
Modeling a Single-stage Hydrocyclone for Potato Starch

Separation

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Abstract

Data from experiments on separation of potato starch by a single stage hydroclone were analyzed based on similarity theory. The stepwise regression was then used to establish mathematical models that best described the effects of a single stage hydroclone on separation of potato starch. The models can be used to predict the separating performance, and as a theoretical basis for scaling-up design, parameter optimization and other applications. Many companies around the world are using hydrocyclones for starch separation because of their simple design and operation.

Key words: Starch, hydrocyclone, mathematical model

1 . Introduction

The literature research shows that studies on hydrocyclones used for separation of potato starch have been conducted only in a few countries including Holland, Poland, the United States and Russia^[1]. There is no evidence indicating that China has been involved in this field of study. The flow of fluid inside a hydrocyclone is a complex process and there are many factors influencing the efficiency of separation. It is therefore necessary to have a thorough understanding of the influencing factors and their relationships to predict the separating performance of a hydrocyclone^[2,3]. Until now, few studies on the underflow concentration have been reported in documents concerning the prediction model, although it has a significant influence on the efficiency of a separation system^[4]. In a separation process, overflows are returned to the previous stage(s) as mother liquid. A too high or low overflow concentration will not only have impacts on the choice of its previous hydrocyclone (s), but also on the determination of separation parameters. Hence, it is appropriate to develop mathematical models for underflow concentration and overflow concentration^[5~8].

In this study, the criteria obtained by dimensional method found in similarity theorem were used to filter experimental data^[9]. A stepwise regression model was developed on the filtered data with mathematical software, Matlab, to predict the performances of hydrocyclone.

2 . Separation Criteria Equation

2.1 Similarity Criteria

Similarity criteria (or similitude parameters) are deduced according to A.O.φeπeπmaH's(Russian language) dimensional analysis^[9]. As the first step, a dimensional analysis is performed on each of the parameters and performance indicators. Physical parameters that have the same dimensions can be expressed as the ratio of one parameter's value to another. The resulting criteria are obviously dimensionless and may be used as similarity criteria.

For structural parameters, we have π_1 :

$$\pi_1 = Du/Do$$

where, Du is the diameter of underflow opening (mm); Do is the diameter of overflow opening (mm).

For physical characteristic parameters, we have π_2 :

$$\pi_2 = Ci$$

where, Ci is the starch concentration of inlet(%).

Another parameter that has significant influence over the performance of separation is the solid content Ci_g , let:

$$\pi_3 = Ci_g/Ci$$

where, Ci_g is the solid content (%).

No and Ni are two additional dimensionless physical parameters, and can be expressed as

$$\pi_4 = Ni \quad \pi_5 = No$$

Let:

$$\pi_6 = Dei/Du \quad \pi_7 = Deo/Dei$$

where, i refers to the inlet, o the overflow and u the underflow.

N and De are two parameters in the noted Rosin-rammler equation^[2], standing for the distributions of particle size. De is a constant that is in direct proportion with X_{50} (i.e., a diameter of particle below which the cumulative distribution of particle size amounts to 50 percent). N is determined by the range of particle size distribution. The greater the value of N , the smaller the range of particle size distribution, showing that the distribution of sampled particle size is more homogeneous.

The other criteria concerned with the separating property is π_8 , let

$$\pi_8 = n/l$$

Where, n is the number of hydrocyclone tubes in parallel within a cyclone.

The operating variables Co and Cu are denoted by:

$$\pi_9 = Co \quad \pi_{10} = Cu$$

Where, Co and Cu are the starch concentration respectively in overflow and underflow (%).

The separation function parameters denoted by $\pi_{11}=S$, $\pi_{12}=Et_1$ and $\pi_{13}=Et_2$ are given by:

$$Et_1 = \frac{(Ci_g - Co_g)Cu_g}{(Cu_g - Co_g)Ci_g} \quad (1)$$

$$Et_2 = \frac{C_u G_u}{C_i G_i} = \frac{(C_i - C_o)C_u}{(C_u - C_o)C_i} \quad (2)$$

Where, S is the split ratio of underflow volume to overflow volume; Et_1 is the separation efficiency of starch; Et_2 is the overall efficiency of separation and; Co_g and Cu_g are concentrations of solids in overflow and underflow respectively (%).

The remaining parameters include D_u , P_i , Q_i , ρ_i and μ_i , on which the similarity criteria are given by dimension analysis method. Among them, D_u , P_i and ρ_i can be regarded as basic variables according to Buckingham's theorem^[3], which can then be integrated with Q_i and μ_i respectively to obtain two dimensionless quantities. Then we have two similitude parameters:

$$\pi_{14} = \frac{2\sqrt{2}}{\pi} \sqrt{\frac{\pi^2 Du^4 P_i}{8 \rho_i Q_i^2}} \quad (3)$$

$$\pi_{15} = \frac{\mu_i}{\sqrt{\rho_i P_i D_u}} \quad (4)$$

Where, μ_i is the viscosity of entrance material (Pa·S); where Pa is in Pascals, s is the time in seconds, ρ_i is the density of inlet material (kg/m³); P_i is the pressure of incoming fluid (Mpa) and; Q_i is the flow of inlet material (m³/h).

Let:

$$Pu = \frac{\mu_i}{\sqrt{\rho_i P_i D_u}} \quad (5)$$

A criteria representing the changes in operating pressure. The Euler number (Eu)

$$Eu = \frac{\pi^2 Du^4 P_i}{8 \rho_i Q_i^2} \quad (6)$$

is

Here, the Euler number (Eu) refers to the relationship between the drop in pressure and the production capacity.

As dimensionless variables will remain dimensionless after any algebra or exponential operation, then π_{14} and π_{15} can be simplified into the following forms:

$$\pi_{14} = Eu \quad \pi_{15} = Pu$$

All the above similarity criteria are re-sorted in terms of settled criteria and pending criteria (table 1). It will not change the behavior of the subsequent modeling

after re-sorting.

Table 1 Similarity Criteria

No.	Criteria	No.	Criteria	No.	Criteria	No.	Criteria
1	$\Pi_1 = D_u/D_o$	5	$\Pi_5 = D_{ei}/D_u$	9	$\Pi_9 = C_o$	13	$\Pi_{13} = S$
2	$\Pi_2 = C_i$	6	$\Pi_6 = Eu$	10	$\Pi_{10} = C_u$	14	$\Pi_{14} = Et$
3	$\Pi_3 = C_i'/C_i$	7	$\Pi_7 = Pu$	11	$\Pi_{11} = N_o$	15	$\Pi_{15} = Et_1$
4	$\Pi_4 = N_i$	8	$\Pi_8 = n$	12	$\Pi_{12} = De_o/De_i$		

2.2 Separation criteria equation

The similarity criteria may fall into two types: the settled and the pended. Those with known physical value in single variable conditions are called settled similarity criteria. In contrast, the similarity criteria with unknown physical value are called pended criteria. Of the above similarity criteria, Π_1 through Π_8 are settled criteria. From the third similarity theorem, it is clear that the prerequisite for similarity of physical phenomenon is that the settled criteria must be equal. For pended criteria to be equal, it is necessary that physical phenomenon must be equal. Therefore, there exists a causal relationship in nature between these two types of criteria. Such a relationship may be expressed as a univalent function or criteria equation. It can be seen from the above analysis that pended criteria Π_9 through Π_{15} can be expressed by a function of settled criteria Π_1 through Π_8 ^[9]. Based on past experience and engineering practice, similitude parameter equation of various kinds can be described in the form of a power function.

In this paper, the experimental data are filtered by using similarity criteria and analyzed with stepwise regression method. The stepwise regression is to select associative variables that have significant influences over the dependent variables in establishing regression equations. For this purpose, the most significant independent variable is always taken from an initial model and put into the regression model in an

iterative way, in order to test the independent variables contained in the original model one by one. Insignificant variables are left out until no variables will be entered into on one hand and left out from the regression equation on the other hand. For the

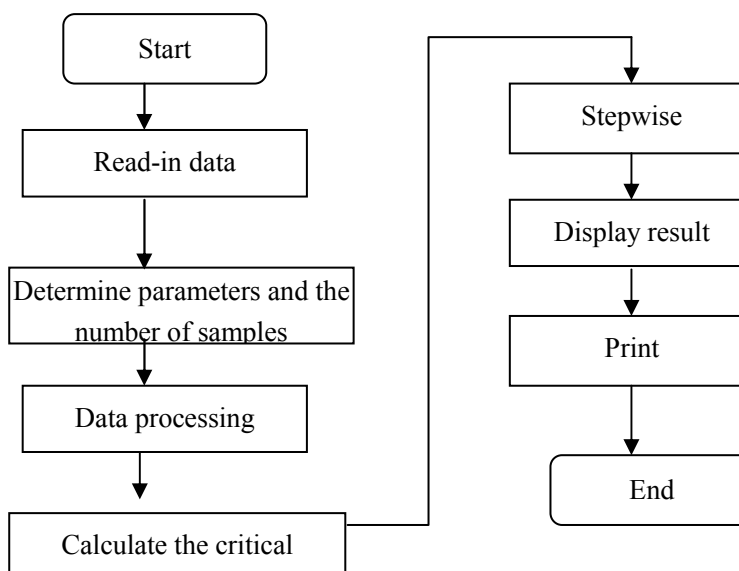


Fig 1. The algorithm of stepwise regression

basic algorithm, see figure 1. Regression analysis is done on experiment data with Matlab. And then, F test is performed on each of the parameters at a significance level of 5 percent. The parameters are accepted or rejected according their ranges of significance. The resulting mathematical regression models of similarity criteria are as follows.

$$Co = 0.028 \left(\frac{D_u}{D_o} \right)^{-1.495} C_i^{1.455} \left(\frac{C_i'}{C_i} \right)^{0.4567} (N_i)^{-1.13} (Eu)^{0.326} \quad (7)$$

$$C_u = 0.2 \left(\frac{D_u}{D_o} \right)^{1.045} C_i^{0.7238} (N_i)^{1.339} \left(\frac{De_i}{D_u} \right)^{1.451} \quad (8)$$

$$N_o = 4.2 \left(\frac{D_u}{D_o} \right)^{0.5767} (N_i)^{0.7164} (Eu)^{-0.3153} (Pu)^{-0.5328} (n)^{0.104} \quad (9)$$

$$\frac{De_o}{De_i} = 0.087 \left(\frac{D_u}{D_o} \right)^{-0.6417} (N_i)^{1.304} (Eu)^{0.2711} (Pu)^{0.5812} (n)^{-0.088} \quad (10)$$

$$S = 2.176 (C_i)^{0.072} \left(\frac{De_i}{D_u} \right)^{-1.059} n^{-0.067} \quad (11)$$

$$Et_1 = 30.9 (C_i)^{0.1302} (N_i)^{0.5696} \quad (12)$$

$$Et_2 = 2.79 \times 10^{-2} C_i^{0.048} (N_i)^{0.405} \left(\frac{De_i}{D_u} \right)^{-0.634} (Eu)^{-0.1347} \quad (13)$$

3. Analysis of Mathematical Models

3.1 Accuracy of Similarity Criteria

As the F-test table shows when the significance level $\alpha=0.05$, the critical value of F-test is:

$$\begin{aligned} F_{0.05} (5, 89) &= 2.32 & F_{0.05} (4, 89) &= 2.47 & F_{0.05} (3, 89) &= 2.71 \\ F_{0.05} (2, 89) &= 3.10 & F_{0.05} (1, 89) &= 3.95 \end{aligned}$$

According to the correlation coefficient test table:

When $\alpha=0.05$, the correlation coefficients are:

$$R_{89,5}^{0.05} = 0.315, R_{89,4}^{0.05} = 0.288, R_{89,3}^{0.05} = 0.254, R_{89,2}^{0.05} = 0.205$$

For the statistics of each model, see table 2.

Table 2 The statistics of models

Model	S ₁	R	F	P	Relative error(%)
C _o	0.548	0.892	64.93	0.000	8.500
C _u	0.427	0.914	106.90	0.000	6.700
No	0.202	0.740	20.04	4.68e-013	4.000
De _o /De _i	0.151	0.731	19.05	1.42e-013	12.00
S	0.269	0.718	30.06	2.41e-013	4.000
Et1	0.301	0.568	20.44	5.45e-008	8.000
Et2	0.170	0.660	16.21	7.13e-010	5.000

It can be observed from table 2 that the residual sum of squares (S) is very small for each of the model and the relative errors are all within the range of acceptance. Moreover, most of the correlation coefficients (R) are above 0.7, showing that a good fitness exists between parameters and indices. The values of F are all greater than the critical values given by the test table, which demonstrates that the models are significant at 95 percent level of confidence. The results calculated with the mathematical-models are given in table 3 in comparison with values of measurements. It is obvious that predictors calculated from the regression models have a high accuracy.

Table 3 Results calculated from mathematical models in comparison with measurements

Co		Cu		No		Deo/Dei		S		Et1		Et2	
(E)	(M)	(E)	(M)	(E)	(M)	(E)	(M)	(E)	(M)	(E)	(M)	(E)	(M)
5.60	5.15	16.8	15.17	2.45	2.32	0.67	0.57	0.66	0.50	67	68.61	57	60.63
2.10	2.85	20.0	21.20	2.68	3.01	0.78	0.81	0.70	0.54	80	81.18	85	78.47
1.08	2.31	16.5	14.58	3.42	2.45	0.89	0.93	0.64	0.53	69	76.88	82	82.54
1.90	2.25	20.2	20.62	3.30	3.20	0.83	0.85	0.42	0.48	87	93.91	82	80.36
0.60	0.79	32.8	28.33	3.35	3.19	0.84	0.84	0.46	0.46	85	86.28	74	79.86
5.00	6.15	32.8	28.85	3.02	2.98	0.82	0.84	0.40	0.35	83	81.84	84	58.93
3.00	3.25	33.3	32.63	3.14	2.71	0.87	0.82	0.35	0.37	85	85.97	58	55.08
4.50	5.73	32.3	34.02	1.80	1.98	0.79	0.89	0.58	0.52	75	75.10	29	57.29
12.00	13.86	33.9	31.80	2.13	1.92	0.85	0.75	0.30	0.36	73	67.73	45	51.99
9.40	11.58	6.9	7.31	2.62	2.51	0.90	0.88	0.60	0.55	90	83.16	46	40.53
10.20	12.58	7.1	6.13	1.63	1.97	0.63	0.66	0.20	0.26	68	76.45	58	58.90

In the table, E denotes the measurement value and M the values calculated from each model.

3.2 Discussion

(1) Model Co

It can be seen from Model Co that the overflow concentration (Co) has a strong relation with the ratio of inlet to outlet opening (Du/Do), the inlet starch concentration (Ci), the concentration ratio of inlet solid to starch, the inlet particle size distribution,

and the inlet pressure. The ratio of inlet to outlet opening, the inlet starch concentration and the distribution of inlet particle size have higher influences than the concentration ratio of inlet solid to starch.

(2) Model C_u

The model is determined by the ratio of inlet to outlet opening, the inlet concentration of starch and the distribution of inlet particle size. The distribution of inlet particle size has the greatest impact on the Co , while the inlet content of starch has the smallest.

(3) Model No

Based on past experience, the smallest size of particle that can be separated may be roughly estimated according to the diameter of a cyclone. However, the applicability of a cyclone for a given material can be determined not only by its separation size, but also by the distributions of particle size in material before and after separation. They will allow for an overall understanding if the cyclone is applicable for separating a given material at relevant operating conditions. It is observed from the model that the distribution of particle size and the ratio of inlet to outlet opening have greater influences than the inlet pressure, working capacity and the number of hydrocyclone tubes connected in parallel.

(4) Model Deo/Dei

The model has a similar form to Model No , and the parameters have similar behaviors.

(5) Model S

In the present practice, the calculation of split ratio is based on an empirical formula whose scope of application is usually restricted by the material properties and the structural parameters of a cyclone. In order to provide a model for split ratio that best describes the separation of potato starch, an empirical formula is worked out through the above regression analysis. It is clear that the model is determined by the starch content and particle size distribution of inlet material and the number of hydrocyclone tubes working in parallel at each separation stage.

(6) Model Et_1

As an important indicator studied in this research, the separation efficiency of starch represents the separating performance of hydrocyclone. Therefore, it is necessary that an accurate prediction of the separating performance should be made to determine the structural parameters and the operating variables of a hydrocyclone. The model is largely determined by the distribution of inlet particle size, and then by the working capacity, operating pressure and the starch content of inlet.

(7) Model Et_2

The separation efficiency of solid content is used for comparison in this study, which has strong relationships with the particle size distribution and starch content of inlet.

It may be concluded from the above discussion that all the models are primarily determined by the ratio of outlet to inlet opening, the particle size distribution of inlet,

the starch concentration of inlet and the operating pressure. It is therefore especially important to regulate these four parameters so as to obtain an optimal status of separation in real production.

4. Conclusion

The mathematical models developed on the basis of experiments, not only allow for a reliable prediction of the performance of a single stage hydroclone for separating potato starch, but also provide an effective way to test the design of hydroclone for optimal performance^[10]. The model can predict the result of separation performance. Thus it can reduce the cost of experiment and debugging.

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