

A MODEL FOR THE INFORMATION CONTENT OF EARNINGS ANNOUNCEMENTS

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Abstract

A Bayesian model is presented for the information content of earnings announcements. The posterior distribution of the payoff parameter, the final liquidating dividend, is derived. The relative precision of earnings is treated by considering the coefficient of earnings in the Bayes estimator of the payoff parameter. The magnitude of belief revision is examined. Interpretations of the results are given.

KEY WORDS: Bayesian estimation; Belief revision; Correlated errors; Relative precision of earnings.

## 1. INTRODUCTION

The information content of a firm's earnings announcements is defined to be the extent of post-announcement revision in the beliefs of market agents concerning firm value. Many empirical studies have examined the issue of the apparent information content of annual or quarterly earnings announcements. Since changes in beliefs are generally unobservable, the tradition in these studies has been to infer a change in beliefs from an observed change in post-announcement price.

Ball and Brown (1968) showed that considerable anticipatory price movement takes place before the announcement of annual earnings. Their results, which have been replicated by numerous subsequent studies for both annual and quarterly earnings, suggest that the information content of annual or quarterly earnings tends to be preempted by more timely information sources. The Bayesian interpretation of such empirical results is as follows. The existence of interim information from nonaccounting sources provides an improved estimate of the payoff parameter, which in a multiperiod world is the vector of all future firm dividends. As the amount of interim information increases, the resulting estimate of future firm dividends becomes increasingly accurate. In the limit, as future firm dividends are known with near certainty, the information value of some marginal accounting signal, such as annual or quarterly earnings, approaches zero.

Beaver (1968) showed that further price movement occurs in the week of announcement of annual earnings. This result, which has also been replicated many times for both annual and quarterly earnings, suggests that

there is still some residual information content (that is, belief revision) associated with the release of annual or quarterly earnings.

An important issue, both theoretically and empirically, is the identification of determinants of earnings information content. Several recent empirical studies have identified firm attributes that are associated with the extent of post-announcement price movement. Grant (1980) showed that post-announcement price movement is, on average, greater for over-the-counter stocks (which tend to be smaller firms) than for listed stocks. Atiase (1980) showed that post-announcement price movement is, on average, greater for smaller firms than for larger firms. These authors interpret their findings as evidence that the (cumulative) precision of prior information is systematically greater for larger firms. McNichols and Manegold (1982) showed that price movement subsequent to the release of annual earnings is, on average, greater for firms with no quarterly earnings reports than for firms with quarterly earnings reports, a result predicted by the theoretical work of Ohlson (1979) discussed below.

As Lev and Ohlson (1982) pointed out in a recent survey of market-based empirical research, better theory is required in order to interpret existing empirical findings such as the above and to develop new empirical questions. This paper develops a Bayesian model relating attributes of the market agents' prior information set to post-announcement belief revision and, consequently, to post-announcement price reaction. The payoff parameter of interest in the model is the final liquidating dividend of the firm.

In a recent theoretical study with a similar objective, Holthausen and Verrecchia (1981) demonstrated that the variance of post-announcement price reaction increases as the precision of the final earnings report increases,

and decreases as the precision of prior beliefs or of the interim earnings signal increases. Precision is defined to be the reciprocal of the variance of the information random variable. The payoff parameter of interest is the final liquidating dividend of the firm. Interim information is represented by an interim earnings release, which precedes both the final earnings announcement and the announcement of the final liquidating dividend. Holthausen and Verrecchia modeled interim and final earnings announcements as having a bivariate normal distribution; they showed that the variance of post-announcement price reaction is greater when the interim and final earnings variables are independent than when they are positively correlated. The interim information signal in Holthausen and Verrecchia's model can be interpreted as a cumulative signal, and their precision of interim earnings can be interpreted as the cumulative precision of all prior interim information.

The model treated in this paper is an extension of the model of Holthausen and Verrecchia. The information set consists of multiple interim information signals, rather than just one; these signals represent information from various nonaccounting sources and a single earnings release. The increased dimensionality allows various attributes of the interim information covariance structure to be analyzed as separate theoretical determinants of the information content of an earnings announcement. We focus on the marginal importance of a firm's earnings release, relative to the importance of information from nonaccounting sources. We demonstrate in the model that the weight attached to earnings in forming posterior beliefs depends upon the following theoretical factors: (1) the precision of earnings, (2) the precision of individual interim signals from nonaccounting sources, (3) the number of interim

signals observed, (4) the correlation between nonaccounting interim signals observed, and (5) the correlation between the interim signals and earnings.

A related theoretical model was given by Ohlson (1979), who demonstrated that the variance of post-announcement price reaction to annual earnings is greater in a coarser disclosure environment (no quarterly earnings reports preceding the annual earnings report) than in a finer disclosure environment (quarterly earnings reports preceding the annual earnings report). Ohlson's interim signals can be interpreted more generally to be all prior interim information. In effect, his model demonstrates that the information content of earnings depends upon the fineness of prior information, a result consistent with our model.

Our model is similar to the models contained in Ohlson (1979) and Holthausen and Verrecchia (1981) in several ways: the models assume homogeneous beliefs; they focus on a single firm; they adopt a partial equilibrium setting - in particular, the alternative investment opportunities of investors are exogenous to the models, and the production and financing decisions of the firm are assumed to be fixed prior to the introduction of information; and the cost of information and the decision to acquire information are exogenous to the models.

We do not model the price formation process directly, unlike the models of Ohlson (1979) and Holthausen and Verrecchia (1981). However, our results extend easily to price reaction if assumptions about agents' preferences similar to those of Holthausen and Verrecchia (1981) and Richardson (1983) (negative exponential utility functions) are made. As the weight attached to earnings in posterior beliefs increases, *ceterus paribus*, price reaction increases also, as Richardson (1983) demonstrated.

The setting of our model is a single-period one. The sequence of interim information arrival from nonaccounting sources does not assume an important role, and the number of such signals can be interpreted as sample size. Earnings are announced by the firm subsequent to the observation by investors of interim nonaccounting signals but prior to the announcement of the liquidating dividend at the end of the period. The draw from nature of the liquidating dividend is assumed to occur at the start of the period but is unknown to investors until the end of the period. The purpose of the model is to analyze the weight attached to earnings in posterior beliefs about future firm dividends, a task for which one period is sufficient.

The remainder of the paper is organized as follows. The model is presented in Section 2. Results on posterior distributions and Bayes estimators of the payoff parameter are given in Section 3. The relative precision of earnings is treated in Section 4, using the coefficient of earnings in the Bayes estimator of the payoff parameter as an indicator of the importance of earnings information. The magnitude of belief revision is examined in Section 5. Interpretation of the results is discussed and conclusions are drawn in Section 6.

## 2. FORMULATION OF THE PROBLEM

Let random variables  $Y_1, \dots, Y_n$  represent the interim information signals from nonaccounting sources. Let  $E$  denote an accounting random variable, whose value is given as the firm's earnings release. Let  $R_T \equiv R$  be the payoff parameter. It will be convenient to define the column vectors  $Y_{n \times 1} \equiv [Y_1, \dots, Y_n]'$ ,  $Y_{(n+1) \times 1}^* \equiv [Y_1, \dots, Y_n, E]'$ ,  $1_{n \times 1} \equiv [1, \dots, 1]'$ , and  $1_{(n+1) \times 1}^* \equiv [1, \dots, 1]'$ .

The following assumptions will be made: (i) the probability density function  $f_R$  of  $R$  is normal  $N(y_0, \sigma_0^2)$ , where the mean  $y_0$  and variance  $\sigma_0^2$  are known; (ii) the conditional density  $f_{\tilde{y}^*|R}(\cdot|r)$  of  $\tilde{y}^*$  given that  $R = r$  is multivariate normal (MVN) with mean vector  $r\mathbf{1}'$  and covariance matrix

$$\Sigma_{(n+1) \times (n+1)}^* \equiv \begin{bmatrix} \Sigma_{n \times n} & \omega \sigma_y \sigma_e \mathbf{1} \\ \omega \sigma_y \sigma_e \mathbf{1}' & \sigma_e^2 \end{bmatrix},$$

where  $\Sigma_{n \times n} \equiv (1 - \rho)\sigma_y^2 \mathbf{I}_{n \times n} + \rho\sigma_y^2 \mathbf{1}\mathbf{1}'$  has intraclass covariance structure. This means that  $\Sigma_{n \times n}^*$  has the pattern

$$\begin{bmatrix} \theta_1 & \theta_2 & \theta_2 & \cdots & \theta_2 & | & \theta_3 \\ \theta_2 & \theta_1 & \theta_2 & \cdots & \theta_2 & | & \theta_3 \\ \theta_2 & \theta_2 & \theta_1 & \cdots & \theta_2 & | & \theta_3 \\ \vdots & \vdots & \vdots & \ddots & \vdots & | & \vdots \\ \theta_2 & \theta_2 & \theta_2 & \cdots & \theta_1 & | & \theta_3 \\ \hline \theta_3 & \theta_3 & \theta_3 & \cdots & \theta_3 & | & \theta_4 \end{bmatrix} \quad (2.1)$$

with  $\theta_1 = \sigma_y^2$ ,  $\theta_2 = \rho\sigma_y^2$ ,  $\theta_3 = \omega\sigma_y\sigma_e$ , and  $\theta_4 = \sigma_e^2$ . Conditional on  $R$ , the variances of the  $n$  interim signals  $Y_i$  have the common value  $\sigma_y^2$ , the covariance of each pair of these signals is  $\rho\sigma_y^2$ , the variance of earnings is  $\sigma_e^2$ , and the covariance of each interim signal and earnings is  $\omega\sigma_y\sigma_e$ . Other covariance structures could be assumed, e.g., the case where  $\Sigma$  is a Toeplitz matrix whose  $(i,j)$ <sup>th</sup> element is  $\rho^{|i-j|}\sigma_y^2$ . The correlation between a pair of interim signals then depends on the number of signals occurring between them in the sequence, which may correspond to a separation time. Such covariance structures can in principle be analyzed by the techniques of Sections 3 to 5 below. The  $\Sigma_{n \times n}^*$  of (2.1) will be used

here. It is a reasonable choice, generalizing previously studied covariance structures while yielding tractable computations.

Useful results involving conditional distributions can be derived by standard Bayesian calculations. Bayes estimators can then be found from the fact that a Bayes estimator minimizes the posterior conditional expected loss. Distributional results and Bayes estimators of R will be given in the next section.

Define the scalar constants

$$\begin{aligned}\phi &\equiv 1 + (n - 1)\rho, \\ \gamma_1 &\equiv (\phi - \rho)[\phi(1 - \rho)\sigma_y^2]^{-1}, \\ \gamma_2 &\equiv -\rho[\phi(1 - \rho)\sigma_y^2]^{-1}.\end{aligned}$$

$\tilde{\Sigma}$  is positive semidefinite if and only if (iff)  $\phi \geq 0$ , or equivalently  $-1/(n-1) \leq \rho$ , and  $\tilde{\Sigma}$  is positive definite iff  $-1/(n-1) < \rho < 1$ . Similarly,  $\tilde{\Sigma}^*$  is positive semidefinite iff  $\phi - n\omega^2 \geq 0$ , or equivalently  $\omega^2 \leq \rho + (1-\rho)/n$ , and  $\tilde{\Sigma}^*$  is positive definite iff  $\omega^2 < \rho + (1-\rho)/n$  and  $\rho < 1$ .

Every diagonal element of  $\tilde{\Sigma}^{-1}$  is  $\gamma_1$ , every off-diagonal element  $\gamma_2$ , i.e.,  $\tilde{\Sigma}^{-1} = (1-\rho)^{-1}\sigma_y^{-2}(\tilde{I}_{n \times n} - \rho\phi^{-1}\tilde{1}\tilde{1}')$ . Similarly,  $\tilde{\Sigma}^{*-1}$  has pattern (2.1) with  $\theta_i = \lambda_i$  for  $i=1, \dots, 4$ , where

$$\begin{aligned}\lambda_1 &\equiv \phi^{-1}\sigma_y^{-2}[\omega^2/(\phi - n\omega^2) + (\phi - \rho)/(1 - \rho)], \\ \lambda_2 &\equiv \phi^{-1}\sigma_y^{-2}[\omega^2/(\phi - n\omega^2) - \rho/(1 - \rho)], \\ \lambda_3 &\equiv -\omega/(\phi - n\omega^2)\sigma_y\sigma_e, \\ \lambda_4 &\equiv \phi/(\phi - n\omega^2)\sigma_e^2.\end{aligned}$$

Define the vector  $\tilde{\Pi}_{(n+1) \times 1} \equiv \sum_{\tilde{\Sigma}^*}^{-1} \tilde{1}^* = [\pi_1, \dots, \pi_1, \pi_2]'$ , where

$$\begin{aligned}\pi_1 &\equiv \lambda_1 + (n - 1)\lambda_2 + \lambda_3, \\ \pi_2 &\equiv n\lambda_3 + \lambda_4.\end{aligned}$$



and the scalars

$$\begin{aligned} \tau_0 &\equiv 1/\sigma_0^2, \\ \tau_1 &\equiv n/\phi\sigma_y^2, \\ \bar{y} &\equiv \sum_{i=1}^n y_i/n, \\ \alpha &\equiv \tau_0 + \Pi' \sum^* \Pi = \tau_0 + n\pi_1 + \pi_2, \\ \beta &\equiv \tau_0 y_0 + \Pi' y^* = \tau_0 y_0 + n\pi_1 \bar{y} + \pi_2 e, \\ \delta &\equiv y^* \sum^{*-1} y^*. \end{aligned}$$

These will appear throughout the remaining sections.

The expectation operator will be denoted by  $E$ , and the variance of  $\bar{Y}$  conditional on  $R$  by  $\sigma_{\bar{y}}^2$ . Note for later use that  $\sigma_{\bar{y}}^2 = \phi\sigma_y^2/n = 1/\tau_1$  and that the covariance of  $\bar{Y}$  and  $E$  conditional on  $R$  is  $\omega\sigma_y\sigma_e$ .

### 3. POSTERIOR DISTRIBUTIONS AND BAYES ESTIMATORS

Conditional and marginal distributions of interest are derived in Theorems 3.1 to 3.4.

Theorem 3.1. *The posterior density of  $R$  given  $\underline{Y} = \underline{y}$  is  $N[(\tau_0 y_0 + \tau_1 \bar{y})/(\tau_0 + \tau_1), 1/(\tau_0 + \tau_1)]$ .*

Proof: Observe that  $f_R(r) \propto \exp[-\frac{1}{2}\tau_0(r - y_0)^2]$  and

$$f_{\underline{Y}|R}(y|r) \propto \exp[-\frac{1}{2}(y' \sum^{-1} y - 2r \underline{1}' \sum^{-1} y + r^2 \underline{1}' \sum^{-1} \underline{1})].$$

Using the relations  $\underline{1}' \sum^{-1} \underline{1} = \tau_1$  and  $\underline{1}' \sum^{-1} y = \tau_1 \bar{y}$ , the joint density of  $\underline{Y}$  and  $R$  is

$$f_{\underline{Y},R}(y,r) = f_{\underline{Y}|R}(y|r) f_R(r) \propto \exp[-\frac{1}{2}\{(\tau_0 + \tau_1)r^2 - 2(\tau_0 y_0 + \tau_1 \bar{y})r\}] \quad (3.1)$$

where terms not involving  $r$  are omitted. Since  $f_{\underline{Y}}(y)$ , which is obtained by

integrating  $r$  out of (3.1), is a constant relative to  $r$ , it follows that  $f_{R|\bar{Y}}(r|y)$  is proportional to the right side of (3.1). Inspection or completing the square finishes the proof. Q.E.D.

Given  $\bar{Y} = y$ , the posterior mean of  $R$  is a weighted average of  $y_0$  and  $\bar{y}$ . The weights are proportional to  $\tau_0$ , the reciprocal of the variance  $\sigma_0^2$  of the prior density of  $R$ , and  $\tau_1$ , the reciprocal of the variance of  $\bar{Y}$  given the value of  $R$ . Thus, a more diffuse prior density for  $R$  results in lower weight on  $y_0$  in the posterior mean of  $R$ . Conversely, greater variability in the density of  $\bar{Y}$  given  $R$ , which could be caused by a higher correlation  $\rho$ , results in lower weight on  $\bar{y}$  in the posterior mean of  $R$ .

Theorem 3.2. *The posterior density of  $R$  given  $\bar{Y}^* = y^*$  is  $N[\beta/\alpha, 1/\alpha]$ .*

Proof: Substituting terms defined in Section 2 into  $f_R$  and  $f_{\bar{Y}^*|R}$  yields

$$\begin{aligned} f_{\bar{Y}^*,R}(y^*, r) &\propto \exp[-\frac{1}{2}\{\alpha r^2 - 2\beta r + \delta + \tau_0 y_0^2\}] \\ &= \exp[-\frac{1}{2}\{\alpha(r - \beta/\alpha)^2 - \beta^2/\alpha + \delta + \tau_0 y_0^2\}] . \end{aligned} \quad (3.2)$$

The terms  $\beta^2/\alpha$ ,  $\delta$ , and  $\tau_0 y_0^2$  involve  $y^*$  but not  $r$ , so

$$f_{R|\bar{Y}^*}(r|y^*) \propto \exp[-\frac{1}{2}\alpha(r - \beta/\alpha)^2] . \quad \text{Q.E.D.}$$

Given  $\bar{Y}^* = y^*$ , the posterior mean  $\beta/\alpha$  of  $R$  is a weighted average of  $y_0$ ,  $\bar{y}$ , and  $e$  with weights proportional to  $\tau_0 = 1/\sigma_0^2$ ,  $n\pi_1$ , and  $\pi_2$ . When  $\omega=0$ ,  $n\pi_1 = \tau_1$  and  $\pi_2 = 1/\sigma_e^2$  are the reciprocals of the variances of  $\bar{Y}$  and  $E$ , respectively, given the value of  $R$ . In general,  $n\pi_1$  and  $\pi_2$  are sums of elements of  $\sum^*{}^{-1}$ .

Under a squared-error loss function, the posterior conditional expected loss is minimized by the posterior mean. Therefore, with squared-error loss, the Bayes estimator of  $R$  given  $Y = y$  is  $(\tau_0 y_0 + \tau_1 \bar{y}) / (\tau_0 + \tau_1)$ , and the Bayes estimator of  $R$  given  $\tilde{Y}^* = \tilde{y}^*$  is  $\beta/\alpha$ , which equals

$$(\tau_0 y_0 + n\pi_1 \bar{y} + \pi_2 e) / (\tau_0 + n\pi_1 + \pi_2) .$$

Under any symmetric loss function, the Bayes estimators of  $R$  given  $Y = y$  and  $R$  given  $\tilde{Y}^* = \tilde{y}^*$  are identical to the corresponding estimators under squared-error loss.

Empirical and theoretical considerations suggest that an asymmetric loss function may be more appropriate than a symmetric one (see Hosomatsu, 1980). The loss that results from overestimating  $R$  may be greater than the loss that results from underestimating  $R$  by the same amount. Under the asymmetric loss function

$$L(r, a) = \begin{cases} c_1 |r-a| & \text{if } a \leq r \\ c_2 |r-a| & \text{if } a > r \end{cases} , \quad (3.3)$$

where constants  $c_1$  and  $c_2$  are positive, the Bayes estimator of  $R$  is the  $p$ th quantile of the posterior distribution of  $R$ , where  $p = c_1 / (c_1 + c_2) = (1 + c_2/c_1)^{-1}$  (see Box and Tiao, 1973, p. 309). A ratio of penalty factors  $c_2/c_1$  exceeding 1 produces a quantile lying below the median of the posterior distribution given by Theorem 3.1 or Theorem 3.2. With this loss function, the Bayes estimator of  $R$  given  $\tilde{Y}^* = \tilde{y}^*$  is  $\beta/\alpha + \Phi^{-1}(p)/\sqrt{\alpha}$ , which equals

$$(\tau_0 y_0 + n\pi_1 \bar{y} + \pi_2 e) / (\tau_0 + n\pi_1 + \pi_2) + \Phi^{-1}(p) / (\tau_0 + n\pi_1 + \pi_2)^{\frac{1}{2}} . \quad (3.4)$$

Another quantity whose distribution is of interest is  $E - \mathcal{E}[R|Y]$ , which measures the extent of surprise in announced earnings. This distri-

bution can be obtained from the marginal distribution of  $\tilde{Y}^*$ , which is given in Theorem 3.3. The proof of this theorem is sketched in the Appendix.

Theorem 3.3. The distribution of  $\tilde{Y}^*$  is

$$MVN[\tilde{y}_0 \tilde{1}^*, \tilde{\Sigma}^* + \sigma_0^2 \tilde{1}^* \tilde{1}^{*'}] .$$

Theorem 3.4. The distribution of  $E - \hat{\epsilon}[R|Y]$  is normal  $N[0, v]$  where

$$v \equiv 1/(\tau_0 + \tau_1) - 2\omega\tau_1\sigma_y\sigma_e/(\tau_0 + \tau_1) + \sigma_e^2 .$$

Proof: Define  $a \equiv \tau_1/n(\tau_0 + \tau_1)$  and the column vector  $\tilde{a}_{(n+1) \times 1} \equiv [-a\tilde{1}', 1]'$ . Then

$$E - \hat{\epsilon}[R|Y] = \tilde{a}'\tilde{Y}^* - \tau_0\tilde{y}_0/(\tau_0 + \tau_1)$$

is a linear combination of normals plus a constant, so it is normally distributed. Its expectation and variance, using Theorem 3.3 and the definitions of  $\phi$ ,  $\tau_0$ , and  $\tau_1$ , are

$$\begin{aligned} \tilde{a}'(\tilde{y}_0 \tilde{1}^*) - \tau_0\tilde{y}_0/(\tau_0 + \tau_1) &= 0 ; \\ \tilde{a}'(\tilde{\Sigma}^* + \sigma_0^2 \tilde{1}^* \tilde{1}^{*'})\tilde{a} &= a^2\{n\sigma_y^2 + n(n-1)\rho\sigma_y^2\} - 2an\omega\sigma_y\sigma_e + \sigma_e^2 + \sigma_0^2(-na+1)^2 \\ &= a^2n\phi\sigma_y^2 - 2an\omega\sigma_y\sigma_e + \sigma_e^2 + \sigma_0^2\tau_0^2/(\tau_0 + \tau_1)^2 \\ &= v . \quad \text{Q.E.D.} \end{aligned}$$

#### 4. RELATIVE PRECISION OF EARNINGS

Under certain conditions involving the model parameters, the earnings  $E$  are preempted by information observed prior to  $E$ . More precisely, the coefficient of  $E$  in the Bayes estimator obtained from the posterior

distribution of Theorem 3.2 is small. The magnitude of this coefficient indicates the importance of earnings as an information source conditional on the other information available concerning R. The coefficient of E in the Bayes estimator (3.4) of R is the relative precision of earnings.

Define  $w_0 \equiv \tau_0/\alpha$ ,  $w_1 \equiv n\pi_1/\alpha$ , and  $w_2 \equiv \pi_2/\alpha$ . These are the coefficients of  $y_0$ ,  $\bar{y}$ , and  $e$  in  $\beta/\alpha$ , so  $w_2$  is the relative precision of earnings, and the Bayes estimator (3.4) is

$$B \equiv w_0 y_0 + w_1 \bar{y} + w_2 e + z \tag{4.1}$$

where

$$z \equiv \Phi^{-1}(p)/\alpha^{\frac{1}{2}} .$$

Theorem 4.1. The coefficient  $w_1$  is positive, zero, or negative according to whether  $\sigma_e^2$  is greater than, equal to, or less than  $\omega\sigma_y\sigma_e$ . The coefficient  $w_2$  is positive, zero, or negative according to whether  $\phi\sigma_y^2/n$  is greater than, equal to, or less than  $\omega\sigma_y\sigma_e$ .

Proof. By the positive definiteness of  $\sum_{\sim}^*$ ,  $\alpha$  and  $\phi - n\omega^2$  must be positive.

Then

$$\begin{aligned} n\pi_1 &= n[\lambda_1 + (n-1)\lambda_2 + \lambda_3] = \frac{n}{\sigma_y(\phi - n\omega^2)} \left\{ \frac{1}{\sigma_y} - \frac{\omega}{\sigma_e} \right\} \\ &= \frac{n}{\sigma_y^2 \sigma_e^2 (\phi - n\omega^2)} \{ \sigma_e^2 - \omega\sigma_y\sigma_e \} , \end{aligned} \tag{4.2}$$

so  $n\pi_1 > (=, <) 0$  iff  $\sigma_e^2 > (=, <) \omega\sigma_y\sigma_e$ , establishing the first result.

Similarly,

$$\begin{aligned} \pi_2 &= n\lambda_3 + \lambda_4 = \frac{1}{\sigma_e(\phi - n\omega^2)} \left\{ -\frac{n\omega}{\sigma_y} + \frac{\phi}{\sigma_e} \right\} \\ &= \frac{1}{\sigma_y^2 \sigma_e^2 (\phi - n\omega^2)} \{ \phi\sigma_y^2 - n\omega\sigma_y\sigma_e \} , \end{aligned} \tag{4.3}$$

so  $\pi_2 > (=, <) 0$  iff  $\phi\sigma_y^2 > (=, <) n\omega\sigma_y\sigma_e$ , establishing the second result.

Q.E.D.

Corollary 4.1. When  $n=1$ ,  $w_1$  is positive, zero, or negative according to whether  $\sigma_e^2$  is greater than, equal to, or less than  $\omega\sigma_y\sigma_e$ ; and  $w_2$  is positive, zero, or negative according to whether  $\sigma_y^2$  is greater than, equal to, or less than  $\omega\sigma_y\sigma_e$ .

This theorem provides a generalization of the sufficiency and redundancy cases of Holthausen and Verrecchia (1981). Define the sufficiency case to occur when  $\pi_1=0$ ; in this case, the distribution of R given  $\tilde{Y}^* = \tilde{y}^*$  is identical to the distribution of R given  $E=e$  alone. This conditional distribution of R is normal

$$N[(\tau_0 y_0 + \pi_2 e)/(\tau_0 + \pi_2), 1/(\tau_0 + \pi_2)] .$$

The earnings report contains all of the information present in the vector of interim signals, and possibly more, pertaining to the final liquidating dividend R. In other words, E is sufficient for determining the behavior of R. The redundancy case occurs when  $\pi_2=0$ ; here, the distribution of R given  $\tilde{Y}^* = \tilde{y}^*$  is identical to the distribution of R given  $\tilde{Y} = \tilde{y}$  (or merely  $\bar{Y} = \bar{y}$ ) alone. This conditional distribution of R is normal

$$N[(\tau_0 y_0 + n\pi_1 \bar{y})/(\tau_0 + n\pi_1), 1/(\tau_0 + n\pi_1)] .$$

The vector of interim signals, or simply the mean of these signals, contains all of the information present in the final earnings report, and possibly more, pertaining to the final liquidating dividend. In other words, E is redundant for determining the behavior of R.

When  $n=1$ , these situations reduce to the sufficiency case and the redundancy case described by Holthausen and Verrecchia. An inspection of Corollary 4.1 indicates that sufficiency and redundancy occur when  $\sigma_e^2 = \omega\sigma_y\sigma_e$  and  $\sigma_y^2 = \omega\sigma_y\sigma_e$ , respectively.

Another interesting feature of these situations (for  $n \geq 1$ ) is that either  $\bar{y}$  or  $e$  can have negative weight in the estimate of  $R$ . Only one of these weights can be negative, since

$$n\pi_1 + \pi_2 = 1 \sum_{\sim}^{*'} \sum_{\sim}^{*-1} 1_{\sim}^* > 0 .$$

The expressions for  $n\pi_1$  and  $\pi_2$  in the proof of Theorem 4.1 can be used to calculate conditions on the model parameters under which the relative precision of earnings,  $w_2$ , is small. The next theorem shows several of these conditions.

Theorem 4.2. In the model of Section 2, fix all parameters except  $n$ ,  $\sigma_y^2$ , or  $\sigma_e^2$ . Let (i)  $n \rightarrow \infty$ , (ii)  $1/\sigma_y^2 \rightarrow \infty$ , or (iii)  $1/\sigma_e^2 \rightarrow 0$ ; then  $w_1 \rightarrow 1$  and  $w_0, w_2, z \rightarrow 0$  in the Bayes estimator  $B$  of (4.1).

Proof: (i) The definitions of  $\lambda_1$  to  $\lambda_4$  yield  $\lambda_1 = O(1)$ ,  $\lambda_2 = O(n^{-1})$ ,  $\lambda_3 = O(n^{-1})$ , and  $\lambda_4 = O(1)$  when  $\rho \neq 0$ ,  $\omega \neq 0$ . Then  $\pi_1 = O(1)$ ,  $\pi_2 = O(1)$ , and  $\alpha = O(n)$ . Consequently,  $\tau_0/\alpha = O(n^{-1})$ ,  $\pi_2/\alpha = O(n^{-1})$ , and  $\phi^{-1}(p)/\sqrt{\alpha} = O(n^{-\frac{1}{2}})$ , and as  $n \rightarrow \infty$

$$n\pi_1/\alpha = 1 - \tau_0/\alpha - \pi_2/\alpha \rightarrow 1 .$$

The calculation differ slightly when  $\rho = 0$ ,  $\omega = 0$ , or both, but the results are the same. Parts (ii) and (iii) are proved similarly. Q.E.D.

As the number of interim information signals increases without bound, the coefficient of their mean  $\bar{y}$  in the Bayes estimator of  $R$  approaches 1 and the relative precision of earnings approaches 0. The same results occur as the common variance of the interim information signals decreases to 0 or as the variance of the annual accounting random variable increases without bound. This generalizes the results of Holthausen and Verrecchia (1981) for the case of  $n=1$ .

The effects of the correlations  $\rho$  and  $\omega$  on the components of B will now be examined. The analysis concerning  $\rho$  is of special interest, since it is not possible to analyze the importance of this parameter in determining information content when  $n=1$ . Moreover, the analysis involving  $\omega$  extends results of Holthausen and Verrecchia, who considered only two limiting cases for  $\omega$ , the cases of sufficiency and redundancy.

Theorem 4.3. In the model of Section 2, fix all parameters except  $\rho$  and consider the Bayes estimator B. As  $\rho$  increases:  $w_0$  increases;  $|w_1|$  decreases;  $w_2$  increases when  $w_1 > 0$  and  $\omega > -1/\tau_0\sigma_y\sigma_e$ , and decreases when  $w_1 < 0$  or  $\omega < -1/\tau_0\sigma_y\sigma_e$ ; and  $z$  increases when  $\rho \neq .5$ . There is one exception to these results:  $w_0$ ,  $w_1$ ,  $w_2$ , and  $z$  are independent of  $\rho$  when the sufficiency case occurs.

Proof. Defining

$$\begin{aligned} D &\equiv (\phi - n\omega^2)(1 + \tau_0\sigma_e^2)\sigma_y^2 + n(\sigma_e - \omega\sigma_y)^2 \\ &= \sigma_y^2(1 + \tau_0\sigma_e^2)\phi + n\sigma_e^2 - 2n\omega\sigma_y\sigma_e - n\omega^2\tau_0\sigma_e^2\sigma_y^2, \end{aligned}$$

observe that  $\phi$  increases with  $\rho$  and that, from (4.2) and (4.3),

$$\begin{aligned} w_0 &= \tau_0\sigma_y^2\sigma_e^2(\phi - n\omega^2)/D, \\ w_1 &= n\sigma_e(\sigma_e - \omega\sigma_y)/D, \\ w_2 &= (\phi\sigma_y^2 - n\omega\sigma_y\sigma_e)/D, \\ \alpha &= D/\sigma_y^2\sigma_e^2(\phi - n\omega^2) = \tau_0/w_0. \end{aligned}$$

Clearly  $w_0$  increases with  $\rho$  unless  $\sigma_e = \omega\sigma_y$ , in which case  $w_0 = \tau_0\sigma_e^2/(1 + \tau_0\sigma_e^2)$  is constant. The numerator of  $w_1$  is constant with respect to  $\rho$  while D is a positive, increasing function of  $\rho$ . Thus  $|w_1|$  is a decreasing



function of  $\rho$  unless  $\sigma_e = \omega\sigma_y$ , in which case  $w_1 = 0$  for any  $\rho$ . Next, for  $w_2$ ,

$$\partial w_2 / \partial \phi = n \sigma_y^2 \sigma_e (\omega \tau_0 \sigma_y \sigma_e + 1) (\sigma_e - \omega \sigma_y) / D^2 .$$

Note that  $w_1 > 0$  iff  $\sigma_e - \omega\sigma_y > 0$ , so  $w_2$  is an increasing function of  $\rho$  when  $w_1 > 0$  and  $\omega > -1/\tau_0 \sigma_y \sigma_e$ . Similarly, if  $w_1 > 0$  and  $\omega < -1/\tau_0 \sigma_y \sigma_e$ , then  $w_2$  decreases as  $\rho$  increases; and  $w_1 < 0$  implies  $\omega > 0$  (since  $\sigma_e^2 - \omega\sigma_y\sigma_e < 0$ ), which again makes  $w_2$  a decreasing function of  $\rho$ . When  $w_1 = 0$ ,  $\partial w_2 / \partial \phi = 0$ , so  $w_2 = 1/(1 + \tau_0 \sigma_e^2)$  for any value of  $\rho$ .

Finally, because  $z = \Phi^{-1}(p) \{w_0/\tau_0\}^{\frac{1}{2}}$ ,  $z$  is a monotonically increasing function of  $w_0$  whenever  $p \neq .5$ . (When  $p = .5$ ,  $z = 0$  for any  $\rho$ .) Thus  $z$  is an increasing function of  $\rho$  except when  $w_1 = 0$ , in which case  $z$  is constant. Q.E.D.

The  $n$  correlated interim information signals can be interpreted as being equivalent to  $n'$  independent interim signals; as  $\rho$  increases,  $n'$  decreases. The magnitude of  $w_1$  decreases, so less weight is given to  $\bar{y}$ , while  $w_0$ , which is always positive, increases. The behavior of  $w_1$  reflects the fact that  $\sigma_y^2$  increases with  $\rho$ . The behavior of  $w_2$ , the relative precision of earnings, is more complex. When  $w_2 > 0$ , it decreases as  $\rho$  increases if either  $w_1$  is negative or  $\omega < -1/\tau_0 \sigma_y \sigma_e$ . When  $w_2 < 0$ , it increases with  $\rho$  if  $\omega > -1/\tau_0 \sigma_y \sigma_e$ . In both of these cases,  $w_2$  is decreasing in magnitude. Similarly, when  $w_2 > 0$ , it increases with  $\rho$  if  $w_1$  is positive and  $\omega > -1/\tau_0 \sigma_y \sigma_e$ ; when  $w_2 < 0$ , it decreases as  $\rho$  increases if  $\omega < -1/\tau_0 \sigma_y \sigma_e$ . In these cases,  $w_2$  is increasing in magnitude. For example, as  $\rho$  approaches 1 with  $\omega = 0$ ,

$$w_0 \uparrow \tau_0 / \tilde{\alpha} , \quad w_1 \downarrow 1 / \sigma_y^2 \tilde{\alpha} , \quad w_2 \uparrow 1 / \sigma_e^2 \tilde{\alpha} , \quad z \uparrow \Phi^{-1}(p) / \tilde{\alpha}^{\frac{1}{2}} ,$$

where

$$\tilde{\alpha} \equiv \tau_0 + 1/\sigma_y^2 + 1/\sigma_e^2 .$$

In the sufficiency case, though, none of these consequences follows from a change in  $\rho$ ; since  $\bar{y}$  has weight  $w_1 = 0$  in B here, it makes sense that  $\rho$  should not affect  $w_0$ ,  $w_1$ ,  $w_2$ , and  $z$ , as Theorem 4.3 shows to be the situation.

Theorem 4.4. In the model of Section 2, fix all parameters except  $\omega$  and consider the Bayes estimator B. Take  $p \neq .5$ . (i) Assume that  $\sigma_{\bar{y}}^2 < \sigma_e^2$ . As  $\omega$  increases,  $w_0$  and  $z$  increase (decrease) iff  $\omega < (>) \phi\sigma_y/n\sigma_e$ , or equivalently  $\omega\sigma_y\sigma_e < (>) \sigma_{\bar{y}}^2$  or  $w_2 > (<) 0$ ;  $w_1$  increases (decreases) iff  $\omega > \omega^*$  ( $\omega < \omega^*$ ), where

$$\omega^* \equiv [\sigma_e^2 - \{(\sigma_e^2 + \sigma_0^2)(\sigma_e^2 - \sigma_{\bar{y}}^2)\}^{\frac{1}{2}}] / \sigma_y\sigma_e ;$$

and  $w_2$  is a decreasing function of  $\omega$ . (ii) Assume that  $\sigma_{\bar{y}}^2 = \sigma_e^2$ . Then  $w_0$  and  $z$  are increasing functions of  $\omega$ , while  $w_1$  and  $w_2$  are decreasing function of  $\omega$ . (iii) Assume that  $\sigma_{\bar{y}}^2 > \sigma_e^2$ . As  $\omega$  increases,  $w_0$  and  $z$  increase (decrease) iff  $\omega < (>) \sigma_e/\sigma_y$ , or equivalently  $w_1 > (<) 0$ ;  $w_1$  is a decreasing function of  $\omega$ ; and  $w_2$  increases (decreases) iff  $\omega > \omega^{**}$  ( $\omega < \omega^{**}$ ), where

$$\omega^{**} \equiv [\sigma_{\bar{y}}^2 - \{(\sigma_{\bar{y}}^2 + \sigma_0^2)(\sigma_{\bar{y}}^2 - \sigma_e^2)\}^{\frac{1}{2}}] / \sigma_y\sigma_e .$$

If  $p = .5$ , then  $z$  is zero for any  $\omega$ .

Proof. Taking derivatives,

$$\partial w_0 / \partial \omega = 2n\tau_0\sigma_y^2\sigma_e^2(\sigma_e - \omega\sigma_y)(\phi\sigma_y - n\omega\sigma_e) / D^2 ,$$

$$\partial w_1 / \partial \omega = n^2\tau_0\sigma_y\sigma_e [(\sigma_e^2 + \sigma_0^2)(\sigma_e^2 - \sigma_{\bar{y}}^2) - (\sigma_e^2 - \omega\sigma_y\sigma_e)^2] / D^2 ,$$

$$\partial w_2 / \partial \omega = n^2\tau_0\sigma_y\sigma_e [(\sigma_{\bar{y}}^2 + \sigma_0^2)(\sigma_{\bar{y}}^2 - \sigma_e^2) - (\sigma_{\bar{y}}^2 - \omega\sigma_y\sigma_e)^2] / D^2 .$$

Because  $z$  is a monotonically increasing function of  $w_0$  for  $p \neq .5$ , it behaves exactly as  $w_0$  does.

From (4.2) and (4.3),  $\partial w_0 / \partial \omega$  has the same sign as  $\pi_1 \pi_2$  and  $w_1 w_2$ . In case (i),  $\text{cov}(\bar{Y}, E) = \omega \sigma_y \sigma_e < \sigma_e^2$ , so by Theorem 4.1,  $w_1$  is positive, and  $w_1 w_2 > 0$  iff  $w_2 > 0$  iff  $\phi \sigma_y^2 / n = \sigma_y^2 > \omega \sigma_y \sigma_e$ . Thus  $\partial w_0 / \partial \omega$  is positive iff  $\omega < \phi \sigma_y / n \sigma_e$ . Reversing inequalities shows that  $\partial w_0 / \partial \omega$  is negative iff  $\omega > \phi \sigma_y / n \sigma_e$ . In case (iii),  $\omega \sigma_y \sigma_e < \sigma_y^2$ , so by Theorem 4.1  $w_2$  is positive, and  $w_1 w_2 > 0$  iff  $w_1 > 0$  iff  $\sigma_e^2 > \omega \sigma_y \sigma_e$ . Then  $\partial w_0 / \partial \omega$  is positive (negative) iff  $\omega < (>) \sigma_e / \sigma_y$ . In case (ii),  $\omega \sigma_y \sigma_e$  is less than both  $\sigma_e^2$  and  $\sigma_y^2$ , so both  $w_1$  and  $w_2$  are positive, as is  $\partial w_0 / \partial \omega$ .

Now consider  $\partial w_1 / \partial \omega$ . It must be negative in cases (ii) and (iii). (It cannot be zero even in (ii), since if  $\sigma_e = \omega \sigma_y$  holds, then  $\sigma_e^2 < \sigma_y^2$  follows from  $\omega^2 < \phi / n$ .) In case (i),  $\partial w_1 / \partial \omega$  is positive (negative) iff  $\omega > \omega^*$  ( $\omega < \omega^*$ ). The same approach suffices for  $w_2$  by symmetry. Q.E.D.

As  $\omega$  increases with all other parameters fixed and case (i),  $\sigma_y^2 < \sigma_e^2$ ,  $w_0$  increases until  $\omega = \phi \sigma_y / n \sigma_e$ , then decreases;  $w_1$  increases until  $\omega = \omega^*$ , then decreases; and  $w_2$  is monotonically decreasing. The relative positions of  $\phi \sigma_y / n \sigma_e$  and  $\omega^*$  depend on the parameter values. On the other hand, as  $\omega$  increases in case (iii),  $\sigma_y^2 > \sigma_e^2$ ,  $w_0$  increases until  $\omega = \sigma_e / \sigma_y$ , then decreases;  $w_1$  decreases monotonically; and  $w_2$  increases until  $\omega = \omega^{**}$ , then decreases. Again the relative positions of the critical points,  $\sigma_e / \sigma_y$  and  $\omega^{**}$ , depend on the parameter values.

The behavior of the relative precision of earnings  $w_2$  when the correlation  $\omega$  changes is determined by the model's remaining parameter values. If  $\sigma_y^2 \leq \sigma_e^2$ ,  $w_2$  decreases as  $\omega$  increases; increasing the covariance between  $E$  and the less variable quantity  $\bar{Y}$  reduces the weight attached to  $E$  in forming the Bayes estimator  $B$ . If  $\sigma_y^2 > \sigma_e^2$ , so  $E$  has

greater precision than  $\bar{Y}$ ,  $w_2$  again decreases as  $\omega$  increases to  $\omega = \omega^{**}$ ; however,  $w_2$  then changes direction and increases as  $\omega$  continues to rise. So for a large value of  $\omega$ , a larger weight is attached to E, the relatively more precise source of information.

### 5. MAGNITUDE OF BELIEF REVISION

When the earnings release E becomes known, the analysts revise their estimates of R by using the posterior density of R given  $\tilde{Y}^* = y^*$  rather than the posterior density of R given  $\tilde{Y} = y$ . The extent of change in the analyst forecast will be treated in this section. For convenience, assume that a squared-error or other symmetric loss function is used, making the posterior mean of R the estimator of choice.

The magnitude of belief revision is

$$\kappa \equiv \underset{\sim}{\mathcal{E}}[R|\tilde{Y}^*] - \underset{\sim}{\mathcal{E}}[R|\tilde{Y}] = \beta/\alpha - (\tau_0 y_0 + \tau_1 \bar{Y})/(\tau_0 + \tau_1) \quad ,$$

the difference between the posterior mean of R given  $\tilde{Y}^*$  and the posterior mean of R given  $\tilde{Y}$ . It is the amount by which the analyst forecast will be changed when E is revealed. It can be expressed as

$$\begin{aligned} \kappa &= (\tau_0 y_0 + n\pi_1 \bar{Y} + \pi_2 E)/(\tau_0 + n\pi_1 + \pi_2) - (\tau_0 y_0 + \tau_1 \bar{Y})/(\tau_0 + \tau_1) \\ &= \pi_2 \{(-\tau_0 - \psi)y_0 + (-\tau_1 + \psi)\bar{Y} + (\tau_0 + \tau_1)E\}/\alpha(\tau_0 + \tau_1) \quad , \end{aligned} \tag{5.1}$$

where  $\psi \equiv \tau_0(n\pi_1 - \tau_1)/\pi_2$  can be thought of as a covariance adjustment term. Its value depends on  $\rho$  and  $\omega$ ;  $\psi = 0$  when  $\omega = 0$ . The methods used to prove Theorem 3.4 show that  $\mathcal{E}[\kappa] = 0$  and

$$\begin{aligned} \text{var}[\kappa] &= \{\pi_2/\alpha(\tau_0 + \tau_1)\}^2 \{(\tau_0 + \psi)^2 \sigma_0^2 + (-\tau_1 + \psi)^2 \phi \sigma_y^2/n \\ &\quad + 2(-\tau_1 + \psi)(\tau_0 + \tau_1)\omega \sigma_y \sigma_e/n + (\tau_0 + \tau_1)^2 \sigma_e^2\} \quad . \end{aligned} \tag{5.2}$$

The distribution of  $\kappa$  is normal, for  $\kappa$  is a linear combination of components of the multivariate normal vector  $\tilde{Y}^*$  plus a constant. This establishes the following theorem.

Theorem 5.1. The magnitude of belief revision  $\kappa$  is normally distributed with mean zero and variance given by equation (5.2).

Special cases of particular value are examined in the next two theorems.

Theorem 5.2. When  $\omega=0$ , equation (5.2) reduces to

$$\text{var}[\kappa] = \tau_e / (\tau_0 + \tau_1)(\tau_0 + \tau_1 + \tau_e)$$

where

$$\tau_e \equiv 1/\sigma_e^2 \quad .$$

Proof: By substitution of  $\omega=0$  into formulas of Section 2 and of these into (5.2). Q.E.D.

Theorem 5.3. Fix all parameters except  $n$ ,  $\sigma_y^2$ , or  $\sigma_e^2$ . As (i)  $n \rightarrow \infty$ , (ii)  $1/\sigma_y^2 \rightarrow \infty$ , or (iii)  $1/\sigma_e^2 \rightarrow 0$ ,

$$\text{var}[\kappa] \rightarrow 0 \quad .$$

Proof: This is an immediate consequence of Theorem 4.2 and an analogous theorem for the Bayes estimator  $(\tau_0 y_0 + \tau_1 \bar{Y}) / (\tau_0 + \tau_1)$ . Q.E.D.

Other cases of interest have yielded expressions that do not simplify (5.2) appreciably. These will not be reproduced here.

The magnitude of belief revision is zero if  $E=e$  results in the equality

$$\frac{\beta}{\alpha} = \frac{\tau_0 y_0 + n\tau_1 \bar{Y} + \tau_2 E}{\tau_0 + n\tau_1 + \tau_2} = \frac{\tau_0 y_0 + \tau_1 \bar{Y}}{\tau_0 + \tau_1} \quad .$$

Somewhat surprisingly, the value of  $E$  that produces  $\kappa=0$  is not  $E=e=(\tau_0 y_0 + \tau_1 \bar{y})/(\tau_0 + \tau_1)$ , but

$$E = e = [(\tau_0 + \psi)y_0 + (\tau_1 - \psi)\bar{y}]/(\tau_0 + \tau_1) \quad . \quad (5.3)$$

The further  $E$  is from the expression in (5.3), the greater  $|\kappa|$  will be.

The magnitude of belief revision is closely related to the extent of surprise in earnings,  $E - \delta[R|Y]$ , defined in Section 3. It follows from Theorem 3.1 and (5.1) that

$$\kappa = (\pi_2/\alpha)\{E - \delta[R|Y] + \psi(\bar{Y} - y_0)/(\tau_0 + \tau_1)\} \quad .$$

Thus when  $\pi_2$  is positive (negative),  $\kappa$  is an increasing (decreasing) function of the extent of surprise in earnings.

## 6. INTERPRETATIONS AND CONCLUSIONS

The above analysis demonstrates that the relative importance of earnings depends upon the information covariance structure. The increased dimensionality of the covariance structure in this paper, relative to that of Holthausen and Verrecchia (1981), provides new insights regarding theoretical determinants of the importance of earnings. The correlation of interim nonaccounting signals is an important determinant of the importance of earnings. As this correlation increases, the relative importance of earnings increases. In effect, there is less prior information available as the correlation increases. The importance of correlation in determining earnings information content has been overlooked in the accounting literature, where extent of prior information availability has been interpreted to be synonymous with the frequency of prior information arrival. Grant (1980), for example, used the number of *Wall Street*

*Journal* articles as a surrogate for extent of prior information availability, and showed that the number of news articles is systematically greater for listed firms than for nonlisted ones. Depending on the extent of correlation among various news articles pertaining to a firm, extent of press coverage may be a weak surrogate for extent of prior information availability.

We also generalize the analysis of Holthausen and Verrecchia (1981) concerning the correlation between interim nonaccounting signals and earnings. They considered the limiting cases of sufficiency and redundancy, which we also consider. Sufficiency occurs when the weight attached to interim information in posterior beliefs is zero. Earnings contain all of the information in interim signals and perhaps more. Redundancy occurs when the weight attached to earnings in posterior beliefs is zero. Interim signals contain all of the information in earnings and perhaps more. We consider sufficiency to be an unlikely possibility, particularly when the analysis treats all interim information rather than just one interim report. Redundancy means, in effect, that earnings are entirely preempted by interim information, a possibility we consider to be more likely, particularly when the analysis is extended to all interim information rather than just one interim report. In addition to the two limiting cases, we analyze a range of cases involving the correlation between interim signals and earnings. Earnings can actually increase in importance as the correlation between interim signals and earnings increases. This occurs when the variance  $\sigma_y^2$  of the sample mean is greater than the variance  $\sigma_e^2$  of earnings, making earnings the more precise source of information. As the correlation between accounting and nonaccounting sources of information

increases, more of the weight in posterior belief formation will be placed on the relatively more precise source of information.

Another interesting theoretical possibility is that earnings could receive a negative weight in the posterior estimate of the liquidating dividend, depending in part on the extent of correlation between the interim signals and earnings. The posterior estimate of the liquidating dividend could conceivably move in the opposite direction from that implied by the earnings release.

Our results concerning the number of prior available signals are consistent with the intuition provided by the models of Ohlson (1979) and Holthausen and Verrecchia (1981). The importance of earnings increases as the cumulative precision of prior information availability decreases.

Our results involving the precision of earnings are consistent with Holthausen and Verrecchia (1981). The importance of earnings decreases as the precision of earnings decreases. In both our model and theirs, precision is the inverse of the variance with which earnings are distributed around the liquidating dividend of the firm. In other words, precision refers to the extent to which earnings are informative regarding future dividends of the firm. The greater the variance, the greater the "noise" in earnings. This interpretation highlights the connection between the model and the empirical domain. In a multiperiod world, the parameter of interest to investors is the vector of uncertain future dividends. The extent to which earnings are informative concerning this vector is a major determinant of the importance of earnings. This view suggests that one possible role for regulatory bodies that set accounting standards is to minimize, wherever possible, the "noise" in earnings as an information source for predicting future cash flows and dividends. For example,



transitory components in earnings, such as large write-offs of bad debts or obsolete inventory, should be disclosed separately in the income statement so that implications for future cash flow and dividend streams can be sorted out by investors. Any disclosure requirement that increases the precision of earnings as an information source for prediction potentially increases the social usefulness of earnings reports.

Finally, our results involving the precision of individual prior information signals are also consistent with Holthausen and Verrecchia (1981). The extent to which individual prior signals from nonaccounting sources are informative regarding future firm dividends is also an important determinant of the importance of earnings. The greater the "noise" in individual prior signals, the greater the potential importance of earnings. In our model, precision is additive and will therefore also depend on the number of prior information signals.

Will the (cumulative) precision of prior information or the precision of earnings differ systematically across different types of firms? This is a difficult question, empirically, since precision of information cannot be observed directly, necessitating surrogate selection. Nevertheless, the answer to the question is important in understanding current empirical results such as those reported by Grant (1980) and Atiase (1980), who showed that the price reaction to earnings reports is systematically greater for smaller firms. A common interpretation of their results in the empirical literature is that the (cumulative) precision of prior information differs systematically between smaller and larger firms. In other words, the extent of availability of nonaccounting information declines with firm size.

We offer an alternative interpretation of the apparent firm-size effect, namely, that the precision of accounting information differs systematically between smaller and larger firms. Feltham (1983) analyzes the importance of firm-specific information, relative to information about economy-wide state variables, in updating beliefs about future firm dividends. It is possible that firm-specific "shocks" or events, rather than economy-wide events, assume systematically greater importance for the valuation of smaller firms, reflecting the tendency for smaller firms to have less diversified asset portfolios. Since accounting information is a major source of information about firm-specific events, it is possible that the precision of accounting information is systematically greater for small firms relative to large firms.

We also demonstrate that the information content of earnings increases as the extent of surprise in earnings increases. By extent of surprise, we mean the extent to which earnings fail to confirm (cumulative) prior information from nonaccounting sources. For example, suppose that economy-wide information tells investors that the overall state of the economy, assumed to be a critical state variable affecting firm cash flows, is improving over the previous year. Consequently, investors will revise their beliefs about dividends (hence, firm value) in a favorable direction. If firm earnings for the year are announced to be unfavorable, we say that the earnings fail to confirm prior information from nonaccounting sources. Depending on the precision of earnings as a source of information, information content and, consequently, price reaction could be dramatic. If the unfavorable earnings are due to some source of "noise" such as a change in accounting policies that has no implications for firm cash flows, posterior

beliefs about future cash flows and dividends will not change significantly and there will be little price reaction.

Empirical researchers in accounting have tended to select surrogates for expected firm earnings using earnings time series models or perhaps the contemporaneous earnings announcements of other firms in the same industry. Our analysis suggests that researchers should measure unexpected earnings using prior information from nonaccounting sources, in order to determine the extent to which earnings fail to confirm (cumulative) prior information. Alternatively, researchers might observe directly the earnings forecasts of professional analysts, which presumably incorporate information from nonaccounting sources.

In conclusion, it has been well established by empirical researchers that earnings possess information content for investors. This paper provides a theoretical framework for analyzing the information content of earnings.

#### APPENDIX. PROOF OF THEOREM 3.3

Integrating  $r$  out of (3.2) gives

$$f_{\tilde{Y}}^*(Y^*) \propto \exp[-\frac{1}{2}\{\delta - \beta^2/\alpha\}] \\ \propto \exp[-\frac{1}{2}\{Y^{*'} \tilde{N} Y^* - 2(\tau_0 y_0/\alpha) \tilde{\Pi}' Y^*\}]$$

where  $\tilde{N} \equiv \sum_{\tilde{}}^{*-1} - (1/\alpha) \tilde{\Pi} \tilde{\Pi}'$ . Therefore

$$\tilde{Y}^* \sim \text{MVN}[(\tau_0 y_0/\alpha) \tilde{N}^{-1} \tilde{\Pi}, \tilde{N}^{-1}] .$$

The theorem follows by noting that  $\tilde{N}^{-1} = \sum_{\tilde{}}^* + (1/\tau_0) \tilde{1} \tilde{1}^{*'}$  and  $\tilde{N}^{-1} \tilde{\Pi} = (\alpha/\tau_0) \tilde{1}^*$ .

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