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A PLANNING MODEL FOR BIOTERRORISM RESPONSE LOGISTICS

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Introduction
The purpose of these notes is to provide a mathematical model that can be used to assess the logistics of the response to a bioterrorism event. The model considers the impact of such an event on a network of medical facilities. The network consists of a set of locations to which the affected population would visit, which we call Emergency Preparedness Centers (EPC's), and a set of hospitals which contain Emergency Departments (ED's), the Floor (patients in beds), and an Intensive Care Unit (ICU).

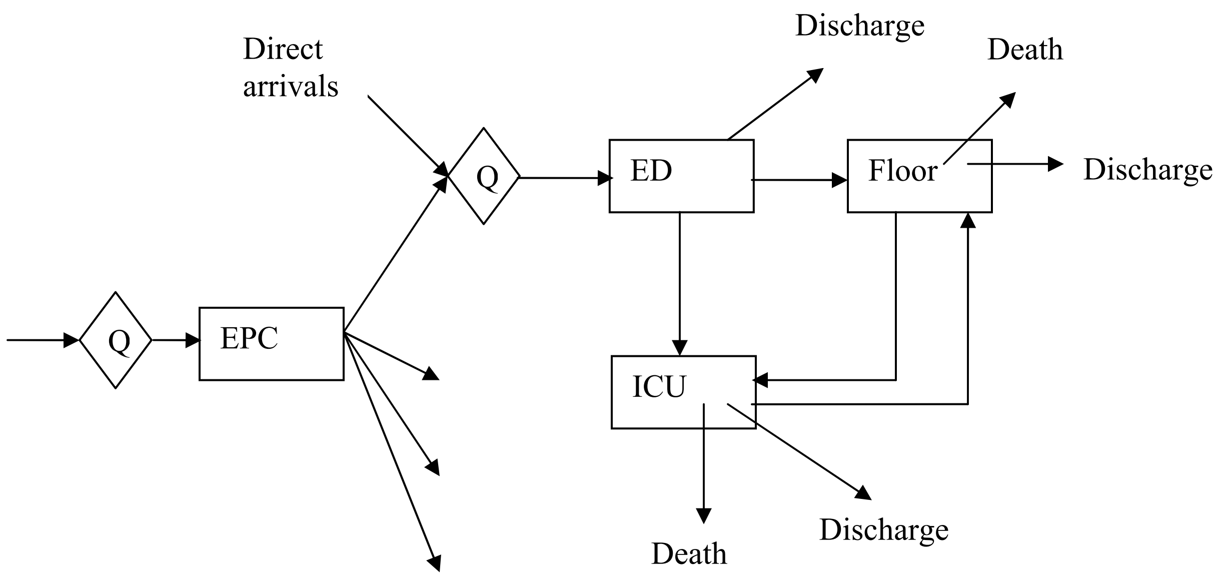
We assume that demand for treatment is a time varying process. We assume that time is divided into periods and that the number of arrivals for treatment provided at each EPC is known for each time period of a finite planning horizon. The model can be used to comprehend the responsiveness of this network given different scenarios for the arrival process as well as for different capacities and other factors that we shall discuss shortly.

Each patient arriving at an EPC has a probability that indicates what type of treatment will be required. The treatment type specifies several factors: resource requirements, duration of care required in each stage of the logistics system, and where the type of care will occur over time. By modeling the system in this manner, we may represent a broad range of care alternatives that could be required. Some patients will have their needs satisfied completely at an EPC. Others may require hospitalization; but, depending on the severity of their medical needs, the length of stay will vary. Furthermore, some patients will require intensive care, again, the duration in an ICU may vary. All such variations are captured in the treatment model that we will employ.

Since we will assume that there are capacities throughout the system, which are time dependent, one key goal of the model is to indicate how demand for treatment should be allocated to hospitals. We will assign costs to waiting for services by patients by treatment type. We will consider delays in moving patients from an EPC to a hospital as well as the cost of doing so. Thus, the model will be used to determine the most cost effective way to meet the requirements of an emergency, and to measure the effect of resources on the cost and responsiveness of the system. Thus the model implicitly can determine the best mix of resources required for each scenario.

System layout
The figure below shows the system layout. Patients arrive at the EPC and enter the EPC queue. After being processed at the EPC, they are transported to one of the available hospitals where they enter the ED queue. From the ED queue, they enter the ED where they spend a predetermined amount of time (dependent on the treatment type). After the end of this period of stay, they are either discharged, or they enter the Floor, or the ICU. They can be on the floor after ED or after ICU. If they are on the floor after ED, they may require ICU treatment at any time before discharge. If they are on the floor after ICU, they will not re-enter ICU. Patients can die in any period while in ICU or on the floor. They are discharged only at the end of their scheduled stay in any of these units. Arrival rates, lengths of stay, transition probabilities, capacities of different units, death rates,

waiting cost, etc. are all deterministic and known. The numbers of patients transferred to each hospital at each point in time are unknown and are the independent variables. Also, the numbers of patients of each treatment type transferred from the ED queue to the ED are unknown and are independent variables. The total cost which includes waiting cost and transportation cost is to be minimized.



Nomenclature
All times are measured in periods.

J = number of EPC's in the system (a subscript j will always refer to the j^{th} EPC)
T = number of periods in the planning horizon (t is used to indicate the period of interest)
$a_j(t)$ = number of patient arrivals at EPC j in period t

$a_k'(t)$ = number of patients arriving directly at hospital k at time t
m = subscript indicating the treatment type required by an arriving patient
k = subscript indicating a hospital
$r_{jm}^{EPC}(t)$ = probability that a patient arriving at EPC j in time period t requires treatment type m.
$r_{km}^{H}(t)$ = probability that a patient directly arriving at hospital k in time period t requires treatment type m.
For each k and m,
L_{km}^{ED} = length of service time in ED
L_{km}^{F} = length of stay on the floor (after ED)
L_{km}^{FI} = length of stay on the floor following ICU care

$L^{I}_{km} =$ length of stay in the ICU

r^{FI}_{km} = probability that a patient on the floor (following ED) requires ICU care in the next period.

$r^{ED,F}_{km}$ = probability that a patient in ED will be transferred to the floor at the end of his ED stay

$r^{ED,I}_{km}$ = probability that a patient in ED will be transferred to the ICU at the end of his ED stay in the next period

$\bar{r}^{F}_{km}$ = probability that a patient on the floor after stay in ED will die in the next period

$\bar{r}^{FI}_{km}$ = probability that a patient on the floor after stay in ICU will die in the next period

$\bar{r}^{I}_{km}$ = probability that a patient in the ICU will die in the next period

$X_{jkm}(t)$ = number of patients with treatment type m sent from EPC j to hospital k in period t

$y_j(t)$ = number of patients processed at EPC j during period t

$N_j(t)$ = number of patients awaiting treatment at EPC j at the end of period t

The various system capacities are given by:

$C_j(t)$ = number of patients that can be processed in period t at EPC j

$\bar{C}^{ED}_{k}(t)$ = capacity of the ED queue of hospital k at time t

$C^{ED}_{k}(t)$ = ED capacity at hospital k at time t

$C^{F}_{k}(t)$ = capacity of floor of hospital k at time t

$C^{I}_{k}(t)$ = capacity of ICU of hospital k at time t

Depending on m, the patient may move directly to the ICU upon entering the ED at an hospital, or be discharged after stay in the ED. The treatment type concept permits this to occur. Thus, some of the stay lengths may be 0 for some values of m.

The number of patients waiting at an EPC can be obtained from

$$N_j(t) = N_j(t-1) + a_j(t) - y_j(t)$$

We assume that the time periods are short in length so that no arriving patient in period t can be processed in that period. If periods are lengthy, then this assumption can be easily modified. Also, once this system is discretized into periods, all events take place at discrete time points 1, 2, 3, …, T. The variables which are indexed by time will refer to the values of different quantities *after* arrivals, transfers, deaths, discharge, etc. have taken place.

The number of patients processed at an EPC is related to the EPC capacity and the queue length by

$$y_j(t) = \min(C_j(t), N_j(t-1))$$

This is consistent with the assumption that patients that have arrived at time t cannot be processed at time t.

We desire to be able to make treatment requirements vary with time at which patients arrive at an EPC. Hence we introduce another set of variables
$Z_j(t,\tau)$ = number of patients processed at EPC j in period t that arrived in period τ, $\tau < t$, by assumption.
Then

$$y_j(t) = \sum_{\tau<t} Z_j(t,\tau)$$

Clearly,

$$\sum_{p=\tau+1}^{T} Z_j(p,\tau) \le a_j(\tau) \qquad (1)$$

This constraint indicates that not all arriving patients will be processed if the system does not have enough capacity. Note that $a_j(\tau) - \sum_{p=\tau+1}^{T} Z_j(p,\tau)$ measures the number of patients that arrived at EPC j in period τ that have not been treated by the end of period t, $t > \tau$.

Next, we relate the number of patients processed at EPC j to the number of patients allocated to each hospital k by the constraint

$$\sum_{t'=1}^{t} \sum_{k} X_{jkm}(t') \le \sum_{t'=1}^{t} \sum_{\tau=1}^{t'} Z_j(t',\tau) r_{jm}^{EPC}(\tau) \qquad (2)$$

This constraint ensures that for each treatment type, the number of patients sent to various hospitals does not exceed the number of patients processed.

Let us now turn our attention to hospital k. We have assumed that some patients will come directly to the hospital. That is, they do not go to an EPC. We assume that it takes L_{jk} periods for a patient to arrive at hospital k if dispatched from EPC j. Then the arrival rate of patients by treatment type at hospital k is

$$\overline{a}_{km}(t) = a'_k(t) r_{km}^{H}(t) + \sum_{j} X_{jkm}(t - L_{jk})$$

Suppose
$N_{km}^{ED}(t)$ = number of patients awaiting treatment of type m in the ED queue at hospital k at the end of period t, and
$E_{km}^{ED}(t,\tau)$ = number of patients entering hospital k's ED at time τ, who require treatment type m, but begin treatment at time t.

Then
$\sum_{m} \sum_{\tau \le t} E_{km}^{ED}(t,\tau)$ = number of patients entering the ED in hospital k at time t
and

$\sum_{m}\sum_{L=0}^{L_{km}^{ED}-1}\sum_{\tau\le t}E_{km}^{ED}(t-L,\tau)$ = number of patients being treated in the ED of hospital k at time t.

Clearly,

$$\sum_{m}\sum_{L=0}^{L_{km}^{ED}-1}\sum_{\tau\le t}E_{km}^{ED}(t-L,\tau)\le C_{k}^{ED}(t) \quad (3)$$

Note, unlike the EPC, patients can immediately enter the ED from the ED queue. Recall that $\bar{a}_{km}(t)$ is the input quantity of patients that require treatment type m at hospital k's ED queue in period t.

Therefore,

$$\sum_{p=\tau}^{t}E_{km}^{ED}(p,\tau)\le \bar{a}_{km}(\tau) \quad (4)$$

and

$$N_{km}^{ED}(t)=N_{km}^{ED}(t-1)+\bar{a}_{km}(t)-\sum_{\tau\le t}E_{km}^{ED}(t,\tau)\ge 0 \quad (5)$$

If $\bar{C}_{k}^{ED}(t)$ is the capacity of the ED queue

$$0\le \sum_{m}N_{km}^{ED}(t)\le \bar{C}_{k}^{ED}(t) \quad (6)$$

The number of patients transferred from the ED to the floor is given by

$$E_{km}^{ED,F}(t)=\sum_{\tau\le t-L_{km}^{ED}}r_{km}^{ED,F}E_{km}^{ED}(t-L_{km}^{ED},\tau)$$

Similarly, the number of patients transferred from the ED to the ICU is given by

$$E_{km}^{ED,I}(t)=\sum_{\tau\le t-L_{km}^{ED}}r_{km}^{ED,I}E_{km}^{ED}(t-L_{km}^{ED},\tau)$$

The number of patients transferred from the floor to the ICU is given by

$$E_{km}^{F,I}(t)=\sum_{\tau=\max(1,t-L_{km}^{F})}^{t-1}E_{km}^{ED,F}(\tau)(1-\bar{r}_{km}^{F}-r_{km}^{FI})^{(t-\tau-1)}r_{km}^{FI}$$

The number of patients transferred from the ICU to the floor is given by

$$E_{km}^{I,F}(t)=\left[E_{km}^{ED,I}(t-L_{km}^{I})+E_{km}^{F,I}(t-L_{km}^{I})\right](1-\bar{r}_{km}^{I})^{L_{km}^{I}}r_{km}^{I,F}$$

We now determine the number of patients on the floor and in ICU. Let $N_{km}^{F}(t)$ denote the number of patients on the floor of hospital k at time t requiring treatment type m. To determine this quantity, we need to consider those patients who were transferred from the ED within the last L_{km}^{F} periods and those patients that were transferred from the ICU in the last L_{km}^{FI} periods. Any patients transferred to the floor outside this range would have left the floor on or before time t. However, we must exclude those patients that either died or were transferred to the ICU during this time.

$$N_{km}^{F}(t)=\sum_{\tau=\max(1,t-L_{km}^{F}+1)}^{t}E_{km}^{ED,F}(\tau)(1-\bar{r}_{km}^{F}-r_{km}^{FI})^{(t-\tau)}+\sum_{\tau=\max(1,t-L_{km}^{FI}+1)}^{t}E_{km}^{I,F}(\tau)(1-\bar{r}_{km}^{FI})^{(t-\tau)}$$

The number of patients on the floor of hospital k should be less than the floor capacity of that hospital, which gives us the inequality

$$\sum_{m} N_{km}^{F}(t) \leq C_{k}^{F}(t) \tag{7}$$

We now turn our attention to the ICU. Let $N_{km}^{I}(t)$ denote the number of patients in the ICU of hospital k at time t requiring treatment type m. To determine this quantity, we need to consider those patients who were transferred from the ED to the ICU or from the floor to the ICU within the last L_{km}^{I} periods. Any patients transferred to the ICU outside this range would have left the ICU on or before time t. However, we must exclude those patients that died during this length of time.

$$N_{km}^{I}(t) = \sum_{\tau=\max(1,t-L_{km}^{I}+1)}^{t} \left(E_{km}^{ED,I}(\tau) + E_{km}^{F,I}(\tau)\right)(1-\overline{r}_{km}^{I})^{(t-\tau)}$$

The number of patients being treated in the ICU of hospital k should be less than the ICU capacity of that hospital, which gives us the inequality

$$\sum_{m} N_{km}^{I}(t) \leq C_{k}^{I}(t) \tag{8}$$

An objective function

There are many possible objective functions that could be constructed. We present one that focuses on three elements. First, the number of patients awaiting treatment at an EPC is an important measure of the responsiveness of the system. The number of patients requiring treatment type m that are waiting at EPC j at time t is given by

$$\sum_{\tau \leq t} \left(a_{j}(\tau) r_{jm}^{EPC}(\tau)\right) - \sum_{\tau \leq t} \sum_{k} X_{jkm}(\tau)$$

Let h_{jm}^{EPC} be the imputed "cost" of each such patient waiting for one period. Then the total cost of waiting at an EPC is given by

$$\sum_{j} \sum_{t} \sum_{m} h_{jm}^{EPC} \left(\sum_{\tau \leq t} \left(a_{j}(\tau) r_{jm}^{EPC}(\tau)\right) - \sum_{\tau \leq t} \sum_{k} X_{jkm}(\tau) \right) \tag{9}$$

Second, we measure the cost of transporting or assigning patients to a hospital given that they were initially processed at an EPC. Let $h_{jk}^{EPC,H}$ measure the "cost" associated with this assignment, that is, the cost per patient assigned to hospital k given the patient was treated at EPC j. Recall that $X_{jkm}(t)$ measures the number of patients with treatment type m processed initially at EPC j and then assigned to hospital k in period t. Hence the total cost of making these assignments over the planning horizon is

$$\sum_{j} \sum_{k} h_{jk}^{EPC,H} \sum_{t=1}^{T} \sum_{m} X_{jk}(t) \tag{10}$$

Third, we measure the cost associated with patients that are waiting for treatment in the ED queue. Let $f_{km}^{ED}(t)$ be the cost of a patient waiting for ED care who requires treatment type m at hospital k in period t. Then the total cost for patients awaiting ED care over the planning horizon is

$$\sum_{k} \sum_{m} \sum_{t} f_{km}^{ED}(t) N_{km}^{ED}(t) \tag{11}$$

Note, at $t = T$, it is debatable whether to include or exclude the waiting and transportation cost for those patients in the queue and those patients that are dispatched.

We prefer to include those costs and keep the expressions and programming simple. If T is large enough so that there are no patients left over in the system, this issue is irrelevant.

Optimization problem

The resulting optimization problem can be written as

minimize (10) + (11) + (12)

subject to (2)-(5), (7)-(9)

and all variables are non-negative. This is a linear programming problem. There are no equality constraints. The intermediate equations are only used to determine values of dependent variables.

Note, that if the number of direct arrivals at a hospital exceeds the ED queue capacity, the problem, as posed, has no solution since there is no feasible point.