VIBRATORY FRUIT HARVESTING: AN EXPERIMENTAL INVESTIGATION OF AN APPLE FRUIT-STEM RESPONSE TO FORCED OSCILLATIONS

A Thesis

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by

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BIOGRAPHY

Peter Parchomchuk was born and raised in Vernon, British Columbia, Canada.

From September 1962 to June 1967 he attended the University of British Columbia where he received a Bachelor of Applied Science Degree in Agricultural Engineering.

After graduation he became employed by the Canada Department of Agriculture at the Summerland, B.C. experiment station where he did design and experimental work with mechanical fruit harvesting equipment.

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1. INTRODUCTION

During the last ten years, mechanical fruit harvesting has become a reality. The impetus for the development of mechanical harvesters has come mainly from the increasing shortage of farm laborers and also from the rising cost of their labor. Where once fruit was picked by hand, mechanical harvesters now vibrate the fruit from the tree onto a catching canopy and then convey it into containers for handling.

Although the advent of mechanical harvesters has been a boon to the fruit growing industry, many problems still remain. For instance, mechanical harvesting usually causes more fruit damage than hand picking. For fruit which is intended for the processing market, a certain amount of this damage can be tolerated; but for fruit which is to be sold in the fresh fruit market, this damage cannot be tolerated and the fruit must still be hand picked.

Depending upon the type of fruit harvested and its ultimate destination, it is desirable to have stems attached in some cases whereas in other cases it is undesirable. Presently, mechanically harvested fruit is removed from the tree both with and without stems. Another problem associated with mechanical harvesting is that in some cases an excessive shaking time and an excessive amplitude are required to remove the fruit and, even so, a substantial amount of fruit is left on the tree. In addition to slowing down the harvesting operation this excessive shaking can cause damage both to the tree and the fruit. It is believed that if the correct shaking frequency range could be predetermined then shaking time could be reduced and much damage could be avoided.

The mathematical model of the fruit-stem system developed by Cooke and Rand (1) has been used to describe mathematically the motion of this system and to predict the correct shaking frequencies. Based on the assumption that maximum bending of the stem is the criterion for fruit detachment, they calculated the forcing frequencies which would produce maximum bending at the stem-branch junction or at the stem-fruit junction. Thus they were able to predict the desired shaking frequency as well as to predict whether fruit would be detached with or without stems intact. In their study they did not examine the stresses developed in the stem.

In this study the motion of an apple fruit-stem system subjected to forced oscillation will be observed and will be compared with the motion predicted from a digital computer simulation of the model. The stem forces caused by the motion will be calculated in order to test the predictions of Cooke and Rand as to the likelihood and location of failure as a function of the forcing frequency and direction.

Before this study can be made it is necessary to know certain physical properties of the apple which affect its motion. A study of the spring stiffness of the stem is made for Golden Delicious Apples as the season progresses. Calculations of viscous damping in the stem are also made.

II. REVIEW OF LITERATURE

Researchers in the mechanical fruit harvesting field have tried a variety of devices either to remove fruit from trees or to aid human pickers. For fruit which is intended for the fresh market, picking aids such as mobile picking platforms and mechanical ladders are being used to increase the harvesting efficiency. For fruit which is to be processed, direct removal and vibratory devices are being used with varying degrees of success.

To remove citrus fruit directly from trees, the use of picking augers was investigated. Coppock (2) found this method impractical because of the low percentage of fruit removal and also because immature fruit was removed along with the mature fruit. Lenker (3), in a later experiment with picking augers, experienced the same problems as well as a large amount of fruit damage.

One vibratory fruit removal device which has been used by several researchers is an oscillating air blast. Adrian (4), reported a low removal efficiency for prunes with a high velocity, low volume blast of pulsating air. Jutras et al. (5) investigated a high velocity, high volume, pulsating air blast to remove oranges. They achieved removal rates from 40 to 95.6 percent but expressed concern about leaf damage and removal of immature fruit. In laboratory experiments, an upward stream of pulsating air was used to vibrate fruit from branches and to reduce the falling velocity of the detached fruit(6,7). It was discovered that the power requirements to use this method on a larger scale would be prohibitive. In another laboratory experiment, Thomas(8) also used an oscillating air blast to investigate apple detachment. Although the laboratory results were encouraging, the method proved ineffective when tried in the field because of large amounts of energy being absorbed by the tree branches and leaves.

On the other hand, numerous researchers have reported better results with vibratory fruit harvesting by using mechanical shakers which attach directly to the tree. Cherries (9,10,11,12), apples(13,14,15), peaches(16,17), citrus fruit (18,19),Concord grapes (20,21), coffee (22,23,24), raspberries (25,26,27) and blueberries(28) are some of the crops for which successful vibratory harvesting has been reported.

Much work has been done to determine the required shaking frequency and stroke for most efficient fruit detachment. (This work has been mostly empirical with a large investment of time, labor and equipment being used to obtain the needed information. In many cases, the information gained could not be utilized for other crops, or even for the same crop in another geographical area, because of the wide variation in the physical properties.)

The predominant belief has been that fruit detachment occurs when the inertia forces due to the motion of the fruit become greater than the axial tensile force necessary to cause removal. It was reasoned that in order to produce large inertial forces, the limb must be shaken at its natural frequency and thus impart a vigorous motion to the fruit. Accordingly, many researchers(29,30,31,32,33,34) have studied the response of the tree limbs to forced vibrations. Although fruit detachment depends on the response of the fruit rather than the response of the limb, this work has been of value in determining how to transmit energy to the fruit. Other researchers have used the fruit detachment force as criterion for determining the desired shaking frequency and stroke combination(6,22,23,15,12,24,25,18,35). Using this criterion, one would expect frequency and stroke combinations which produce the same inertial force to be equally effective in removing fruit. Markwardt et al. (11), however, found a high frequency, short stroke combination to be more effective in removing cherries than a longer stroke and lower frequency. Opposite results were found for olives by Lamouria and Brewer(36)who found a frequency of 1200 c.p.m. and a 1 inch stroketo be more effective than a frequency of 2400 c.p.m. and a 1/4 inch stroke. They concluded that detachment depends not only upon the magnitude of the applied inertial force but also upon the rate of application of the force. Wang et al. (37) explained these findings in terms of cumulative damage caused by cyclic stresses. It has also been reported that fruit detachment force is decreased if the force is applied at an angle (38,39,40,41), and that detachment occurs after a number of cycles rather than being solely due to an axial tensile force (20,8,37,12,13,15,42,43).

In order to understand more fully the mechanics of fruit detachment, many researchers have focused their attention on the fruit-stem system and its response to forced vibrations and repeated bending. Singley, et al.,(44) investigated the relative fruit detachment effectiveness of tension, shear, bending and torsion of fruit stems. They found that only shear or bending produced satisfactory detachment of apples at the abscission zone. The abscission zone is a layer of corky material which develops between the stem and branch as the fruit matures. The failure angle for bending varied from 24 to 144 degrees and did not change as the fruit matured. In 1962 (42) they investigated the static effect of frequency and amplitude of repeated bending of an apple stem about the abscission zone. They found that increasing the amplitude of the bending decreased the number of cycles required for failure and also found that the frequency of bending had little effect. These tests were conducted under static conditions with the apple held stationary and with the stem flexed relative to the fruit.The results do not contradict the later findings that the liklihood of failure is frequency-dependent under dynamic conditions. Below a certain amplitude of bending they found that it took an excessive number of cycles for failure to occur. From this they speculated that there is a threshold value of bending angle which must be exceeded for repeated bending to be effective. It was later reported (8) that this threshold value for apples is $\pm$ 60 degrees.

Thomas (8) modelled the apple fruit-stem system as a pendulum with a restoring torque at the pivot point. To test the model he oscillated the apple with an air blast at frequencies between 0.6 and 1.4 of the basic natural frequency (which he had measured for small deflections). For large amplitudes (greater then 80 degrees), the maximum bending and greatest percent of fruit detachment occurred when the forcing frequency was .85 of the basic natural frequency. Between .85 and .90 of the basic natural frequency he noticed the jump phenomenom common to most non-linear systems, ie. two distinct amplitudes existing for the same forcing frequency.

Diener et al. (13) noticed that a vibrating apple has several distinct motions or mode shapes depending on the forcing frequency. As the frequency increased, the three most predominant modes observed by Diener et al. were: a) pendulur motion about the abscission zone, b) tilting of the fruit about its mass center, and c) twisting about the stem-calyx axis. The tilting and twisting modes were judged to be the most effective for fruit detachment. Similar modes of oscillation were later observed and extensively documented by Rumsey (38, 41) for oranges and grapefruit.

Wang (22,23) modelled the coffee cherry and its stem as a cantilever beam with an end mass. In order to solve for the first natural frequency of the system, this model was simplified to an equivalent spring-mass system.

In 1967 Wang et al. (37), applied the theory of cumulative damage due to cyclic stresses to explain the process of fruit detachment by vibration. They postulated that:

a) failure of biological materials is related to the total energy absorbed by the material,

b) the total energy required to cause failure decreases with the maximum value of strain,

c) the energy absorbed per cycle is proportional to the maximum strain amplitude during the cycle raised to a positive power.

Using this theory they were able to explain observations made by other researchers.

Studer (20,21,45) modelled the fruit-stem system of grapes, olives and cherries as a pendulum with a vertical, sinusoidal forcing displacement at the pivot point. From the model he was able to classify the resulting motion as stable or unstable depending upon the equivalent stem length, fruit mass,forcing stroke and frequency. In a range of forcing frequencies from 450 to 750 c.p.m. he found that pendular motion of cherries becomes unstable as stroke length is increased or as stem length and forcing frequency are decreased.

The response of grapefruit and orange fruit-stem systems to forced vibrations was mathematically investigated by Rumsey (38) who also assumed the fruit-stem system to be equivalent to a cantilever beam with a lumped end mass. Assuming small deflections and the boundary conditions he used the Bernouilli-Euler beam equation to solve for the natural frequencies and also for the steady-state response of the system due to a forcing displacement of the fixed end. Using high-speed photography he was able to observe the modes of vibration similar to those described by Diener et al. but was unable to compare with his theoretical results because of a lack of physical data.

To describe mathematically the three principal modes of vibration observed by Diener et al., Cooke and Rand (1) used a double physical pendulum with torsion springs at the pivots as a model of the fruit-stem system. The support was subjected to simultaneous horizontal and vertical forcing displacements with a phase angle which can be adjusted to describe any elliptic path, of which straight line motion is a special case. Using Lagrange's equations they derived the three second-order, non-linear, non-homogenous, differential equations describing the motion of the system. Upon linearizing the equations of motion they solve for the natural frequencies and related the natural frequencies to the associated mode shapes. For forced vibrations with a vertical component, the widest range of forcing frequencies which cause unstable motion were found to be at twice the pendulum and tilting mode natural frequencies. The pendulum and tilting modes of response occur if the system is oscillated at their respective natural frequencies as well as at twice their respective natural frequencies. For the pendulum mode, the maximum relative bending occurs between the branch and stem, whereas for the tilting mode bending is maximum between the stem and fruit. From this analysis they were able to conclude that in order to induce bending at the fruit-branch junction, and thus cause fruit removal with stems intact, the forcing frequency should be twice the pendulum mode natural frequency. Forcing at twice the tilting mode natural frequency induces bending at the stem-fruit junction and causes fruit separation without stems. In a further analysis of this model Rand and Cooke (46) examined the free vibration motion predicted by the non-linear equations of motion. They found that the tilting mode free vibration natural frequency decreased rapidly with an increase in amplitude of oscillation. They speculated that to maintain resonance conditions for the tilting mode it would be necessary to fluctuate the forcing frequency after oscillations were induced. Since the resonance frequency decreased with increasing amplitude, the decreasing and increasing of the forcing frequency should sustain resonance. However they suggest that the forcing frequency should not be dropped so low as to excite the pendulum mode if the fruit is to be harvested without stems. For the pendulum mode, the frequency of free vibration is essentially independent of the amplitude of oscillation for angles less than 90 degrees. This suggests that the range of forcing frequencies for harvesting fruit with stems intact is relatively small.

Garman et al. (47), using an artificial apple attached to a branch, were able, with a force transducer and high speed photography, to measure the axial tensile force and motion produced by vertical and horizontal vibration at two different frequencies. As expected, they found that the maximum tensile force was produced by vertical shaking.

III. STATEMENT OF PROBLEM

It can be seen from the above studies that fruit detachment depends upon the response of the fruit-stem system to forced vibrations. The response, in turn, is determined by the physical properties of the system as well as the forcing function. The stresses induced by the motion can be used to indicate the desired forcing frequency for a particular forcing function. If the motion can be shown to be mathematically predictable from the fruit parameters and forcing function, then the desired forcing frequency can be calculated directly from a mathematical model.

This study examines the model of the fruit-stem system proposed by Cooke and Rand in order to determine whether it can be used to predict the motion of an apple fruit-stem system when it is subjected to forced vibrations. From the response of the system, the induced stresses in the stem are evaluated in order to predict the desired shaking frequency. The question of which type of motion causes the largest stem stresses is believed to be of great importance in determining the shaking frequency. For instance, is a shaking frequency that causes large bending amplitude in the stem more desirable than a shaking frequency which causes large tensile or shear forces; or is there a frequency where the combination of these stresses is maximum? To predict the location of failure, which determines whether the fruit will detach with or with--out stems, the stresses at each junction must be examined in terms of not only the bending, but also the tension, shear and relative strengths of attachment at each junction.

IV. THEORETICAL ANALYSIS

Model for Predicting Motion

In their analysis of the fruit-stem system, Cooke and Rand(1) assumed that fruit is removed due to bending stresses caused by large deflections of the fruit relative to the stem or the stem relative to the branch. To achieve large localized deflections they reasoned that the fruit should be vibrated at a frequency which will cause instability of the fruit-stem system. This, then, should also be the desired frequency to shake the tree limbs since the entire tree structure will vibrate with the same frequency. The model used by Cooke and Rand to predict the motion and maximum bending stresses of a hanging fruit is shown in figure 1.

The model consists of a double compound pendulum with torsion springs at the joints to account for the flexibility of the stem. The torsion spring constants at the stem-branch junction and the stem-fruit junction are S and K respectively. A torsional spring which resists twisting of the fruit about its vertical axis is represented by the spring constant, C. The stem is represented by a thin rigid rod of mass, μ, length, ℓ, and the fruit by a sphere of mass, M, radius, R. To take into account the three modes of vibration observed by Diener (13) the model has three degrees of freedom corresponding to

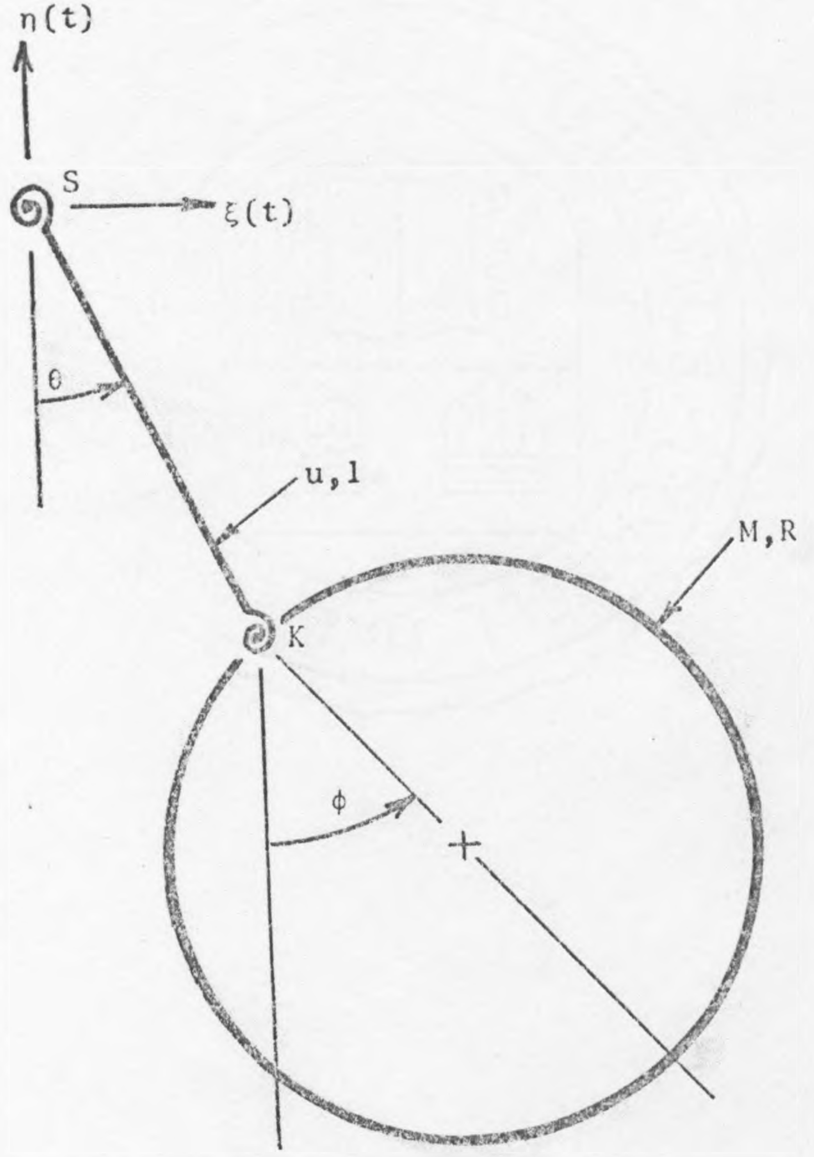


Figure 1

Mathematical Model of Fruit-Stem System Proposed by Cooke and Rand

a) the angle of deflection, θ , of the stem from the vertical, b) the angle of deflection, ϕ , of the stem-calyx axis of the fruit from the vertical and c) the angle of twist, ψ , of the fruit about its stem-calyx axis. The point of support is subjected to simultaneous horizontal, $\xi(t)$, and vertical, $\eta(t)$, forcing displacements, of which the amplitudes and phase angles can be adjusted to describe any planar elliptical path including straight line and circular motion.

Using Lagrange's equations, the equations of motion of this system can be derived. These equations are non-linear and probably cannot be solved analytically. However, if they are linearized, an analytical solution for the natural frequencies is possible as detailed by Cooke and Rand. The linear analysis was also used to solve for the widest range of forcing frequencies which cause unstable motion. To solve the non-linear forced vibration equations for the motion of the co-ordinates as a function of time, numerical methods with the use of digital computer are convenient.

The model presented is a general model meant to encompass a wide variety of hanging fruit. In order to describe more accurately the behavior of an apple, the model is refined to include the characteristics of apples not included in the general model. For apples the point of attachment of the stem to the fruit (rather than on the surface) lies within the sphere representing the fruit and the model is altered to account for this.

Even though various researchers have observed the twisting mode of oscillation of the fruit about the stem axis, it was not judged to be important on the basis of observations (44) which reported an average twist angle of 839° was necessary to cause stem failure. Even though the twisting mode could be accounted for in the model, the physical properties of the fruit which cause it to occur are quite variable and unpredictable. In subsequent vibration tests conducted, this mode was not observed and was thus eliminated from the model.

It has been shown(1) that the moment of inertia of an apple about its mass canter does not differ greatly from that of a homogenous sphere having the same volume and weight as the apple. Thus little error is introduced by representing the apple as a sphere.

Even though the forcing displacement of the upper end of the stem is restricted by the model to planar motion, this motion describes most of the forcing displacements which are induced at the stem-branch junction by a mechanical shaker.

The effects of friction (or damping), though not important in calculating resonant frequencies, become important if the steady-state amplitude and form of response of non-homogenous mechanical systems are to be calculated by numerical methods. The reason for this is readily apparent from a study of a linear, non-homogenous system. The analytical solution of such a system consists of a transient plus a steady-state solution. If damping is included the transient solution dies out in time and the steady-state solution remains. In all mechanical systems there is always a certain amount of damping which causes this transient portion to die out, ususlly in a very short time. However, if the system is solved by numerical methods and damping is not included, the transient portion of the solution will not die out and the computed solution will not resemble the observed motion because of the superimposed transient. By inferring that this aspect of the non-linear, non-homogenous system is similar to the linear system, which in fact it is for small oscillations (46), it can be seen that damping must be included in the numerical solution of the non-homogenous equations.

With these modifications the model is now represented in figure 2. The apple is represented by an equivalent sphere with moment of inertia I_c about its mass center. The distance between the center of mass of the apple and the attachment point of the stem is the distance r. The torsional coefficeint of viscous damping c due to the relative motion in the joints is included and is assumed to be the same in both joints. The effects of air friction are neglected because the velocity of a vibrating apple is small realtive to the velocity necessary to cause a substantial drag force. The equations of motion can not be derived by using a modified form of Lagrange's equations (48) which include the Rayleigh dissipation function. These equations have the form:

$$\frac{d}{dt}\left[\frac{\partial L}{\partial q_i}\right] - \frac{\partial L}{\partial q_i} + \frac{\partial P}{\partial q_i} = 0 \qquad (1)$$

where,

P= the Rayleigh dissipation function

L= the Lagrangian function

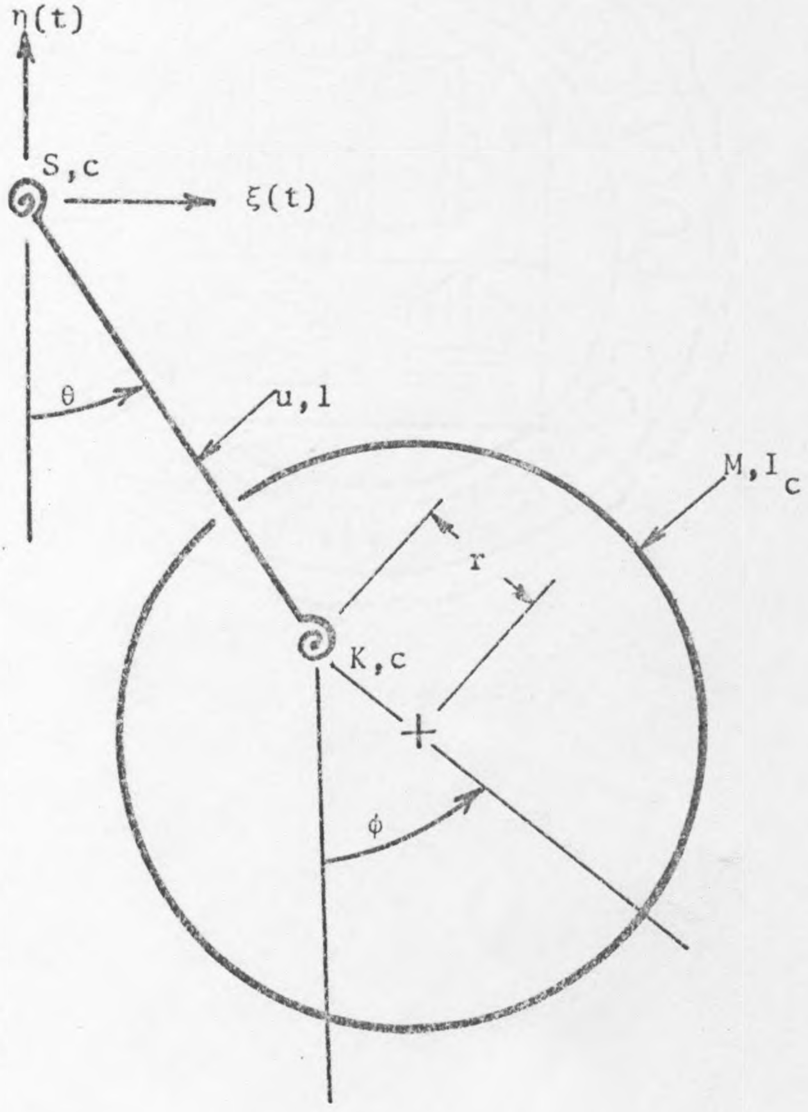


Figure 2

Model Modified to Include Damping at Joints, and Stem Attachment Point Within the Equivalent Sphere

q_i = the generalized co-ordinates, θ and ϕ.

To derive the equations of motion, the kinetic energy due to the velocity of the fruit is

$$T = \dot{\theta}^2\left[\frac{\mu\ell^2}{6} + \frac{M\ell^2}{2}\right] + \frac{\dot{\phi}^2}{2}\left[I_c + Mr^2\right]$$
$$+ \dot{\theta}\left[\frac{\mu\ell}{2}\ (\dot{\xi}\cos\theta + \dot{\eta}\sin\theta)\right] + \dot{\phi}\left[Mr(\dot{\xi}\cos\phi + \dot{\eta}\sin\phi)\right]$$
$$+ M\ell r\dot{\theta}\dot{\phi}\cos(\theta-\phi) + (\dot{\eta}^2 + \dot{\xi}^2)\left(\frac{\mu+M}{2}\right) \qquad (2)$$

and the potential energy due to the spring deflection at upper and lower pivots as well as the gravitational potential is

$$V = \frac{1}{2}S\theta^2 + \frac{1}{2}K(\phi-\theta)^2 - \left(\frac{\mu+2M}{2}\right)g\ell\cos\theta - Mgr\cos\phi \qquad (3)$$

where g is the gravitational constant. The potential energy due to the upward displacement of the support point can be neglected because "...the Lagrangian is defined only to within an additive total time derivative of any function of coordinates and time."(57)

The Rayleigh dissipation function for viscous damping in the joints can be expressed as (49)

$$P = \frac{1}{2}\left[c\dot{\theta}^2 + c(\dot{\phi}-\dot{\theta})^2\right] \qquad (4)$$

where $c\dot{\theta}^2$ and $c(\dot{\phi}-\dot{\theta})^2$ is the energy dissipated due to damping at the upper and lower joints respectively.

Upon performing the required operations, the equations of motion are

$$a_1\ddot{\theta} + a_2\ddot{\phi}\cos(\theta-\phi) + a_2\dot{\phi}^2 \sin(\theta-\phi)$$

$$+ a_3 \left[\ddot{\xi}\cos\theta + (g+\ddot{\eta})\sin\theta\right] + K(\theta-\phi) + S\theta + c(2\dot{\theta}-\dot{\phi})=0 \qquad (5\text{-}1)$$

and

$$a_4\ddot{\phi} + a_2\ddot{\theta}\cos(\theta-\phi) - a_2\dot{\theta}^2 \sin(\theta-\phi)$$

$$+ a_5\left[\ddot{\xi}\cos\phi + (g+\ddot{\eta})\sin\phi\right] - K(\theta-\phi) - c(\dot{\theta}-\dot{\phi})=0 \qquad (5\text{-}2)$$

where

$$a_1 = \frac{\mu\ell^2}{3} + M\;{}^2$$

$$a_2 = M\ell r$$

$$a_3 = \frac{\ell(\mu + 2M)}{2}$$

$$a_4 = I_c + Mr^2$$

$$a_5 = Mr$$

To express the equations in the standard form suitable for a digital computer solution they are written in terms of four first order equations. This is accomplished by letting

$$x_1 = \dot{\theta} \qquad\qquad \dot{x}_1 = \ddot{\theta}$$

or

$$x_2 = \dot{\phi} \qquad\qquad \dot{x}_2 = \ddot{\phi}$$

Now the equations are

$$a_1\dot{x}_1 + a_2\dot{x}_2 \cos(\theta-\phi) = -a_2 x_2^2 \sin(\theta-\phi)$$

$$-a_3[\ddot{\xi}\cos\theta + (g+\ddot{\eta})\sin\theta] - K(\theta-\phi) - S\theta - c(2x_1-x_2) = b_1 \quad (6\text{-}1)$$

$$a_4\dot{x}_2 + a_2\dot{x}_1\cos(\theta-\phi) = a_2 x_1^2 \sin(\theta-\phi)$$

$$-a_5[\ddot{\xi}\cos\phi + (g+\ddot{\eta})\sin\phi] + K(\theta-\phi) + c(x_1-x_2) = b_2 \quad (6\text{-}2)$$

$$\dot{\theta} = x_1 \quad (6\text{-}3)$$

$$\dot{\phi} = x_2 \quad (6\text{-}4)$$

Using Cramer's rule to eliminate $\dot{x}_2$ from the first equation and $\dot{x}_1$ from the second equation, the result is

$$\dot{x}_1 = B_1/D \quad (7\text{-}1)$$

$$\dot{x}_2 = B_2/D \quad (7\text{-}2)$$

$$\dot{\theta} = x_1 \quad (7\text{-}3)$$

$$\dot{\phi} = x_2 \quad (7\text{-}4)$$

where B_1, B_2 and D are the determinants

$$B_1 = \begin{vmatrix} b_1 & a_2\cos(\theta-\phi) \\ b_2 & a_4 \end{vmatrix}$$

$$B_2 = \begin{vmatrix} a_1 & b_1 \\ a_2\cos(\theta-\phi) & b_2 \end{vmatrix}$$

$$D = \begin{vmatrix} a_1 & a_2\cos(\theta-\phi) \\ a_2\cos(\theta-\phi) & a_4 \end{vmatrix}$$

In performing this operation care must be taken that the determinant D is not ill-conditioned otherwise the numerical solution is upset. According to Forsythe and Moler (50), the system of equations is ill-conditioned if the normalized value of the determinant times the normalized value of the determinant of the inverse of the matrix of D is very large compared to one. A check of the conditioning by the method described reveals that it is well conditioned for the range of values anticipated.

The equations of motion have now been derived and expressed in a form convenient for a numerical solution. In order to simulate the forced vibration of an apple, the values of the physical parameters of the fruit must first be measured and substituted into the equations along with initial conditions and the forcing function. The physical parameters of the fruit could be inferred from the values which make the calculated motion best fit the observed motion. This, however, is a weak test of the model. A better test is made if the fruit parameters to be used in the equations are measured independently.

Determination of the Viscous Damping Coefficient c

The viscous damping coefficient c due to internal friction at the stem joints cannot be measured directly from the fruit-stem system. Under dynamic conditions it can, however, be calculated in terms of certain other measurable properties. It has been shoen (46) that for small deflections of apples the pendulum mode motion can be approximated by the motion of a one-degree-of-freedom pendulum. Using a linear, one-degree-of-freedom analysis, the coefficient of viscous damping at the stem-branch junction can be calculated by measuring the damping factor, ζ, and by expressing the critical damping coefficient, c_c, in terms of the fruit parameters which can be measured.

If the value of ζ is smaller than one, it can be measured experimentally by observing the rate of amplitude decay of the oscillating fruit and by using the logarithmic decrement to calculate its value (51,52). The value of c_c can be calculated from the linearized, homogenous equation of motion of the pendulum:

$$\ddot{\theta} + \frac{c\dot{\theta}}{[M(\ell+r)^2+I_c]} + \theta\frac{[Mg(\ell+r)+S]}{[M(\ell+r)^2+I_c]} = 0 \qquad (8)$$

From this equation the undamped natural frequency w_n is given by:

$$w_n = \left[\frac{Mg(\ell+r) + S}{M(\ell+r)^2 + I_c}\right]^{1/2} \qquad (9)$$

and the critical damping coefficient is seen to be

$$c_c = 2w_n[M(\ell+r) + I_c] \qquad (10)$$

If the damping factor is small, the undamped natural frequency w_n is approximately the same as the damped natural frequency which can be determined experimentally by observing the period of oscillation of the apple or from equation (9). By measuring the fruit parameters and calculating c_c from equation (10), the damping coefficient can be calculated from equation (11) which defines this coefficient to be (51)

$$c = \zeta c_c \tag{11}$$

The above analysis has been based on the assumption that the value of the damping factor, ζ, is small compared to one. Subsequent calculations have in fact shown this to be an acceptable approximation.

Calculation of the Stem Forces

The stresses induced in the stem due to forced vibrations include not only those due to bending but also stresses due to tensile and shear forces. The bending stresses at the upper and lower joints are readily apparent from the solution of the equations of motion and are proportional to θ and $\phi-\theta$ respectively. In a similar manner the tensile and shear forces can also be expressed in terms of the co-ordinates of the model.

To solve for the tensile and shear forces in the stem, the method of Lagrange multipliers is used (48,49). For the calculation of the tensile force in the stem, the stem length

ℓ is assumed to be a "variable" of the system. A superfluous co-ordinate δ at right angles to the stem is introduced for calculating the shear force. Lagrange's equations are now used to solve for the generalized constraint forces in terms of four generalized co-ordinates; θ, ϕ, ℓ, and δ; and two constraint equations, namely:

$$\dot{\ell} = 0$$

and

$$\dot{\delta} = 0$$

With the introduction of these new co-ordinates the kinetic energy becomes

$$T = \frac{1}{2} I_c \dot{\phi}^2 + \frac{1}{2} M(\dot{x}^2 + \dot{y}^2)$$

where

$$\dot{x} = \dot{\xi} + \dot{\ell}\sin\theta + \ell\dot{\theta}\cos\theta + \dot{\delta}\cos\theta - \delta\dot{\theta}\sin\theta + r\dot{\phi}\cos\phi$$

and

$$\dot{y} = \dot{\eta} - \dot{\ell}\cos\theta + \ell\dot{\theta}\sin\theta + \dot{\delta}\sin\theta + \delta\dot{\theta}\cos\theta + r\dot{\phi}\sin\phi$$

The potential energy becomes

$$V = \frac{1}{2} S\theta^2 + \frac{1}{2} K(\phi - \theta)^2 - Mg(\ell\cos\theta - \delta\sin\theta + r\cos\phi)$$

Since the mass of the stem is small compared to the mass of the fruit, the stem has a negligible effect upon the dynamics of the system (1) and was not included in the above calculations of kinetic and potential energy. When the differentiations of T and V with respect to $\ell, \dot{\ell}$, $\delta, \dot{\delta}$ and t are performed in accordance with Lagrange's equations and $\dot{\ell}, \ell, \delta$, $\dot{\delta}$, $\ddot{\delta}$ are set equal to zero, the equations for the tensile force τ and the shear force Q are

$$\tau = M[(g+\ddot{\eta})\cos\theta - \ddot{\xi}\sin\theta + \ell\dot{\theta}^2 + r\dot{\phi}^2\cos(\phi-\theta) + r\ddot{\phi}\sin(\phi-\theta)] \quad (12)$$

and

$$Q = M[(g+\ddot{\eta})\sin\theta + \ddot{\xi}\cos\theta + \ell\ddot{\theta} - r\dot{\phi}^2\sin(\phi-\theta) + r\ddot{\phi}\cos(\phi-\theta)] \quad (13)$$

As expected the previously derived equations of motion are unaffected by the addition of the superfluous co-ordinates. These second-order equations can be reduced to first-order equations by substitution from the previously derived equations of motion. The $\ddot{\phi}$ term is eliminated from equation (12) by substitution of equation (7-2):

$$\tau = M[(g+\ddot{\eta})\cos\theta - \ddot{\xi}\sin\theta + \ell\dot{\theta}^2 + r\dot{\phi}^2\cos(\phi-\theta) + r\frac{B_1}{D}\sin(\phi-\theta)] \quad (14)$$

To eliminate the second-order derivatives from equation (13), equation (6-1) is substituted to give the following expression for the shear force in terms of the magnitude and velocity of relative bending at the joints:

$$Q = \frac{K}{\ell}(\phi-\theta) - \frac{S}{\ell}\theta + \frac{C}{\ell}(\dot{\phi}-2\dot{\theta}) \quad (15)$$

The stem forces are now expressed in terms of the motion of the system. The motion, in turn, can be calculated by numerical methods as outlined previously. Thus, knowing the response of the system to forced vibrations, the induced stem forces for a particular forcing function can be determined from these equations.

V. EXPERIMENTAL PROCEDURE

On the basis of the mathematical model,the motion and resulting forces can be obtained by numerical methods if the physical parameters of the fruit are known. The problem is now refined to one of a) experimentally determining the values of the physical properties of the apple which affect its motion, b) measuring and recording the forced vibratory motion of an apple, and c) calculating and comparing the predicted motion with the observed motion. Golden Delicious apples were used for this investigation because of their availability.

Measurement of Spring Constants

A device for measuring the torsional spring constants of the stem-fruit and stem-branch junctions was constructed (Figure 3). The device consists of a pulley, string and weights for applying a torque; a protractor for measuring the angle of deflection; and clamps for holding the apple. To measure the spring constant K, the apple(along with the stem and a piece of the branch)is positioned in the device so that the stem-fruit junction is in line with the axis of rotation of the pulley. A torque is applied by adding weights to the pan on the end of the string and the resulting deflection is

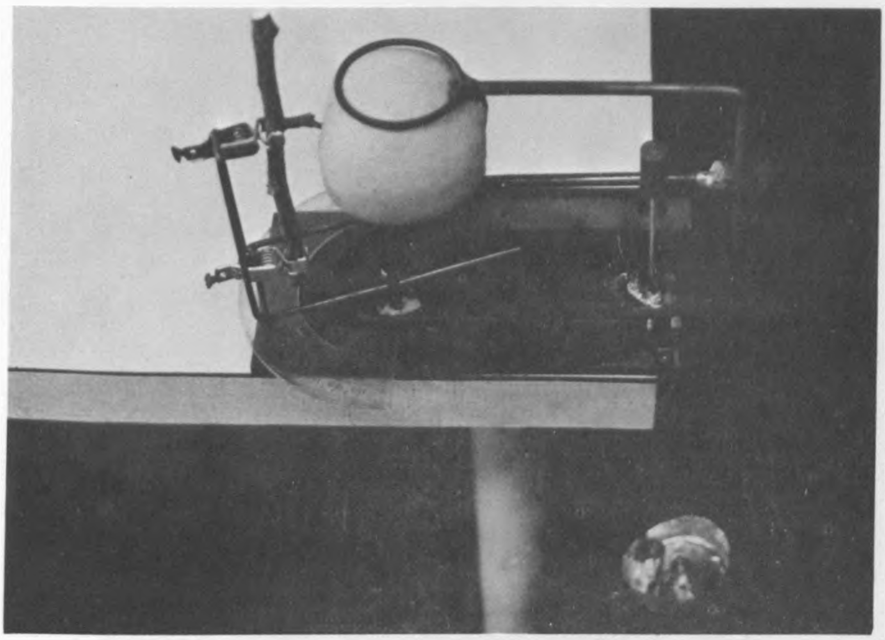


Figure 3

Device for Measuring Spring Stiffness

measured with a pointer on the protractor. To measure the spring constant S, the apple is repositioned so that the stem-branch junction is in line with the axis of rotation of the pulley and the procedure is repeated. The slope of the torque vs. deflection curve is the desired spring constant. A least squares linear regression analysis of the torque vs. deflection data is used to calculate the slope of the best fit line.

To find the effect of maturity upon the stiffness of the stem joints measurements of spring constants were made on six samples at one week intervals for the four weeks preceding harvest. To reduce the effects of variations between trees and the location on the tree from which the sample is taken, samples were taken from two trees. Each tree was divided into thirds and a sample was taken from each third. The mean of the six measurements was taken as the value at that date.

Measurement of Damping and Natural Frequency

The system shown in figure 4 was used to measure the natural frequency and the logarithmic decrement of amplitude decay of the vibrating apple. The system consists of a cantilever beam to which a portion of the branch with the stem and apple intact is attached. The apple is set in motion by a small initial displacement. The resulting motion causes a small movement of the cantilever beam which is measured by a L.V.D.T. (Linearly Variable Differential Transducer). The a.c. output voltage of the L.V.D.T. is converted to d.c. by a diode bridge(52) and then fed into a Brush strip chart recorder to give a graphical representation of the free vibration motion

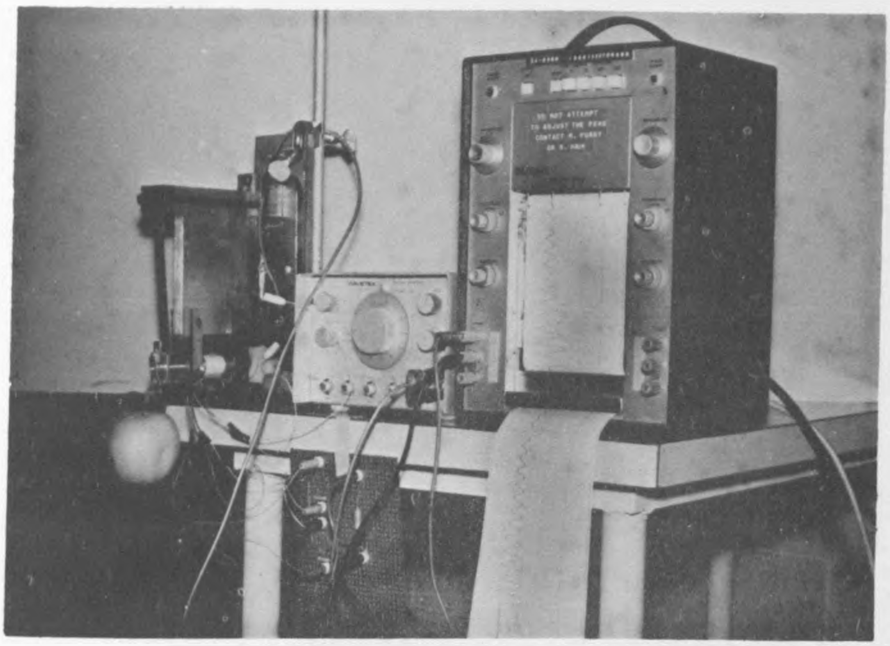


Figure 4

System for Measuring Damping and Free Vibration
Natural Frequency

of the pendulum mode of the apple.

The natural frequency of the pendulum mode can now be calculated from the period of cyclic motion of the fruit; and the damping factor, ζ, can be calculated by measuring the rate of amplitude decay.

To obtain an average value of the torsional damping coefficient in the joints, for use in solving the equations of motion, measurements of damping were made on thirty-six samples. The natural frequencies obtained by this procedure were compared with those calculated from the linearized, homogenous equations of motion and with those observed for forced vibrations. These values are tabulated in Tables 111 and 1V of chapter V1.

Vibration Tests

A vibrator (figure 5) for studying the response of an apple to forced vibrations was constructed. The vibrator is capable of producing the wide variety of elliptic motions described previously, and consists of two chain-driven cranks and connecting rods. The ends of the connecting rods are joined together with a bearing so that they are approximately at right angles to each other. If the connecting rods are long in comparison with the crank length, the resulting motion of the junction of the connecting rods is a good approximation of sinusoidal motion. The crank lengths are independently variable from 0 to 1.5 inches to give any desired stroke length from 0 to 3 inches peak to peak in any direction of straight line motion from horizontal to vertical. To generate

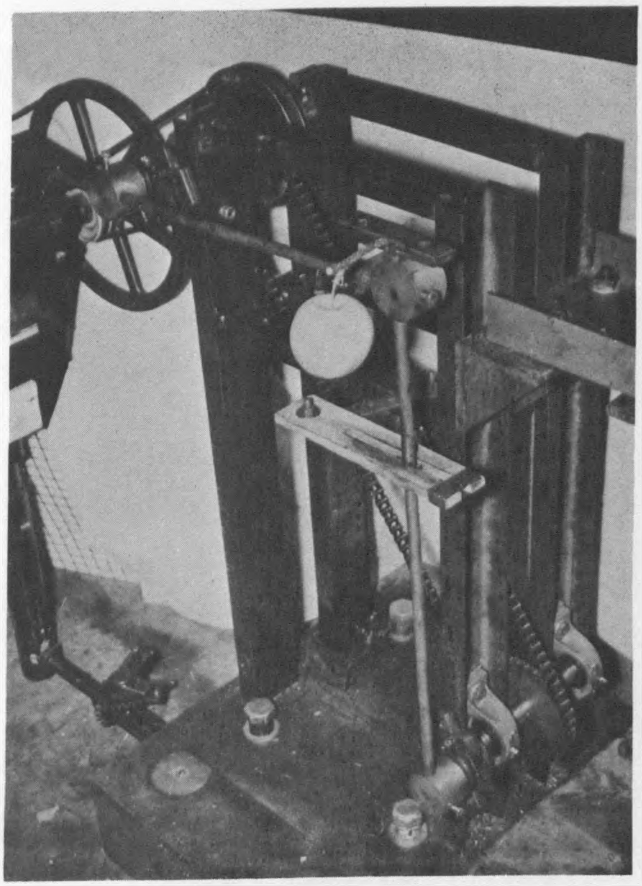


Figure 5

Mechanical Vibrator

any circular or elliptical motion, the phase angles between the cranks can be adjusted by removing the drive chain and rotating one crank relative to the other. The size and shape of the elliptical motion can be varied by adjusting the crank lengths. The vibrator is driven with a variable speed electric motor through a system of belts and pulleys to give a continuously adjustable frequency range of 80 to 600 c.p.m. Higher driving frequencies are possible by changing the drive pulleys.

Since the resonant frequencies are expected to be important factors in determining the correct shaking frequencies, it is desirable to know how they are affected by the maturity of the apple and by various forms of forcing displacements. The effects of maturity upon the natural frequencies of the apple were studied by conducting vibration tests on six samples each week for the three weeks preceding harvest. The samples were cut from the tree with a 6 inch portion of the branch intact and the tests were conducted within 5 hours.

In order to observe the vibrating sample as long as possible without detachment occuring, a short horizontal stroke of 0.5 inches peak to peak was selected. The portion of branch with the apple attached was clamped to the vibrator at the junction of the two connecting rods and the forcing frequency was slowly increased. The resulting motion of the fruit-stem system was observed and the forcing frequencies which caused resonance to occur were measured with a hand held tachometer for the range of forcing frequencies from

80 to 600 c.p.m.

Although the amplitude and direction of shake is not expected to have any effect upon the frequency at which resonance occurs, it is expected to have an effect upon the range of frequencies which causes resonance. It is also of interest to observe the response of the fruit-stem system at frequencies other than the resonant frequencies to see how the response is affected by different forcing displacements. The effect of stroke length and direction of shake were investigated by vibrating three samples each at stroke lengths of 0.5, 1.0 and 1.5 inches for both horizontal and vertical displacements. The effects of a 0.5 inch stroke at directions of 20°,45° and 70° from the horizontal and the effects of elliptical motion at phase angles of 20°,60° and 90° were also investigated by conducting vibration tests on three samples for each particular motion. Before the tests were conducted, the pendulum mode natural frequency of each sample was measured by the method described previously. These values were compared with the observed resonant frequencies and also with those calculated from the linearized non-homogenous equations of motion(see Tables 111 and 1V in chapter V1).

High Speed Film Tests

A test of the validity of the mathematical model can be made by comparing the observed resonant frequencies with those calculated from the model. This, however, is a partial test since only resonant frequencies are compared. A more complete test of the model can be made if the observed motion is compared with the predicted motion over a wide range of forcing frequencies. Since it is difficult to observe the motion of a vibrating apple, particulary at higher frequencies, high-speed photography was used to record the motion of the apple.

A Waddell high-speed motion picture camera with 100 ft. rolls of Kodak 4X reversal type film were used. To provide a reference for measuring the motion of the apple, a white cardboard panel with ruled one-inch square was placed in the background, and a horizontal black line was drawn around the circumference of the apple. It was calculated that if an adequate number of frames per cycle were to be obtained (at least 10), the camera speed would have to be such that the filming time would be no greater than 30 seconds. To gain as much information as possible from a 100 foot roll of film in 30 seconds of exposure time, the forcing frequency of the vibrator was gradually increased from 100 to approximately 550 c.p.m. as the resulting motion was being photographed. Thus it was possible to record the response of the apple to a wide range of forcing frequencies. The motion of the vibrating apple for several different forcing displacements was recorded on eight rolls of film: (All strokes are reported with peak to peak amplitudes)

A. 0.5 inch horizontal stroke

B. 1.0 inch horizontal stroke

C. 0.5 inch horizontal stroke starting at a high frequency and gradually reducing to a low frequency

D. two 15 second runs at constant forcing frequencies of 140 c.p.m. and 400 c.p.m. with a 0.5 inch horizontal stroke

E. 0.5 inch vertical stroke

F. 0.5 inch stroke at a 45° angle from the horizontal

G. 0.5 inch circular stroke (phase angle=90°)

H. repeat of roll A

Since the calculated motion of the apple is to be presented in a graphical form with the co-ordinates, θ and ϕ, expressed as functions of time; the data contained on the high-speed film must be transposed into a similar form if a comparison between the observed and the predicted motion is to be made. A method similar to that used by Davis (53) was employed for transposing the film data into a convenient graphical form.

The picture was projected onto the underside of a horizontal opaque screen about five feet above the floor surface. To give an adequate picture size and to invert the image so that it would appear correct when viewed from the top side of the opaque screen the picture was first projected onto a mirror on the floor then reflected to the underside of the screen (figure 6). The angle of projection and the angle of the mirror were adjusted so that the distortion of the picture was minimum. The top side of the screen was used as a working surface for measuring the angles of deflection from the picture.

To electrically measure and record the angles of deflection, a Bruning drafting machine was adapted with an angular potentiometer as shown in figures 7 and 8. The potentiometer

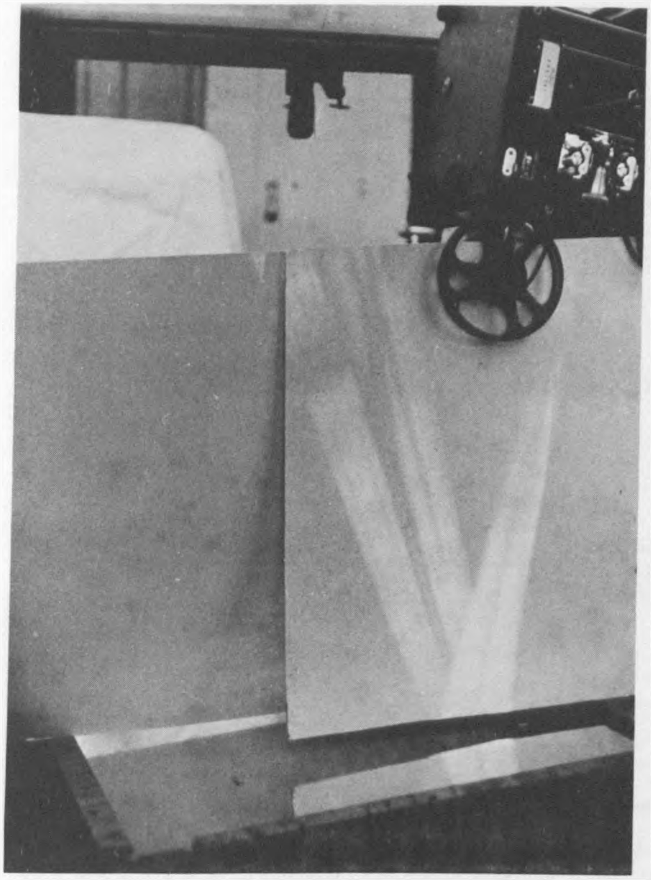


Figure 6

System for Analyzing the High-Speed Film. The Projector, Mirror and Path of the Projected Beam are Shown

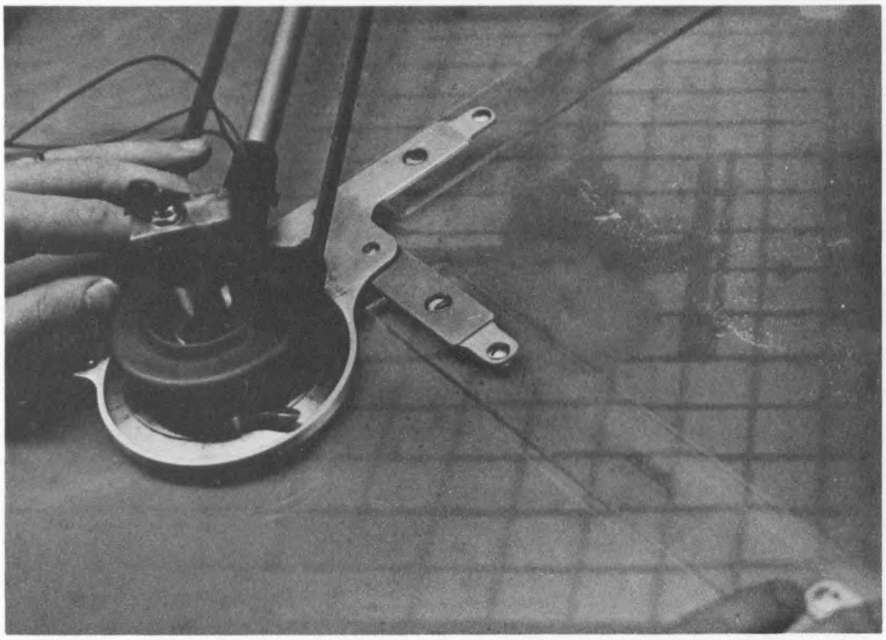


Figure 7

Drafting Machine with Potentiometer Attached

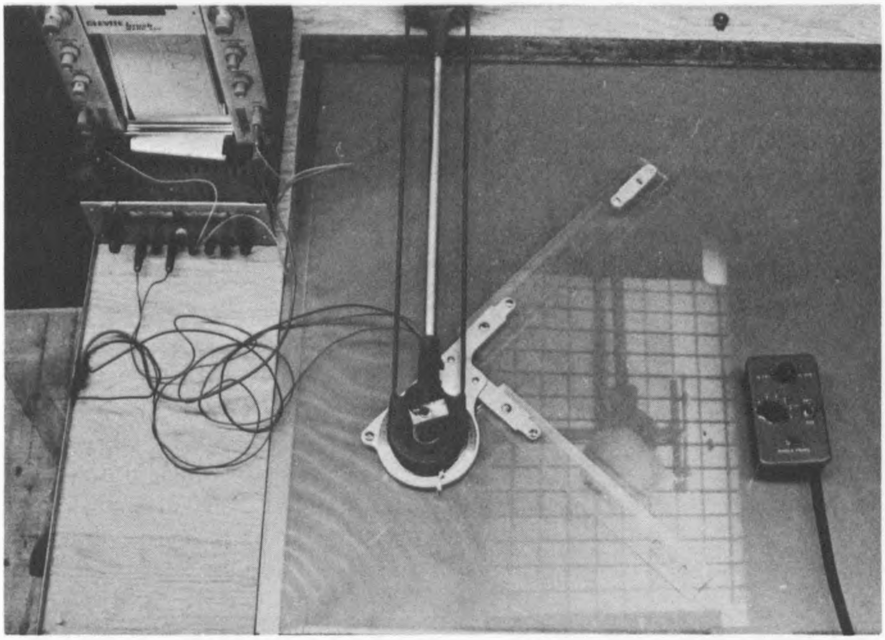


Figure 8

System Used for Transposing High-Speed Film Data into Graphical Form

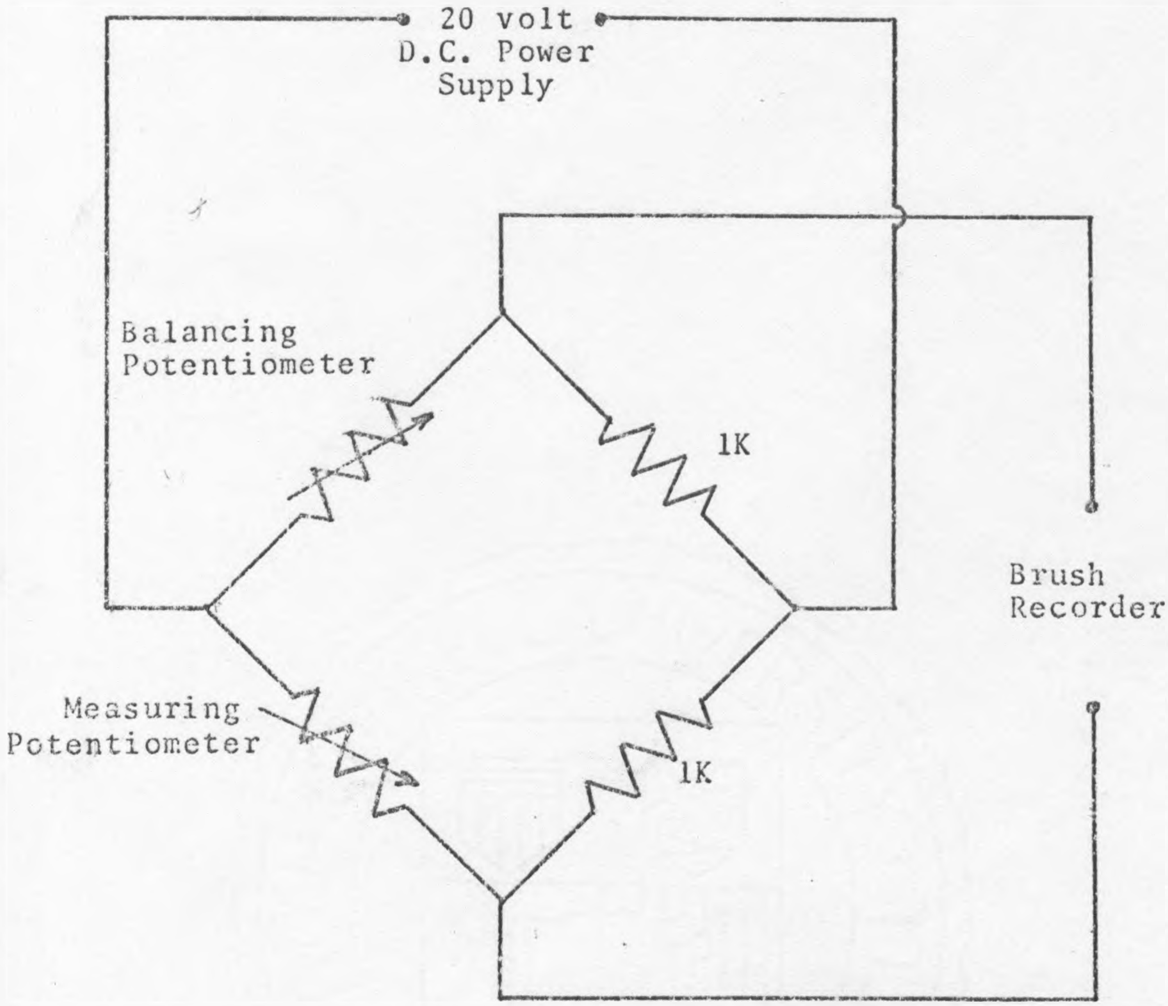


Figure 9

Wheatstone Bridge Circuit for Curve Tracing System is one arm of the Wheatstone bridge circuit of figure 9. Another potentiometer in the adjacent arm is used for balancing the bridge. A rotation of the arms of the drafting machine, and thus the potentiometer, produces an imbalance of the bridge which is proportional to the angle of rotation. The output of the bridge is fed into a Brush strip-chart recorder and is calibrated directly in degrees per vertical division.

The following procedure was used to record the motion of the fruit as a function of time. After zeroing the bridge circuit along one of the grid lines; the pictures were projected onto the screen at a rate of 2 frames per second and the strip chart recorder was run at a constant speed of 2 mm. per second.(This gives a calibration of 1 frame per millimeter on the horizontal axis of the strip chart.) By manually following with one arm of the drafting machine the motion of the stem relative to the support, a graph of θ vs frame number was obtained. When the graph for the θ co-ordinate was completed the film was rewound and the procedure was repeated for the ϕ co-ordinate by following the black line on the apple. If the same arm of the drafting machine is used to follow this line, the angle measured would be $(90-\phi)^{\circ}$. If the arm perpendicular to the one used for measuring θ is used, the measured angle is ϕ.

The resulting graphs are on two separate traces with frame number rather than time as the abscissa. To observe the phase relationship between the two co-ordinates, the two graphs were superimposed by tracing both onto a third sheet of paper. To convert the abscissa to a time scale the frame number vs. time relationship must first be determined. It is also necessary to know the forcing frequency vs. time relationship for use in the planned digital computer simulation.

The time vs. frame number relationship is easily obtained from timing marks which are imprinted onto the edge of the film at 1/10 second intervals. By counting the number of timing marks from an arbitrary reference point the time elapsed for any frame number can be evaluated.

The forcing frequency vs. time relationship is obtained by establishing the relationships between film speed and frame number and between frames per forcing cycle and frame number. These relationships can be obtained in graphical form by counting the number of frames between timing marks and by counting the number of frames for each forcing cycle. Knowing the film speed (frames per minute) and the number of frames per forcing cycle at any frame number as well as the frame number vs. time relationship, the forcing frequency at any time can be estimated from

$$\frac{\text{cycle}}{\text{min}} = \frac{\text{frame}}{\text{min}} \bigg/ \frac{\text{frame}}{\text{cycle}}$$

Since the high-speed camera does not photograph at a constant speed, the frame number vs. time relationship is non-linear (figure 10) and thus the time scale of the graphs is also non-linear. The acceleration of the film is slow, however, and the time scale can be considered to be piecewise

Typical Frame Speed Vs. Time Curve for the High-Speed Camera

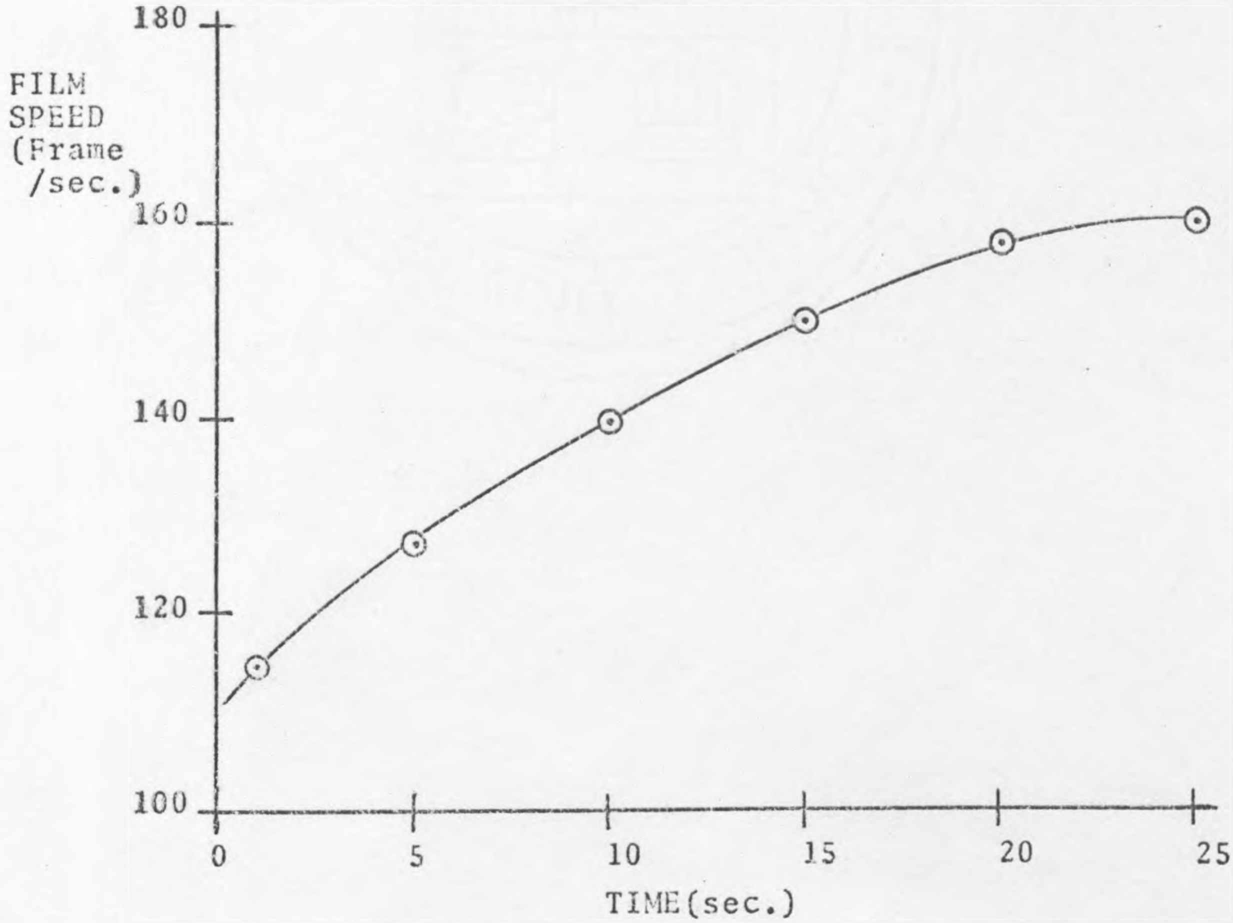


Figure 10

linear for time intervals up to a few cycles with little loss of accuracy (see figure 10).

Computer Simulation of Forced Vibration

A Fortran IV computer program for the IBM 360/65 computer was written to simulate the forced vibration of an apple. The forcing displacements, ξ and η , were assumed to be of the form

$$\xi = A\cos(\omega_o t + \frac{1}{2}\gamma t^2 + \alpha_x)$$

$$\eta = B\cos(\omega_o t + \frac{1}{2}\gamma t^2 + \alpha_y)$$

By taking the second derivatives of the displacement equations the forcing accelerations, $\ddot{\xi}$ and $\ddot{\eta}$, to be used in solving the equations of motion were obtained in terms of time, t, and angular acceleration, γ.

$$\ddot{\xi} = -A(\omega_o + \gamma t)^2\sin(\omega_o t + \frac{1}{2}\gamma t^2 + \alpha_x) + A\gamma\cos(\omega_o t + \frac{1}{2}\gamma t^2 + \alpha_x)$$

$$\ddot{\eta} = -B(\omega_o + \gamma t)^2\sin(\omega_o t + \frac{1}{2}\gamma t^2 + \alpha_y) + B\gamma\cos(\omega_o t + \frac{1}{2}\gamma t^2 + \alpha_y)$$

where A and B are the half amplitude horizontal and vertical displacements, respectively. The initial forcing frequency is ω_o and the initial phase angles of the horizontal and vertical displacements relative to their zero position are α_x and α_y. To simulate as closely as possible the actual forcing conditions the forcing frequency vs. time data as measured from the film is read in by the computer program and is used to calculate the angular acceleration, γ, as a function of time.

The four first-order simultaneous differential equations are solved by the Hamming predictor-corrector method contained in the IBM Scientific Subroutine Package (54). The input consists of the values of fruit parameters M, μ, ℓ, r, S, K, c and I_c ; the values of forcing frequency vs. time data; and the initial conditions $\theta, \phi, \dot{\theta}$ and $\dot{\phi}$. The values af M and ℓ were measured from the fruit which was being simulated. The values of r,S,K, and c were average values calculated from a number of samples. The moment of inertia, I_c, of the fruit about its mass center was calculated from the mass and density of the fruit. The initial conditions were determined from the graphical output of the observed motion.

The output includes printed numerical values of $\theta, \phi, \dot{\theta}$ and $\dot{\phi}$ vs time as well as punched card output of θ and ϕ vs. time values. A program for the Calcomp plotter was written to read the punched data and to plot the results in a graphical form.

Because of the cost of computer simulation, (2 minutes of running time per 30 seconds of simulation), only three of the eight high-speed films were simulated. The film rolls with 0.5 inch horizontal and vertical forcing displacements (rolls A and D) and the roll with two 15 second runs at two different constant forcing frequencies (roll E) were chosen for simulation and for comparison with the predicted motion. A computer simulution was made of the sample used in roll A with zero initial conditions and a constant forcing frequency for several different forcing frequencies. This simulation was used to compare the response due to a gradually increasing frequency with the response due to a constant frequency.

VI. ANALYSIS OF DATA AND RESULTS

Spring Constants

The spring constants calculated from a linear regression, least squares analysis of the torque vs. deflection data are tabulated in Table I for the four weeks preceding harvest. The value of the spring constant K at the fruit-stem junction was evaluated with sufficiently small angles so that the stem does not come in contact with the cheek of the apple. This usually occurs between 30° and 50° of stem deflection relative to the fruit. After the stem comes in contact with the fruit, the apparent spring constant becomes much stiffer. The value of the spring constant S between the stem and the branch was calculated for deflections beyond 90° or until failure occured.

In all cases the correlation coefficient of the torque vs. deflection data was between 0.86 and 1.00 with a mean value of 0.98. The high values of correlation coefficients indicate that the spring constants were linear as was assumed for the mathematical model. It is also of interest to note that in subsequent high-speed films of the vibrating apple, the stem remained rigid and the bending took place at the stem junctions. This observation gives greater confidence in the mathematical representation of the stem by a rigid

Table I

Values of Spring Constants vs. Date (1970)

a) Values of S (ft.-lb./rad.)

Date	Sept.22	Sept.29	Oct.6	Oct.13
	.0040	.0229	.0112	.0128
	.0063	.0126	.0160	.0161
	.0040	.0094	.0090	.0169
	.0170	.0146	.0093	.0112
	.0080	.0073	.0244	.0157
	.0030	.0043	.0112	.0150
Mean	.0071	.0119	.0135	.0146
Std.Dev.	±.0052	±.0065	±.0059	±.0022

b) Values of K (ft.-lb./rad.)

Date	Sept.22	Sept.29	Oct.6	Oct.13
	.0084	.0219	.0067	.0266
	.0061	.0281	.0132	.0169
	.0050	.0219	.0084	.0256
	.0116	.0227	.0112	.0233
	.0376	.0178	.0113	.0249
	.0078	.0071	.0129	.0223
Mean	.0128	.0199	.0106	.0233
Std.Dev.	±.0124	±.0071	±.0026	±.0035

Table I
(continuation)

c) Analysis of Variance Table for Spring Constants S

Source of Variation	Degrees of Freedom	Sums of Squares	Mean Square	F
Amongst Dates	3	$2.01x10^{-4}$	$0.67x10^{-4}$	2.46
Within Dates	20	$5.46x10^{-4}$	$0.27x10^{-4}$	
Total	23	$7.47x10^{-4}$		

Signifigant at 90% level

Not Signifigant at 95% level

d) Analysis of Variance Table for Spring Constants K

Source of Variation	Degrees of Freedom	Sums of Squares	Mean Square	F
Amongst Dates	3	$6.38x10^{-4}$	$2.13x10^{-4}$	3.83
Within Dates	20	$11.11x10^{-4}$	$0.56x10^{-4}$	
Total	23	$17.49x10^{-4}$		

Signifigant at 95% level

Not Signifigant at 97.5% level

rod with a spring at either end.

From the analysis of variance (Table I-c and I-d), the maturity is seen to have a signifigant effect upon the spring constants at the 90% confidence level. The values of K are in general larger than the values of S for the same sample.

Damping Coefficient

The damping factor ζ for 36 samples was calculated from (52)

$$\zeta = \frac{\ln x_1/x_n}{2\pi n}$$

where x_1/x_n is the amplitude ratio over n cycles of oscillation. The damping coefficient was calculated using the measured value of ζ and equations (10) and (11). The calculated dimensionless values of ζ ranged from .022 to .046 with a mean of .033. Thomas(8) reported a range of ζ values for a similar variety, Red Delicious, from .033 to .085. The calculated values of damping coefficients ranged from .00015 to .00058 ft.-lb./rad./sec. with a mean of .00030 ft.-lb./rad./sec.

Even though Thomas reported higher values of the damping factor than those measured, it is believed that even the measured values are higher than the correct value because of an error in measurement with the system of figure 4. Nevertheless, the measured values are still useful in that they give an estimate of the upper bound of the value of the damping factor. The error in measuring ζ is due to the motion of the oscillating apple not remaining in the direction of the cantilever beam deflection. Thus the L.V.D.T. picked up only the component of apple motion in the direction of the beam deflection. The resulting graphical trace of the motion indicated that the amplitude of the motion decayed faster than it actually did. The value of the damping coeffieient at this low level, it was discovered from trial computer simulation, does not greatly affect the predicted motion. Even though the magnitude of the damping coefficient is not important, it is important that some damping be included to indicate the actual transient effects; thus a value for c of .0001 ft.-lb./rad./sec was used. Of course, if a large damping coefficient were used then its effects on the 'steady-state' output would be apparent.

Values of Fruit and Stem Dimensions

The values of stem length and fruit mass were measured from each of the samples used in the vibration and high-speed film tests. The stem length ranged from .075 to .150 ft. with a mean of .102 ft. and standard deviation of .021 ft.; and the mean fruit mass was .0102 slugs with a standard deviation of .00155 slug and a range of extreme values from .0078 to .0144 slugs.

To calculate the moment of inertia I_c of the apple about an axis through the mass center, it is necessary to first calculate the radius R of a sphere having the same volume and weight as the apple. Knowing the mass density ρ (slug/ft.^{3}) and mass M (slug) of the apple, the radius R (ft.) can be calculated from the formula for the volume of a sphere

$$R = \left[\frac{3M}{4\Pi\rho} \right]^{\frac{1}{3}}$$

The average value of the specific gravity at harvest time of Golden Delicious apples is given by Westwood (55) as .81 (or a mass density of 1.57 slug/ft.^{3}). The moment of inertia of a sphere about its center is

$$I_c = \frac{2}{5} MR^2 (\text{slug-ft.}^2)$$

which upon substituting for R becomes

$$I_c = \frac{2M}{5} \left[\frac{3M}{4\Pi\rho} \right]^{\frac{2}{3}} (\text{slug-ft.}^2)$$

Thus, from the measured value of M the moment of inertia is calculated by the computer program for each sample.

The average distance r of the stem-fruit junction from the center of mass of the apple was determined for 12 samples. Vis et al (56) measured the position of the center of mass to be at 0.48 of the height of the apple from the stem end. Using this information and measuring the distance from the stem end to the point of stem attachment, the mean value of r was found to be 0.46 of the equivalent radius R.

Resonant Frequencies

Table II shows the forcing frequency of a 0.5 inch

Table II

Values of Forcing Frequencies (c.p.m.)Causing Resonance vs. Date (1970)

(0.5 inch peak-to peak horizontal stroke)

a) Pendulum Mode

Date	Sept.29	Oct.6	Oct.13
	142	148	145
	138	158	138
	135	142	145
	132	142	145
	138	138	145
	138	138	148
Mean	137	144	144
Std.Dev.	±3	±8	±3

b) Tilting Mode

Date	Sept.29	Oct.6	Oct.13
	360	420	460
	382	480	480
	370	400	420
	347	390	382
	362	432	435
	352	430	432
Mean	362	425	435
Std.Dev.	±13	±32	±34

horizontal displacement at which resonance was observed for the three weeks preceding harvest. Although resonance occurred over a 30-50 c.p.m. range for the pendulum mode and over a 100-150 c.p.m. range for the tilting mode, the tabulated values represent the mid-values of the observed range. It can be seen from Table II that the tilting mode natural frequency is more sensitive to changes caused by maturity than the pendulum mode. The resonance frequency of both modes is seen to increase with maturity.

Table III shows a comparison of the measured values of the pendulum mode natural frequency and the values of forcing frequencies which caused resonance of this mode. Contrary to observations made by Thomas (8) and calculations made by Rand and Cooke (46) the tabulated values of the forcing frequencies which cause resonance are slightly higher than the free vibration natural frequencies. As mentioned previously, it is difficult to observe the motion of the oscillating fruit by eye and the tabulated values are the mid-values of a wide range of frequencies causing resonance. It is quite possible that the maximum amplitude occurs before the mid-point of the frequency range. This could explain why the forcing frequency is slightly higher than the free vibration frequency when, in fact, the opposite results had been expected.

A comparison of the measured free vibration pendulum mode natural frequency with the corresponding calculated frequency is also shown in Table III. The natural frequen-

Table III

Comparison of Measured, Observed and Calculated Pendulum Mode Natural Frequencies (c.p.m.)

	Measured from Period of Free Oscillation	Forcing Frequency which Causes Resonance	Calculated from Original Model(Fig.1)	Calculated from Modified Model(Fig.2)
	146	140	124	143
	138	145	123	141
	138	140	115	131
	130	132	110	124
	156	160	124	142
	134	140	111	126
	136	145	114	130
	135	148	117	132
	132	125	114	129
	141	155	116	132
	126	148	107	121
	127	142	118	135
	133	142	114	129
	146	162	126	146
	130	145	113	128
	150	155	127	147
	141	158	119	137
	138	140	119	135
	152	150	125	145
	140	142	119	137
	143	140	128	149
	144	155	125	144
	135	135	112	128
	160	162	133	155
	142	148	118	135
	141	152	121	139
	136	135	113	129
	148	155	121	139
	144	145	121	139
	141	152	116	132
	152	155	129	149
	150	-	127	147
	143	148	127	147
	150	140	115	130
	143	140	112	127
Mean	141	147	119	137
Std. Dev.	±8	±9	±6	±8

cies are calculated by linearizing the equations of motion of the mathematical model as outlined by Cooke and Rand (1). If the stem-fruit junction is at the edge of the sphere as in the original model (figure 1), there is a 16% difference between the measured and predicted frequencies. If the modified model (figure 2) is used with the same stem length and with the pivot point the measured distance, 0.46 of R, from the center of the sphere, the difference between measured and predicted frequencies is only 3%. Average values of the spring constants were used in the calculations and this may account for the small discrepancies between the observed and calculated values.

Table IV compares the calculated free vibration tilting mode natural frequency with the forcing frequency causing resonance of this mode. Once again it can be seen that the modified model gives better agreement with the observed values than the original model. The observed values of forcing frequencies which cause resonance are expected to be lower than the calculated free vibration natural frequencies on the basis of the calculations of Rand and Cooke (46) which show that the resonance frequency decreases slightly with amplitude for the pendulum mode and appreciably for the tilting mode. The tabulated values of the large amplitude resonant frequencies, however, are slightly larger than the calculated low amplitude natural frequencies. From observations of the high speed films there is reason to believe that these tabulated values may actually include values of the

Table IV

Comparison of Observed and Calculated Tilting Mode Natural Frequencies (c.p.m.)

	Forcing Frequency which Causes Resonance	Calculated from Original Model (Fig.1)	Calculated from Modified Model (Fig.2)
	470	687	476
	455	695	472
	450	581	397
	305	530	370
	460	1103	465
	420	547	382
	-	572	390
	395	1027	409
	-	567	390
	475	598	406
	440	510	356
	360	610	425
	400	580	390
	-	710	492
	385	573	410
	-	743	503
	-	766	520
	-	635	436
	560	640	433
	500	704	488
	-	642	437
	435	748	516
	570	718	490
	500	557	389
	515	829	566
	455	648	435
	500	669	453
	498	563	390
	505	678	457
	425	660	452
	460	591	407
	455	750	518
	-	738	505
	-	743	509
	465	577	396
	-	557	374
Mean	456	668	442
Std. Dev.	±58	±126	±53

second tilting mode which occurs at twice the frequency of the first tilting mode and was mistakenly identified as such. The inclusion of these values in the table would naturally result in a higher average value for the basic resonance frequency than is actually true. Recall, again, that the tabulated values are the mid-points of a wide range but the maximum amplitude may not necessarily be at the mid-point.

Vibration Tests

The vibration tests were conducted in order to observe the effects of different types of forcing displacements upon the dynamic response of the fruit-stem system. For all the forcing displacements, the two basic natural frequencies were found, and the response of the fruit-stem system was seen to vary according to the type of forcing displacement. The observations are qualitative in nature and are corroborated by the high-speed films.

For a .5 inch peak-to-peak horizontal forcing displacement the following forms of response were observed as the forcing frequency increased:

1. Pendulum mode- the stem and the apple oscillate in phase. Observed in a range from 115 to 170 c.p.m.
2. Circular- the stem and apple describe a circular path about the support point in a manner similar to a spherical pendulum. This motion is observed in some cases at the high frequency end of the pendulum mode and exists in a very short frequency range.

3. Beat- the stem and fruit exhibit a 'stop and go'type of motion. The fruit-stem system may be stationary for a brief period and then undergo a few cycles of oscillation before again becoming stationary. The oscillation of the stem and the fruit may be in phase at some times and out of phase at other times. This form of response is observed from the end of the perdulum mode to the beginning of the tilting mode in a range from 170 to 250 c.p.m. This type of motion most likely is composed of a combination of oscillations whose frequencies are the basic natural frequencies of the fruit-stem system.
4. Tilting- the stem and fruit oscillate 180° out of phase. This form of oscillation starts out at a low amplitude them quickly increases in amplitude as the frequency increases. A further increase in forcing frequency causes the response amplitude to gradually decrease. In some cases, at the height of the response the center of mass of the fruit is observed to behave as a spherical pendulum about the stem-fruit junction.

This observed motion is similar to the response for oranges and grapefruit observed by Rumsey (38) and the response observed by Diener et al (13) for apples. One exception is that the twisting mode which was observed by Diener was not observed in these tests.

The main effect of increasing the horizontal stroke length is to increase the range of frequencies over which the pendulum mode occurs and to increase the amplitude of the response. For a 0.5 inch stroke the pendulum mode occurs over a range of about 50 c.p.m. whereas for a 1.5 inch stroke the range is increased to about 90 c.p.m. The range of the tilting mode is apparently unaffected by an increase in stroke length; however, the amplitude of tilting increases.

The most noticeable difference between the horizontal and vertical shaking directions is the reduced amplitudes of vibration for vertical oscillation. The frequency ranges over which the pendulum and tilting modes occur are also reduced. Instead of the beat type of response which was noticed between modes for the horizontal forcing displacement, the fruit-stem system remains remarkably stationary relative to the support point for the vertical forcing displacement. Another response peculiar to the vertical forcing displacement (as predicted by Studer (21) and Cooke and Rand (1)) is the occurrence of a second pendulum mode at twice the frequency of the first pendulum mode. The response frequency is only one-half of the forcing frequency, or the same frequency as the first pendulum mode. The second pendulum mode is usually of longer duration and of greater amplitude than the first pendulum mode. Following the second pendulum mode, the tilting mode is observed followed by stable motion. Increasing the vertical stroke length increases the amplitude of response.

Changing the direction of straight line motion from a horizontal to a vertical shaking direction causes the response of the fruit-stem system to be a combination of the characteristic responses observed for purely horizontal and vertical oscillations. For instance, as the angle is increased from horizontal to vertical, the second pendulum mode appears and is of longer duration for larger angles. The beat phenomenon, which is characteristic of the horizontal oscillation, becomes less pronounced as the angle is increased and, instead, the motion is more nearly stable.

When the phase angle between the two cranks on the vibrator is changed to generate various forms of elliptical motion, there is no appreciable effect on the response over the response caused by straight line motion.

When the forcing frequency is increased beyond 500 c.p.m. it becomes increasingly difficult to observe the response of the fruit-stem system. However from the high-speed motion pictures a second tilting mode of larger amplitude than the first one was observed at frequencies over 500 c.p.m.

Even though it was not the purpose of the vibration tests to cause fruit detachment, when detachment did occur the point of separation and the mode of oscillation at the time was noted. In almost all cases detachment occurred at the stem-branch junction when oscillating in the pendulum mode and at the stem-fruit junction when oscillating in the tilting mode.

Comparison of Observed and Predicted Motion

The most convenient method of comparing the actual with the predicted motion was judged to be a graphical comparison. By using a graphical representation of the response of the fruit-stem system, it is easy to compare the frequency and waveforms of the observed and predicted responses. The phase relationships and relative amplitudes of the two coordinates, for use in making an estimate of stresses, are also readily observed from a graphical representation. Table V shows the values of physical data and natural frequencies for each sample used in the high-speed film tests.

Figures 11 and 12 show the observed responses for the high-speed film rolls A, D and E along with the corresponding responses predicted by the computer simulation. As explained previously, the time scales for the observed motion graphs are not linear due to the increasing camera speed and one should be aware of this when comparing responses at a particular time. The forcing feequency corresponding to each time division is also included.

Figures 11-a and 11-b can be used to compare the observed and predicted motion for a 0.5 inch peak-to-peak horizontal forcing displacement. The simulated response agrees closely with the observed response, and the times at which the pendulum and tilting modes appear agree almost exactly. The types of characteristic motion described previously for a 0.5 inch horizontal forcing displacement are displayed by both graphs. Note that even the flattened portions at the peaks of the ϕ curve at the pendulum mode of the observed motion are also evident on the simulated curve.

The actual and simulated response for a 0.5 inch peak to peak vertical forcing displacement are shown in figures 11-c and 11-d, respectively. Both graphs exhibit an initial small amplitude pendulum mode oscillation followed by a stable region. A second pendulum mode oscillation of larger amplitude and longer duration than the first is observed at approximately twice the first pendulum mode frequency. The response frequency is one-half the forcing frequency. Following the second pendulum mode a tilting mode is observed for the actual case;whereas,the predicted motion is stable. In order for the tilting mode to occur for vertical displacement, there must be some initial deflection of the fruit-stem system. Apparently the initial deflection was not large enough to cause the tilting mode to occur in the simulated case.

The steady state response for two constant forcing frequencies is compared with the predicted response in figure 12. Figure 12-a is the actual steady-state response when the forcing frequency is near the pendulum mode natural frequency and figure 12-b shows the corresponding predicted response. The predicted motion of the θ and ϕ co-ordinates is similar to the observed motion,having a slightly larger amplitude and higher frequency than the observed motion.

Table V

Physical Data and Natural Frequencies of Samples Used in High-Speed Film Tests

Roll	M (slug)	L (ft.)	Measured Pendulum Mode Natural Frequency (c.p.m.)	Calculated Pendulum Mode Natural Frequency (c.p.m.)	Calculated Tilting Mode Natural Frequency (c.p.m.)
A	.00906	.1250	133	128	330
B	.00973	.0958	153	133	430
C	.00906	.1250	133	128	330
D	.00973	.0958	153	133	430
E	.00779	.1083	138	154	430
F	.00908	.1416	130	133	350
G	.00908	.1416	130	133	350

Spring Constants S = .0146 ft.-lb./rad.
K = .0233 ft.-lb./rad.
Damping Coefficient c = .0001 ft.-lb./rad./sec
Stem Mass u = 0 slug

Figure 11

Comparison of the Observed and Simulated Response of the Golden Delicious Apple Fruit-Stem System Due to a Gradually Increasing Forcing Frequency.

a) Observed motion for a 0.5 inch peak to peak horizontal forcing displacement(Film Roll A)

b) Simulated motion for a 0.5 inch peak to peak horizontal forcing displacement

c) Observed motion for a 0.5 inch peak to peak vertical forcing displacement(Film Roll C)

d) Simulated motion for a 0.5 inch peak to peak vertical forcing displacement

Note:

1. The observed motion was measured with high-speed photography for an unaltered fruit and stem cut from the tree at harvest time with a portion of branch intact.

2. The time axes for the graphs of the observed and simulated motion are not linear.

3. The values on the frequency axis refer to the forcing frequency at the corresponding time on both graphs.

4. The θ co-ordinate indicates the bending of the stem relative to the branch; $\phi-\theta$ indicates the bending of the stem relative to the fruit. The values of θ and $\phi-\theta$ are both maximum when the motion of θ and ϕ are in phase (pendulum mode) and when they are 180° out of phase (tilting mode). However the value of θ is greater than $\phi-\theta$ at the pendulum mode. The opposite is true for the tilting mode.

5. See table V for values of fruit mass, stem length, spring constants, damping coefficients, and calculated natural frequencies.

Figure 11

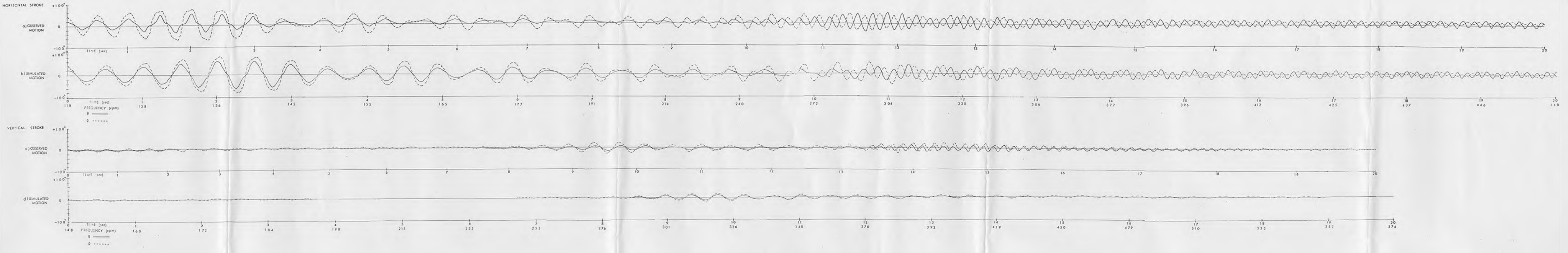

HORIZONTAL STROKE
a) OBSERVED MOTION
b) SIMULATED MOTION
TIME (sec)
FREQUENCY (cpm)
VERTICAL STROKE
c) OBSERVED MOTION
d) SIMULATED MOTION

Figure 12

Comparison of the Observed and Simulated Response of the Golden Delicious Apple Fruit-Stem System Due to Steady Forcing Frequencies Near the Natural Frequencies for a 0.5 Inch Peak to Peak Horizontal Forcing Displacement

a) Observed response for forcing frequency (140 c.p.m.) near the pendulum mode natural frequency (Film roll D)

b) Simulated response for forcing frequency (140 c.p.m.) near the pendulum mode natural frequency

c) Observed response for forcing frequency (400 c.p.m.) near the tilting mode natural frequency (Film roll D)

d) Simulated response for forcing frequency (400 c.p.m.) near the tilting mode natural frequency

See Note on Figure 11

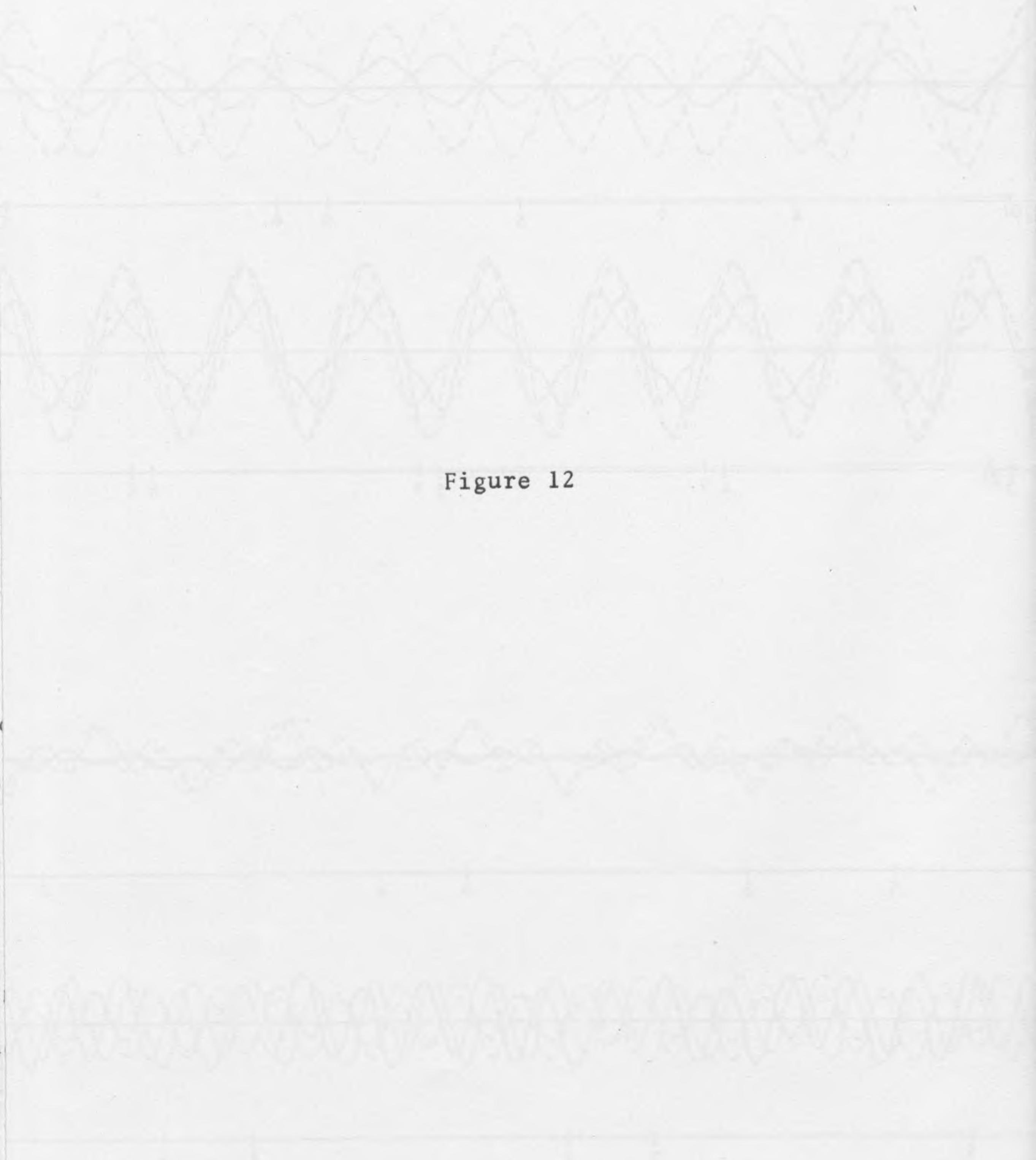
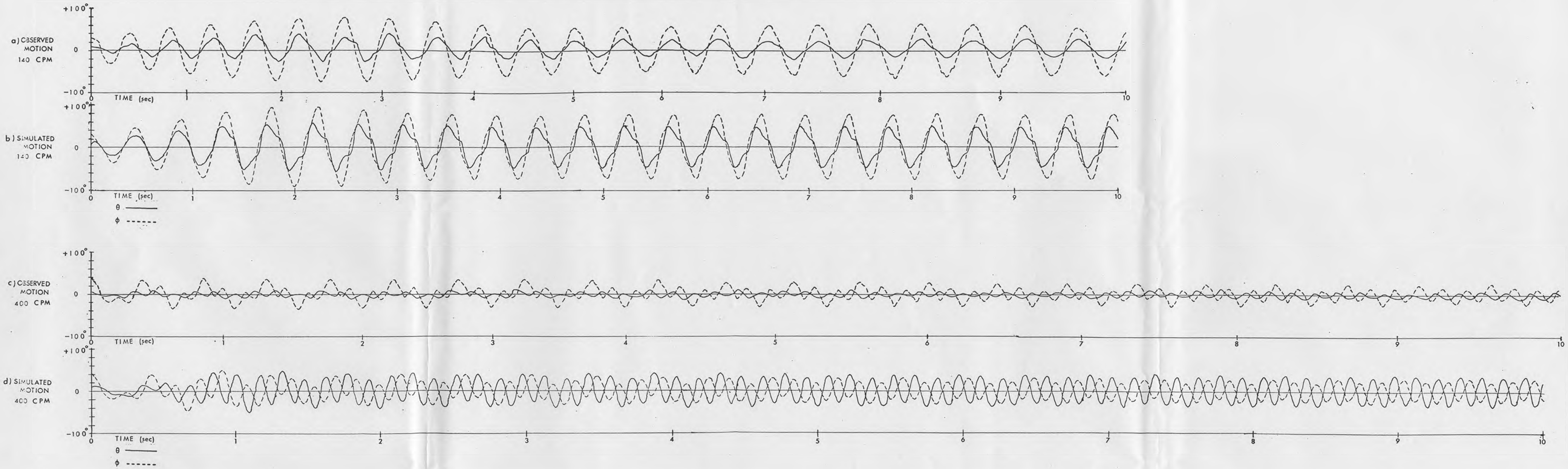


Figure 12


+100°
a) OBSERVED
MOTION
140 CPM
0
-100°
TIME (sec)
0 1 2 3 4 5 6 7 8 9 10
+100°
b) SIMULATED
MOTION
140 CPM
0
-100°
TIME (sec)
0 1 2 3 4 5 6 7 8 9 10
θ
ϕ
+100°
c) OBSERVED
MOTION
400 CPM
0
-100°
TIME (sec)
0 1 2 3 4 5 6 7 8 9 10
+100°
d) SIMULATED
MOTION
400 CPM
0
-100°
TIME (sec)
0 1 2 3 4 5 6 7 8 9 10
θ
ϕ

This slight discrepancy is probably caused by using average values of spring constants in the calculations of the motion rather than the actual values.

The observed and calculated response when the forcing frequency is near the tilting mode natural frequency is illustrated in figures 12-c and 12-d, respectively. The calculated motion of the ϕ co-ordinates is seen to be in close agreement with the observed motion; however, the calculated motion of the θ co-ordinate differs by about 40° at the peaks from the observed motion. The motion of the fruit-stem system near the tilting mode natural frequency is highly sensitive to the values of the spring constants. The difference between the actual values and the average values used in the calculations may have been sufficient to cause this difference in response. Note that only one or two cycles of oscillation are necessary before the response reaches steady-state.

The response caused by the gradually increasing forcing frequency is compared with the simulated response for a constant frequency in figures 13-16. The motion of the sample used in film roll A was simulated for zero initial conditions at constant forcing frequencies of 122,180,318 and450 c.p.m. and a few cycles of the resulting steady-state motion was compared with the predicted and the observed motion for increasing frequency at these particular forcing frequencies. These four frequencies were chosen because the motion represented is typical of four distinct types of

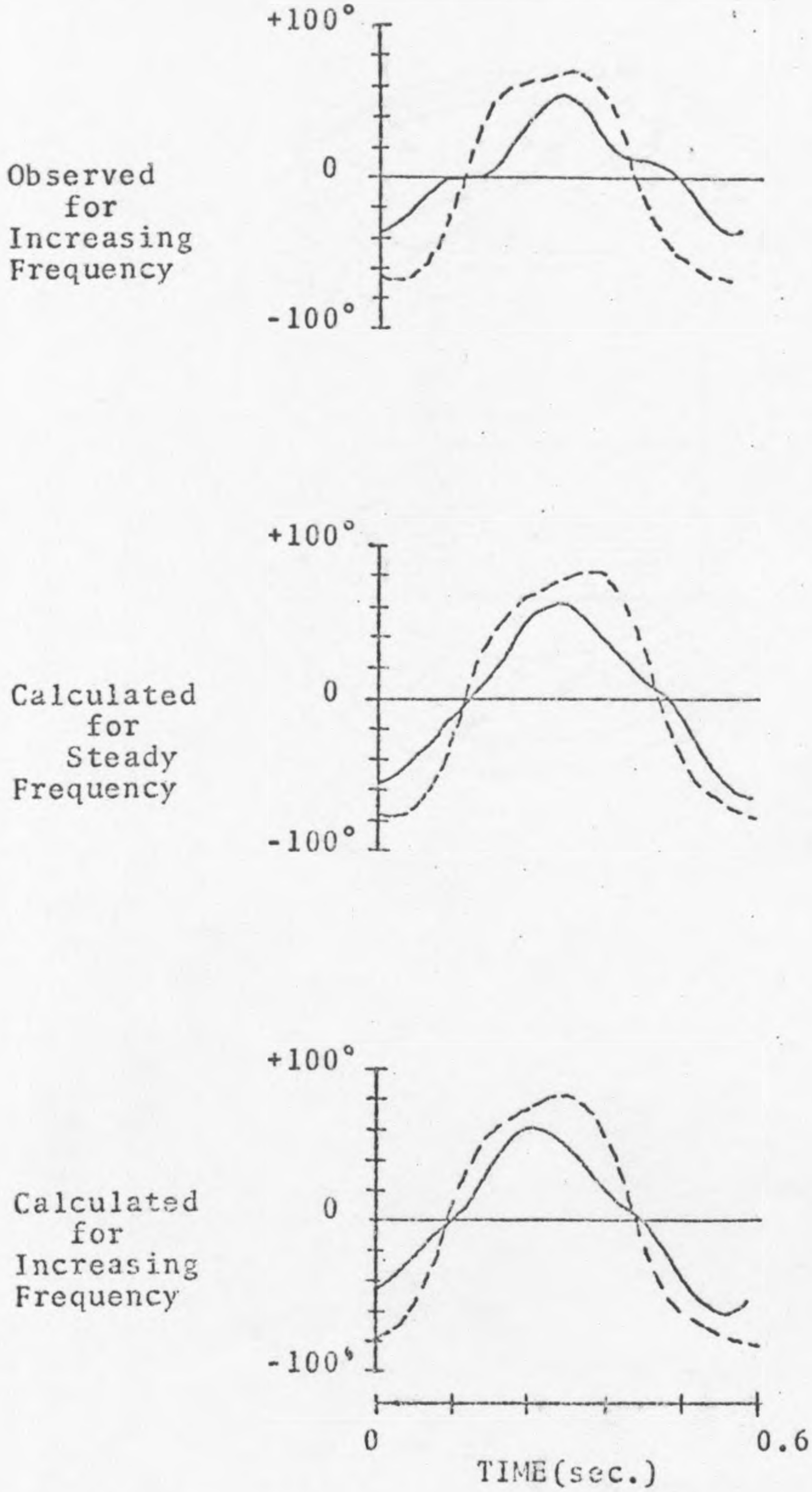


Figure 13 Response at 122 c.p.m.
------ ϕ (0.5 inch peak to peak horizontal stroke)
——— θ

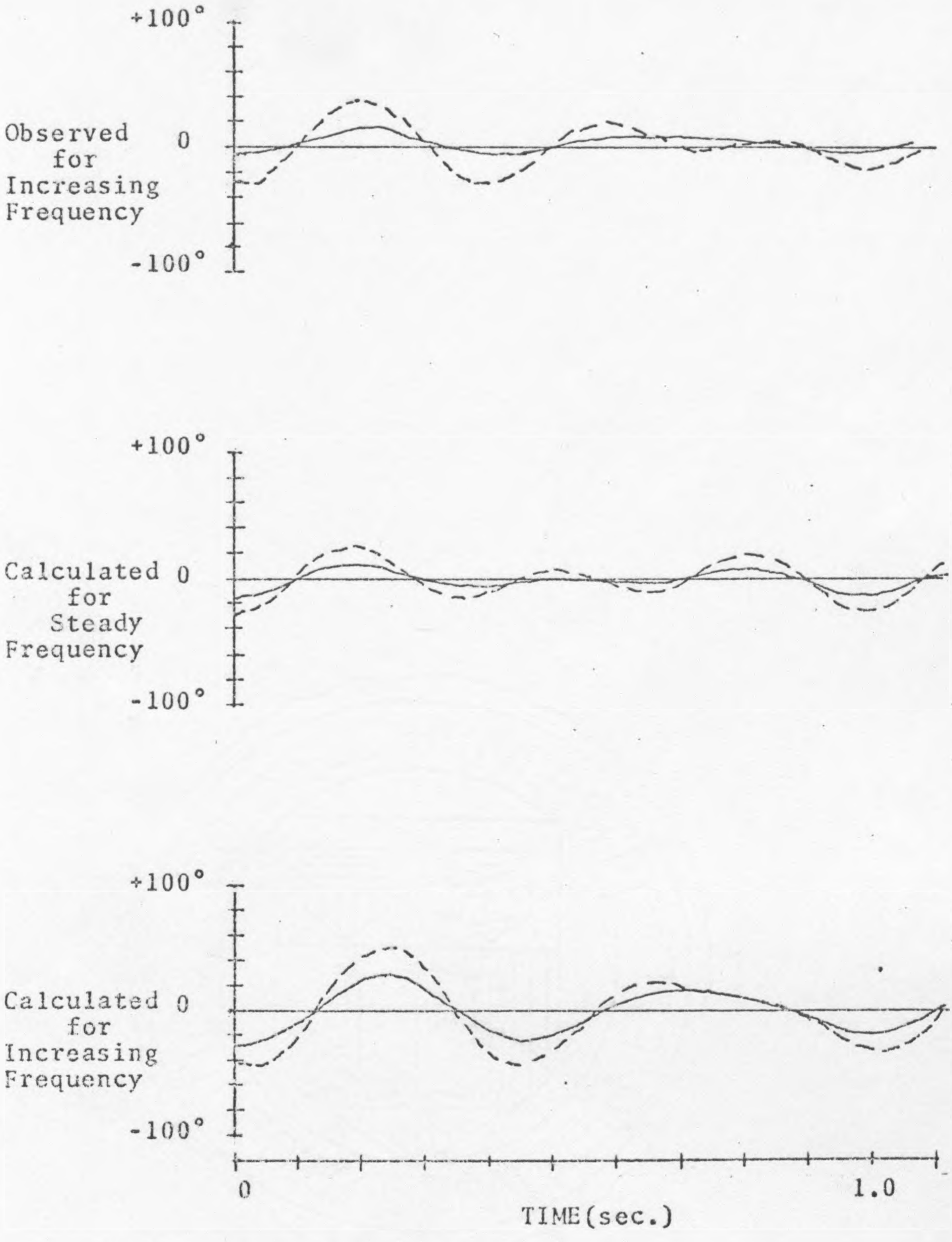


Figure 14 Response at 180 c.p.m.
------ ϕ (0.5 inch peak to peak horizontal stroke)
———— θ

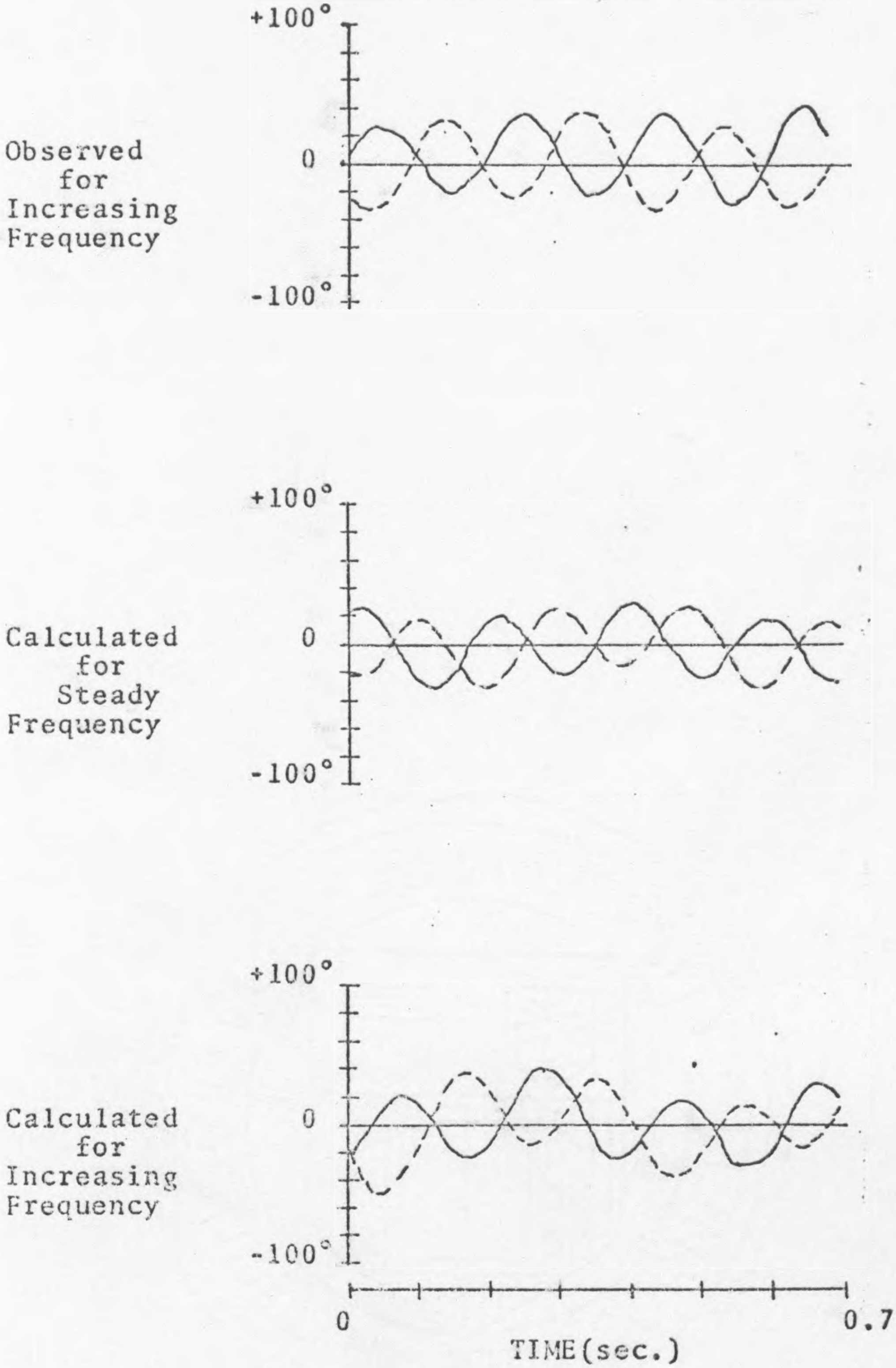


Figure 15 Response at 318 c.p.m.
------ ϕ (0.5 inch peak to peak horizontal stroke)
———— θ

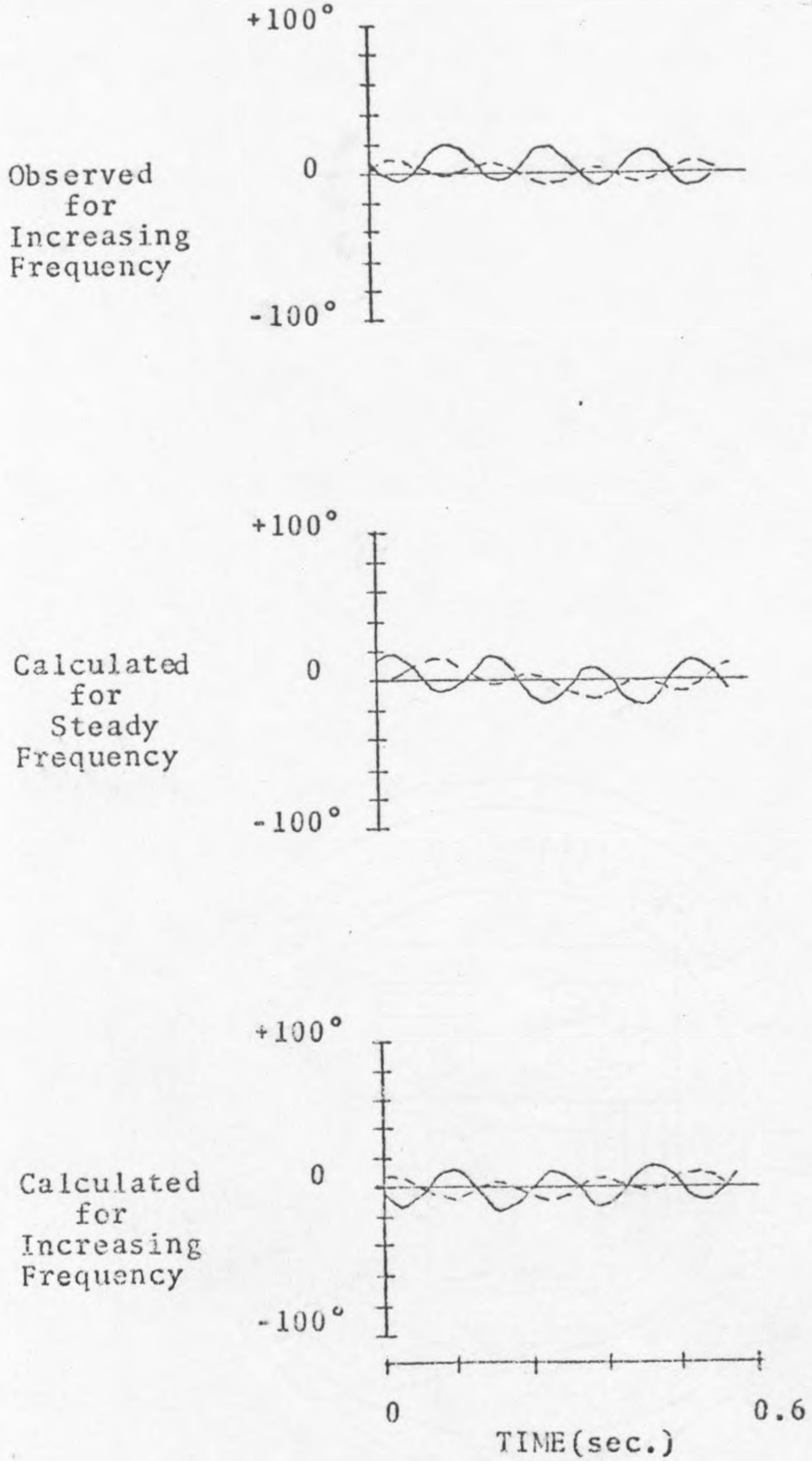


Figure 16 Response at 450 c.p.m.
------ ϕ (0.5 inch peak to peak horizontal stroke)
______ θ

responses; pendulum, beat, tilting and relatively stable motion. As can be seen from these diagrams there is little difference between the response for the increasing forcing frequency and for the steady forcing frequency at a particular frequency.

The motions measured from the remaining high speed films not used for comparison with the simulated motion are shown in the Appendix.Graph number 1 represents the motion for a 0.5 inch peak-to-peak stroke at a 45° angle with the horizontal (roll F) and graph number 2 represents the motion caused by a corresponding circular displacement(roll G). It can be seen that the response due to both forcing functions is quite similar,with the pendulum and beat type of motion being evident in both cases. The second pendulum mode is also discernible; however, the first tilting mode appears at approximately the same frequency and the displayed motion is probably a superposition of the two modes. Following a period of relative stability, a large amplitude tilting motion is observed at approximately twice the frequency of the first tilting mode. The frequency of response is one-half the forcing frequency.

Graph number 3 of Appendix A illustrates the effect on the motion caused by starting the vibrator at a high frequency and gradually reducing to a low frequency (roll C). In certain non-linear systems it is known that the initial conditions and the direction of changing frequency has a major effect on the response of the system. By comparing graph number 3 with figure 11-a it can be seen that at a particular forcing frequency the response caused by gradually increasing the forcing frequency is similar to the response caused by gradually reducing the forcing frequency.

The effect of increasing the horizontal stroke length is seen from graph number 4 of the Appendix which shows the response for a 1.0 inch peak to peak horizontal stroke (roll B). The increased amplitude of oscillation and the reduction of the beat type of motion is noticed. The amplitude of oscillation of the ϕ co-ordinate remains relatevely large and constant, and the amplitude of the θ co-ordinate increases in amplitude at the not-well-defined tilting mode.

These filmed observations were made from single samples and, of course, the motion of the particular sample may not be typical for all samples in general. There is, however, reason to believe that the filmed responses were typical because similar responses were observed by eye for the previous vibration tests.

Stem Stresses

A qualitative judgement of stresses developed in the stem can be made from the motion of the fruit-stem system which has been measured and recorded on the graphs of figures 11,12 andthe Appendix. The bending stresses at the stem-branch junction depend upon the angle of deflection θ, and the bending stresses at the stem-fruit junction depend upon the angle of deflection $\phi-\theta$. To observe the effects of the various forcing frequencies and displacements upon the bending stresses, it is necessary to observe the magnitudes of these deflections. In all cases, the magnitudes of θ and of $\phi-\theta$ observed from the graphs are both greatest for the pendulum and tilting modes of oscillation. The magnitude of θ is greater than the magnitude of $\phi-\theta$ at the pendulum mode and the opposite is true at the tilting mode. This results in maximum bending stresses at the stem-branch junction for the pendulum mode and at the stem-fruit junction for the tilting mode as predicted by Cooke and Rand (1).

An approximation of the shear force in the stem can be made from equation (15). By neglecting the damping term and by making the substitution

$$S = \frac{2}{3} K$$

(approximately true when mean values are compared, Table I) the shear force is

$$Q = \frac{K}{\ell} (\phi - \frac{5}{3}\theta)$$

By inspection of the graphs it can be seen that $(\phi-\frac{5}{3}\theta)$ is smallest at the pendulum mode and largest at the tilting mode. Thus the stresses due to the shear force are maximum for the tilting mode. Unlike the bending stresses, the stresses due to shear forces are equal at both stem junctions.

An analysis of the tensile force in the stem can be made by studying equation (12)

$$\tau = M[(g+\ddot{\eta})\cos\theta - \ddot{\xi}\sin\theta + \ell\dot{\theta}^2 + r\dot{\phi}^2\cos(\phi-\theta) + r\ddot{\phi}\sin(\phi-\theta)] \quad (12)$$

In general, large values of θ and ϕ will result in large values of $\dot{\theta}, \dot{\phi}, \ddot{\theta}$, and $\ddot{\phi}$. For a given forcing frequency, the large angular velocities and accelerations of the fruit-stem system relative to the support will result in large tensile forces. Tensile forces are also induced due to the forcing acceleration of the support. Since the tensile forces are due not only to the forcing displacement but are also due to the response of the fruit-stem system, the maximum tensile force is not necessarily induced by large forcing accelerations but is caused by forcing accelerations which result in large amplitudes of oscillation. For instance, it is possible that tensile forces induced by a forcing frequency of 400 c.p.m. may be smaller than tensile forces induced by the same amplitude of displacement at a forcing frequency of 300 c.p.m. if the lower forcing frequency causes resonance.

VII. DISCUSSION

For ease of experimentation, the samples which were cut from the tree and used for this analysis were chosen because they hung vertically and were attached to a rigid branch, much as depicted by the model. This configuration is not always the case under field conditions. The stem may grow out horizontally from the branch or the fruit may be supported on branches not much stiffer than the stem itself. Nearby branches, leaves and other fruit may severely restrict the motion of the fruit. While the usefulness of this analysis is of limited value for such cases, it does serve as a basis for insight into the more complex situation.

The analysis does not account for impulse forces, either applied by a shaker or caused by buckling of the stem. Even though the shaking direction and stroke length were controlled for the laboratory tests,(except for some observed twig deflections)the stroke length and the shaking direction of the point of support of the fruit are not controlled when shaking with mechanical tree shakers. The amplitude of the resultant forcing displacement depends upon the location of the particular fruit relative to the attachment location of the shaker. The small amplitude displacement was used in the laboratory tests merely to prolong the shaking time before detachment occurred in order to observe the response. The displacement of the support point under field conditions would most likely be greater than 0.5 inches. The effect of a greatly increased stroke length is not known, but is believed to result in broader regions of resonance. The arbitrary shaking direction under field conditions indicates that the motion observed on film rolls F and G (see description on pages 37-38) most closely represents the actual forcing conditions.

The motion of the fruit-stem system at the second tilting mode was observed to be very violent and fruit damage could easily result from this motion. Shaking at twice the tilting mode natural frequency is not reccommended for harvesting apples for this reason and also because separation most likely will occur without the stem. Present day harvesters employ a frequency of 400-600 c.p.m. which is the range of frequencies necessary to excite the second tilting mode. To cause the large amplitudes which are necessary for separation, and still not cause the violent type of motion which was observed at the second tilting mode, shake at twice the pendulum mode naturalfrequency(250-350c.p.m.).At this frequency the range of frequencies which cause resonance of the pendulum mode is the widest, thus a greater liklihood of inducing resonance exists. However, for apples, the first tilting mode was observed to also occur near this frequency and it may not be possible to selectively harvest the fruit with stems intact.

At present, research is being conducted with chemical looseners which weaken the abscission zone and thus cause easier fruit separation. The result of such chemicals would be to weaken the spring constant S at the stem-branch junction and to lower the pendulum mode natural frequency. This would cause a wider separation between the occurrence of the second pendulum mode and the first tilting mode, thus improving the chances of selectively harvesting the fruit with stems.

The stiffening of the spring constant K, due to the stem contacting the cheek of the apple, was not accounted for in the model and its omission had no apparent effect on the accuracy of the simulation. However, at large deflections, damage may occur under dynamic conditions due to the contact of the stem and the apple. This suggests that the tilting mode motion should be avoided if fruit damage is to be reduced.

From the analysis of stem forces it can be seen that tensile and shear forces act equally at both joints and only stresses caused by bending are different. The stresses caused by shear forces are greatest when the fruit is oscillating in the tilting mode and the stresses caused by tensile forces, for a given forcing function, are greatest when the amplitudes of oscillation are greatest (ie. at the pendulum and tilting mode natural frequencies).

In addition to the forces developed in the stem, the relative strengths of the stem joints should be considered in determining where failure will occur and whether detachment occurs with or without stems intact. For instance, if the stem-fruit junction is much weaker than the stem-branch junction, detachment could occur at the weak lower junction due to tensile forces even though the analysis would otherwise indicate failure at the upper junction for pendulum mode oscillation.

The response of the fruit-stem system to forcing frequencies below 100 c.p.m. and above 600 c.p.m. was not studied. Forcing frequencies below 100 c.p.m. are not expected to be effective in removal of apples. Above 600 c.p.m., violent motion of the fruit-stem system which is likely to cause fruit damage is expected. At these high frequencies, more power is required to induce vibration of the tree limbs than would be required at lower forcing frequencies which are nearer the resonant frequency of the limb (about 140 c.p.m.)

An extension of this study should include a Fourier analysis of the calculated and observed motion in order to express the response explicitly as a function of time for a particular forcing function. Numerical values for the stem forces would be desirable for comparing the relative effectiveness of tensile, shear and bending stresses as well as determining more closely the dependence of the shear and

and tensile forces upon the forcing frequency. Not considered in this analysis was the shaking time necessary to cause failure. An experimental evaluation of the shaking time vs. forcing frequency and amplitude relationship would be desirable.

This study applies to the Golden Delicious variety of apple; the model, however, can be used advantageously for making calculations for any variety upon determination of the relevant physical parameters.

VIII. CONCLUSIONS

This analysis has shown that the model as depicted in figure 2 is an excellent representation of the fruit-stem system of the apples used in this study. It can be used to closely predict the motion, not only at the natural frequencies, but for any forcing frequency. Stem forces caused by the motion of the fruit-stem system can be calculated from the model.

Observations of the response of the fruit-stem system to forced vibrations have shown that the response is not random but is highly predictable and repeatable. The form of the response is useful in determining the correct shaking frequency.

For the Golden Delicious variety of apples, the desired forcing frequency is 250-350 c.p.m. For any variety in general, the desired forcing frequency is twice the pendulum mode natural frequency. Because of variations in the physical parameters of the fruit, the forcing frequencies which cause the pendulum mode-motion to occur are distributed over a wide range. To induce resonance for all the fruit, it may be desirable to gradually increase the forcing frequency over this range. By increasing the frequency instead of decreasing it, the pendulum mode is induced before the tilting mode and the chance of causing detachment without stems is reduced.

The equation for the tensile force in the stem shows that the largest tensile force is not necessarily caused by the largest forcing acceleration but also depends upon the motion of the fruit-stem system relative to the support point.

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APPENDIX

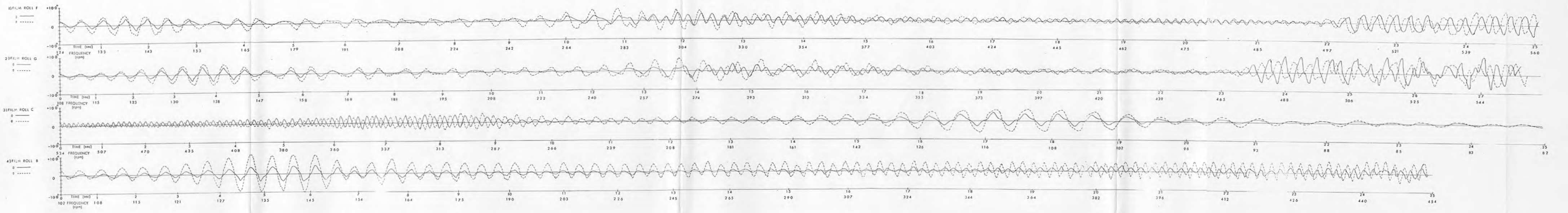
Figure 17

Graphs of the Observed Response of the Golden Delicious Fruit-Stem System for

1) Film Roll F - 0.5 inch peak to peak linear stroke at 45° from the horizontal
2) Film Roll G - 0.5 inch peak to peak circular stroke (phase angle=90°)
3) Film Roll C - 0.5 inch peak to peak horizontal stroke, starting at high frequency and gradually reducing to low frequency
4) Film Roll B - 1.0 inch peak to peak horizontal stroke

See Notes on Figure 11

Figure 17


1) FILM ROLL F
+100
0
-100
TIME (sec)
FREQUENCY (cpm)
124 133 143 153 165 179 191 208 224 242 264 283 304 330 354 377 403 424 445 462 475 485 497 521 539 560
2) FILM ROLL G
+100
0
-100
TIME (sec)
FREQUENCY (cpm)
108 115 123 130 138 147 158 169 181 195 208 222 240 257 274 293 313 334 353 373 397 420 439 463 488 506 525 544
3) FILM ROLL C
+100
0
-100
TIME (sec)
FREQUENCY (cpm)
534 507 470 435 408 380 360 337 313 287 266 239 208 181 161 142 126 116 108 102 98 93 88 85 83 82
4) FILM ROLL B
+100
0
-100
TIME (sec)
FREQUENCY (cpm)
102 108 115 121 127 135 145 154 164 175 190 203 226 245 265 290 307 324 344 364 382 396 412 426 440 454