SOME CONFOUNDED ARRANGEMENTS

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For a time and motion study* in a dairy barn a $2^4 x 3^2$ factorial arrangement of the treatments was selected. Two weights of a milk pail (a), two types of pail (b), two methods of carrying the pail (c), two heights of pouring (d), three distances of carrying the pail (e), and three types of paths (straight path, a path with a right angle turn, and a path with stairs) (f) represented the six factors making up the 144 treatments for the $2^4 x 3^2$ factorial arrangement. Since the number of treatments is large it was not possible to carry out all 144 treatments within a short time period. Also, six subjects were available to carry out the experiment. The character measured was the oxygen consumption of the subject performing the task.

Various schemes of confounding for this experiment are available. A relatively simple scheme of confounding would involve three pairs of subjects with each pair performing 48 treatments. If one of the main effects E or F is completely confounded with pairs of subjects three sets of 48 treatments would be performed by each pair. Then, the ABCD effect could be confounded with subjects within a pair. The 24 treatments performed by each subject are given in figure 1. The analysis of variance would yield full information on all main effects, all two-factor interactions and all three-factor interactions among the 5 factors a, b, c, d, and e. The residual sum of squares would contain 10 degrees of

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freedom. However, it is doubtful that the three-factor interactions have real meaning in an experiment of this kind, and they may be pooled with the residual sum of squares resulting in 26 degrees of freedom for the error mean square.

A second scheme of confounding is presented in figure 2. Four subjects are used, and each subject performs 36 treatments. The arrangement given confounds the AD, the ABC, and the BCD interactions. Other interactions may be completely confounded with the differences among subjects if the above ones are considered important. In the analysis of variance for the confounded arrangement given in figure 2 full information is available on all main effects, on all two-factor interactions except AD, and on all three-factor interactions except ABC and BCD. The residual sum of squares is associated with 65 degrees of freedom.

The third arrangement described (figure 3) involves partial confounding of the EF component of the E x F interaction and complete confounding of ABCD. The scheme also partially confounds some three- and higher- factor interactions but since three-factor interactions are not considered important this is of no consequence in the present design. In the analysis of variance 104 degrees of freedom are associated with the residual sum of squares. If meaningful the three-factor interactions may be partitioned out of the residual sum of squares.

Since the experiment could be performed in an hour's time with all six subjects and since it was desired to utilize the services of the available subjects the experimental plan presented in figure 3 appeared to be the desired one. However, other schemes of confounding are possible and the one sacrificing the least amount of information will depend upon the nature of the experimental material and the factors involved.

REFERENCE

Yates, F. The design and analysis of factorial experiments. Imperial Bureau of Soil Science, Tech. Comm. 35, 1937.

Figure 1. Experimental plan and analysis of variance for a $2^4 \times 3$ factorial in incomplete blocks of 24.

abcde	abcde
00000	00010
00001	00011
00002	00012
00110	00100
00111	00101
00112	00102
01010	01000
01011	01001
01012	01002
01100	01110
01101	01111
01102	01112
10010	10000
10011	10001
10012	10002
10100	10110
10101	10111
10102	10112
11000	11010
11001	11011
11002	11012
11110	11100
11111	11101
11112	11102

Level of effects

$(ABCD)_0$	$(ABCD)_1$
subject 1	subject 2

Source of variation	df	
Blocks (ABCD)	1	
Main effects	6	
A		1
B		1
C		1
D		1
E		2
Two-factor interactions	14	
AB		1
AC		1
AD		1
AE		2
BC		1
BD		1
BE		2
CD		1
CE		2
DE		2
Three-factor interactions	16	
ABC		1
ABD		1
ABE		2
ACD		1
ACE		2
ADE		2
BCD		1
BCE		2
BDE		2
CDE		2
Residual	10	
Total	47	

Figure 2. Experimental plan and analysis of variance for a $2^4 x 3^2$ factorial in incomplete blocks of 36.

abcdef	abcdef	abcdef	abcdef
000000	001000	000100	001100
000001	001001	000101	001101
000002	001002	000102	001102
000010	001010	000110	001110
000011	001011	000111	001111
000012	001012	000112	001112
000020	001120	000120	001120
000021	001021	000121	001121
000022	001022	000122	001122
011000	010000	011100	010100
011001	010001	011101	010101
011002	010002	011102	010102
011010	010010	011110	010110
011011	010011	011111	010111
011012	010012	011112	010112
011020	010020	011120	010120
011021	010021	011121	010121
011022	010022	011122	010122
101100	100100	101000	100000
101101	100101	101001	100001
101102	100102	101002	100002
101110	100110	101010	100010
101111	100111	101011	100011
101112	100112	101012	100012
101120	100120	101020	100020
101121	100121	101021	100021
101122	100122	101022	100022
110100	111100	110000	111000
110101	111101	110001	111001
110102	111102	110002	111002
110110	111110	110010	111010
110111	111111	110011	111011
110112	111112	110012	111012
110120	111120	110020	111020
110121	111121	110021	111021
110122	111122	110022	111022

Level of effects

$(ABC)_0$	$(ABC)_1$	$(ABC)_0$	$(ABC)_1$
$(AD)_0$	$(AD)_0$	$(AD)_1$	$(AD)_1$
$(BCD)_0$	$(BCD)_1$	$(BCD)_1$	$(BCD)_0$
subject 1	subject 2	subject 3	subject 4

Source of variation	df	
Blocks (AD, ABC, BCD)	3	
Main effects	8	
A		1
B		1
C		1
D		1
E		2
F		2
Two-factor interactions	25	
AB		1
AC		1
AE		2
AF		2
BC		1
BD		1
BE		2
BF		2
CD		1
CE		2
CF		2
DE		2
DF		2
EF		4
Three-factor interactions	42	
ABD		1
ABE		2
ABF		2
ACD		1
ACE		2
ACF		2
ADE		2
ADF		2
AEF		4
BCE		2
BCF		2
BDE		2
BDF		2
BEF		4
CDE		2
CDF		2
CEF		4
DEF		4
Residual	65	
Total	143	

Figure 3. Experimental plan and analysis of variance for a $2^4 x 3^2$ factorial in incomplete blocks of 24.

no.	abcdef	abcdef	abcdef	abcdef	abcdef	abcdef
1	000000	000010	000020	000100	000110	000120
2	000001	000011	000021	000101	000111	000121
3	000002	000012	000022	000102	000112	000122
4	001110	001120	001100	001010	001020	001000
5	001111	001121	001101	001011	001021	001001
6	001112	001122	001102	001012	001022	001002
7	010120	010100	010110	010020	010000	010010
8	010121	010101	010111	010021	010001	010011
9	010122	010102	010112	010022	010002	010012
10	011000	011001	011002	011100	011101	011102
11	011012	011010	011011	011112	011110	011111
12	011021	011022	011020	011121	011122	011120
13	100100	100110	100120	100000	100010	100020
14	100101	100111	100121	100001	100011	100021
15	100102	100112	100122	100002	100012	100022
16	101010	101020	101000	101110	101120	101100
17	101011	101021	101001	101111	101121	101101
18	101012	101022	101002	101112	101122	101102
19	110020	110000	110010	110120	110100	110110
20	110021	110001	110011	110121	110101	110111
21	110022	110002	110012	110122	110102	110112
22	111101	111102	111100	111001	111002	111000
23	111110	111111	111112	111010	111011	111012
24	111122	111120	111121	111022	111020	111021

Level of effects

$(ABCD)_0$	$(ABCD)_0$	$(ABCD)_0$	$(ABCD)_1$	$(ABCD)_1$	$(ABCD)_1$
$(EF)_0$ on 10-12	$(EF)_1$ on 10-12	$(EF)_2$ on 10-12	$(EF)_0$ on 10-12	$(EF)_1$ on 10-12	$(EF)_2$ on 10-12
$(EF)_1$ on 22-24	$(EF)_2$ on 22-24	$(EF)_0$ on 22-24	$(EF)_1$ on 22-24	$(EF)_2$ on 22-24	$(EF)_0$ on 22-24
subject 1	subject 2	subject 3	subject 4	subject 5	subject 6

Figure 3 continued.

Analysis of variance

Source of variation	df	
Blocks	5	
Main effect	8	
A		1
B		1
C		1
D		1
E		2
F		2
Two-factor interaction	26	
AB		1
AC		1
AD		1
AE		2
AF		2
BC		1
BD		1
BE		2
BF		2
CD		1
CE		2
CF		2
DE		2
DF		2
(EF^2)		2
$(EF)'$ (part.conf.)		2
Residual	104	
Total	143	