F-Square Geometries for n = 3, 4, 5, and 6

by

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ABSTRACT

Through the use of complete sets of mutually orthogonal F-squares, the concept of F-square geometries has been introduced. This follows from the one-to-one correspondence of complete sets of mutually orthogonal latin squares and projective geometry. The cases of n = 3, 4, 5, and 6 as the order of the F-square are considered. The case for n = 3 is completely resolved where it is shown that there is only one geometry, the projective. The case for n = 4 is partially resolved and four F-square geometries have been found. It is not known if there are more. The case for n = 5 has not been investigated, but one geometry for the complete set of orthogonal latin squares does exist. No one has as yet found an F-square geometry for n = 6. A study of <u>all</u> F-square geometries for these cases will be useful for considering other values of n.

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1. INTRODUCTION

It is well known that for latin squares of order three,

(i) a complete set of orthogonal latin squares, denoted by OL(3,2) exist, and

(ii) there is a single transformation set.

With the introduction and development of F-square design theory by Hedayat [1969] and Hedayat and Seiden [1970] and from section XV of a paper by Federer et al. [1971], where A. Hedayat shows the equivalence of various combinatorial systems starting with an OL(n,n-1) set, the question arises as to the use of F-square design theory in a one-to-one correspondence with other combinatorial systems. As a first step we shall look at all possible complete sets of F-squares. We shall call each one an F-square geometry and shall be studying complete sets of F-square geometries for $n = 3, 4, 5$, and 6. The case for $n = 3$ is very simple. The case for $n = 4$ becomes considerably more difficult and the difficulty increases with n since the number of possible cases becomes increasingly large.

First of all, an F-square of order n with m symbols is denoted as $F(n;\ \lambda_1,\cdots,\lambda_m)$-square. The λ_i are integers and refer to the frequency of any given symbol in a row or in a column. When the λ_i are ones, a latin square of order n is indicated. Also, $\sum_{i=1}^{m} \lambda_i = n$ for any F-square. A set of t mutually orthogonal F-squares with the same number, m, of symbols will be denoted as $OF(n;\lambda_1,\cdots,\lambda_m;t)$ to correspond to the notation OL(n,t) for t orthogonal latin

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squares. If the number of symbols in the complete set of orthogonal F-squares varies, then we use the notation

$$\sum_{i=1}^{n} OF(n;\lambda_1,\cdots,\lambda_i;N_{i\lambda}) \quad \text{for all} \quad \lambda_h , \quad h=1,\cdots,i ,$$

to indicate that there are $N_{i\lambda}$ F-squares with i symbols for each possible set of λ_h.

Note that there are $(n-1)^2$ degrees of freedom associated with the row $\times$ column interaction and that these are the only degrees of freedom available for constructing F-squares. In an F-square with i symbols there are (i-1) degrees of freedom among the i symbols. Hence, $\sum_{i=2}^{n} N_{i\lambda}(i-1) = (n-1)^2$ for all possible sets of λ_i, for a complete set of F-squares.

The idea of many complete sets for each n may be somewhat new for most people, but a discussion for n = 3, 4, 5, and 6 below should clarify what is meant by the complete set of F-square geometries of order n.

2. THE CASE FOR n = 3

The possible sets of λ_h, $h=1,\cdots,i \leq 3$ in an $F(3;\lambda_1,\cdots,\lambda_i)$-square are 1,1,1 and 2,1. Note that 1,2 is merely a permutation of the set 2,1. A complete set of orthogonal $F(3;\lambda_.,\cdots,\lambda_i)$-squares is given by the terms of the summation

$$OF(3;2,1;N_1) + OF(3;1,1,1;N_2) .$$

The possible values for N_1 and N_2, given that $N_1(2-1) + N_2(3-1) = (3-1)^2 = 4$ are:

N_1	N_2	F-square geometry given by
0	2	OL(3,2) set
2	1	does not exist
4	0	does not exist

The members of an OL(3,2) set are

$$L_1 = \boxed{\begin{matrix} A & B & C \\ B & C & A \\ C & A & B \end{matrix}} \qquad L_2 = \boxed{\begin{matrix} a & b & c \\ c & a & b \\ b & c & a \end{matrix}} \quad .$$

A permutation of the last two rows of L_2 produces L_1.

The problem of producing a complete set of orthogonal F-squares for $N_1 = 2$ and $N_2 = 1$ resolves itself if one is able to decompose a latin square of order three into two orthogonal F(3;2,1)-squares. Hence, the following theorem:

<u>Theorem 2.1.</u> <u>It is impossible to decompose a latin square of order three into an orthogonal pair of</u> F(3;2,1)-<u>squares</u>.

<u>Proof</u>. It is immaterial whether one uses L_1 or L_2 so we shall show that L_2 cannot be decomposed into two orthogonal F(3;2,1)-squares. Consider the following set of orthogonal single degree of freedom contrasts for a 3 × 3 square:

Contrast										
1.	mean	+	+	+	+	+	+	+	+	+
2.	row 1 versus row 2	+	+	+	-	-	-	0	0	0
3.	row 1+2 vs. row 3	+	+	+	+	+	+	-2	-2	-2
4.	col. 1 vs. col. 2	+	-	0	+	-	0	+	-	0

5. columns 1+2 vs. 3	+	+	-2	+	+	-2	+	+	-2
6. A versus B	+	-	0	-	0	+	0	+	-
7. A + B vs. C	+	+	-2	+	-2	+	-2	+	+
8. a + b vs. c	+	+	-2	-2	+	+	+	-2	+
9. unknown = (?)	a_{11}	a_{12}	a_{13}	a_{21}	a_{22}	a_{23}	a_{31}	a_{32}	a_{33}

Contrast 8 forms an F(3;2,1)-square if we put a symbol, say x, where the pluses occur in the contrast, and a second symbol, say y, where the minus two occurs. This F-square follows as does the unknown in contrast 9:

Contrast 8

x	x	y
y	x	x
x	y	x

Contrast 9

a_{11}	a_{12}	a_{13}
a_{21}	a_{22}	a_{23}
a_{31}	a_{32}	a_{33}

Note that for contrast 8 we could have taken a + c vs. b or b + c vs. a to obtain the F(3;2,1)-square and that these three ways exhaust the possibilities for forming F(3;2,1)-squares. Since the sum of the coefficients must equal zero and since the sum of products of coefficients in any two rows must be zero the only possible values for the a_{rs} are given below:

$a_{11} = 1$	$a_{12} = -1$	$a_{13} = 0$
$a_{21} = 0$	$a_{22} = 1$	$a_{23} = -1$
$a_{31} = -1$	$a_{32} = 0$	$a_{33} = 1$

There is no way to form an $F(3;2,1)$-square from the above since there are three, not two, coefficients, i.e., 1, -1, and 0. Thus, the complete set of F-squares for $N_1 = 2$ and $N_2 = 1$ does not exist.

Consider now the case where $N_1 = 4$ and $N_2 = 0$. Since the orthogonal $F(3;2,1)$-squares must be formed by contrasts of the form a + b versus c and A + B versus C (or some permutation of the symbols), from the full set of 9 orthogonal contrasts, seven will be specified as above. The remaining cannot take on any other values than +1, -1, and 0 as described above. Hence, it is impossible to form an $OF(3;2,1;4)$ set, resulting in the following theorem:

Theorem 2.2. *The* $OF(3;2,1;4)$ *set does not exist.*

It is possible to form a pair of orthogonal $F(3;2,1)$-squares by taking one square from L_1 and one from L_2 above. It is not possible to obtain more than two.

3. THE CASE FOR n = 4

The possible configurations of the λ_h, $h=1,\cdots,i \leq 4$ in $F(4;\lambda_1,\cdots,\lambda_i)$-squares are: 1,1,1,1; 2,1,1; 2,2; and 3,1. Note that $\sum_{h=1}^{i} \lambda_h = 4$. A complete set of mutually orthogonal F-squares of order 4 is indicated as follows:

$$OF(4;3,1;N_1) + OF(4;2,2;N_2) + OF(4;2,1,1;\ N_3) + OF(4;1,1,1,1;N_4)\ .$$

Subject to the constraint that $\sum_{i=1}^{4} N_i = (4-1)^2 = 9$, the possible values for the N_i are given below:

N_1	N_2	N_3	N_4	Complete set given by
0	0	0	3	OL(4,3)-set
0	1	1	2	does not exist (see below)
1	0	1	2	does not exist (see below
0	3	0	2	given below
1	2	0	2	does not exist (see below)
2	1	0	2	does not exist (see below)
3	0	0	2	does not exist (see below)
0	0	3	1	
0	2	2	1	
1	1	2	1	
2	0	2	1	
0	4	1	1	
1	3	1	1	
2	2	1	1	
3	1	1	1	
4	0	1	1	
0	6	0	1	Mandeli [1975]
1	5	0	1	
2	4	0	1	
3	3	0	1	
4	2	0	1	
5	1	0	1	
6	0	0	1	
0	1	4	0	
1	0	4	0	
0	2	3	0	
1	1	3	0	
2	0	3	0	
0	5	2	0	
1	4	2	0	
2	3	2	0	
3	2	2	0	

4	1	2	0	
5	0	2	0	
0	7	1	0	
1	6	1	0	
2	5	1	0	
3	4	1	0	
4	3	1	0	
5	2	1	0	
6	1	1	0	
7	0	1	0	
0	9	0	0	Hedayat, Raghavarao and Seiden [1975]
1	8	0	0	
2	7	0	0	
3	6	0	0	
4	5	0	0	
5	4	0	0	
6	3	0	0	
7	2	0	0	
8	1	0	0	
9	0	0	0	

3.1. Solution for $N_4 = 2$. For latin squares of order 4 there are two transformation sets, one of which is mateless and one which can be used to construct an OL(4,3) set such as the following:

$$L_1 = \begin{bmatrix} A & B & C & D \\ B & A & C & D \\ C & D & A & B \\ D & C & B & A \end{bmatrix} \qquad L_2 = \begin{bmatrix} a & b & c & d \\ d & c & b & a \\ b & a & d & c \\ c & d & a & b \end{bmatrix} \qquad L_3 = \begin{bmatrix} \alpha & \beta & \gamma & \delta \\ \gamma & \delta & \alpha & \beta \\ \delta & \gamma & \beta & \alpha \\ \beta & \alpha & \delta & \gamma \end{bmatrix}$$

If a and α are set equal to A, b and β to B, c and γ to C, and d and δ to D, one may observe that L_2 and L_3 can be converted into L_1 by a simple row permutation of the last three rows. In addition, it is known that <u>any</u> pair of orthogonal latin squares of order 4 can be extended to form an OL(4,3) set. Thus, any two of L_1, L_2, or L_3 may be used and the problem is to show how to decompose the remaining latin square into combinations of F(4;2,1,1)-, F(4;2,2)-, and/or F(4;3,1)-squares. Suppose that L_1 and L_2 are the latin squares in the set for $N_4 = 2$. Then, our problem is to decompose L_3 into F-squares. The <u>only</u> F-squares with two symbols that are possible are the F(4;3,1)-square and the F(4;2,2)-square. The former implies the contrast $3\alpha - \beta - \gamma - \delta$ and the latter implies the contrast $\alpha + \beta - \gamma - \delta$ among the four symbols. Note that although there are an infinite number of sets of contrasts for n = 4, these two from Helmert polynomials and from the 2^2 factorial are the <u>only</u> ones giving rise to F-squares. Therefore, one needs only to investigate the following two cases to determine if L_3 can be decomposed into three F-squares with two symbols:

$$\begin{array}{|cccc|}\hline + & + & + & + \\ 1 & 1 & 1 & -3 \\ a_1 & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \\ \hline \end{array} \qquad \begin{array}{|cccc|}\hline + & + & + & + \\ + & + & - & - \\ c_1 & c_2 & c_3 & c_4 \\ d_1 & d_2 & d_3 & d_4 \\ \hline \end{array}$$

M_1 $\qquad$ M_2

For M_1, note that

$$a_1 + a_2 + a_3 + a_4 = 0$$

$$a_1 + a_2 + a_3 - 3a_4 = 0 .$$

The only solution for a_4 is $a_4 = 0$, and if all 16 cells of a 4 x 4 square are used, one cannot form a F-square. Hence, M_1 cannot be completed to form a set of three orthogonal F-squares with two symbols. Likewise, the same holds for the b_g coefficients.

In M_2,

$$c_1 + c_2 + c_3 + c_4 = 0$$

$$c_1 + c_2 - c_3 - c_4 = 0 \quad .$$

Therefore,

$$c_1 + c_2 = 0 \quad \text{and} \quad c_3 + c_4 = 0$$

are solutions for these two conditions. Possible solutions for c_1 and c_2 are +1 and -1, or 0 and 0, or multiples thereof. Likewise, these are the possible solutions for c_3 and c_4. Therefore, the possible sets of solutions are:

$$\begin{array}{rrrr} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{array} \qquad \begin{array}{rrrr} 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{array} \; .$$

The first set does not produce F-squares, but the second one does. Hence, the only decomposition of L_3 into F-squares with two symbols is into three $F(4;2,2)$-squares.

Now consider the decomposition of L_3 into an $F(4;2,1,1)$-square plus an F-square with two symbols. First combine any two symbols of L_3 into a single symbol to form the $F(4;2,1,1)$-square, e.g., let $\alpha = \delta = \alpha$. Then, form the contrast of $2\alpha - \beta - \gamma$. The only contrast orthogonal to this contrast is $\beta - \gamma$. The remaining orthogonal contrast would be α versus the original δ. This last contrast does not form an F-square.

Another way of looking at this problem probably could be using a result due to S. S. Shrikhande (personal communication from A. Hedayat, 8/12/76).

He showed that if a matrix contains the first $4t-2$ rows of a Hadamard matrix the only way to make this an orthogonal matrix is to complete the Hadamard matrix. This implies the existence of $F(4;2,2)$-squares only. The above then leads to the following theorem:

Theorem 4.1. The only decomposition of a latin square from the $OL(4,3)$ set is into three $F(4;2,2)$-squares.

3.2. Solution for $N_4 = 1$. Here one needs to consider the solution for a latin square from the set $OL(4,3)$ and a latin square from the other transformation set which is an orthogonally mateless latin square. The only solution for the 16 cases is the one for which $N_4 = 1$, $N_3 = N_1 = 0$, and $N_2 = 6$. Mandeli [1975] has given the solution for both transformation sets. The solution for the remainder of the cases is an open problem.

3.3. Solution for $N_4 = 0$. Of the 29 possibilities for complete sets of F-squares when $N_4 = 0$, only one has been solved, and that is for the $OF(4;2,2;9)$ set. Some decomposition and composition theorems are needed for these solutions.

4. THE CASE FOR n = 5

The possible configurations of the λ_h, $h=1,\cdots,i \le 5$ in $F(5;\lambda_1,\cdots,\lambda_i)$ are: 1,1,1,1,1; 2,1,1,1; 2,2,1; 3,1,1; 4,1; and 3,2. Note that the $\sum_{h=1}^{i} \lambda_h = 5$. Consider a complete set of mutually orthogonal $F(5;\lambda_1,\cdots,\lambda_i)$-squares such that there are N_i of the i^{th} type and denoted as $OF(5,\lambda_1,\cdots,\lambda_i;N_i)$, where $\sum_{i=2}^{5} N_i(i-1) = (5-1)^2 = 16$. A complete set of mutually orthogonal F-squares will have the following numbers of types: $OF(5;3,2;N_1) + OF(5;4,1;N_2) + OF(5;3,1,1;N_3) + OF(5;2,2,1;N_4) + OF(5;2,1,1,1;N_5) + OF(5;1,1,1,1,1;N_6)$. The possible values for the N_i are:

N_1	N_2	N_3	N_4	N_5	N_6	Complete set given by
0	0	0	0	0	4	OL(5,4)-set
0	1	0	0	1	3	
1	0	0	0	1	3	
0	0	0	2	0	3	
0	0	1	1	0	3	
0	0	2	0	0	3	
0	2	0	1	0	3	
1	1	0	1	0	3	
2	0	0	1	0	3	
0	2	1	0	0	3	
1	1	1	0	0	3	
2	0	1	0	0	3	
0	4	0	0	0	3	
1	3	0	0	0	3	
2	2	0	0	0	3	
3	1	0	0	0	3	
4	0	0	0	0	3	
0	0	1	0	2	2	
0	2	0	0	2	2	
1	1	0	0	2	2	
2	0	0	0	2	2	
0	1	0	2	1	2	
1	0	0	2	1	2	
0	1	2	0	1	2	
1	0	2	0	1	2	
0	1	1	1	1	2	
1	0	1	1	1	2	
0	7	0	1	1	2	
1	6	0	1	1	2	
2	5	0	1	1	2	
3	4	0	1	1	2	

4	3	0	1	1	2
5	2	0	1	1	2
6	1	0	1	1	2
7	0	0	1	1	2
0	7	1	0	1	2
1	6	1	0	1	2
2	5	1	0	1	2
3	4	1	0	1	2
4	3	1	0	1	2
5	2	1	0	1	2
6	1	1	0	1	2
7	0	1	0	1	2
0	9	0	0	1	2
1	8	0	0	1	2
2	7	0	0	1	2
3	6	0	0	1	2
4	5	0	0	1	2
5	4	0	0	1	2
6	3	0	0	1	2
7	2	0	0	1	2
8	1	0	0	1	2
9	0	0	0	1	2
0	0	0	4	0	2
0	0	1	3	0	2
0	2	0	3	0	2
1	1	0	3	0	2
2	0	0	3	0	2
0	0	2	2	0	2
0	2	1	2	0	2
1	1	1	2	0	2
2	0	1	2	0	2
0	4	0	2	0	2
1	3	0	2	0	2
2	2	0	2	0	2
3	1	0	2	0	2

4	0	0	2	0	2
0	0	3	1	0	2
0	2	2	1	0	2
1	1	2	1	0	2
2	0	2	1	0	2
0	4	1	1	0	2
1	3	1	1	0	2
2	2	1	1	0	2
3	1	1	1	0	2
4	0	1	1	0	2
0	6	0	1	0	2
1	5	0	1	0	2
2	4	0	1	0	2
3	3	0	1	0	2
4	2	0	1	0	2
5	1	0	1	0	2
6	0	0	1	0	2
0	0	4	0	0	2
0	2	3	0	0	2
1	1	3	0	0	2
2	0	3	0	0	2
0	4	2	0	0	2
1	3	2	0	0	2
2	2	2	0	0	2
3	1	2	0	0	2
4	0	2	0	0	2
0	6	1	0	0	2
1	5	1	0	0	2
2	4	1	0	0	2
3	3	1	0	0	2
4	2	1	0	0	2
5	1	1	0	0	2
6	0	1	0	0	2
0	8	0	0	0	2
1	7	0	0	0	2

2	6	0	0	0	2	
3	5	0	0	0	2	
4	4	0	0	0	2	
5	3	0	0	0	2	
6	2	0	0	0	2	Those above for $N_6 = 2,3$
7	1	0	0	0	2	that exist should be obtainable
8	0	0	0	0	2	from the OL(5,4)-set
0	1	0	1	3	1	
1	0	0	1	3	1	
0	1	1	0	3	1	
1	0	1	0	3	1	
0	3	0	0	3	1	
1	2	0	0	3	1	
2	1	0	0	3	1	
3	0	0	0	3	1	
0	0	0	3	2	1	
0	0	1	2	2	1	
0	2	0	2	2	1	
1	1	0	2	2	1	
2	0	0	2	2	1	
0	0	2	1	2	1	
0	2	1	1	2	1	
1	1	1	1	2	1	
2	0	1	1	2	1	
0	4	0	1	2	1	
1	3	0	1	2	1	
2	2	0	1	2	1	
3	1	0	1	2	1	
4	0	0	1	2	1	
0	0	3	0	2	1	
0	2	2	0	2	1	
1	1	2	0	2	1	
2	0	2	0	2	1	
0	4	1	0	2	1	
1	3	1	0	2	1	

2	2	1	0	2	1
3	1	1	0	2	1
4	0	1	0	2	1
0	6	0	0	2	1
1	5	0	0	2	1
2	4	0	0	2	1
3	3	0	0	2	1
4	2	0	0	2	1
5	1	0	0	2	1
6	0	0	0	2	1
				1	1
				1	1
				1	1
				1	1
				1	1
				1	1
				1	1
				1	1
				1	1
				1	1
				1	1
				1	1
				1	1
				1	1
				1	1
				1	1
				1	1
				⋮	⋮

etc. for other combinations of the N_i down to the last case where $N_1 = 16$, $N_2 = N_3 = N_4 = N_5 = N_6 = 0$. Because of the very large number of cases, some decomposition and composition theorems are needed to obtain the solution for classes rather than single cases. Note that if $N_6 \geq 2$, the F-squares under consideration must come from a decomposition of latin squares from the OL(5,4) set. For $N_6 = 1$, there are two transformation sets, one of mateless latin squares of order 5 and the other which is a member of an OL(5,4) set. Note that only one case, i.e., for the OL(5,4) set, has been solved in the complete set of F-square geometries.

5. THE CASE FOR n = 6

No one has as yet obtained a complete set of orthogonal $F(6;\lambda_1,\cdots,\lambda_i)$-squares, for $i=2,3,\cdots,6$. The maximum number so far obtained is an OF(6;2,2,2;7) + OF(6;1,1,1,1,1,1;1) set. Two F(6;2,2,1,1)-squares, if orthogonal to the above, would be needed to complete the set. Likewise, the addition of six $OF(6;\lambda_1,\lambda_2)$-squares would also complete the set. Many such combinations are possible, but so far a complete set of mutually orthogonal F-squares has not been obtained.

In this connection there are ten possible F-squares of order 6. These are:

F(6;5,1)	F(6;3,2,1)	F(6;2,1,1,1,1)
F(6;4,2)	F(6;2,2,2)	F(6;1,1,1,1,1,1)
F(6;3,3)	F(6;3,1,1,1)	
F(6;4,1,1)	F(6;2,2,1,1)	

A complete set should be obtainable as some combination of the following: $OF(6;5,1;N_1) + OF(6;4,2;N_2) + OF(6;3,3;N_3) + OF(6;4,1,1;N_4) + OF(6;3,2,1;N_5) + OF(6;2,2,2;N_6) + OF(6;3,1,1,1;N_7) + OF(6;2,2,1,1;N_8) + OF(6;2,1,1,1,1;N_9) + OF(6;1,1,1,1,1,1;N_{10})$. We know, for example, that N_{10} must be one or zero since no pair of orthogonal latin squares of order six is possible.

In order to reduce the possible combinations of N_i such that $\sum_{i=1}^{10} N_i(i-1) = 25 = (6-1)^2$, some results of composition and decomposition would be desirable to eliminate certain combinations of the N_i. For example, is it possible to decompose a latin square of order six into one $F(6;3,3)$-square and two $F(6;2,2,2)$-squares? One could do the following for a latin square of order six:

LS(6)

1 - 01	2 - 00	3 - 11	4 - 10	5 - 21	6 - 20
2 - 00	1 - 01	4 - 10	3 - 11	6 - 20	5 - 21
3 - 11	4 - 10	5 - 21	6 - 20	1 - 01	2 - 00
4 - 10	3 - 11	6 - 20	5 - 21	2 - 00	1 - 01
5 - 21	6 - 20	1 - 01	2 - 00	3 - 11	4 - 10
6 - 20	5 - 21	2 - 00	1 - 01	4 - 10	3 - 11

In the above the following representation to a 2 X 3 factorial has been made:

				Symbol in F(6;3,3)
	1 = 01	3 = 11	5 = 21	1
	2 = 00	4 = 10	6 = 20	0
Symbol in F(6;2,2,2)	0	1	2	

Thus any 6 X 6 square can be decomposed, via 2 X 3 factorial representation, into an $F(6;3,3)$-square and an $F(6;2,2,2)$-square. But, can another square of the latter type be formed from the interaction contrast coefficients? This has not yet been done. It is, however, simple to form another $F(6;2,2,2)$-square orthogonal to the previous two as follows: Form all possible tetrads in the above 2 X 3 table; these are:

$$\boxed{\begin{matrix}1 & 3\\ 2 & 4\end{matrix}}\ ,\ \boxed{\begin{matrix}1 & 5\\ 2 & 6\end{matrix}}\ ,\ \text{and}\ \boxed{\begin{matrix}3 & 5\\ 4 & 6\end{matrix}}\ .$$

Considering all interaction contrasts, we form an F(6;2,2,2)-square as below. If in each pair of rows in the original latin square, we set the symbols as follows:

Rows		Symbols	
1 & 2	1 & 4 = 0	2 & 3 = 1	5 & 6 = 2
3 & 4	1 & 6 = 0	2 & 5 = 1	3 & 4 = 2
5 & 6	3 & 6 = 0	4 & 5 = 1	1 & 2 = 2

Although this procedure makes use of interaction contrasts, this is not a correct decomposition of the original latin square of order six.

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