1	0	1	2	3	3	0	1	0	0	2
23	9	3	7	0	2	1	2	1	2	3	3	1	2	0	1	3	1	3	0	2	0	1	0	1
24	9	3	8	0	2	1	3	1	3	0	3	2	0	3	2	1	0	3	1	0	3	2	1	0
25	120	4	1	0	3	0	0	0	0	0	3	3	3	3	3	3	3	3	3	3	3	3	3	3
26	59	4	2	0	3	0	1	0	1	1	3	1	1	2	0	1	2	3	0	1	2	0	0	2
27	43	4	3	0	3	0	2	0	2	2	3	2	3	1	1	3	1	3	1	3	1	1	0	1
28	43	4	4	0	3	0	3	0	3	3	3	3	1	0	2	1	0	3	2	1	0	2	1	0
29	15	4	5	0	3	1	0	1	0	1	0	3	3	3	0	0	0	0	3	3	3	0	1	0
30	10	4	6	0	3	1	1	1	1	2	0	1	1	2	1	2	3	0	0	1	2	1	0	3
31	2	4	7	0	3	1	2	1	2	3	0	2	3	1	2	0	2	0	1	3	1	2	0	2
32	49	4	8	0	3	1	3	1	3	0	0	3	1	0	3	2	1	0	2	1	0	3	2	1
33	49	5	1	1	0	0	0	1	1	1	0	0	0	0	0	0	0	1	1	1	1	1	1	1
34	64	5	2	1	0	0	1	1	2	2	0	1	2	3	1	2	3	1	2	3	0	2	3	0
35	25	5	3	1	0	0	2	1	3	3	0	2	0	2	2	0	2	1	3	1	3	3	0	3
36	25	5	4	1	0	0	3	1	0	0	0	3	2	1	3	2	1	1	0	3	2	0	0	2

37	24	5	5	1	0	1	0	0	1	2	1	0	0	0	1	1	1	2	1	1	1	2	2	2
38	8	5	6	1	0	1	1	0	2	3	1	1	2	3	2	3	0	2	2	3	0	3	0	1
39	7	5	7	1	0	1	2	0	3	0	1	2	0	2	3	1	3	2	3	1	3	0	0	0
40	7	5	8	1	0	1	3	0	0	1	1	3	2	1	0	3	2	2	0	3	2	1	0	3
41	134	6	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2
42	60	6	2	1	1	0	1	1	2	2	1	2	3	0	2	3	0	2	3	0	1	3	0	1
43	52	6	3	1	1	0	2	1	3	3	1	3	1	3	3	1	3	2	0	2	0	0	0	0
44	20	6	4	1	1	0	3	1	0	0	1	1	3	2	0	3	2	2	1	0	3	1	0	3
45	28	6	5	1	1	1	0	0	1	2	2	1	1	1	2	2	2	3	2	2	2	3	3	3
46	11	6	6	1	1	1	1	0	2	3	2	2	3	0	3	0	1	3	3	0	1	0	1	2
47	11	6	7	1	1	1	2	0	3	0	2	3	1	3	0	2	0	3	0	2	0	1	0	1
48	28	6	8	1	1	1	3	0	0	1	2	1	3	2	1	0	3	3	1	0	3	2	1	0
49	20	7	1	1	2	0	0	1	1	1	2	2	2	2	2	2	2	3	3	3	3	3	3	3
50	52	7	2	1	2	0	1	1	2	2	2	3	0	1	3	0	1	3	0	1	2	0	1	2
51	60	7	3	1	2	0	2	1	3	3	2	1	2	0	0	2	0	3	1	3	1	1	0	1
52	134	7	4	1	2	0	3	1	0	0	2	2	0	3	1	0	3	3	2	1	0	2	1	0
53	7	7	5	1	2	1	0	0	1	2	3	2	2	2	3	3	3	0	3	3	3	0	0	0
54	24	7	6	1	2	1	1	0	2	3	3	3	0	1	0	1	2	0	0	1	2	1	0	3
55	25	7	7	1	2	1	2	0	3	0	3	1	2	0	1	3	1	0	1	3	1	2	0	2
56	25	7	8	1	2	1	3	0	0	1	3	2	0	3	2	1	0	0	2	1	0	3	2	1
57	64	8	1	1	3	0	0	1	1	1	3	3	3	3	3	3	3	0	0	0	0	0	0	0
58	49	8	2	1	3	0	1	1	2	2	3	1	1	2	0	1	2	0	1	2	3	1	0	3
59	60	8	3	1	3	0	2	1	3	3	3	2	3	1	1	3	1	0	2	0	2	2	0	2
60	52	8	4	1	3	0	3	1	0	0	3	3	1	0	2	1	0	0	3	2	1	3	2	1
61	20	8	5	1	3	1	0	0	1	2	0	3	3	3	0	0	0	1	0	0	0	1	0	1
62	28	8	6	1	3	1	1	0	2	3	0	1	1	2	1	2	3	1	1	2	3	2	0	0
63	20	8	7	1	3	1	2	0	3	0	0	2	3	1	2	0	2	1	2	0	2	3	1	3
64	11	8	8	1	3	1	3	0	0	1	0	3	1	0	3	2	1	1	3	2	1	0	3	2

The ANOVA for the four factor factorial for n = 8 is:

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	63	53985.93750	856.91964	.	.
Error	0	0.00000	.		
Corrected Total	63	53985.93750			

Source	DF	Type I SS	Mean Square	F Value	Pr > F
A	1	162.56250	162.56250	.	.
B	3	677.56250	225.85417	.	.
A*B	3	1895.81250	631.93750	.	.
C	1	20306.25000	20306.25000	.	.
D	3	3382.31250	1127.43750	.	.
C*D	3	3122.37500	1040.79167	.	.
A*C	1	272.25000	272.25000	.	.
A*D	3	857.81250	285.93750	.	.
A*C*D	3	2075.37500	691.79167	.	.
B*C	3	446.12500	148.70833	.	.
B*D	9	4518.06250	502.00694	.	.
B*C*D	9	5550.25000	616.69444	.	.
A*B*C	3	651.87500	217.29167	.	.
A*B*D	9	7005.81250	778.42361	.	.
A*B*C*D	9	3061.50000	340.16667	.	.

The ANOVA for the 17 F-squares is given below. As may be observed there is partial confounding of effects among the F-squares resulting in a loss of 7 degrees of freedom as shown for the Error term. This should be zero if the set had been sum-of-squares orthogonal.

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	56	51112.77044	912.72804	2.22	0.1338
Error	7	2873.16706	410.45244		
Corrected Total	63	53985.93750			

Source	DF	Type I SS	Mean Square	F Value	Pr > F
ROW	7	2735.93750	390.84821	0.95	0.5249
COL	7	26810.93750	3830.13393	9.33	0.0043
F1	1	272.25000	272.25000	0.66	0.4422
F2	3	857.81250	285.93750	0.70	0.5830
F3	3	2075.37500	691.79167	1.69	0.2561
F4	3	446.12500	148.70833	0.36	0.7825
F5	3	1909.31250	636.43750	1.55	0.2843
F6	2	1251.06250	625.53125	1.52	0.2822
F7	2	742.62500	371.31250	0.90	0.4473
F8	3	3643.62500	1214.54167	2.96	0.1072
F9	2	470.56250	235.28125	0.57	0.5881
F10	2	1054.25000	527.12500	1.28	0.3349
F11	3	651.87500	217.29167	0.53	0.6762
F12	3	4122.81250	1374.27083	3.35	0.0851
F13	2	601.31250	300.65625	0.73	0.5142
F14	2	171.62500	85.81250	0.21	0.8162
F15	3	1939.62500	646.54167	1.58	0.2788
F16	3	794.79389	264.93130	0.65	0.6101
F17	2	560.85155	280.42577	0.68	0.5357

N=10

The 21 F-squares constructed from the interactions are:

F1 = A + C
F2 = A + D
F3 = A + C + D
F4 = B + C
F5 = B + D
F6 = B + 2*D
F7 = B + 3*D
F8 = B + 4*D
F9 = B + C + D
F10 = B + C + 2*D
F11 = B + C + 3*D
F12 = B + C + 4*D
F13 = A + B + C
F14 = A + B + D
F15 = A + B + 2*D
F16 = A + B + 3*D
F17 = A + B + 4*D
F18 = A + B + C + D
F19 = A + B + C + 2*D
F20 = A + B + C + 3*D
F21 = A + B + C + 4*D

These 21 F-squares are given below:

y	ROW	COL	A	C	D	F1	F2	F3	F4	F5	F6	F7	F8	F9	F10	F11	F12	F13	F14	F15	F16	F17	F18	F19	F20	F21
92	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
66	1	2	0	0	1	0	1	1	0	1	2	3	4	1	2	3	4	0	1	2	3	4	1	2	3	4
19	1	3	0	0	2	0	2	2	0	2	4	1	3	2	4	1	3	0	2	4	1	3	2	4	1	3
19	1	4	0	0	3	0	3	3	0	3	1	4	2	3	1	4	2	0	3	1	4	2	3	0	4	2
19	1	5	0	0	4	0	4	4	0	4	3	2	1	4	3	2	1	0	4	3	2	1	4	0	2	1
29	1	6	0	0	1	0	1	0	1	1	0	0	0	0	1	1	1	1	1	0	0	0	0	1	1	1
16	1	7	0	0	1	1	1	1	2	1	1	2	3	4	2	3	4	0	1	1	2	3	4	2	3	4	0
25	1	8	0	0	1	2	1	2	3	1	2	4	1	3	3	0	2	4	1	2	4	1	3	3	1	2	4
25	1	9	0	0	1	3	1	3	4	1	3	1	4	2	4	2	0	3	1	3	1	4	2	4	0	0	3
25	1	10	0	0	1	4	1	4	0	1	4	3	2	1	0	4	3	2	1	4	3	2	1	0	0	3	2

60	2	1	0	1	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
46	2	2	0	1	0	1	0	1	1	1	2	3	4	0	2	3	4	0	1	2	3	4	0	2	3	4	0
35	2	3	0	1	0	2	0	2	2	1	3	0	2	4	3	0	2	4	1	3	0	2	4	3	1	2	4
35	2	4	0	1	0	3	0	3	3	1	4	2	0	3	4	2	0	3	1	4	2	0	3	4	0	0	3
35	2	5	0	1	0	4	0	4	4	1	0	4	3	2	0	4	3	2	1	0	4	3	2	0	0	3	2
10	2	6	0	1	1	0	1	0	1	2	1	1	1	1	2	2	2	2	2	1	1	1	1	2	2	2	2
11	2	7	0	1	1	1	1	1	2	2	2	3	4	0	3	4	0	1	2	2	3	4	0	3	4	0	1
5	2	8	0	1	1	2	1	2	3	2	3	0	2	4	4	1	3	0	2	3	0	2	4	4	0	3	0
46	2	9	0	1	1	3	1	3	4	2	4	2	0	3	0	3	1	4	2	4	2	0	3	0	0	1	4
46	2	10	0	1	1	4	1	4	0	2	0	4	3	2	1	0	4	3	2	0	4	3	2	1	0	4	3
46	3	1	0	2	0	0	0	0	0	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
81	3	2	0	2	0	1	0	1	1	2	3	4	0	1	3	4	0	1	2	3	4	0	1	3	4	0	1
17	3	3	0	2	0	2	0	2	2	2	4	1	3	0	4	1	3	0	2	4	1	3	0	4	0	3	0
22	3	4	0	2	0	3	0	3	3	2	0	3	1	4	0	3	1	4	2	0	3	1	4	0	0	1	4
22	3	5	0	2	0	4	0	4	4	2	1	0	4	3	1	0	4	3	2	1	0	4	3	1	0	4	3
22	3	6	0	2	1	0	1	0	1	3	2	2	2	2	3	3	3	3	3	2	2	2	2	3	3	3	3
16	3	7	0	2	1	1	1	1	2	3	3	4	0	1	4	0	1	2	3	3	4	0	1	4	1	1	2
9	3	8	0	2	1	2	1	2	3	3	4	1	3	0	0	2	4	1	3	4	1	3	0	0	0	4	1
9	3	9	0	2	1	3	1	3	4	3	0	3	1	4	1	4	2	0	3	0	3	1	4	1	0	2	0
9	3	10	0	2	1	4	1	4	0	3	1	0	4	3	2	1	0	4	3	1	0	4	3	2	1	0	4
12	4	1	0	3	0	0	0	0	0	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
59	4	2	0	3	0	1	0	1	1	3	4	0	1	2	4	0	1	2	3	4	0	1	2	4	1	1	2
43	4	3	0	3	0	2	0	2	2	3	0	2	4	1	0	2	4	1	3	0	2	4	1	0	0	4	1
43	4	4	0	3	0	3	0	3	3	3	1	4	2	0	1	4	2	0	3	1	4	2	0	1	0	2	0
43	4	5	0	3	0	4	0	4	4	3	2	1	0	4	2	1	0	4	3	2	1	0	4	2	1	0	4
15	4	6	0	3	1	0	1	0	1	4	3	3	3	3	4	4	4	4	4	3	3	3	3	4	4	4	4
10	4	7	0	3	1	1	1	1	2	4	4	0	1	2	0	1	2	3	4	4	0	1	2	0	0	2	3
2	4	8	0	3	1	2	1	2	3	4	0	2	4	1	1	3	0	2	4	0	2	4	1	1	0	0	2
49	4	9	0	3	1	3	1	3	4	4	1	4	2	0	2	0	3	1	4	1	4	2	0	2	0	3	1
49	4	10	0	3	1	4	1	4	0	4	2	1	0	4	3	2	1	0	4	2	1	0	4	3	2	1	0
49	5	1	0	4	0	0	0	0	0	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
64	5	2	0	4	0	1	0	1	1	4	0	1	2	3	0	1	2	3	4	0	1	2	3	0	0	2	3
25	5	3	0	4	0	2	0	2	2	4	1	3	0	2	1	3	0	2	4	1	3	0	2	1	0	0	2
25	5	4	0	4	0	3	0	3	3	4	2	0	3	1	2	0	3	1	4	2	0	3	1	2	0	3	1
25	5	5	0	4	0	4	0	4	4	4	3	2	1	0	3	2	1	0	4	3	2	1	0	3	2	1	0
24	5	6	0	4	1	0	1	0	1	0	4	4	4	4	0	0	0	0	0	4	4	4	4	0	1	0	0
8	5	7	0	4	1	1	1	1	2	0	0	1	2	3	1	2	3	4	0	0	1	2	3	1	0	3	4
7	5	8	0	4	1	2	1	2	3	0	1	3	0	2	2	4	1	3	0	1	3	0	2	2	0	1	3
7	5	9	0	4	1	3	1	3	4	0	2	0	3	1	3	1	4	2	0	2	0	3	1	3	1	4	2
7	5	10	0	4	1	4	1	4	0	0	3	2	1	0	4	3	2	1	0	3	2	1	0	4	3	2	1
34	6	1	1	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
60	6	2	1	0	0	1	1	2	2	0	1	2	3	4	1	2	3	4	1	2	3	4	0	2	3	4	0
52	6	3	1	0	0	2	1	3	3	0	2	4	1	3	2	4	1	3	1	3	0	2	4	3	0	2	4
20	6	4	1	0	0	3	1	4	4	0	3	1	4	2	3	1	4	2	1	4	2	0	3	4	1	0	3
20	6	5	1	0	0	4	1	0	0	0	4	3	2	1	4	3	2	1	1	0	4	3	2	0	0	3	2
28	6	6	1	0	1	0	0	1	2	1	0	0	0	0	1	1	1	1	2	1	1	1	1	2	2	2	2
11	6	7	1	0	1	1	0	2	3	1	1	2	3	4	2	3	4	0	2	2	3	4	0	3	4	0	1
11	6	8	1	0	1	2	0	3	4	1	2	4	1	3	3	0	2	4	2	3	0	2	4	4	0	3	0
28	6	9	1	0	1	3	0	4	0	1	3	1	4	2	4	2	0	3	2	4	2	0	3	0	0	1	4
28	6	10	1	0	1	4	0	0	1	1	4	3	2	1	0	4	3	2	2	0	4	3	2	1	0	4	3
20	7	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2
52	7	2	1	1	0	1	1	2	2	1	2	3	4	0	2	3	4	0	2	3	4	0	1	3	4	0	1
60	7	3	1	1	0	2	1	3	3	1	3	0	2	4	3	0	2	4	2	4	1	3	0	4	0	3	0
34	7	4	1	1	0	3	1	4	4	1	4	2	0	3	4	2	0	3	2	0	3	1	4	0	0	1	4
34	7	5	1	1	0	4	1	0	0	1	0	4	3	2	0	4	3	2	2	1	0	4	3	1	0	4	3
17	7	6	1	1	1	0	0	1	2	2	1	1	1	1	2	2	2	2	3	2	2	2	2	3	3	3	3
24	7	7	1	1	1	1	0	2	3	2	2	3	4	0	3	4	0	1	3	3	4	0	1	4	0	1	2

25	7	8	1	1	1	2	0	3	4	2	3	0	2	4	4	1	3	0	3	4	1	3	0	0	1	4	1
25	7	9	1	1	1	3	0	4	0	2	4	2	0	3	0	3	1	4	3	0	3	1	4	1	0	2	0
25	7	10	1	1	1	4	0	0	1	2	0	4	3	2	1	0	4	3	3	1	0	4	3	2	1	0	4
64	8	1	1	2	0	0	1	1	1	2	2	2	2	2	2	2	2	2	3	3	3	3	3	3	3	3	3
49	8	2	1	2	0	1	1	2	2	2	3	4	0	1	3	4	0	1	3	4	0	1	2	4	0	1	2
60	8	3	1	2	0	2	1	3	3	2	4	1	3	0	4	1	3	0	3	0	2	4	1	0	1	4	1
52	8	4	1	2	0	3	1	4	4	2	0	3	1	4	0	3	1	4	3	1	4	2	0	1	0	2	0
52	8	5	1	2	0	4	1	0	0	2	1	0	4	3	1	0	4	3	3	2	1	0	4	2	1	0	4
20	8	6	1	2	1	0	0	1	2	3	2	2	2	2	3	3	3	3	4	3	3	3	3	4	4	4	4
28	8	7	1	2	1	1	0	2	3	3	3	4	0	1	4	0	1	2	4	4	0	1	2	0	0	2	3
20	8	8	1	2	1	2	0	3	4	3	4	1	3	0	0	2	4	1	4	0	2	4	1	1	0	0	2
11	8	9	1	2	1	3	0	4	0	3	0	3	1	4	1	4	2	0	4	1	4	2	0	2	0	3	1
11	8	10	1	2	1	4	0	0	1	3	1	0	4	3	2	1	0	4	4	2	1	0	4	3	2	1	0
11	9	1	1	3	0	0	1	1	1	3	3	3	3	3	3	3	3	3	4	4	4	4	4	4	4	4	4
20	9	2	1	3	0	1	1	2	2	3	4	0	1	2	4	0	1	2	4	0	1	2	3	0	0	2	3
28	9	3	1	3	0	2	1	3	3	3	0	2	4	1	0	2	4	1	4	1	3	0	2	1	0	0	2
20	9	4	1	3	0	3	1	4	4	3	1	4	2	0	1	4	2	0	4	2	0	3	1	2	0	3	1
52	9	5	1	3	0	4	1	0	0	3	2	1	0	4	2	1	0	4	4	3	2	1	0	3	2	1	0
60	9	6	1	3	1	0	0	1	2	4	3	3	3	3	4	4	4	4	0	4	4	4	4	0	0	0	0
59	9	7	1	3	1	1	0	2	3	4	4	0	1	2	0	1	2	3	0	0	1	2	3	1	1	3	4
58	9	8	1	3	1	2	0	3	4	4	0	2	4	1	1	3	0	2	0	1	3	0	2	2	0	1	3
57	9	9	1	3	1	3	0	4	0	4	1	4	2	0	2	0	3	1	0	2	0	3	1	3	1	4	2
46	9	10	1	3	1	4	0	0	1	4	2	1	0	4	3	2	1	0	0	3	2	1	0	4	3	2	1
45	10	1	1	4	0	0	1	1	1	4	4	4	4	4	4	4	4	4	0	0	0	0	0	0	0	0	0
54	10	2	1	4	0	1	1	2	2	4	0	1	2	3	0	1	2	3	0	1	2	3	4	1	1	3	4
53	10	3	1	4	0	2	1	3	3	4	1	3	0	2	1	3	0	2	0	2	4	1	3	2	0	1	3
52	10	4	1	4	0	3	1	4	4	4	2	0	3	1	2	0	3	1	0	3	1	4	2	3	1	4	2
51	10	5	1	4	0	4	1	0	0	4	3	2	1	0	3	2	1	0	0	4	3	2	1	4	3	2	1
50	10	6	1	4	1	0	0	1	2	0	4	4	4	4	0	0	0	0	1	0	0	0	0	1	0	1	1
49	10	7	1	4	1	1	0	2	3	0	0	1	2	3	1	2	3	4	1	1	2	3	4	2	0	4	0
48	10	8	1	4	1	2	0	3	4	0	1	3	0	2	2	4	1	3	1	2	4	1	3	3	0	2	4
45	10	9	1	4	1	3	0	4	0	0	2	0	3	1	3	1	4	2	1	3	1	4	2	4	2	0	3
69	10	10	1	4	1	4	0	0	1	0	3	2	1	0	4	3	2	1	1	4	3	2	1	0	4	3	2

The ANOVA for the four factor factorial for n = 10 is given below:

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	99	37746.75000	381.28030	.	.
Error	0	0.00000	.		
Corrected Total	99	37746.75000			

Source	DF	Type I SS	Mean Square	F Value	Pr > F
A	1	1755.610000	1755.610000	.	.
B	4	832.700000	208.175000	.	.
A*B	4	3146.140000	786.535000	.	.
C	1	5140.890000	5140.890000	.	.
D	4	1035.600000	258.900000	.	.
C*D	4	2943.760000	735.940000	.	.
A*C	1	1056.250000	1056.250000	.	.
A*D	4	1678.640000	419.660000	.	.
A*C*D	4	2072.800000	518.200000	.	.
B*C	4	3867.660000	966.915000	.	.
B*D	16	3834.200000	239.637500	.	.
B*C*D	16	2396.440000	149.777500	.	.
A*B*C	4	2666.700000	666.675000	.	.

A*B*D	16	2472.360000	154.522500	.	.
A*B*C*D	16	2847.000000	177.937500	.	.

The Type I ANOVA for the 21 F-squares is given below. As may be observed, the sum of the sums of squares for each set of F-squares formed from an interaction, adds to that for the interaction, indicating sum-of-squares orthogonality.

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	99	37746.75000	381.28030	.	.
Error	0	0.00000	.		
Corrected Total	99	37746.75000			

The ANOVA for the 21 F-squares for n = 10 is:

Source	DF	Type I SS	Mean Square	F Value	Pr > F
ROW	9	5734.450000	637.161111	.	.
COL	9	9120.250000	1013.361111	.	.
F1	1	1056.250000	1056.250000	.	.
F2	4	1678.640000	419.660000	.	.
F3	4	2072.800000	518.200000	.	.
F4	4	3867.660000	966.915000	.	.
F5	4	276.900000	69.225000	.	.
F6	4	1267.200000	316.800000	.	.
F7	4	1629.400000	407.350000	.	.
F8	4	660.700000	165.175000	.	.
F9	4	828.860000	207.215000	.	.
F10	4	724.160000	181.040000	.	.
F11	4	95.560000	23.890000	.	.
F12	4	747.860000	186.965000	.	.
F13	4	2666.700000	666.675000	.	.
F14	4	667.540000	166.885000	.	.
F15	4	920.240000	230.060000	.	.
F16	4	820.240000	205.060000	.	.
F17	4	64.340000	16.085000	.	.
F18	4	1180.300000	295.075000	.	.
F19	4	125.356655	31.339164	.	.
F20	4	291.449103	72.862276	.	.
F21	4	1249.894242	312.473561	.	.

N = 12

Considering the row numbers to be represented by the levels of the 2-level factor A and the 6-level factor B and the columns by the levels of the 2-level factor C and the 6-level factor D, the 25 F-sqiuares constructed from the row by colmn interactions are

F1 = A + C	F8 = B + 4*D	F15 = A + B + C	F22 = A + B + C + 2*D
F2 = A + D	F9 = B + 5*D	F16 = A + B + D	F23 = A + B + C + 3*D
F3 = A + C + D	F10 = B + C + D	F17 = A + B + 2*D	F24 = A + B + C + 4*D
F4 = B + C	F11 = B + C + 2*D	F18 = A + B + 3*D	F25 = A + B + C + 5*D
F5 = B + D	F12 = B + C + 3*D	F19 = A + B + 4*D	
F6 = B + 2*D	F13 = B + C + 4*D	F20 = A + B + 5*D	
F7 = B + 3*D	F14 = B + C + 5*D	F21 = A + B + C + D	

F1 is summed modulo two and the remaining are summed modulo six. Although some of the F-squares are SOSOFs, most are not. As indicated in the ANOVA table for the F-squares, there is considerable partial

confounding among them. A total of 40 degrees of freedom are lost because of the partial confounding of effects among the 25 F-squares.

The ANOVA for the four factor factoial for n = 12 is:

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	143	52773.75000	369.04720	.	.
Error	0	0.00000	.		
Corrected Total	143	52773.75000			

Source	DF	Type I SS	Mean Square	F Value	Pr > F
ROW	11	10951.75000	995.61364	.	.
COL	11	14252.58333	1295.68939	.	.
A*C	1	625.00000	625.00000	.	.
A*D	5	1414.47222	282.89444	.	.
A*C*D	5	2008.58333	401.71667	.	.
B*C	5	4835.80556	967.16111	.	.
B*D	25	5687.66667	227.50667	.	.
B*C*D	25	3178.44444	127.13778	.	.
A*B*C	5	2946.16667	589.23333	.	.
A*B*D	25	3109.02778	124.36111	.	.
A*B*C*D	25	3764.25000	150.57000	.	.

The Type I ANOVA for the 25 F-squares is given below. As may be observed these F-squares are not sum-of-squares orthogonal. There is enough partial confounding so that 40 degrees of freedom are associated with the confounding. In order for this to be sum-of-squares orthogonal, there would need to be zero degrees of freedom associated with the error term.

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	103	46288.41667	449.40210	2.77	0.0002
Error	40	6485.33333	162.13333		
Corrected Total	143	52773.75000			

Source	DF	Type I SS	Mean Square	F Value	Pr > F
ROW	11	10951.75000	995.61364	6.14	<.0001
COL	11	14252.58333	1295.68939	7.99	<.0001
F1	1	625.00000	625.00000	3.85	0.0566
F2	5	1414.47222	282.89444	1.74	0.1467
F3	5	2008.58333	401.71667	2.48	0.0479
F4	5	4835.80556	967.16111	5.97	0.0003
F5	5	719.66667	143.93333	0.89	0.4983
F6	4	510.05556	127.51389	0.79	0.5408
F7	2	234.29167	117.14583	0.72	0.4918
F8	2	150.22222	75.11111	0.46	0.6326
F9	2	1046.79167	523.39583	3.23	0.0502
F10	5	798.05556	159.61111	0.98	0.4392
F11	4	662.61111	165.65278	1.02	0.4078
F12	2	251.43056	125.71528	0.78	0.4673
F13	2	39.38889	19.69444	0.12	0.8859
F14	2	633.34722	316.67361	1.95	0.1551
F15	5	2946.16667	589.23333	3.63	0.0084
F16	5	718.47222	143.69444	0.89	0.4993
F17	4	174.11111	43.52778	0.27	0.8965
F18	2	354.34722	177.17361	1.09	0.3451
F19	2	206.88889	103.44444	0.64	0.5336
F20	2	96.84722	48.42361	0.30	0.7434

F21	5	620.41667	124.08333	0.77	0.5802
F22	4	183.66667	45.91667	0.28	0.8871
F23	2	86.43056	43.21528	0.27	0.7674
F24	2	402.16667	201.08333	1.24	0.3002
F25	2	1364.84722	682.42361	4.21	0.0219

N = 15

Considering the row numbers to be represented by the 3-level factor A and the 5-level factor B and the column numbers by the 3-level factor C and the 5-level factor D, 50 F-squares are formed from the row by column interactions. These are constructed as follows:

F1 = A + C
F2 = A + 2*C
F3 = A + D
F4 = A + 2*D
F5 = A + C + D
F6 = A + C + 2*D
F7 = A + 2*C + D
F8 = A + 2*C + 2*D
F9 = B + C
F10 = 2*B + C
F11 = B + D
F12 = B + 2*D
F13 = B + 3*D
F14 = B + 4*D
F15 = B + C + D
F16 = B + C + 2*D
F17 = B + C + 3*DA
F18 = B + C + 4*D
F19 = B = 2*C + D
F20 = B + 2*C + 2*D
F21 = B + 2*C + 3*D
F22 = B + 2*C + 4*D
F23 = A + B + C
F24 = A +2*B + C
F25 = A + B + 2* C
F26 = A + 2*B + 2*C
F27 = A + B + D
F28 = A + B + 2*D
F29 = A + B + 3*D
F30 = A + B + 4*D
F31 = A + 2*B + D
F32 = A + 2*B + 2*D
F33 = A + 2*B + 3*D
F34 = A + 2*B + 4*D
F35 = A + B + C + D
F36 = A + B + C + 2*D
F37 = A + B + C + 3*D
F38 = A + B + C + 4*D
F39 = A + 2*B + C +D
F40 = A + 2*B + C + 2*D
F41 = A + 2*B + C + 3*D
F42 = A + 2*B + C + 4*D
F43 = A + B + 2*C + D
F44 = A + B + 2*C + 2*D
F45 = A + B + 2*C + 3*D
F46 = A + B + 2*C + 4*D
F47 = A + 2*B + 2*C + D
F48 = A + 2*B + 2*C + 2*D
F49 = A + 2*B + 2*C + 3*D
F50 = A + 2*B + 2*C + 4 *D

F1 ad F2 are taken modulus 3 and the remaining 48 F-squares are taken modulus 5.

The 50 sum-of-squares orthogonal F-squares for n = 15 = 3 × 5 are given below:.

0		R	C														F	F	F	F	F	F	F	F	F	F	F
b		0	0					F	F	F	F	F	F	F	F	F	1	1	1	1	1	1	1	1	1	2	2
s	y	W	L	A	B	C	D	1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9	0	1
1	92	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	66	1	2	0	0	0	1	0	0	1	2	1	2	1	2	0	0	1	2	3	4	1	2	3	4	1	2	3
3	19	1	3	0	0	0	2	0	0	2	4	2	4	2	4	0	0	2	4	1	3	2	4	1	3	2	4	1
4	19	1	4	0	0	0	3	0	0	3	1	3	1	3	1	0	0	3	1	4	2	3	1	4	2	3	1	4
5	19	1	5	0	0	0	4	0	0	4	3	4	3	4	3	0	0	4	3	2	1	4	3	2	1	4	3	2
6	29	1	6	0	0	1	0	1	2	0	0	1	1	2	2	1	1	0	0	0	0	1	1	1	1	2	2	2
7	16	1	7	0	0	1	1	1	2	1	2	2	3	3	4	1	1	1	2	3	4	2	3	4	0	3	4	0
8	25	1	8	0	0	1	2	1	2	2	4	3	0	4	1	1	1	2	4	1	3	3	0	2	4	4	1	3
9	25	1	9	0	0	1	3	1	2	3	1	4	2	0	3	1	1	3	1	4	2	4	2	0	3	0	3	1
10	25	1	10	0	0	1	4	1	2	4	3	0	4	1	0	1	1	4	3	2	1	0	4	3	2	1	0	4
11	29	1	11	0	0	2	0	2	1	0	0	2	2	4	4	2	2	0	0	0	0	2	2	2	2	4	4	4
12	16	1	12	0	0	2	1	2	1	1	2	3	4	0	1	2	2	1	2	3	4	3	4	0	1	0	1	2
13	25	1	13	0	0	2	2	2	1	2	4	4	1	1	3	2	2	2	4	1	3	4	1	3	0	1	3	0
14	25	1	14	0	0	2	3	2	1	3	1	0	3	2	0	2	2	3	1	4	2	0	3	1	4	2	0	3
15	25	1	15	0	0	2	4	2	1	4	3	1	0	3	2	2	2	4	3	2	1	1	0	4	3	3	2	1
16	60	2	1	0	1	0	0	0	0	0	0	0	0	0	0	1	2	1	1	1	1	1	1	1	1	1	1	1
17	46	2	2	0	1	0	1	0	0	1	2	1	2	1	2	1	2	2	3	4	0	2	3	4	0	2	3	4
18	35	2	3	0	1	0	2	0	0	2	4	2	4	2	4	1	2	3	0	2	4	3	0	2	4	3	0	2

19	35	2	4	0	1	0	3	0	0	3	1	3	1	3	1	1	2	4	2	0	3	4	2	0	3	4	2	0
20	35	2	5	0	1	0	4	0	0	4	3	4	3	4	3	1	2	0	4	3	2	0	4	3	2	0	4	3
21	10	2	6	0	1	1	0	1	2	0	0	1	1	2	2	2	3	1	1	1	1	2	2	2	2	3	3	3
22	11	2	7	0	1	1	1	1	2	1	2	2	3	3	4	2	3	2	3	4	0	3	4	0	1	4	0	1

0	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
b	2	2	2	2	2	2	2	2	3	3	3	3	3	3	3	3	3	3	4	4	4	4	4	4	4	4	4	5
s	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9	0

1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	4	0	0	0	0	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
3	3	0	0	0	0	2	4	1	3	2	4	1	3	2	4	1	3	2	4	1	3	2	4	1	3	2	4	1	3
4	2	0	0	0	0	3	1	4	2	3	1	4	2	3	1	4	2	3	1	4	2	3	1	4	2	3	1	4	2
5	1	0	0	0	0	4	3	2	1	4	3	2	1	4	3	2	1	4	3	2	1	4	3	2	1	4	3	2	1
6	2	1	1	2	2	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2
7	1	1	1	2	2	1	2	3	4	1	2	3	4	2	3	4	0	2	3	4	0	3	4	0	1	3	4	0	1
8	0	1	1	2	2	2	4	1	3	2	4	1	3	3	0	2	4	3	0	2	4	4	1	3	0	4	1	3	0
9	4	1	1	2	2	3	1	4	2	3	1	4	2	4	2	0	3	4	2	0	3	0	3	1	4	0	3	1	4
10	3	1	1	2	2	4	3	2	1	4	3	2	1	0	4	3	2	0	4	3	2	1	0	4	3	1	0	4	3
11	4	2	2	4	4	0	0	0	0	0	0	0	0	2	2	2	2	2	2	2	2	4	4	4	4	4	4	4	4
12	3	2	2	4	4	1	2	3	4	1	2	3	4	3	4	0	1	3	4	0	1	0	1	2	3	0	1	2	3
13	2	2	2	4	4	2	4	1	3	2	4	1	3	4	1	3	0	4	1	3	0	1	3	0	2	1	3	0	2
14	1	2	2	4	4	3	1	4	2	3	1	4	2	0	3	1	4	0	3	1	4	2	0	3	1	2	0	3	1
15	0	2	2	4	4	4	3	2	1	4	3	2	1	1	0	4	3	1	0	4	3	3	2	1	0	3	2	1	0
16	1	1	2	1	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2
17	0	1	2	1	2	2	3	4	0	3	4	0	1	2	3	4	0	3	4	0	1	2	3	4	0	3	4	0	1
18	4	1	2	1	2	3	0	2	4	4	1	3	0	3	0	2	4	4	1	3	0	3	0	2	4	4	1	3	0
19	3	1	2	1	2	4	2	0	3	0	3	1	4	4	2	0	3	0	3	1	4	4	2	0	3	0	3	1	4
20	2	1	2	1	2	0	4	3	2	1	0	4	3	0	4	3	2	1	0	4	3	0	4	3	2	1	0	4	3
21	3	2	3	3	4	1	1	1	1	2	2	2	2	2	2	2	2	3	3	3	3	3	3	3	3	4	4	4	4
22	2	2	3	3	4	2	3	4	0	3	4	0	1	3	4	0	1	4	0	1	2	4	0	1	2	0	1	2	3

23	5	2	8	0	1	1	2	1	2	2	4	3	0	4	1	2	3	3	0	2	4	4	1	3	0	0	2	4
24	46	2	9	0	1	1	3	1	2	3	1	4	2	0	3	2	3	4	2	0	3	0	3	1	4	1	4	2
25	46	2	10	0	1	1	4	1	2	4	3	0	4	1	0	2	3	0	4	3	2	1	0	4	3	2	1	0
26	10	2	11	0	1	2	0	2	1	0	0	2	2	4	4	3	4	1	1	1	1	3	3	3	3	0	0	0
27	11	2	12	0	1	2	1	2	1	1	2	3	4	0	1	3	4	2	3	4	0	4	0	1	2	1	2	3
28	5	2	13	0	1	2	2	2	1	2	4	4	1	1	3	3	4	3	0	2	4	0	2	4	1	2	4	1
29	46	2	14	0	1	2	3	2	1	3	1	0	3	2	0	3	4	4	2	0	3	1	4	2	0	3	1	4
30	46	2	15	0	1	2	4	2	1	4	3	1	0	3	2	3	4	0	4	3	2	2	1	0	4	4	3	2
31	46	3	1	0	2	0	0	0	0	0	0	0	0	0	0	0	2	4	2	2	2	2	2	2	2	2	2	2	2
32	81	3	2	0	2	0	1	0	0	1	2	1	2	1	2	2	4	3	4	0	1	3	4	0	1	3	4	0
33	17	3	3	0	2	0	2	0	0	2	4	2	4	2	4	2	4	4	1	3	0	4	1	3	0	4	1	3
34	22	3	4	0	2	0	3	0	0	3	1	3	1	3	1	2	4	0	3	1	4	0	3	1	4	0	3	1
35	22	3	5	0	2	0	4	0	0	4	3	4	3	4	3	2	4	1	0	4	3	1	0	4	3	1	0	4
36	22	3	6	0	2	1	0	1	2	0	0	1	1	2	2	3	0	2	2	2	2	3	3	3	3	4	4	4
37	16	3	7	0	2	1	1	1	2	1	2	2	3	3	4	3	0	3	4	0	1	4	0	1	2	0	1	2
38	9	3	8	0	2	1	2	1	2	2	4	3	0	4	1	3	0	4	1	3	0	0	2	4	1	1	3	0
39	9	3	9	0	2	1	3	1	2	3	1	4	2	0	3	3	0	0	3	1	4	1	4	2	0	2	0	3
40	9	3	10	0	2	1	4	1	2	4	3	0	4	1	0	3	0	1	0	4	3	2	1	0	4	3	2	1
41	10	3	11	0	2	2	0	2	1	0	0	2	2	4	4	4	1	2	2	2	2	4	4	4	4	1	1	1
42	11	3	12	0	2	2	1	2	1	1	2	3	4	0	1	4	1	3	4	0	1	0	1	2	3	2	3	4
43	5	3	13	0	2	2	2	2	1	2	4	4	1	1	3	4	1	4	1	3	0	1	3	0	2	3	0	2
44	46	3	14	0	2	2	3	2	1	3	1	0	3	2	0	4	1	0	3	1	4	2	0	3	1	4	2	0

23	1	2	3	3	4	3	0	2	4	4	1	3	0	4	1	3	0	0	2	4	1	0	2	4	1	1	3	0	2
24	0	2	3	3	4	4	2	0	3	0	3	1	4	0	3	1	4	1	4	2	0	1	4	2	0	2	0	3	1

25	4	2	3	3	4	0	4	3	2	1	0	4	3	1	0	4	3	2	1	0	4	2	1	0	4	3	2	1	0
26	0	3	4	0	1	1	1	1	1	2	2	2	2	3	3	3	3	4	4	4	4	0	0	0	0	1	1	1	1
27	4	3	4	0	1	2	3	4	0	3	4	0	1	4	0	1	2	0	1	2	3	1	2	3	4	2	3	4	0
28	3	3	4	0	1	3	0	2	4	4	1	3	0	0	2	4	1	1	3	0	2	2	4	1	3	3	0	2	4
29	2	3	4	0	1	4	2	0	3	0	3	1	4	1	4	2	0	2	0	3	1	3	1	4	2	4	2	0	3
30	1	3	4	0	1	0	4	3	2	1	0	4	3	2	1	0	4	3	2	1	0	4	3	2	1	0	4	3	2
31	2	2	4	2	4	2	2	2	2	4	4	4	4	2	2	2	2	4	4	4	4	2	2	2	2	4	4	4	4
32	1	2	4	2	4	3	4	0	1	0	1	2	3	3	4	0	1	0	1	2	3	3	4	0	1	0	1	2	3
33	0	2	4	2	4	4	1	3	0	1	3	0	2	4	1	3	0	1	3	0	2	4	1	3	0	1	3	0	2
34	4	2	4	2	4	0	3	1	4	2	0	3	1	0	3	1	4	2	0	3	1	0	3	1	4	2	0	3	1
35	3	2	4	2	4	1	0	4	3	3	2	3	0	1	0	4	3	3	2	1	0	1	0	4	3	3	2	1	0
36	4	3	0	4	1	2	2	2	2	4	4	4	4	3	3	3	3	0	0	0	0	4	4	4	4	1	1	1	1
37	3	3	0	4	1	3	4	0	1	0	1	2	3	4	0	1	2	1	2	3	4	0	1	2	3	2	3	4	0
38	2	3	0	4	1	4	1	3	0	1	3	0	2	0	2	4	1	2	4	1	3	1	3	0	2	3	0	2	4
39	1	3	0	4	1	0	3	1	4	2	0	3	1	1	4	2	0	3	1	4	2	2	0	3	1	4	2	0	3
40	0	3	0	4	1	1	0	4	3	3	2	3	0	2	1	0	4	4	3	2	1	3	2	1	0	0	4	3	2
41	1	4	1	1	3	2	2	2	2	4	4	4	4	4	4	4	4	1	1	1	1	1	1	1	1	3	3	3	3
42	0	4	1	1	3	3	4	0	1	0	1	2	3	0	1	2	3	2	3	4	0	2	3	4	0	4	0	1	2
43	4	4	1	1	3	4	1	3	0	1	3	0	2	1	3	0	2	3	0	2	4	3	0	2	4	0	2	4	1
44	3	4	1	1	3	0	3	1	4	2	0	3	1	2	0	3	1	4	2	0	3	4	2	0	3	1	4	2	0

45	46	3	15	0	2	2	4	2	1	4	3	1	0	3	2	4	1	1	0	4	3	3	2	1	0	0	4	3
46	12	4	1	0	3	0	0	0	0	0	0	0	0	0	0	3	1	3	3	3	3	3	3	3	3	3	3	3
47	59	4	2	0	3	0	1	0	0	1	2	1	2	1	2	3	1	4	0	1	2	4	0	1	2	4	0	1
48	43	4	3	0	3	0	2	0	0	2	4	2	4	2	4	3	1	0	2	4	1	0	2	4	1	0	2	4
49	43	4	4	0	3	0	3	0	0	3	1	3	1	3	1	3	1	1	4	2	0	1	4	2	0	1	4	2
50	43	4	5	0	3	0	4	0	0	4	3	4	3	4	3	3	1	2	1	0	4	2	1	0	4	2	1	0
51	15	4	6	0	3	1	0	1	2	0	0	1	1	2	2	4	2	3	3	3	3	4	4	4	4	0	0	0
52	10	4	7	0	3	1	1	1	2	1	2	2	3	3	4	4	2	4	0	1	2	0	1	2	3	1	2	3
53	2	4	8	0	3	1	2	1	2	2	4	3	0	4	1	4	2	0	2	4	1	1	3	0	2	2	4	1
54	49	4	9	0	3	1	3	1	2	3	1	4	2	0	3	4	2	1	4	2	0	2	0	3	1	3	1	4
55	49	4	10	0	3	1	4	1	2	4	3	0	4	1	0	4	2	2	1	0	4	3	2	1	0	4	3	2
56	10	4	11	0	3	2	0	2	1	0	0	2	2	4	4	0	3	3	3	3	3	0	0	0	0	2	2	2
57	11	4	12	0	3	2	1	2	1	1	2	3	4	0	1	0	3	4	0	1	2	1	2	3	4	3	4	0
58	5	4	13	0	3	2	2	2	1	2	4	4	1	1	3	0	3	0	2	4	1	2	4	1	3	4	1	3
59	46	4	14	0	3	2	3	2	1	3	1	0	3	2	0	0	3	1	4	2	0	3	1	4	2	0	3	1
60	46	4	15	0	3	2	4	2	1	4	3	1	0	3	2	0	3	2	1	0	4	4	3	2	1	1	0	4
61	49	5	1	0	4	0	0	0	0	0	0	0	0	0	0	4	3	4	4	4	4	4	4	4	4	4	4	4
62	64	5	2	0	4	0	1	0	0	1	2	1	2	1	2	4	3	0	1	2	3	0	1	2	3	0	1	2
63	25	5	3	0	4	0	2	0	0	2	4	2	4	2	4	4	3	1	3	0	2	1	3	0	2	1	3	0
64	25	5	4	0	4	0	3	0	0	3	1	3	1	3	1	4	3	2	0	3	1	2	0	3	1	2	0	3
65	25	5	5	0	4	0	4	0	0	4	3	4	3	4	3	4	3	3	2	1	0	3	2	1	0	3	2	1
66	24	5	6	0	4	1	0	1	2	0	0	1	1	2	2	0	4	4	4	4	4	0	0	0	0	1	1	1

45	2	4	1	1	3	1	0	4	3	3	2	3	0	3	2	1	0	0	4	3	2	0	4	3	2	2	1	0	4
46	3	3	1	3	1	3	3	3	3	1	1	1	1	3	3	3	3	1	1	1	1	3	3	3	3	1	1	1	1
47	2	3	1	3	1	4	0	1	2	2	3	4	0	4	0	1	2	2	3	4	0	4	0	1	2	2	3	4	0
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207	43	14	12	2	3	2	1	1	0	3	4	0	1	2	3	0	3	4	0	1	2	1	2	3	4	3	4	0	
208	44	14	13	2	3	2	2	1	0	4	1	1	3	3	0	0	3	0	2	4	1	2	4	1	3	4	1	3	
209	56	14	14	2	3	2	3	1	0	0	3	2	0	4	2	0	3	1	4	2	0	3	1	4	2	0	3	1	
210	54	14	15	2	3	2	4	1	0	1	0	3	2	0	4	0	3	2	1	0	4	4	3	2	1	1	0	4	
211	43	15	1	2	4	0	0	2	2	2	2	2	2	2	2	4	3	4	4	4	4	4	4	4	4	4	4	4	
212	56	15	2	2	4	0	1	2	2	3	4	3	4	3	4	4	3	0	1	2	3	0	1	2	3	0	1	2	
213	44	15	3	2	4	0	2	2	2	4	1	4	1	4	1	4	3	1	3	0	2	1	3	0	2	1	3	0	
214	43	15	4	2	4	0	3	2	2	0	3	0	3	0	3	4	3	2	0	3	1	2	0	3	1	2	0	3	
215	34	15	5	2	4	0	4	2	2	1	0	1	0	1	0	4	3	3	2	1	0	3	2	1	0	3	2	1	
216	43	15	6	2	4	1	0	0	1	2	2	3	3	4	4	0	4	4	4	4	4	0	0	0	0	1	1	1	
217	44	15	7	2	4	1	1	0	1	3	4	4	0	0	1	0	4	0	1	2	3	1	2	3	4	2	3	4	
218	32	15	8	2	4	1	2	0	1	4	1	0	2	1	3	0	4	1	3	0	2	2	4	1	3	3	0	2	
219	33	15	9	2	4	1	3	0	1	0	3	1	4	2	0	0	4	2	0	3	1	3	1	4	2	4	2	0	
220	43	15	10	2	4	1	4	0	1	1	0	2	1	3	2	0	4	3	2	1	0	4	3	2	1	0	4	3	
199	0	0	3	0	3	3	1	4	2	1	4	3	0	3	1	4	2	1	4	2	0	3	1	4	2	1	4	2	0
200	4	0	3	0	3	4	3	2	1	2	1	0	4	4	3	2	1	2	1	0	4	4	3	2	1	2	1	0	4
201	0	1	4	2	0	0	0	0	0	3	3	3	3	1	1	1	1	4	4	4	4	2	2	2	2	0	0	0	0
202	4	1	4	2	0	1	2	3	4	4	0	1	2	2	3	4	0	0	1	2	3	3	4	0	1	1	2	3	4
203	3	1	4	2	0	2	4	1	3	0	2	4	1	3	0	2	4	1	3	0	2	4	1	3	0	2	4	1	3
204	2	1	4	2	0	3	1	4	2	1	4	3	0	4	2	0	3	2	0	3	1	0	3	1	4	3	1	4	2
205	1	1	4	2	0	4	3	2	1	2	1	0	4	0	4	3	2	3	2	1	0	1	0	4	3	4	3	2	1
206	2	2	0	4	2	0	0	0	0	3	3	3	3	2	2	2	2	0	0	0	0	4	4	4	4	2	2	2	2
207	1	2	0	4	2	1	2	3	4	4	0	1	2	3	4	0	1	1	2	3	4	0	1	2	3	3	4	0	1
208	0	2	0	4	2	2	4	1	3	0	2	4	1	4	1	3	0	2	4	1	3	1	3	0	2	4	1	3	0
209	4	2	0	4	2	3	1	4	2	1	4	3	0	0	3	1	4	3	1	4	2	2	0	3	1	0	3	1	4
210	3	2	0	4	2	4	3	2	1	2	1	0	4	1	0	4	3	4	3	2	1	3	2	1	0	1	0	4	3
211	4	1	0	1	0	1	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0
212	3	1	0	1	0	2	3	4	0	1	2	3	4	2	3	4	0	1	2	3	4	2	3	4	0	1	2	3	4
213	2	1	0	1	0	3	0	2	4	2	4	3	3	3	0	2	4	2	4	1	3	3	0	2	4	2	4	1	3
214	1	1	0	1	0	4	2	0	3	3	1	3	2	4	2	0	3	3	1	4	2	4	2	0	3	3	1	4	2
215	0	1	0	1	0	0	4	3	2	4	3	2	1	0	4	3	2	4	3	2	1	0	4	3	2	4	3	2	1
216	1	2	1	3	2	1	1	1	1	0	0	0	0	2	2	2	2	1	1	1	1	3	3	3	3	2	2	2	2
217	0	2	1	3	2	2	3	4	0	1	2	3	4	3	4	0	1	2	3	4	0	4	0	1	2	3	4	0	1
218	4	2	1	3	2	3	0	2	4	2	4	3	3	4	1	3	0	3	0	2	4	0	2	4	1	4	1	3	0
219	3	2	1	3	2	4	2	0	3	3	1	3	2	0	3	1	4	4	2	0	3	1	4	2	0	0	3	1	4
220	2	2	1	3	2	0	4	3	2	4	3	2	1	1	0	4	3	0	4	3	2	2	1	0	4	1	0	4	3
221	45	15	11	2	4	2	0	1	0	2	2	4	4	1	1	1	0	4	4	4	4	1	1	1	1	3	3	3	
222	54	15	12	2	4	2	1	1	0	3	4	0	1	2	3	1	0	0	1	2	3	2	3	4	0	4	0	1	
223	55	15	13	2	4	2	2	1	0	4	1	1	3	3	0	1	0	1	3	0	2	3	0	2	4	0	2	4	
224	43	15	14	2	4	2	3	1	0	0	3	2	0	4	2	1	0	2	0	3	1	4	2	0	3	1	4	2	
225	56	15	15	2	4	2	4	1	0	1	0	3	2	0	4	1	0	3	2	1	0	0	4	3	2	2	1	0	
221	3	3	2	0	4	1	1	1	1	0	0	0	0	3	3	3	3	2	2	2	2	0	0	0	0	4	4	4	4
222	2	3	2	0	4	2	3	4	0	1	2	3	4	4	0	1	2	3	4	0	1	1	2	3	4	0	1	2	3
223	1	3	2	0	4	3	0	2	4	2	4	3	3	0	2	4	1	4	1	3	0	2	4	1	3	1	3	0	2

```
224 0  3  2  0  4  4  2  0  3  3  1  3  2  1  4  2  0  0  3  1  4  3  1  4  2  2  0  3  1
225 4  3  2  0  4  0  4  3  2  4  3  2  1  2  1  0  4  1  0  4  3  4  3  2  1  3  2  1  0
```

The ANOVA for the four factor factorial is given below:

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	224	69326.59556	309.49373	.	.
Error	0	0.00000	.		
Corrected Total	224	69326.59556			

Source	DF	Type I SS	Mean Square	F Value	Pr > F
A	2	10785.07556	5392.53778	.	.
B	4	1563.88444	390.97111	.	.
A*B	8	5056.96889	632.12111	.	.
C	2	4950.14222	2475.07111	.	.
D	4	2683.26222	670.81556	.	.
C*D	8	8944.25778	1118.03222	.	.
A*C	4	3216.89778	804.22444	.	.
A*D	8	2099.99111	262.49889	.	.
A*C*D	16	5898.16889	368.63556	.	.
B*C	8	2838.43556	354.80444	.	.
B*D	16	2892.11556	180.75722	.	.
B*C*D	32	2389.16444	74.66139	.	.
A*B*C	16	5191.19111	324.44944	.	.
A*B*D	32	3783.29778	118.22806	.	.
A*B*C*D	64	7033.74222	109.90222	.	.

The Type I ANOVA for the 50 F-squares is given below. As may be observed, the sum of the sums of squares for each of the sets of F-squares formed from an interaction adds to the sum of squares for the interaction, indicating that the F-squares are sum-of-squares orthogonal.

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	224	69326.59556	309.49373	.	.
Error	0	0.00000	.		
Corrected Total	224	69326.59556			

Source	DF	Type I SS	Mean Square	F Value	Pr > F
ROW	14	17405.92889	1243.28063	.	.
COL	14	16577.66222	1184.11873	.	.
F1	2	365.66222	182.83111	.	.
F2	2	2851.23556	1425.61778	.	.
F3	4	1217.47717	304.36929	.	.
F4	4	882.51394	220.62848	.	.
F5	4	1507.65781	376.91445	.	.
F6	4	111.09466	27.77366	.	.
F7	4	1385.48914	346.37229	.	.
F8	4	2893.92727	723.48182	.	.
F9	4	1045.48283	261.37071	.	.
F10	4	1792.95273	448.23818	.	.
F11	4	367.88444	91.97111	.	.
F12	4	400.19556	100.04889	.	.
F13	4	1152.68444	288.17111	.	.
F14	4	971.35111	242.83778	.	.
F15	4	108.48889	27.12222	.	.

F16	4	490.65051	122.66263	.	.
F17	4	91.10707	22.77677	.	.
F18	4	221.41010	55.35253	.	.
F19	4	529.14667	132.28667	.	.
F20	4	261.60727	65.40182	.	.
F21	4	158.79515	39.69879	.	.
F22	4	527.95879	131.98970	.	.
F23	4	699.24312	174.81078	.	.
F24	4	2369.47965	592.36991	.	.
F25	4	1393.12289	348.28072	.	.
F26	4	729.34545	182.33636	.	.
F27	4	132.56404	33.14101	.	.
F28	4	643.48929	160.87232	.	.
F29	4	648.11556	162.02889	.	.
F30	4	143.07313	35.76828	.	.
F31	4	395.12000	98.78000	.	.
F32	4	465.47152	116.36788	.	.
F33	4	85.76015	21.44004	.	.
F34	4	1269.70409	317.42602	.	.
F35	4	291.08444	72.77111	.	.
F36	4	273.50795	68.37699	.	.
F37	4	127.13624	31.78406	.	.
F38	4	438.51475	109.62869	.	.
F39	4	335.88067	83.97017	.	.
F40	4	846.26211	211.56553	.	.
F41	4	284.47201	71.11800	.	.
F42	4	392.16914	98.04229	.	.
F43	4	475.39607	118.84902	.	.
F44	4	764.39482	191.09871	.	.
F45	4	257.09542	64.27385	.	.
F46	4	969.39223	242.34806	.	.
F47	4	489.34545	122.33636	.	.
F48	4	620.98182	155.24545	.	.
F49	4	350.94545	87.73636	.	.
F50	4	117.16364	29.29091	.	.

$N = 18 = 2 \times 3^2$

The complete set of 137 sum-of-squares orthogonal F-squares is:

F1=A+D
F2=A+E
F3=A+F
F4=A+D+E
F5=A+D+F
F6=A+E+F
F7=A+E+2F
F8=A+D+E+F
F9=A+D+E+2F
F10=B+D
F11=B+E
F12=B+2E
F13=B+F
F14=B+2F
F15=B+D+E
F16=B+D+2E
F49=A+B+D+E
F50=A+B+D+2E
F51=A+B+D+F
F52=A+B+D+2F
F53=A=B+E+F
F54=A+B+E+2F
F55=A+B+2E+F
F56=A+B+2E+2F
F57=A+B+D+E+F
F58=A+B+D+E+2F
F59=A+B+D+2E+F
F60=A+B+D+2E+2F
F61=A+C+D
F62=A+C+E
F63=A+C+2E
F64=A+C+F
F97=B+C+E+2F
F98=B+C+2E+F
F99=B+C+2E+2F
F100=B+2C+E+F
F101=B+2C+E+2F
F102=B+2C+2E+F
F103=B+2C+2E+2F
F104=B+C+D+E+F
F105=B+C+D+E+2F
F106=B+C+D+2E+F
F107=B+C+D+2E+2F
F108=B+2C+D+E+F
F109=B+2C+D+E+2F
F110=B+2C+D+2E+F
F111=B+2C+D+2E+2F
F112=A+B+C+D

F17=B+D+F
F18=B+D+2F
F19=B+E+F
F20=B+E+2F
F21=B+2E+F
F22=B+2E+2F
F23=B+D+E+F
F24=B+D+E+2F
F25=B+D+2E+F
F26=B+D+2E+2F
F27=C+D
F28=C+E
F29=C+2E
F30=C+F
F31=C+2F
F32=C+D+E
F33=C+D+2E
F34=C+D+F
F35=C+D+2F
F36=C+E+F
F37=C+E+2F
F38=C+2E+F
F39=C+2E+2F
F40=C+D+E+F
F41=C+D+E+2F
F42=C+D+2E+F
F43=C+D+2E+2F
F44=A+B+D
F45=A+B+E
F46=A+B+2E
F47=A+B+F
F48=A+B+2F
F65=A+C+2F
F66=A+C+D+E
F67=A+C+D+2E
F68=A+C+D+F
F69=A+C+D+2F
F70=A+C+E+F
F71=A+C+E+2F
F72=A+C+2E+F
F73=A+C+2E+2F
F74=A+C+D+E+F
F75=A+C+D+E+2F
F76=A+C+D+2E+F
F77=A+C+D+2E+2F
F78=B+C+D
F79=B+2C+D
F80=B+C+E
F81=B+C+2E
F82=B+2C+E
F83=B+2C+2E
F84=B+C+F
F85=B+C+2F
F86=B+2C+F
F87=B+2C+2F
F88=B+C+D+E
F89=B+C+D+2E
F90=B+2C+D+E
F91=B+2C+D+2E
F92=B+C+D+F
F93=B+C+D+2F
F94=B+2C+D+F
F95=B+2C+D+2F
F96=B+C+E+F
F113=A+B+2C+D
F114=A+B+C+E
F115=A+B+C+2E
F116=A+B+2C+E
F117=A+B+2C+2E
F118=A+B+C+F
F119=A+B+C+2F
F120=A+B+2C+F
F121=A+B+2C+2F
F122=A+B+C+D+E
F123=A+B+C+D+2E
F124=A+B+2C+D+E
F125=A+B+2C+D+2E
F126=A+B+C+D+F
F127=A+B+C+D+2F
F128=A+B+2C+D+F
F129=A+B+2C+D+2F
F130=A+B+C+D+E+F
F131=A+B+C+D+E+2F
F132=A+B+C+D+2E+F
F133=A+B+C+D+2E+2F
F134=A+B+2C+D+E+F
F135=A+B+2C+D+E+2F
F136=A+B+2C+D+2E+F
F137=A+B+2C+D+2E+2F

LITERATURE CITED

Schwager, S. J., W. T. Federer, and J. P. Mandeli (1984). Embedding cyclic Latin squares of order 2^n in a complete set of orthogonal F-squares. Journal of Statistical Planning and Inference 10:207-218.

APPENDIX A-- n = 6

The data and code for for constructing one F(6, 3) and 12 F(6, 2) sum-of-squares orthogonal squares are given below:

```
DATA FSS6;
INPUT Y ROW COL A B C D;
F1=A+C; F2 = A+D; F3=A+C+D;
F4=B+C; F5=B+D; F6=B+2*D;F7=B+C+D; F8=B+C+2*D;
F9=A+B+C; F10=A+B+D;F11=A+B+2*D; F12=A+B+C+D; F13=A+B+C+2*D;
IF F1 > 1 THEN F1 = A+C -2;
IF F2 > 2 THEN F2 = A+D -3;
IF F3 > 2 THEN F3 = A+C+D - 3;
IF F4 > 2 THEN F4=B+C -3;
IF F5 > 2 THEN F5=B+D - 3;
IF F6>5 THEN F6=B+2*D - 6;IF F6 > 2 THEN F6=B+2*D-3;
```

```
IF F7 > 2 THEN F7 = B+C+D-3;
IF F8 > 8 THEN F8 = B+C+2*D - 9;IF F8 > 5 THEN F8=B+C+2*D - 6;
IF F8 > 2 THEN F8=B+C+2*D - 3;
IF F9 > 2 THEN F9=A+B+C-3;
IF F10 > 2 THEN F10=A+B+D-3;
IF F11 > 5 THEN F11=A+B+2*D-6;IF F11 > 2 THEN F11=A+B+2*D-3;
IF F12 > 8 THEN F12= A+B+C+D - 9; IF F12 > 5 THEN F12=A+B+C+D-6;
IF F12 > 2 THEN F12=A+B+C+D-3;
IF F13 > 8 THEN F13=A=B+C+2*D-9; IF F13 > 5 THEN F13=A+B+C+2*D-6;
IF F13 > 2 THEN F13=A+B+C+2*D-3;

DATALINES;
92 1 1 0 0 0 0
66 1 2 0 0 0 1
19 1 3 0 0 0 2
29 1 4 0 0 1 0
16 1 5 0 0 1 1
25 1 6 0 0 1 2
60 2 1 0 1 0 0
46 2 2 0 1 0 1
35 2 3 0 1 0 2
10 2 4 0 1 1 0
11 2 5 0 1 1 1
 5 2 6 0 1 1 2
46 3 1 0 2 0 0
81 3 2 0 2 0 1
17 3 3 0 2 0 2
22 3 4 0 2 1 0
16 3 5 0 2 1 1
 9 3 6 0 2 1 2
120 4 1 1 0 0 0
59  4 2 1 0 0 1
43  4 3 1 0 0 2
15  4 4 1 0 1 0
10  4 5 1 0 1 1
 2  4 6 1 0 1 2
49  5 1 1 1 0 0
64  5 2 1 1 0 1
25  5 3 1 1 0 2
24  5 4 1 1 1 0
 8  5 5 1 1 1 1
 7  5 6 1 1 1 2
134 6 1 1 2 0 0
60  6 2 1 2 0 1
52  6 3 1 2 0 2
20  6 4 1 2 1 0
28  6 5 1 2 1 1
11  6 6 1 2 1 2
RUN;
PROC PRINT ;
RUN;
PROC GLM DATA=FSS6;
  CLASS ROW COL A B C D;
  MODEL Y = ROW COL A*C A*D A*C*D B*C B*D B*C*D A*B*C A*B*D A*B*C*D;
RUN;

PROC GLM DATA=FSS6;
```

```
  CLASS ROW COL F1 F2 F3 F4 F5 F6 F7 F8 F9 F10 F11 F12 F13;
  MODEL Y = ROW COL F1 F2 F3 F4 F5 F6 F7 F8 F9 F10 F11 F12 13;
RUN;
```

APPENDIX B

The data and code for constructing one F(8, 4) and 16 F(8, 2) partially sum-of-squares orthogonal squares are given below:

```
DATA FSS8;
INPUT y ROW COL A B C D;
F1=A+C; F2 = A+D; F3=A+C+D;
F4=B+C; F5=B+D;F6=B+2*D;F7=B+3*D; F8=B+C+D; F9=B+C+2*D;F10=B+C+3*D;
F11=A+B+C;F12=A+B+D;F13=A+B+2*D;F14=A+B+3*D;
F15=A+B+C+D; F16=A+B+C+2*D;F17=A+B+C+3*D;
IF F1 > 1 THEN F1 = A+C -2;
IF F2 > 3 THEN F2 = A+D -4;
IF F3 > 3 THEN F3 = A+C+D - 4;
IF F4 > 3 THEN F4 = B+C - 4;
IF F5 > 3 THEN F5=B+D - 3;
IF F6>7 THEN F6=B+2*D - 8;IF F6 > 3 THEN F6=B+2*D-4;
IF F7 > 11 THEN F7 = F7 - 12; IF F7>7 THEN F7= F7-8; IF F7>3 THEN
F7=F7-4;
IF F8 > 7 THEN F8 = F8 - 8; IF F8 > 3 THEN F8 = B+C+D-4;
IF F9 > 7 THEN F9 = B+C+2*D - 8;IF F9 > 3 THEN F9=B+C+2*D - 4;
IF F10>11 THEN F10=F10-12;IF F10>7 THEN F10=F10-8;IF F10>3 THEN
F10=F10-4;
IF F11 > 3 THEN F11=A+B+C+ - 4;
IF F12>7 THEN F12=F12-8;IF F12>3 THEN F12=A+B+D-4;
IF F13 > 7 THEN F13=A+B+2*D-8;IF F13>3 THEN F13=F13-4;
IF F14 >11 THEN F14=A+B+3*D-12;IF F14 > 7 THEN F14=A+B+3*D-8;
IF F14 > 3 THEN F14 = F14 - 4;
IF F15 > 7 THEN F15= A+B+C+D - 8; IF F15 > 3 THEN F15=A+B+C+D-4;
IF F16 > 11 THEN F16=A+B+C+2*D-12;IF F16 > 7 THEN F16 = F16 - 8;
IF F16 > 3 THEN F16=A=B+C+3*D-4; IF F17 > 11 THEN F17=A+B+C+3*D-12;
IF F17 > 7 THEN F17=A+B+C+3*D-8;IF F17>3 THEN F17= F17 - 4;

DATALINES;
92 1 1 0 0 0 0
66 1 2 0 0 0 1
19 1 3 0 0 0 2
19 1 4 0 0 0 3
29 1 5 0 0 1 0
16 1 6 0 0 1 1
25 1 7 0 0 1 2
25 1 8 0 0 1 3
60 2 1 0 1 0 0
46 2 2 0 1 0 1
35 2 3 0 1 0 2
35 2 4 0 1 0 3
10 2 5 0 1 1 0
11 2 6 0 1 1 1
 5 2 7 0 1 1 2
46 2 8 0 1 1 3
46 3 1 0 2 0 0
81 3 2 0 2 0 1
17 3 3 0 2 0 2
```

```
22 3 4 0 2 0 3
22 3 5 0 2 1 0
16 3 6 0 2 1 1
 9 3 7 0 2 1 2
 9 3 8 0 2 1 3
120 4 1 0 3 0 0
59  4 2 0 3 0 1
43  4 3 0 3 0 2
43  4 4 0 3 0 3
15  4 5 0 3 1 0
10  4 6 0 3 1 1
 2  4 7 0 3 1 2
49  4 8 0 3 1 3
49  5 1 1 0 0 0
64  5 2 1 0 0 1
25  5 3 1 0 0 2
25  5 4 1 0 0 3
24  5 5 1 0 1 0
 8  5 6 1 0 1 1
 7  5 7 1 0 1 2
 7  5 8 1 0 1 3
134 6 1 1 1 0 0
60  6 2 1 1 0 1
52  6 3 1 1 0 2
20  6 4 1 1 0 3
28  6 5 1 1 1 0
11  6 6 1 1 1 1
11  6 7 1 1 1 2
28  6 8 1 1 1 3
20  7 1 1 2 0 0
52  7 2 1 2 0 1
60  7 3 1 2 0 2
134 7 4 1 2 0 3
7   7 5 1 2 1 0
24  7 6 1 2 1 1
25  7 7 1 2 1 2
25  7 8 1 2 1 3
64  8 1 1 3 0 0
49  8 2 1 3 0 1
60  8 3 1 3 0 2
52  8 4 1 3 0 3
20  8 5 1 3 1 0
28  8 6 1 3 1 1
20  8 7 1 3 1 2
11  8 8 1 3 1 3
RUN;
PROC PRINT ;
RUN;
PROC GLM DATA=FSS8;
  CLASS ROW COL A B C D;
  MODEL Y = A B A*B C D C*D A*C A*D A*C*D B*C B*D B*C*D A*B*C A*B*D
  A*B*C*D;
RUN;

PROC GLM DATA=FSS8;
  CLASS ROW COL F1 F2 F3 F4 F5 F6 F7 F8 F9 F10 F11 F12 F13 F14
  F15 F16 F17;
```

```
  MODEL Y = ROW COL F1 F2 F3 F4 F5 F6 F7 F8 F9 F10 F11 F12 F13 F14
  F15 F16 F17;
RUN;
```

APPENDIX C-- n = 10

The code for constructing one F(10, 5) and 20 F(10, 2) sum-of-squares orthogonal squares is:

```
DATA FSS10;
INPUT y ROW COL A B C D;
F1=A+C; F2 = A+D; F3=A+C+D;
F4=B+C; F5=B+D;F6=B+2*D;F7=B+3*D; F8 = B+4*D;
F9=B+C+D; F10=B+C+2*D;F11=B+C+3*D;F12 = B+C+4*D;
F13=A+B+C;F14=A+B+D;F15=A+B+2*D;F16=A+B+3*D;F17=A+B+4*D;
F18=A+B+C+D;F19=A+B+C+2*D;F20=A+B+C+3*D;F21=A+B+C+4*D;
IF F1 > 1 THEN F1 = A+C -2;
IF F2 > 4 THEN F2 = A+D -5;
IF F3 > 4 THEN F3 = A+C+D - 5;
IF F4 > 4 THEN F4 = B+C - 5;
IF F5 >4 THEN F5=B+D - 5;
IF F6>9 THEN F6=B+2*D - 10;IF F6 > 4 THEN F6=B+2*D-5;
IF F7 > 14 THEN F7 = F7 - 15; IF F7>9 THEN F7= F7-10; IF F7>4 THEN
F7=F7-5;
IF F8 > 19 THEN F8 = F8 - 20; IF F8 > 14 THEN F8 = B+4*D-15;
IF F8 > 9 THEN F8=F8-10; IF F8 > 4 THEN F8=F8 - 5;
IF F9 > 4 THEN F9= B+C+D - 5;
IF F10 > 9 THEN F10 = B+C+2*D - 10;IF F10 > 4 THEN F10=B+C+2*D - 5;
IF F11>14 THEN F11=B+C+3*D-15;
IF F11>9 THEN F11=F11-10; IF F11>4 THEN F11=F11-5;
IF F12 > 19 THEN F12 = B+C+4*D -20; IF F12 > 14 THEN F12=F12-15;
IF F12 > 9 THEN F12=F12 - 10; IF F12> 4 THEN F12=F12 - 5;
IF F13 > 4 THEN F13=A+B+C - 5;
IF F14>4 THEN F14=A+B+D-5;
IF F15 > 9 THEN F15=A+B+2*D-10;IF F15 >4 THEN F15=F15 - 5;
IF F16 >14 THEN F16=A+B+3*D-15;IF F16 > 9 THEN F16=A+B+3*D-10;
IF F16 > 4 THEN F16 = F16 - 5;
IF F17 > 19 THEN F17 = A+B+4*D - 20; IF F17 > 14 THEN F17 =F17-15;
IF F17 > 9 THEN F17 = F17 - 10; IF F17 > 4 THEN F17=F17 - 5;
IF F18 > 9 THEN F18= A+B+C+D - 10; IF F18 > 4 THEN F18=A+B+C+D-5;
IF F19 > 14 THEN F19=A+B+C+2*D-15;IF F19 > 9 THEN F19 = F19 - 10;
IF F19 > 4 THEN F19=A=B+C+2*D-5;
IF F20 > 14 THEN F20=A+B+C+3*D-15;
IF F20 > 9 THEN F20=A+B+C+3*D-10;IF F20>4 THEN F20= F20 - 5;
IF F21 > 19 THEN F21 = A+B+C+4*D -20; IF F21> 14 THEN F21=F21-15;
IF F21 > 9 THEN F21=F21 - 10; IF F21 > 4 THEN F21 = F21 - 5;

DATALINES;
92 1  1 0 0 0 0
66 1  2 0 0 0 1
19 1  3 0 0 0 2
19 1  4 0 0 0 3
19 1  5 0 0 0 4
29 1  6 0 0 1 0
16 1  7 0 0 1 1
25 1  8 0 0 1 2
```

25	1	9	0	0	1	3
25	1	10	0	0	1	4
60	2	1	0	1	0	0
46	2	2	0	1	0	1
35	2	3	0	1	0	2
35	2	4	0	1	0	3
35	2	5	0	1	0	4
10	2	6	0	1	1	0
11	2	7	0	1	1	1
5	2	8	0	1	1	2
46	2	9	0	1	1	3
46	2	10	0	1	1	4
46	3	1	0	2	0	0
81	3	2	0	2	0	1
17	3	3	0	2	0	2
22	3	4	0	2	0	3
22	3	5	0	2	0	4
22	3	6	0	2	1	0
16	3	7	0	2	1	1
9	3	8	0	2	1	2
9	3	9	0	2	1	3
9	3	10	0	2	1	4
12	4	1	0	3	0	0
59	4	2	0	3	0	1
43	4	3	0	3	0	2
43	4	4	0	3	0	3
43	4	5	0	3	0	4
15	4	6	0	3	1	0
10	4	7	0	3	1	1
2	4	8	0	3	1	2
49	4	9	0	3	1	3
49	4	10	0	3	1	4
49	5	1	0	4	0	0
64	5	2	0	4	0	1
25	5	3	0	4	0	2
25	5	4	0	4	0	3
25	5	5	0	4	0	4
24	5	6	0	4	1	0
8	5	7	0	4	1	1
7	5	8	0	4	1	2
7	5	9	0	4	1	3
7	5	10	0	4	1	4
34	6	1	1	0	0	0
60	6	2	1	0	0	1
52	6	3	1	0	0	2
20	6	4	1	0	0	3
20	6	5	1	0	0	4
28	6	6	1	0	1	0
11	6	7	1	0	1	1
11	6	8	1	0	1	2
28	6	9	1	0	1	3
28	6	10	1	0	1	4
20	7	1	1	1	0	0
52	7	2	1	1	0	1
60	7	3	1	1	0	2
34	7	4	1	1	0	3
34	7	5	1	1	0	4

```
17 7  6 1 1 1 0
24 7  7 1 1 1 1
25 7  8 1 1 1 2
25 7  9 1 1 1 3
25 7 10 1 1 1 4
64 8  1 1 2 0 0
49 8  2 1 2 0 1
60 8  3 1 2 0 2
52 8  4 1 2 0 3
52 8  5 1 2 0 4
20 8  6 1 2 1 0
28 8  7 1 2 1 1
20 8  8 1 2 1 2
11 8  9 1 2 1 3
11 8 10 1 2 1 4
11 9  1 1 3 0 0
20 9  2 1 3 0 1
28 9  3 1 3 0 2
20 9  4 1 3 0 3
52 9  5 1 3 0 4
60 9  6 1 3 1 0
59 9  7 1 3 1 1
58 9  8 1 3 1 2
57 9  9 1 3 1 3
46 9 10 1 3 1 4
45 10 1 1 4 0 0
54 10 2 1 4 0 1
53 10 3 1 4 0 2
52 10 4 1 4 0 3
51 10 5 1 4 0 4
50 10 6 1 4 1 0
49 10 7 1 4 1 1
48 10 8 1 4 1 2
45 10 9 1 4 1 3
69 10 10 1 4 1 4
RUN;
PROC PRINT ;
RUN;
PROC GLM DATA=FSS10;
  CLASS ROW COL A B C D;
  MODEL Y = A B A*B C D C*D A*C A*D A*C*D B*C B*D B*C*D A*B*C A*B*D
  A*B*C*D;
RUN;

PROC GLM DATA=FSS10;
  CLASS ROW COL F1 F2 F3 F4 F5 F6 F7 F8 F9 F10 F11 F12 F13 F14
  F15 F16 F17 F18 F19 F20 F21;
  MODEL Y = ROW COL F1 F2 F3 F4 F5 F6 F7 F8 F9 F10 F11 F12 F13 F14
  F15 F16 F17 F18 F19 F20 F21;
RUN;
```

APPENDIX D--n = 15 = 3 × 5

The code for constructing two F(15, 5) and 48 F(15, 3) sum-of-squares orthogonal squares is:

```
DATA FSS15;
```

```
INPUT y ROW COL A B C D;
F1=A+C;F2 = A+2*C;F3 = A+D;F4 = A+2*D;F5=A+C+D;F6=A+C+2*D;
F7=A+2*C+D;F8=A+2*C+2*D;F9 = B+C; F10 = C+2*B;F11=B+D;F12=B+2*D;
F13=B+3*D; F14 = B+4*D;
F15=B+C+D; F16=B+C+2*D;F17=B+C+3*D;F18 = B+C+4*D;
F19=B+2*C+D;F20 = B+2*C+2*D;F21 = B+2*C+3*D;F22 = B+2*C+4*D;
F23=A+B+C;F24 = A+C+2*B; F25 = A+2*C+B; F26 = A+2*C+2*B;
F27 =A+B+D;F28 =A+B+2*D;F29 =A+B+3*D;F30 =A+B+4*D;
F31 = A+2*B+D;F32= A+2*B+2*D;F33 = A+2*B+3*D;F34= A+2*B+4*D;
F35 =A+B+C+D;F36 =A+B+C+2*D;F37 =A+B+C+3*D;F38=A+B+C+4*D;
F39=A+2*B+C+D;F40=A+C+2*B+2*D;F41=A+2*B+C+3*D;F42=A+2*B+C+4*D;
F43=A+B+2*C+D;F44=A+B+2*C+2*D;F45=A+B+2*C+3*D;F46=A+B+2*C+4*D;
F47=A+2*B+2*C+D;F48=A+2*B+2*C+2*D;F49=A+2*B+2*C+3*D;F50=A+2*B+2*C+4*D;
IF F1>2 THEN F1=A+C-3;IF F2>5 THEN F2=F2-6;IF F2>2 THEN F2=F2-3;
IF F3 > 4 THEN F3 = F3-5;IF F4>9 THEN F4=F4-10;
IF F4 > 4 THEN F4=F4 - 5;
IF F5 > 4 THEN F5 = F5 - 5;
IF F6 > 9 THEN F6 = F6 - 10;IF F6 > 4 THEN F6 = F6 - 5;
IF F7 > 9 THEN F7 = F7 - 10;IF F7 > 4 THEN F7 = F7 - 5;
IF F8> 9 THEN F8 = F8 - 10;IF F8 > 4 THEN F8 = F8 - 5;
IF F9 > 4 THEN F9 = F9 - 5;
IF F10 >9 THEN F10 = F10 - 10;IF F10 > 4 THEN F10 = F10 - 5;
IF F11 > 4 THEN F11 = F11 - 5;
IF F12 > 9 THEN F12 = F12 - 10;IF F12 > 4 THEN F12 = F12 - 5;
IF F13 > 14 THEN F13 = F13 - 15;
IF F13 > 9 THEN F13 = F13 - 10 ;
IF F13 > 4  THEN F13 = F13 - 5;
IF F14 > 19 THEN F14 = F14 - 20;IF 14> 14 THEN F14 =F14-15;
IF F14 > 9 THEN F14 = F14 - 10;IF F14 > 4 THEN F14 =F14 - 5;
IF F15 > 9 THEN F15 = F15 - 10;IF F15 > 4 THEN F15 =F15 - 5;
IF F16 > 9 THEN F16 = F16 - 10;IF F16 > 4 THEN F16 =F16 - 5;
IF F17 > 14 THEN F17 =F17 - 15;IF F17 > 9 THEN F17 =F17 - 10;
IF F17 > 4 THEN F17 = F17 - 5;
IF F18 > 19 THEN F18 =F18 - 20;IF F18>14 THEN F18= F18 - 15;
IF F18 > 9 THEN F18 = F18 - 10;IF F18> 4 THEN F18= F18 - 5;
IF F19 > 9 THEN F19 = F19 - 10; IF F19 > 4 THEN F19 = F19 - 5;
IF F20 > 14 THEN F20 = F20 - 15;
IF F20 > 9 THEN F20 = F20 - 10;IF F20 > 4 THEN F20=F20 - 5;
IF F21 >19 THEN F21 = F21 - 20;IF F21> 14 THEN F21=F21 - 15;
IF F21 > 9 THEN F21 = F21 - 10; IF F21 > 4 THEN F21=F21 - 5;
IF F22 >19 THEN F22 = F22 - 20;IF F22> 14 THEN F22=F22 - 15;
IF F22 > 9 THEN F22 = F22 - 10; IF F22> 4 THEN F22=F22 - 5;
IF F23 > 4 THEN F23 = F23 - 5;
IF F24 > 9 THEN F24 = F24 - 10;IF F24> 4 THEN F24 = F24 - 5;
IF F25 > 9 THEN F25 = F25 - 10;IF F25 >4 THEN F25 = F25 - 5;
IF F26 > 9 THEN F26 = F26 - 10;IF F26> 4 THEN F26 = F26 - 5;
IF F27 >9 THEN F27 = F27 - 10;IF F27 > 4 THEN F27 = F27 - 5;
IF F28 >9 THEN F28 = F28 - 10;IF F28 > 4 THEN F28 = F28 - 5;
IF F29 >14 THEN F29 = F29 - 15;IF F29 > 9 THEN F29=F29 - 10;
IF F29 > 4 THEN F29 = F29 - 5;
IF F30 >19 THEN F30 = F30 - 20;IF F30 > 14 THEN F30 =F30-15;
IF F30 > 9 THEN F30 = F30 - 10; IF F30 > 4 THEN F30=F30 - 5;
IF F31 > 9 THEN F31= F31 - 10; IF F31 > 4 THEN F31=F31-5;
IF F32 > 14 THEN F32=F32-15;IF F32 > 9 THEN F32 = F32 - 10;
IF F32 > 4 THEN F32=F32-5;
IF F33 > 19 THEN F33 = F33 - 20;IF F33 > 14 THEN F33=33-15;
IF F33 > 9 THEN F33=F33-10;IF F33>4 THEN F33= F33 - 5;
```

```
IF F34 >24 THEN F34= F34 - 25;IF F34 > 19 THEN F34= F34 -20;
IF F34>14 THEN F34=F34 - 15;IF F34 > 9 THEN F34=F34 - 10;
IF F34 > 4 THEN F34 = F34 - 5;
IF F35 > 9 THEN F35=F35 - 10; IF F35 > 4 THEN F35 = F35 - 5;
IF F36 > 14 THEN F36 = F36 - 15;IF F36 >9 THEN F36=F36 - 10;
IF F36 > 4 THEN F36 = F36 - 5;
IF F37 >19 THEN F37 = F37-20;IF F37 > 14 THEN F37 = F37-15;
IF F37>9 THEN F37 = F37 - 10; IF F37 > 4 THEN F37=F37-5;
IF F38 > 19 THEN F38 = F38-20;
IF F38 > 14 THEN F38 = F38 - 15;
IF F38 > 9 THEN F38 = F38 - 10; IF F38 > 4 THEN F38 = F38 - 5;
IF F39 > 14 THEN F39 = F39 - 15 ;
IF F39 > 9 THEN F39 = F39 - 10 ;
IF F39 > 4 THEN F39 = F39 - 5;
IF F40 > 19 THEN F40 = F40-20;IF F40 > 14 THEN F40 = F40-15;
IF F40 > 9 THEN F40 = F40-10; IF F40 > 4 THEN F40 = F40 - 5;
IF F41 > 19 THEN F41 = F41-20; IF F41>14 THEN F41 = F41-15;
IF F41> 9 THEN F41= F41 - 10; IF F41>4 THEN F41 = F41 - 5;
IF F42 > 24 THEN F42=F42-25; IF F42>19 THEN F42 = F42 - 20;
IF F42 >14 THEN F42=F42 - 15;IF F42> 9 THEN F42 = F42-10;
IF F42 > 4 THEN F42 = F42 - 5;
IF F43>9 THEN F43 = F43 - 10;IF F43 > 4 THEN F43 = F43 - 5;
IF F44>14 THEN F44 = F44-15;
IF F44 > 9 THEN F44=F44 - 10; IF F44>4 THEN F44 = F44 - 5;
IF F45 > 19 THEN F45=F45 - 20;
IF F45 >14 THEN F45=F45 - 15;IF F45 > 9 THEN F45=F45 - 10;
IF F45 > 4 THEN F45 = F45 - 5;
IF F46>24 THEN F46= F46 - 25;
IF F46>19 THEN F46=F46 - 20; IF F46>14 THEN F46 = F46 - 15;
IF F46> 9 THEN F46 = F46 - 10; IF F46>4 THEN F46 = F46 - 5;
IF F47 > 14 THEN F47=F47 - 15;
IF F47 > 9 THEN F47=F47 - 10; IF F47 > 4 THEN F47=F47 - 5;
IF F48>19 THEN F48 = F48 - 20;
IF F48>14 THEN F48 = F48 - 15; IF F48>9 THEN F48 = F48 - 10;
IF F48 > 4 THEN F48 = F48 - 5;
IF F49 > 24 THEN F49 = F49 - 25;
IF F49 >19 THEN F49 = F49-20;IF F49 > 14 THEN F49=F49 - 15;
IF F49 > 9 THEN F49 = F49 - 10;IF F49 > 4 THEN F49 = F49 - 5;
IF F50>29 THEN F50=F50 - 30;
IF F50>24 THEN F50=F50 - 25;IF F50>19 THEN F50=F50 - 20;
IF F50>14 THEN F50=F50 -15;IF F50>9 THEN F50 = F50 - 10;
IF F50 > 4 THEN F50 = F50 - 5;
DATALINES;
92  1  1 0 0 0 0
66  1  2 0 0 0 1
19  1  3 0 0 0 2
19  1  4 0 0 0 3
19  1  5 0 0 0 4
29  1  6 0 0 1 0
16  1  7 0 0 1 1
25  1  8 0 0 1 2
25  1  9 0 0 1 3
25  1 10 0 0 1 4
29  1 11 0 0 2 0
16  1 12 0 0 2 1
25  1 13 0 0 2 2
25  1 14 0 0 2 3
```

```
25  1 15 0 0 2 4
60  2  1 0 1 0 0
46  2  2 0 1 0 1
35  2  3 0 1 0 2
35  2  4 0 1 0 3
35  2  5 0 1 0 4
10  2  6 0 1 1 0
11  2  7 0 1 1 1
 5  2  8 0 1 1 2
46  2  9 0 1 1 3
46  2 10 0 1 1 4
10  2 11 0 1 2 0
11  2 12 0 1 2 1
 5  2 13 0 1 2 2
46  2 14 0 1 2 3
46  2 15 0 1 2 4
46  3  1 0 2 0 0
81  3  2 0 2 0 1
17  3  3 0 2 0 2
22  3  4 0 2 0 3
22  3  5 0 2 0 4
22  3  6 0 2 1 0
16  3  7 0 2 1 1
 9  3  8 0 2 1 2
 9  3  9 0 2 1 3
 9  3 10 0 2 1 4
10  3 11 0 2 2 0
11  3 12 0 2 2 1
 5  3 13 0 2 2 2
46  3 14 0 2 2 3
46  3 15 0 2 2 4
12  4  1 0 3 0 0
59  4  2 0 3 0 1
43  4  3 0 3 0 2
43  4  4 0 3 0 3
43  4  5 0 3 0 4
15  4  6 0 3 1 0
10  4  7 0 3 1 1
 2  4  8 0 3 1 2
49  4  9 0 3 1 3
49  4 10 0 3 1 4
10  4 11 0 3 2 0
11  4 12 0 3 2 1
 5  4 13 0 3 2 2
46  4 14 0 3 2 3
46  4 15 0 3 2 4
49  5  1 0 4 0 0
64  5  2 0 4 0 1
25  5  3 0 4 0 2
25  5  4 0 4 0 3
25  5  5 0 4 0 4
24  5  6 0 4 1 0
 8  5  7 0 4 1 1
 7  5  8 0 4 1 2
 7  5  9 0 4 1 3
 7  5 10 0 4 1 4
10  5 11 0 4 2 0
```

```
11   5 12  0  4  2  1
 5   5 13  0  4  2  2
46   5 14  0  4  2  3
46   5 15  0  4  2  4
34   6  1  1  0  0  0
60   6  2  1  0  0  1
52   6  3  1  0  0  2
20   6  4  1  0  0  3
20   6  5  1  0  0  4
28   6  6  1  0  1  0
11   6  7  1  0  1  1
11   6  8  1  0  1  2
28   6  9  1  0  1  3
28   6 10  1  0  1  4
10   6 11  1  0  2  0
11   6 12  1  0  2  1
 5   6 13  1  0  2  2
46   6 14  1  0  2  3
46   6 15  1  0  2  4
20   7  1  1  1  0  0
52   7  2  1  1  0  1
60   7  3  1  1  0  2
34   7  4  1  1  0  3
34   7  5  1  1  0  4
17   7  6  1  1  1  0
24   7  7  1  1  1  1
25   7  8  1  1  1  2
25   7  9  1  1  1  3
25   7 10  1  1  1  4
10   7 11  1  1  2  0
11   7 12  1  1  2  1
 5   7 13  1  1  2  2
46   7 14  1  1  2  3
46   7 15  1  1  2  4
64   8  1  1  2  0  0
49   8  2  1  2  0  1
60   8  3  1  2  0  2
52   8  4  1  2  0  3
52   8  5  1  2  0  4
20   8  6  1  2  1  0
28   8  7  1  2  1  1
20   8  8  1  2  1  2
11   8  9  1  2  1  3
11   8 10  1  2  1  4
10   8 11  1  2  2  0
11   8 12  1  2  2  1
 5   8 13  1  2  2  2
46   8 14  1  2  2  3
46   8 15  1  2  2  4
11   9  1  1  3  0  0
20   9  2  1  3  0  1
28   9  3  1  3  0  2
20   9  4  1  3  0  3
52   9  5  1  3  0  4
60   9  6  1  3  1  0
59   9  7  1  3  1  1
58   9  8  1  3  1  2
```

```
57  9  9 1 3 1 3
46  9 10 1 3 1 4
10  9 11 1 3 2 0
11  9 12 1 3 2 1
 5  9 13 1 3 2 2
46  9 14 1 3 2 3
46  9 15 1 3 2 4
45 10  1 1 4 0 0
54 10  2 1 4 0 1
53 10  3 1 4 0 2
52 10  4 1 4 0 3
51 10  5 1 4 0 4
50 10  6 1 4 1 0
49 10  7 1 4 1 1
48 10  8 1 4 1 2
45 10  9 1 4 1 3
69 10 10 1 4 1 4
50 10 11 1 4 2 0
49 10 12 1 4 2 1
48 10 13 1 4 2 2
45 10 14 1 4 2 3
69 10 15 1 4 2 4
51 11  1 2 0 0 0
50 11  2 2 0 0 1
40 11  3 2 0 0 2
49 11  4 2 0 0 3
48 11  5 2 0 0 4
47 11  6 2 0 1 0
46 11  7 2 0 1 1
45 11  8 2 0 1 2
44 11  9 2 0 1 3
43 11 10 2 0 1 4
42 11 11 2 0 2 0
41 11 12 2 0 2 1
41 11 13 2 0 2 2
42 11 14 2 0 2 3
43 11 15 2 0 2 4
44 12  1 2 1 0 0
45 12  2 2 1 0 1
46 12  3 2 1 0 2
46 12  4 2 1 0 3
27 12  5 2 1 0 4
43 12  6 2 1 1 0
44 12  7 2 1 1 1
45 12  8 2 1 1 2
46 12  9 2 1 1 3
43 12 10 2 1 1 4
45 12 11 2 1 2 0
46 12 12 2 1 2 1
43 12 13 2 1 2 2
46 12 14 2 1 2 3
46 12 15 2 1 2 4
23 13  1 2 2 0 0
34 13  2 2 2 0 1
43 13  3 2 2 0 2
55 13  4 2 2 0 3
45 13  5 2 2 0 4
```

```
43 13  6 2 2 1 0
33 13  7 2 2 1 1
44 13  8 2 2 1 2
54 13  9 2 2 1 3
34 13 10 2 2 1 4
44 13 11 2 2 2 0
43 13 12 2 2 2 1
44 13 13 2 2 2 2
45 13 14 2 2 2 3
46 13 15 2 2 2 4
65 14  1 2 3 0 0
55 14  2 2 3 0 1
44 14  3 2 3 0 2
44 14  4 2 3 0 3
56 14  5 2 3 0 4
44 14  6 2 3 1 0
45 14  7 2 3 1 1
43 14  8 2 3 1 2
46 14  9 2 3 1 3
46 14 10 2 3 1 4
34 14 11 2 3 2 0
43 14 12 2 3 2 1
44 14 13 2 3 2 2
56 14 14 2 3 2 3
54 14 15 2 3 2 4
43 15  1 2 4 0 0
56 15  2 2 4 0 1
44 15  3 2 4 0 2
43 15  4 2 4 0 3
34 15  5 2 4 0 4
43 15  6 2 4 1 0
44 15  7 2 4 1 1
32 15  8 2 4 1 2
33 15  9 2 4 1 3
43 15 10 2 4 1 4
45 15 11 2 4 2 0
54 15 12 2 4 2 1
55 15 13 2 4 2 2
43 15 14 2 4 2 3
56 15 15 2 4 2 4
RUN;
PROC PRINT;
RUN;
PROC GLM DATA=FSS15;
  CLASS ROW COL A B C D;
  MODEL Y = A B A*B C D C*D A*C A*D A*C*D B*C B*D B*C*D A*B*C A*B*D
  A*B*C*D;
RUN;

PROC GLM DATA=FSS15;
  CLASS ROW COL F1 F2 F3 F4 F5 F6 F7 F8 F9 F10 F11 F12 F13 F14
  F15 F16 F17 F18 F19 F20 F21 F22 F23 F24 F25 F26 F27 F28
  F29 F30 F31 F32 F33 F34 F35 F36 F37 F38 F39 F40 F41
  F42 F43 F44 F45 F46 F47 F48 F49 F50;
  MODEL Y=ROW COL F1 F2 F3 F4 F5 F6 F7 F8 F9 F10 F11 F12 F13 F14
  F15 F16 F17 F18 F19 F20 F21 F22 F23 F24 F25 F26 F27 F28
  F29 F30 F31 F32 F33 F34 F35 F36 F37 F38 F39 F40 F41 F42
```

```
  F43 F44 F45 F46 F47 F48 F49 F50;
RUN;
```