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ON THE NONEXISTENCE OF KNUT VIK DESIGNS FOR ALL EVEN ORDERS*

by

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ABSTRACT

A Knut Vik design of order n can be defined as an $n \times n$ array of elements, chosen from a set of n elements (treatments) such that with respect to rows and columns the array is a Latin square and in addition each treatment appears once in each of the n left and right diagonals. These designs are useful for eliminating sources of variation in four directions. This paper is concerned with the existence and nonexistence of these designs. Specifically, (i) it is shown that no such design exists for n even, (ii) these designs exist for all odd orders except possibly for $n \equiv 0 \pmod 3$, (iii) the Kronecker product of two Knut Vik designs is a Knut Vik design and (iv) the concept of semi Knut Vik design is also defined and it is shown that while these designs do not exist for even orders, they exist for all odd orders.

Key words and phrases. Knut Vik design, Latin square design, block design.

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1. INTRODUCTION. The Knut Vik design for n = 5 treatments has been known since about 1872. The statistical analysis of experiments designed in this manner has been discussed by a number of individuals, e.g. Vik (1924), Tedin (1931), Fisher (1935), Dorph-Petersen (1942), Nissen (1951), Kempthorne (1952), and Federer (1955). The main purpose of this note is to indicate the nonexistence of Knut Vik designs for n even and to demonstrate the construction of these designs for values other than n = 5.

A Knut Vik design K of order n can be defined as an $n \times n$ array of elements, chosen from a set of n elements (treatments) such that (i) with respect to rows and columns the design is a Latin square of order n and (ii) for each $j=0,1,\cdots,n-1$ each of the following n cells (forming the right and the left diagonals of K) contains each symbol once.

$$\begin{array}{ll} \text{Right diagonals:} & (i,\ j+i) \\ \text{Left diagonals:} & (i,j-i-1) \end{array} \quad i=0,1,\cdots,n-1 \quad .$$

The rows and columns of K are indexed from 0 to n-1. In other words, the right and the left diagonals of K form 2n transversals for K. We shall shortly show that these designs cannot be constructed for n even. It is easy to verify that no such design exists for n = 3. Therefore, the smallest Knut Vik design has order 5. An example of such a design for n = 5 is one whose successive rows are (12345), (45123), (23451), (51234) and (34512). Thus, for removing variation due to rows, columns, left diagonals, and right diagonals, a Knut Vik design would be an appropriate design. Such a situation could arise in check planting material where there are row and column gradients and in addition left and right diagonal gradients which result from cultivating and spraying operations. Such gradients may also be encountered in experiments conducted over certain types of ridges or valleys and in experiments where insect infestations occur in certain types of waves from a corner of the experiment. There are certain experiments of the above type for which the Knut Vik design is not appropriate.

2. EXISTENCE AND NONEXISTENCE OF KNUT VIK DESIGNS.

To be brief we summarize our findings in the sequel.

Theorem 2.1. Knut Vik designs cannot be constructed for even orders.

Assume to the contrary and suppose K is a Knut Vik design of order n. Now consider the $n \times n$ square $A = (a_{ij})$ with $a_{ij} \equiv (i+j) \bmod n$. Clearly A is a Latin square of order n and $\{A,K\}$ forms a pair of orthogonal Latin squares of order n. However, note that A is a multiplication table of the cyclic group of order n and it is well known that A cannot have an orthogonal mate [see for example Hedayat and Federer (1969)]. The above argument indicates the nonexistence of Knut Vik designs of even orders with even less stringent conditions.

The following Lemma is easy to verify.

Lemma 2.1. If K_i is a Knut Vik design of order n_i, i=1,2. Then the Kronecker product of K_1 and K_2 is a Knut Vik design of order n_1n_2.

Whether or not Knut Vik designs exist for all odd orders is not known to date. It can be shown that a Knut Vik design of order 3 does not exist. For other odd orders we have the following partial answer.

Theorem 2.2. Let p be a prime ($p \neq 2,3$) and let α be a positive integer. Then a Knut Vik design of order $n=p^{\alpha}$ can be constructed.

If $\alpha = 1$ then let $K = (k_{ij}) = [(n-2)i+j] \bmod n$. It is not difficult to show that K is a Knut Vik design. This, together with Lemma 2.1, provides a proof for the theorem.

Corollary 2.1. If n is an odd number such that $n \not\equiv 0 \pmod 3$ then there exists a Knut Vik design of order n.

3. SEMI KNUT VIK DESIGNS. A Latin square of order n is said to be a semi Knut Vik design of order n if each treatment appears once in each of the n right (or left) diagonals. Such a design is useful for eliminating three sources of variation. The proof of Theorem 2.1 clearly shows that no semi Knut Vik design exists for even orders. However, these designs exist for all odd orders. One way to construct them is by putting $[(n-2)i+j] \bmod n$ in the (i,j) entry of an $n \times n$ square, $i,j=0,1,\cdots,n-1$.

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