Analytic Methods for Estimating Labor Requirements at a Parts Distribution Center

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Abstract

Picking and put-away operations must be executed quickly and efficiently in warehouses within multi-echelon supply chains. The ability to perform these picking and put-away tasks in a timely fashion is a function of the distance the workers must travel within the warehouse. Two analytic methods for estimating the labor requirements associated with these activities are presented. The first method is a stochastic model in which demand at each location is described by a random variable. The second method is based on the assumption that demand is always equal to its expected value.

1 Introduction

Multi-echelon inventory systems exist to provide excellent customer service in a timely manner without requiring each retailer or other stocking location to hold high levels of inventory. In one such environment, automotive service parts are produced, packaged, stocked, and distributed. In this instance there are four echelons in the system. At the lowest level, car dealers stock parts to repair automobiles for customers who bring their vehicles to dealer service departments. The dealers must either have the required parts in their stock, obtain

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the parts from a local supplier very quickly, or obtain parts from one of the higher echelons in the inventory system with some delay. Usually they receive their replenishment stocks for sold parts from the second echelon in the system, called Parts Distribution Centers (PDCs). These locations in turn are re-supplied by parts plants, which constitute the third echelon. Each parts plant may package or perform some operation to incoming parts in addition to stocking them. The final or fourth echelon consists of a network of component manufacturing plants. These plants may or may not be owned by the car manufacturer. Thus not all echelons are under the direct control of the car company. Our objective is to describe how the design and operational rules of one echelon in this system, the PDCs, affect the requirements for PDC labor. PDC labor is a major cost in the system, so estimating labor requirements as a function of the PDC's design and operational rules is of great importance to the car company. Specifically, our goal is to develop simple analytic methods that provide estimates of the amount of labor required at the PDC given the facility layout, inventory policies, material handling equipment, replenishment lead times, customer demand, and loading patterns of materials incoming to the PDC from parts plants. The models we will present are particularly well suited to estimate labor requirements for the PDCs operated by Toyota in the U.S. and the newest ones going into operation at General Motors.

The number of PDCs found in a given company's system reflects the number and locations of its dealers. Each PDC may vary greatly in size and configuration, and productivity may also vary among these PDCs. One PDC that we visited stocks over 70,000 part numbers and consumes half a million square feet of space; other PDCs operated by the same company are even larger. Each day, some 30,000 "lines" are ordered by car dealers served by this PDC; a line is an order by one dealer for some quantity of one specific part number. These orders must be picked quickly and prepared for shipment on trucks which must leave at set times to meet dealer delivery time requirements. In addition, receipts at a PDC from the various parts plants must be put away in a timely fashion (or in some cases, shipped immediately to waiting dealers). The sheer size and complexity of a PDC make managing one a difficult task.

To operate efficiently, each modern PDC is divided into zones. Parts which have common characteristics are stocked in the same zone. One zone might contain fragile items; another binned items; another engines; and another bulky items, for example. Zones are further divided into subzones, roughly corresponding to aisles or a set of bins. The location of zones and subzones within a PDC reflects both the physical attributes of parts, such as size and weight, as well as demand volume. For our purposes we will assume that a zone consists of a fixed collection of subzones, and a subzone contains a fixed set of part numbers that are stored in it.

There are two main activities performed at a PDC, put-away and picking. Put-away activities are those associated with the receipt, processing, and storing of material received from a parts plant or an outside supplier. Picking activities are those associated with retrieving parts from a subzone within the PDC and then preparing those parts for shipment to dealers. The picking activities and put-away activities are done in different portions of a day, so they may be analyzed independently.

The material handling equipment used within a PDC varies among the zones and among different PDCs. While the largest items are almost always transported by lift trucks, and the smallest items may be stored in an automatic bin retrieval system, the majority of the mid-sized parts are stored in bins and may be conveyed by automatic systems with conveyor belts, or by manual systems such as workers pushing hand carts. In the PDCs that we have visited, including the newest GM and Toyota sites, there is a trend towards reducing automation and relying on manual material handling systems. As binned parts comprise the majority of the volume handled at the PDC, we will focus our attention on them. Binned parts are transported by hand carts in these new facilities.

We assume that the following sequence of events occurs when picking orders for dealers. The picking process is divided into a number of waves. Each wave consists of a set of orders that are picked at approximately the same time. This material is shipped to dealers on the same outgoing truck. At the beginning of each wave, workers are given a list of parts to be picked that are loaded on the same truck. This list of parts is sequenced in a way so as to minimize walking time within the warehouse.

Each worker has a conveyance device for carrying parts, which we will generically term a cart. These carts will vary in type and size depending on the parts they carry, and thus, by the zone to which they travel. The cart leaves the dock, travels to the zone in which material is stored, and enters the first subzone in the sequence. The worker must then locate the bin in which the part to be picked is stored. Once the worker locates the bin, he picks the part. The worker then continues through the subzone, locating the next part number and picking this item. When done with this subzone, the worker moves the cart to the next subzone in the list and continues the process, until the cart is filled with parts. The worker and cart then return to the dock for reassignment, while another empty cart initiates the picking operation where the previous one left off. This process repeats until all parts are picked within a zone and the wave is completed. This sequence of steps is repeated for each subsequent wave.

The put-away process is similar. The PDC receives parts by truck or by train from various parts plants and other suppliers. Depending on the procedures and the physical constraints of the PDC, some number of trucks are unloaded simultaneously, the material is placed on the dock, and then the material is loaded onto a cart. These carts will vary in size depending on the type of parts they carry, and thus, by the zone to which they travel. Because each parts plant produces a limited range of similar parts, the parts from any given parts plant will likely be stored in only a few zones within the PDC. These parts will not always be pre-sorted by PDC subzone, however.

The total amount of labor required is affected by a number of factors including the trip work content, and the inventory reorder point and the reorder quantity rules. Each of these decisions represents a trade-off between two or more factors. To explore these trade-offs, two different approaches may be used: an analytic model or a simulation model. Darmawan et al. (1998) used a computer simulation to estimate labor requirements at a Parts Distribution Center. In contrast, this paper provides computationally tractable and easily understood analytic approaches for estimating the amount of labor required for picking and put-away of parts in a PDC. While a simulation model has the strength of being able to capture the dynamics of day-by-day variations in operational requirements and can be much more detailed, an analytic model is much simpler, quicker, and may give clearer insight as to the relationship between input factors and performance measures.

One trade-off that exists and must be evaluated is between throughput rates and the capacity of a cart. Each cart trip is assumed to contain a fixed maximum amount of work based on this capacity. This amount may be governed by the physical limitations of the cart or of the person operating the cart. Alternatively, the amount of work assigned to a cart may be a policy decision governed by workload considerations; for example, each cart trip may be constrained to contain a certain number of minutes worth of work corresponding to the amount of time available to load a truck. Increasing the capacity of each cart trip would result in fewer trips being made. However, each trip would take more time to walk through the zone and to put away or pick the parts.

The sorting of parts has an effect on the amount of distance traveled. If, for the put-away phase, parts are not sorted before they are loaded into each cart, then it is likely that a cart will have to travel through much more of a zone to complete its put-aways. Moreover, it is likely that a subsequent cart will have to visit some of the same subzones during its trip. If parts are sorted before they are loaded onto the carts, then each cart trip may be as short as possible, visiting only a few subzones located close together before returning to the dock. Subsequent carts would begin their trips where the previous cart finished. The amount of labor saved during the trips by sorting beforehand can be compared against the effort necessary to perform the sorting operation. The sorting may be done at the loading dock after unloading the trucks and before loading the carts, or it may have been done back at the parts plant before loading the trucks bound for shipment to the PDC. Thus a trade-off exists between the amount of labor required to pre-sort material and the amount of labor needed for put-away activities.

Finally, there is the trade-off of labor hours versus inventory. For example, (S, s) policies are used to manage inventories in the PDCs we studied. One factor in this tradeoff is the magnitude of $S - s$. If the value of S is reduced, there will be less inventory on the shelves in the warehouse. This reduces inventory investment. In addition, each part number would require less storage space. Thus the density of part storage would increase. That is, the number of different parts stored per lineal foot would increase. Hence the distances between bin locations for different part numbers would be reduced. If trips are shorter in distance, the walking time will be decreased, thereby decreasing the labor requirement. However, there is a trade-off with order frequency. Smaller inventories mean that each part number would be ordered more frequently. The total number of visits to a subzone would thus increase for the put-away task, with each visit to a subzone requiring a visit to more part number bin locations. The reduction in walking time for each put-away trip may not be enough to offset the increase in number of put-away trips and the increase in time needed to locate bins; however the reduction in walking time required for picking orders for shipment may be large enough to reduce the total number of labor hours that are required. Thus one goal is to understand the tradeoff among inventory investment, space investment and operating costs, and the number of labor hours required for put-away and picking activities.

The total labor hour requirement in the PDC is the sum of the labor hour requirement for the picking phase and the labor hour requirement for the put-away phase. These requirements may be estimated independently since they are performed at different times during a day: the put-away phase occurs during the day, while the picking phase occurs late at night or early morning after orders from dealers have been received at the PDC during the day. Since the picking and put-away processes are similar in many ways, models for the picking process may be modified to represent the put-away process and vice-versa.

The rest of the paper is organized as follows. We will present two analytic models to evaluate the trade-offs we have discussed. The first model treats demand on each day as a random variable and calculates the expected labor hours necessary to complete the picking requirements. We will introduce this model in the context of the picking process. The second model assumes that demands are the same each day and that they are equal to their average values. This latter model can be modified easily to evaluate the effect of time-varying demand due to either a day-of-the-week phenomenon or some seasonal requirement for picking or put-away. We will introduce this second model in the context of the put-away process.

2 Literature Review

Cormier and Gunn (1992) provide a review of models for optimizing warehouse design and operations. They divide the literature into three categories: throughput capacity models, storage capacity models, and warehouse design models. The objective of a throughput capacity model is to minimize material handling costs or to maximize throughput. Throughput models include storage assignment policies, which focus on the assignment of parts to locations within a warehouse; batching policies, which optimize the assignment of orders to picking tours; and picking policies, which optimize the sequencing and routing picking tours. Storage capacity models seek to find the optimal warehouse size with minimizing cost as the objective, or maximize the usage of space within an existing warehouse. Warehouse design models consider the design of storage devices such as aisles, and the use of retrieval technologies such as automated cranes. Papers such as ours which consider the problem of analyzing picking and put-away throughput as a function of warehouse size would fall under the category of throughput capacity models, but could be considered to be a storage capacity model as well.

A number of papers apply variations of the economic order quantity (EOQ) model. Hodgson and Lowe (1982) present a model for production lot sizing with an objective function which simultaneously considers production setup, inventory holding, and material handling costs, in contrast to other models which use a two stage approach, in which lot sizes are set first and then space for the resulting inventory is assigned. Hodgson and Lowe model material handling costs for picking only, and assume the use of a crane which returns to its origin between picking each item. Cormier and Gunn (1996) consider setup and inventory holding costs along with warehouse building costs which are proportional to the lot size. They do this for both the single item and the multiple item case. The result is a square-root formula similar to the EOQ formula. Cormier and Gunn conclude that the inventory costs are at least as important, and in some cases dominate, the warehouse building costs.

Gray et al. (1992) conduct a study of an automotive parts warehouse. Rather than modeling just one aspect of the warehouse, the authors model a wide range of design and operational decisions, from facility layout to picking procedures. Since building and solving one huge model is impractical, they decompose the problem into a hierarchy of decision levels. The three levels are facility design and equipment selection, allocation of items within the facility, and order batch sizes and worker assignments. Of particular interest to this paper is their treatment of assigning main warehouse space to items. Minimizing costs for picker travel time, costs for replenishment of parts from secondary storage areas, and costs for facility space result in a square root formula for the optimal number of units of a part that should be placed in the main picking area.

Darmawan et al. (1998) conduct a simulation study of a typical General Motors Parts Distribution Center. They examine the effects of supplier lead time, replenishment batch size, facility layout and size, and work dispatching rules on labor requirements and customer fill rates. The study also examines the dynamic behavior of the system over the course of time, from the point when a change in system operations occurs to the point when the performance of the system stabilizes.

Additional references on the design and layout of a facility may be found in Chapter 2 of Francis et al. (1992).

3 A Probabilistic Model for Picking

We begin our analysis with a model of picking operations in which we assume the demand for each part varies from day-to-day and the magnitude of the demand is given by the realization of a stochastic process. By adjusting the parameters of the demand distribution, we may also use this model to evaluate the effect of day-of-the-week or other seasonal phenomena on labor requirements.

The following data are required:

$p_k(d)$, the probability that the requirement for picks in a day in subzone k is equal to d units,

l_k, the length of subzone k, in feet,

w, the distance to the next subzone, in feet,

x_k, the x-coordinate of entry to subzone k, relative to the loading dock, in feet,

y_k, the y-coordinate of entry to subzone k, relative to the loading dock, in feet,

N, the number of subzones,

g_k, the average time to pick a unit in subzone k, in seconds,

v, the walking speed, in feet per second,

B, the capacity of a cart, in units, and

M, the shift length per worker, in seconds.

The model essentially follows the movement of carts traveling through the zone, one subzone at a time. When a cart enters the first subzone, it is empty. If there is demand for parts from this subzone, those parts will be picked, requiring some amount of time proportional to the number of items. When all lines for a subzone have been picked, the cart then proceeds to the next subzone. When the cart is full, the cart returns to the dock from the subzone. While in reality many carts are dispatched simultaneously, we assume for ease of exposition that a new empty cart starts where the old cart left off, and the process continues. Observe that this assumption does not affect the total labor requirement.

In this model, demand for each part is described by a random variable. Therefore, on any given day the number of parts that must be picked in any subzone is a random quantity. Consequently, if we count the number of parts on a cart as it goes from subzone to subzone, we will observe a random process. As mentioned, a cart begins with no parts on it. When it goes to a subzone it picks the required parts, thus leaving the subzone with some random number of parts. The number of parts on the cart will increase as the cart moves along and parts are picked, until the cart has B units of parts, where B is the capacity of the cart. At this point a new cart is brought in, which continues the picking operation in that subzone. Thus, prior to returning to the dock, the number of parts in a cart takes on values in the following set: $\{0, \ldots, B-1\}$.

The labor required to meet picking needs can be modeled using a dynamic program-like recursion. The program consists of N stages, with each stage corresponding to a subzone. Within each stage, there are B states, indexed from $\{0, \ldots, B-1\}$ corresponding to the number of parts in the cart when the cart has finished with the subzone. When a cart moves from subzone to subzone, and goes from having i parts on the cart to having j parts on the cart, it is equivalent to making a transition from state i in one stage to state j in the next stage. Each transition has an associated probability, which measures the chance that the cart's operator would be required to pick a certain number of parts, as well as an associated time, which corresponds to the number of labor hours that would be required for picking and walking. We can use the times and probabilities of each transition to calculate the total expected time to travel through the zone.

We will compute the following values:

z_k,	the distance from the dock to subzone k, in feet,
$\mathcal{P}(i, j, k, n)$,	the probability that the last cart to depart subzone k carries j parts on it, and that n carts also visited this subzone, given that the cart which entered subzone k had i parts on it.
$C(i, j, k, n)$,	the total time associated with the transition described above,
$T_{Z,k}$,	the expected total time per day during the picking process spent moving between the dock and subzone k, in seconds,
$T_{G,k}$,	the expected total time per day during the picking process spent picking units in subzone k, in seconds,
$T_{L,k}$,	the expected total time per day during the picking process spent moving within subzone k, in seconds,
$T_{W,k}$,	the expected total time per day during the picking process spent traveling between subzones, in seconds,
T_{put},	the expected total labor requirement per day, in seconds, and
R_{put},	the expected total number of workers required per day.

The distance z_k from the dock to subzone k can be calculated in a number of ways. One way is the rectilinear distance, which assumes movement is in a grid, and serves as an upper bound:

$$z_k = |x_k| + |y_k|.$$

Another way is to use the Euclidean distance, which assumes movement is by the shortest direct path between two points:

$$z_k = \sqrt{x_k^2 + y_k^2}.$$

In practice, the actual distance will be some combination of moving in a direct path and moving in a grid. We will use the Euclidean distance for simplicity; however, any appropriate method could be used to find z_k.

Let us examine the transition from state i to state j as a cart moves from subzone $k-1$ to subzone k. This represents leaving subzone $k-1$ with i units in the cart, entering subzone k, performing picks, and leaving the subzone k with j units in the cart. The probabilities and the costs of transitions from each state i to each state j can be determined by considering three cases.

Case 3.1 $i = j$

This subcase may occur in one of two ways. The first occurs when there is no picking requirement in subzone k. The probability that this occurs is equal to the probability of there being zero units demanded from this subzone. The cost incurred is the time to skip the subzone and walk to the next subzone. Another set of ways that this case may happen would be if there are exactly n full cart loads of demand, where $n = \{1, 2, \ldots\}$. Thus, the probability that this transition occurs is the probability that there are $d_k = nB$ demands in subzone k, where $n = \{0, 1, 2, \ldots\}$,

$$\mathcal{P}(i, j, k, n) = p_k(d_k) = p_k(nB) \tag{1}$$

and the time is computed by adding the time of several components. First, there is the time for n round trips from the dock,

$$T_{Z,k} = \frac{2nz_k}{v}.$$

Then there is the time to pick nB items,

$$T_{G,k} = d_k g_k = (nB)g_k.$$

To this, we add the time for n carts to move through the subzone,

$$T_{L,k} = \frac{nl_k}{v},$$

where we assume as an upper bound that each cart must traverse the length of the subzone. Other assumptions about how far each cart must travel within the subzone may be made, and $T_{L,k}$ would be modified accordingly. Finally we add the time for the last cart to walk to the next subzone,

$$T_{W,k} = \frac{w}{v}.$$

This yields a total time of

$$C(i, j, k, n) = T_{Z,k} + T_{G,k} + T_{L,k} + T_{W,k}, \tag{2}$$

where $n = \{0, 1, 2, \ldots\}$.

Case 3.2 $i < j$

As in the preceding section, two subcases must be considered. One way for a cart to go from having i parts in it at the end of subzone $k-1$ to having j parts in the cart at the end of subzone k would be for there to have been $j-i$ items which had to be picked in subzone k. It is also possible that there are some number n of full cart loads demanded in addition to the $j-i$ units. Thus the probability of this transition is the probability that are $d_k = j-i+nB$ units to be picked in subzone k, where $n = \{0, 1, 2, \ldots\}$,

$$\mathcal{P}(i, j, k, n) = p_k(d_k) = p_k(j - i + nB) \tag{3}$$

and the corresponding time to perform the operations is composed of the time for n round trips from the dock,

$$T_{Z,k} = \frac{2nz_k}{v},$$

plus the picking time of $j - i + nB$ items,

$$T_{G,k} = d_k g_k = (j - i + nB)g_k,$$

plus the time for $n + 1$ carts to move through the subzone,

$$T_{L,k} = \frac{(n+1)l_k}{v},$$

and the time for the last cart to move to the next subzone,

$$T_{W,k} = \frac{w}{v},$$

which yields a total time of

$$C(i,j,k,n) = T_{Z,k} + T_{G,k} + T_{L,k} + T_{W,k}, \tag{4}$$

where $n = \{0, 1, 2, \ldots\}$.

Case 3.3 $i > j$

For the number of parts in a cart at the end of subzone k to be less than it was at the end of subzone $k-1$ would mean that the cart which entered subzone k became full, returned to the dock, and at least one new cart was started. As before, besides this base case we must consider that there may be an additional n full cart loads required. Thus the probability of this transition is the probability that there are $d_k = (B-i)+j+nB$ units to be picked, or equivalently, the probability of $j-i+(n+1)B$ units requiring picking, where $n = \{0, 1, 2, \ldots\}$, is

$$\mathcal{P}(i,j,k,n) = p_k(d_k) = p_k\left(j-i+(n+1)B\right). \tag{5}$$

The time to complete $n+1$ round trips from the dock is

$$T_{Z,k} = \frac{2(n+1)z_k}{v},$$

the picking time of $j-i+(n+1)B$ items is

$$T_{G,k} = d_k g_k = \left(j-i+(n+1)B\right) g_k,$$

the time for $n+1$ carts to move through the subzone is

$$T_{L,k} = \frac{(n+1)l_k}{v},$$

and the time for the last cart to move to the next subzone,

$$T_{W,k} = \frac{w}{v},$$

which yields a total picking time of

$$C(i,j,k,n) = T_{Z,k} + T_{G,k} + T_{L,k} + T_{W,k}, \tag{6}$$

where $n = \{0, 1, 2, \ldots\}$.

Thus each transition from state i to state j represents not just one possible value of demand, but rather a family of possible demand values. Each instance of demand has a corresponding probability and a corresponding time, depending on the value of n. To calculate the expected time to make the transition from state i to state j in general, one must calculate the expectation over n. What we seek, however, is not just the time of one particular transition or even the time for a set of transitions, but rather, the expected time to travel through the entire network. Let us define

N,	the number of subzones in the zone,
k,	the stage index, and
$T_k(i)$,	the expected time to complete picking when beginning in stage k in state i and completing all picking tasks in stages k through N.

The expected time to travel through the network, starting at state i in stage k, until all remaining stages have been picked, can then be calculated using the following recursion:

$$T_k(i) = \sum_j \sum_n \mathcal{P}(i,j,k,n)[C(i,j,k,n) + T_{k+1}(j)], \tag{7}$$

where

$$T_{N+1}(j) = 0, \quad \forall j.$$

The total expected time to travel through the network, which is the total labor required for picking in the zone, is

$$T_{pick} = T_0(0), \tag{8}$$

and the expected number of workers needed in the zone is

$$R_{pick} = \left\lceil \frac{T_{pick}}{M} \right\rceil, \tag{9}$$

where $\lceil x \rceil$ is the smallest integer greater than or equal to x.

Let $n_{\max}$ be an upper bound on the number of full carts n which may be needed in a subzone. If $d_{\max}$ is an upper bound on the demand in a subzone, then $n_{\max} \leq \frac{d_{\max}}{B}$ The run time of this procedure described above is proportional to the number of states B, the number of stages N, and the number of arcs $B * n_{\max} \leq B\frac{d_{\max}}{B}$, for a total run time of $O(BNd_{max})$. In addition, at each of the N stages, the probability distribution for demand must be computed.

4 An Expected Value Model

A second and simpler model for computing labor requirements is based on the assumption that demand in each subzone is equal to its expected value. Due to Jensen's inequality the estimate of labor requirements obtained from this model will be a lower bound on the actual amount required.

As stated before, the models for the picking process and for the put-away process are very similar. To illustrate the similarities and differences, we will introduce this model in the context of the put-away process.

We will assume that the list of parts to be put away has been sorted by bin location to minimize travel distance. If the parts are not sorted, the model would have to be modified. We also assume that trips have been constructed so that each trip is as short as possible while making full use of the cart. Carts leave the dock full of parts, and proceed from subzone to subzone, putting parts away. When a cart is empty, it returns to the dock for reassignment and another cart full of parts continues where the previous one left off. Therefore, in describing the state of a cart, we will measure the amount of material remaining on the cart.

Another difference in the put-away process is that the expected replenishments of a part number depend not only on expected demand but also on the replenishment batch size. A part with a larger batch size will require replenishment less frequently than a part with a smaller batch size, assuming equal demand rates.

In addition to the differences in the models associated with the put-away versus picking process, this model is also different in that we will measure cart capacity in terms of cubic volume. The previous (probabilistic) model measured cart capacity in terms of number of parts. Space is a more accurate measure of capacity than units, since parts in different subzones may be of different sizes. However, in the probabilistic model, it would have been much more difficult to work with volume measures in the state space and transition probability calculations.

The following data are required to estimate the time and labor hour requirements for each zone:

l_k,	the length of subzone k (aisle), in feet,
w,	the average distance between subzones, in feet,
x_k,	the x-coordinate of entry to subzone k, relative to the loading dock, in feet,
y_k,	the y-coordinate of entry to subzone k, relative to the loading dock, in feet,
g_k,	the average time to put away a unit in subzone k, in seconds,
K,	the fixed time for locating each part number, in seconds,
v,	the walking speed, in feet per second,
B,	the capacity of a cart, in cubic feet,
f_k,	the average volume per part in subzone k, in cubic feet,
d_k,	the average daily demand in units, totaled across all part numbers in subzone k,
h_k,	the number of different part numbers in subzone k,
q_k,	the average lot size in subzone k, measured in days of supply, and
M,	the shift length per worker, in seconds.

From these data, we will compute the following values:

c_k, the total number of cart loads worth of expected daily demand in subzone k,

s_k, the total number of carts which must be started in subzone k,

z_k, the distance from the dock to subzone k, in feet,

p_k, the expected number of different part numbers put away per day in subzone k,

$T_{Z,k}$, the total average time spent per day moving between the dock and subzone k to complete put-away operations, in seconds,

$T_{K,k}$, the total average time spent per day on locating part numbers in subzone k to complete put-away operations, in seconds,

$T_{G,k}$, the total average time spent per day on putting away units in subzone k, in seconds,

$T_{L,k}$, the total average time spent per day moving within subzone k to complete put-away operations, in seconds,

$T_{W,k}$, the total average time spent per day traveling between subzones to complete put-away operations, in seconds,

T_{put}, the total average put-away labor requirement per day in the zone, in seconds, and

R_{put}, the total number of workers required per average day in the zone to complete put-away operations.

The number of trips may be calculated in one of two ways. The average daily demand, measured in units, in subzone k is d_k. Consequently, the average amount which must be put away per day in subzone k, or the requirement for put-aways, is d_k. The space volume required is $d_k f_k$. Thus the number of carts needed to meet put-away requirements in subzone k may be determined by dividing the space volume required for the subzone by the capacity of the cart. This value is:

$$c_k = \frac{d_k f_k}{B},$$

which is not likely to be integer valued.

An alternate way to measure trips is to determine how many carts are *started* in each subzone. We term this number s_k. To calculate s_k requires keeping track of the remaining capacity in each cart as it travels sequentially through the subzones. Unlike c_k, s_k would be an integer. We now show how s_k may be calculated.

Let m_k be the amount of material, measured in cubic feet, remaining in the cart just before visiting subzone k, or the amount of material remaining in the cart just after visiting subzone $k-1$. Let us follow what happens to a cart which enters subzone k. The average amount which must be put away per day in the subzone, in cubic feet, is $d_k f_k$. The entering cart carries as much put-away material as it can, up to m_k. The amount remaining to be put-away by other carts is then $d_k f_k - m_k$. If this number is negative it may be interpreted as the amount of material remaining on this cart destined for other subzones. The cart proceeds to the next subzone; no new carts must be started. If, however, the amount remaining to be put away is positive, then new carts must be sent to this subzone. The number of new carts s_k which must be started for subzone k would thus be:

$$s_k = \begin{cases} 0 & \text{if } d_k f_k - m_k < 0 \\ \left\lceil \frac{d_k f_k - m_k}{B} \right\rceil & \text{if } d_k f_k - m_k \geq 0 \end{cases}$$

and the material remaining in the last cart leaving subzone k, before entering subzone $k+1$, would be:

$$m_{k+1} = s_k B - (d_k f_k - m_k)$$

Note that if s_k carts start in subzone k, then a total of $s_k + 1$ carts visit subzone k. The following example shows that the number of carts started s_k is not found by simply rounding the number of cart loads worth of material c_k to the next highest integer.

Example 4.1 *If the amount to be put away in subzone k was 0.2 cart loads, and the remaining material in the cart after subzone $k-1$ was 0.1 cart loads, then both $\lceil c_k \rceil = 1$ and $s_k = 1$. But if the amount to be put away in subzone k was 0.2 cart loads, and the remaining material in the cart after subzone $k-1$ was 0.9 cart loads, then $\lceil c_k \rceil = 1$ while $s_k = 0$.*

In the appendix we show that in general, either $s_k = \lceil c_k \rceil$ or $s_k = \lceil c_k \rceil - 1$ depending on the relative values of the amount of material to be put away in a subzone and the amount of material remaining on the cart.

As before, we will calculate the distance from the dock to the subzone by using the Euclidean distance,

$$z_k = \sqrt{x_k^2 + y_k^2}.$$

There are h_k part numbers in subzone k. Since q_k is the average lot size, measured in days of supply, for each part number in the subzone, the time between receipts of a part number is $\frac{1}{q_k}$. That is, we expect a replenishment for a part every q_k days. Hence the average number of part numbers replenished per day is

$$p_k = \frac{h_k}{q_k}.$$

The total time required for a subzone per day can be thought of as the sum of the following components. First, we have the dock travel time for carts which finish or start their journeys at this subzone. This includes the time for a finishing cart to return to the dock, plus the round trip journeys of $s_k - 1$ carts which start and finish their journeys at this subzone, plus the time for the last cart to come from the dock to start its journey. In total, this is:

$$T_{Z,k} = \frac{2s_k z_k}{v}.$$

Second, there is the total put-away time in a subzone. Put-away time has two components. There is a fixed component associated with locating and putting away each new part number in subzone k:

$$T_{K,k} = p_k K = \frac{h_k K}{q_k}.$$

The other component is associated with the total number of units put away in subzone k:

$$T_{G,k} = d_k g_k.$$

Third, the intra-subzone travel time within subzone k is approximately

$$T_{L,k} = \frac{(s_k + 1) l_k}{v}.$$

Fourth, by assumption in this model, each day there is only one trip in which a cart moves from subzone k to subzone $k+1$. Hence the total time spent in a day, traveling from subzone k to subzone $k+1$ is:

$$T_{W,k} = \frac{w}{v}.$$

The total average labor requirement per day, across all subzones in the zone, is then estimated to be

$$T_{put} = \sum_k (T_{Z,k} + T_{K,k} + T_{G,k} + T_{L,k} + T_{W,k}), \tag{10}$$

and correspondingly the estimated number of workers required per day for put-away in the zone is

$$R_{put} = \left\lceil \frac{T_{put}}{M} \right\rceil. \tag{11}$$

5 Conclusion

Analytic models may be used to predict labor requirements in response to changes in facility design or operating policies. In this paper we have presented two models which differ in level of detail and in complexity. The first model treats picking requirements as a random variable, while the second model uses the average requirements. We have presented the probabilistic model in the context of the picking operation, and the expected value in the context of the put-away operation.

The expected value model presented for the put-away phase may be easily modified to model a typical wave of the picking phase. The equations are essentially identical to the notation used in the put-away version, with a few exceptions. Definitions which referred to put-away rates would now refer to picking rates. One further change would be in the calculation of the expected number of part numbers that would be picked in each wave. In the put-away phase this number, h_k, was a function of the lot size. In the picking phase, however the number of part numbers we expect to pick in each wave does not depend on the lot size. Instead, we must estimate it directly from prior demand history.

Likewise, using the probabilistic model for the put-away phase requires a few changes. First, the transitions of the state space must be modified. Whereas in the picking phase carts go from subzone to subzone, accumulating more and more parts until the cart is full, in the put-away phase, carts carry material that will be put away as they travel the zone, resulting in fewer and fewer parts on the cart until the cart is empty.

Second, definitions referring to quantities or rates of picking are replaced by quantities or rates of put-aways. The modeling of put-away requirements using a random variable presents some difficulties, however. While the number of units to be picked in each subzone was modeled with a probability distribution describing the demand process, the number of units to be put-away in each subzone depends not only on the random demand but also on the replenishment lot-sizing policies. If each subzone followed a one-for-one replenishment rule, the replenishment requirements would follow the same distribution as the demand process. Otherwise, a probability distribution for replenishments would have to be constructed.

It should be noted that the difficulty in constructing the probability distribution is the reason that the probabilistic model does not explicitly model the number of part numbers picked or put away per day, nor does it model the space consumed by each part number in the cart. It is possible to do so, but it would require the convolution of the demand distributions for each part number. In addition, modeling the state of the system in terms of volume of the cart would result in a much larger state space.

The analytic models presented in this paper provide intuition into the relationship between demand characteristics, operational characteristics such as inventory policy and wave construction, and warehouse characteristics such as layout and material handling equipment. The strength of these analytic models is that they are fast and easy to use. The results given by the models provide directions for further exploration of decisions using more complicated and more detailed methods including simulation and pilot projects.

Appendix A: Relationship between c_k and s_k

Recall that the average daily demand, measured in units, in subzone k is d_k. If f_k is the average volume of a unit in subzone k, and B is the capacity of the cart, measured in volume, then the minimum number of carts required to meet put-away requirements in subzone k is

$$c_k = \frac{d_k f_k}{B}.$$

The value c_k is a lower bound on the number of carts which must visit the subzone. The number of carts which visit subzone k is equal to one plus the number of carts s_k which start their trips at subzone k, thus

$$c_k \leq 1 + s_k,$$

where s_k, the number of carts which start in subzone k, is calculated by

$$s_k = \begin{cases} 0 & \text{if } d_k f_k - m_k < 0 \\ \left\lceil \frac{d_k f_k - m_k}{B} \right\rceil & \text{if } d_k f_k - m_k \geq 0. \end{cases}$$

Define $I = \lfloor \frac{d_k f_k}{B} \rfloor$, the integral portion of c_k. Let $\alpha = \frac{d_k f_k}{B} - I \geq 0$ be the fractional portion of c_k; note that $\alpha < 1$. Also let $\hat{m} = \frac{m_k}{B} \leq 1$. The relative values of α and $\hat{m}$ determine the value s_k as a function of c_k.

Case 1 Suppose $\alpha > \hat{m} \geq 0$. Recall that $\frac{d_k f_k}{B} - \frac{m_k}{B} = I + \alpha - \hat{m} > I$. Furthermore, $\alpha - \hat{m} < 1$ and therefore $I < I + \alpha - \hat{m} < I + 1$. Since $\lceil c_k \rceil = \lceil I + \alpha \rceil = I + 1$ and $s_k = \lceil I + \alpha - \hat{m} \rceil = I + 1$, $\lceil c_k \rceil = s_k$ in this case.

Case 2 Suppose $\alpha = \hat{m} > 0$. Then $\lceil c_k \rceil = \lceil I + \alpha \rceil = I + 1$ and $s_k = \lceil I \rceil = I$ and $s_k + 1 = \lceil c_k \rceil$.

Case 3 Suppose $\alpha = \hat{m} = 0$. Then $\lceil c_k \rceil = \lceil I \rceil = I$ and $s_k = \lceil I \rceil = I$ and $s_k = \lceil c_k \rceil$.

Case 4 Suppose $0 < \alpha < \hat{m}$. Then $I - 1 < I + \alpha - \hat{m} < I$. Then $\lceil c_k \rceil = I + 1$ and $s_k = I$.

Case 5 Suppose $\alpha = 0$, $1 > \hat{m} > 0$. Then $\lceil c_k \rceil$ and $s_k = I$.

Case 6 Suppose $\alpha = 0$, $\hat{m} = 1$. Then $\lceil c_k \rceil$ and $s_k = I - 1$.

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