BU-557-M

ASA Study on Modular Instruction - Position Paper

Constance L. Wood*

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1. Introduction. At Cornell University, most graduate students are required to take at least one introductory Statistical Methods course. A large number of students, the majority coming from the College of Agriculture and Life Sciences, previously enrolled in a two-semester sequence given in the Department of Plant Breeding and Biometry. The first course essentially presented the basics of normal theory estimation and hypothesis testing procedures at a pre-calculus level, while its sequel went beyond the introductory level, preparing the student for analysis of thesis data. Due to the needs of students in the past, the latter course was devoted mainly to the analysis of data from planned experiments. Consequently, many of the examples were taken from field, greenhouse, and laboratory experiments.

In the most recent past, the course was also drawing students from disciplines requiring vastly different techniques for analysis of research data and it was becoming increasingly apparent that the scope of topics was not meeting the diverse needs of the student. Therefore, it was decided to change the format of the second semester course to that of modular courses, retaining emphasis on planned experiments in some modules and also offering new topics in others.

To be more specific, in Spring 1975, the second semester course was replaced by modules, each consisting of fourteen lectures (or equivalent), each lasting for approximately four weeks, and most requiring a weekly laboratory session. These modules were titled Design and Analysis I, Design and Analysis II, and Regression I. In addition, three other short courses were introduced: Regression II, Sampling from Biological Populations, and Non-parametric Statistical Methods.

2. Modular Format. The spring semester of 1975 was divided into three equal parts for the scheduling of the six modular courses. Design and Analysis I and Sampling from Biological Populations ran concurrently from January 28, 1975 to February 26, 1975; Design and Analysis II and Regression I both ran from February 28, 1975 to April 7, 1975; while Regression II and Non-parametric Statistical Methods occupied the remaining one-third of the semester.

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* Biometrics Unit, Department of Plant Breeding and Biometry, Cornell University.

Each of the above courses was self-contained, except for the continuation courses, and each carried a prerequisite of at least a one-semester introductory statistical methods course. The students were assumed to have no other exposure to statistical concepts, except through direct research interest and activity in their own subject matter area.

Brief course descriptions are given below and complete course syllabuses are given in the appendix.

Design and Analysis I. Basic statistical designs will be analyzed and compared with respect to efficiency and applicability in biological experimentation. This includes hypothesis testing for balanced and unbalanced data. Emphasis will also be placed on both point and interval estimation of treatment effects. Single degree of freedom tests will be stressed in the context of data analysis. Simultaneous confidence intervals and multiple comparisons are also considered. In addition, two-factor Factorial Experiments will be considered with emphasis on the interpretation of interaction. Lecturer: C. L. Wood, Guest Lecturer: W. T. Federer.

Design and Analysis II. Continuation of Design and Analysis I. Emphasis will be placed on the analysis and use of Factorial Experiments within each of the basic designs considered in Design and Analysis I. In addition, Latin Square Designs and some Incomplete Block Designs will be considered. Fractional Factorials and Split-plot experiments are also included. Lecturer: C. L. Wood, Guest Lecturer: W. T. Federer.

Regression Analysis I. Basic topics in Multiple Linear Regression will be covered, using a matrix formulation. These include selection of the regression model, least squares estimation of the regression coefficients, and regression approach to both Analysis of Variance and Analysis of Covariance. Emphasis will be placed on model selection, hypothesis testing, and confidence intervals. Lecturer: C. L. Wood, Guest Lecturer: F. B. Cady.

Regression Analysis II. Continuation of Regression Analysis I. Data analysis and interpretation using standard multiple regression programs will be emphasized with special attention given to the interpretation of partial regression coefficients and R^2. Variable selection procedures include forward selection, backward elimination, stepwise and PRESS with RIDGE regression an alternative to unbiased esti-

mation. Estimated residuals are plotted and tested leading to model reformulation, transformations, or generalized least squares estimation. Covariance is presented through a multiple regression model and the relationship to analysis of variance calculations shown. Lecturer: F. B. Cady.

Sampling Biological Populations. Standard methods of socio-economic sample survey design and estimation will be presented, including stratified-random sampling, cluster sampling, double sampling, and variable probability sampling. Special emphasis will be given to methods of particular utility or specifically designed for biological sampling. Examples will be taken from forestry, fisheries, and other biological areas. Lecturer: D. S. Robson.

Non-parametric and Distribution-Free Statistical Methods. The basic principles of non-parametric and distribution-free inference will be introduced through specific testing procedures. Emphasis will be placed on alternatives to normal-theory testing procedures. These include Randomization Tests, Kruskal-Wallis Tests, and Friedman Rank Sum Tests for Analysis of Variance. Other topics to be discussed are non-parametric multiple comparisons, product-moment correlations, goodness-of-fit testing, and analysis of categorical data. Lecturer: C. L. Wood.

Most of the modular courses also required a weekly laboratory session in which application to current or classical biological problems were discussed and computation methods taught.

Each course had a final exam, all of which were $2\frac{1}{2}$-hour, open book exams, except for Regression I, which was a take-home exam. Copies of the exams for Design and Analysis I, Design and Analysis II, Regression I, and Sampling from Biological Populations are also included in the appendix.

3. Advantages: Proposed and Realized. The six modules discussed above were tailored to the needs, as we saw them, of the graduate students at Cornell University. The evaluation of any modular program must lean heavily on the criterion of successfully meeting this need.

3.1. Flexibility in Scheduling. As mentioned earlier, the requirements of the students we are trying to reach are varied. Many students, especially those involved in field and greenhouse experiments, need to emphasize design of experiments, including such topics as split-plot designs and fractional replicates of

factorial experiments. Students in other areas, such as fisheries, require a working knowledge of the more basic experimental and treatment designs, and also an introduction to the techniques of biological sampling. Students in Human Development and Family Studies, for example, require not only the basics of classical analysis of planned experiments, but also the use of ranking and other distribution-free methods; while students in Agricultural Economics are mainly interested in studying regression analysis.

It is our feeling that offering two modules on design and analysis of experiments, two modules on regression analysis, and one each on sampling and non-parametric methods allows the student to tailor his program to study those areas most relevant to his own interests. This makes the most efficient use of a graduate student's time – a graduate student who is required to take courses outside his own area of specialization.

The scheduling of the modules was also done with the student curriculum in mind. Students, especially those in the plant sciences, are heavily involved in field trips and planning summer experiments at the end of the spring semester. Most of these students are advised to take the design and analysis sequence and either Regression I or Sampling Biological Populations. The present scheduling leaves the last third of the semester open for field trips, planting, etc. For first-year graduate students this has the added advantage that the design courses are completed before the student begins his experiments.

It is our intention that these six modules be offered at least every spring semester. Therefore a student need not take all of the modules in his first year of graduate study. After a student has selected a research topic, he may then enroll in those modules which he feels to be the most beneficial. This not only is an efficient use of student time, but also increases the motivation for taking such courses and hence alleviating some of the student's hang-ups over taking required statistics courses.

This flexibility is also beneficial to graduate students in the fields of Biometry and Statistics, who enter the graduate program with vastly differing backgrounds. These students need take only those modules on topics in which they are deficient, thus eliminating duplication.

3.2. Statistical Computing. Due to previous class size and to the broad interests of the students in our semester courses, it was not possible to justify the time needed to introduce computer techniques. This past semester, students in Regression II were introduced to the Statistical Analysis System (SAS) programs, and actually had a relatively free rein on the use of the system to analyze data; e.g., plotting, variable selection, etc. This was done with a great deal of input and direction from both the instructor, F. B. Cady, and a graduate teaching assistant without whose help such a setup would have been impossible. We have found that the laboratory sessions are not only useful, but also essential in this regard. Next year, it is our intention to also utilize the available computing facilities in Regression I, but on a much smaller scale.

3.3. Increased Faculty Input. One of our main objectives was to increase the number of faculty involved in the teaching of the introductory course. To be more specific, we wanted to have faculty teaching topics in their own research areas. This has to be one of the most successful facets of our program. Professor D. S. Robson taught the sampling module and Professor F. B. Cady taught the second regression module along with guest lecturing in Regression I. Professor W. T. Federer was a frequent guest lecturer in both of the design and analysis modules. The benefit to the student is incalculable. Not only are the topics which are taught current, but also the student gains a greater insight into both the motivation for and the application of the methods presented.

3.4. New Modules. As new research techniques evolve in subject matter areas, the topics offered in a research oriented statistical methods course should change accordingly. The modular format is ideal for meeting this need. First, if student interest warrants, topics currently taught in the modules can be extended in other modules. Secondly, interest in more specialized topics can lead to modules which are taught more infrequently than those already in progress. Topics for some modules which are being considered include bioassay, sequential experimentation, correlated and repeated measurements, multiple comparisons, reliability and life-testing, biological and econometric models, and linear programming and operations research. The number of topics which can be taught in modules is limited only by student interest and the availability of faculty and teaching assistants.

4. Disadvantages and Proposed Solutions. Some of the disadvantages which we recognize at this time are inherent in the concept of modular instruction; some are confounded with the problems encountered in teaching any new course. When comparing the modular system to the traditional semester course, many disadvantages can be noted. However, we feel that some, if not most, of these drawbacks can be corrected by a change in approach by both the instructor and the students.

4.1. Student Approach. Students should be made aware of not only the benefits but also the consequences of the short duration of a modular course. This necessitates a change in a student's study habits; a fact which should be obvious but one which we found not to be widely recognized. There are several ramifications - all of which can lead to problems. Because many topics are taught in a relatively short period, each building on the previous ones, the material presented in lecture must be studied daily and mastered before the next class meeting. Also there can be no lag between material presented in lecture and the applications of such concepts in the laboratory. The student is gaining flexibility in the choice of topics and in scheduling, but for a short time is sacrificing some freedom in personal study habits.

Another consequence of this approach is that the student must be able to very quickly ascertain if he actually understands the concepts presented. A traditional semester course usually includes one or more preliminary examinations, which serve as checks for both the student and the instructor. Due to time restrictions, this obviously cannot be done in a modular course and thus places greater responsibility and pressure on the student.

Several strategies can alleviate these problems. First, daily problem assignment, due at the first of each lecture, would force the student to keep up with the lecture material. This is being tried in one of our later modules, Regression II, and is meeting with very good success. Of course this is very demanding on both the instructor and graduate teaching assistants. Another way is the preparation of a complete set of class notes which are distributed at the beginning of each module. This will be discussed in more detail later.

It is also very important that the student view each module as a separate course. If this notion is not fully realized, it can be especially frustrating for the student taking two modules concurrently. It is critical that both students

and their advisers be made acutely aware of this fact, not only from the standpoint of grading but also time allocation. From experience last semester, we feel that the problem can be greatly minimized by having different instructors for courses which are running simultaneously.

4.2. Faculty Approach. In building the modular courses, there seems to be a tendency to include too many topics. One reason for this is that 14 lectures do not have the flexibility of 42; it is nigh on to impossible to make up time in a short course. Topics should be carefully considered as to the time involved for presentation and the need for students having assimilated previously presented material in order to understand the concept.

In the process of teaching the modules, we found that it was efficient to identify each lecture with a specific topic. Great care was taken in order to insure that the material was not continued (or finished up) at the beginning of the next lecture. In some cases, this necessitated taking one period as a "clean-up" period, but it was found to be more efficient than the alternative.

Another way to alleviate this problem is the preparation of a set of class lecture notes to be handed out to the students at the beginning of each course. By doing this, the student can actually study the lecture notes before coming to class. This frees a great deal of time which is tied up in both introduction and motivation for topics. Last semester this goal was not realized, but in most of the modules, lecture notes were either given to the students at the lecture or were available a couple of days later. In this manner, we developed a set of notes which can be revised and used next year.

5. Evaluation and Summary. At the present time, the last two of the six modules are in the final stages of completion. The evaluations which can be made at this time are basically subjective. Despite the substantial increase in administrative details and the disorganization accompanying six new courses, we feel that the program was successful. Feedback from other departments largely substantiates these conclusions and also reinforces our feeling of the need for the changes discussed in the preceding section.

In Design and Analysis I, student evaluations were solicited. Four of the questions and the responses on the evaluation forms are given below. The questions which have been omitted concerned biographical data and evaluation of both the texts and laboratory sessions.

(1) Did the course fulfill your expectations in terms of what you wished to learn?

Response	1*	2	3	4	5**	Total
Frequency	0	5	15	28	18	66

* 1 = not at all

** 5 = course fully met my expectations

(2) As a result of this course are you interested in taking Design and Analysis II?

Response	1*	2	3	4	5**	Total
Frequency	6	8	13	18	22	67

* 1 = not at all

** 5 = very definitely

(3) As a result of this course are you interested in taking more of the Statistics and Biometry modules?

Response	1*	2	3	4	5**	Total
Frequency	3	8	19	20	17	67

* 1 = not at all

** 5 = very definitely

(4) Would you have taken another semester statistical methodology course if the modules were not offered?

Response	1*	2	3	4	5**	Total
Frequency	5	11	10	18	23	67

* 1 = would not consider it

** 5 = definitely would

Several interesting points can be made. Comparing Question (1) with Question (2), the percentage of responses in the lowest two categories increases from 7.6% in Question (1) to 20.9% in Question (2). One interpretation of this fact is that

some students, while pleased with the present course, felt they had no need for the material to be covered in Design and Analysis II. This reinforces our move to split the material in this topic, previously contained in our one-semester course, into two separate modules.

The response to Question (4) was somewhat disappointing in that one main object of the modular format was to attract students who would not be interested in taking a full-semester course. However, 23.9% of the responses did fall into the lower two categories, denoting reluctance to take the semester course. In the future, it is our hope to increase the relative number of students in these categories.

In summary, a cursory and subjective analysis implies that in part we are meeting our objectives of tailoring our courses to the needs of the students. Many aspects of the program need improvement. The amount of faculty input into this program, however, was phenomenal – requiring the equivalent of two full-time instructors. At this point, we feel that this increase is not only justified, but also necessary.

Appendix

Included in the pages which follow are course syllabuses for all of the six modules. Also included are the final exams given in the Spring of 1975 for the following courses:

Design and Analysis I (Statistics and Biometry 502);
Design and Analysis II (Statistics and Biometry 503);
Regression I (Statistics and Biometry 504);

and

Sampling Biological Populations (Statistics and Biometry 506).

STATISTICS AND BIOMETRY 502

DESIGN AND ANALYSIS I

Lecturer: C. L. Wood Guest Lecturer: W. T. Federer

Dates: Jan. 27 - Feb. 26

Outline:

Lecture	Lab	Topic
		Completely Randomized Design
1		Fixed and Random effects models: Expected mean squares
2		Confidence Intervals for variance components
3	1	Multiple Comparisons: Scheffé Method and Multiple T-test
		Hierarchical Design
4		Model equation and Sums of Squares identity; Expected Mean Squares
5-6	2	Tests of Significance and Confidence Intervals
		Randomized Block Design
7		What is blocking? What is design? (W. T. Federer)
8		Model equation and analysis with one observation per cell
9		Missing data and Single Degree of Freedom Tests
10		Subsampling in a Randomized Block Design
11	3	Analysis with unbalanced data
		Two-factor Factorial Experiments in Completely Randomized Design
12		Two-factor Factorials with no interaction
13	4	Meaning of Interaction and Mean Square Expectation
14		Partitioning Sums of Squares

Texts: Snedecor and Cochran: Statistical Methods

D. R. Cox: Planning of Experiments

W. T. Federer: Statistics and Society, Chapters VI and VII.

STATISTICS AND BIOMETRY 503

DESIGN AND ANALYSIS II

Lecturer: C. L. Wood Guest Lecturer: W. T. Federer

Dates: Feb. 28 - Apr. 7

Outline:

Lecture	Lab	Topic
1-2		Higher-Order Factorial Experiments in Completely Randomized Designs
		Latin Square and Rectangle Designs
3		The Design (W. T. Federer)
4	1	Analysis and Confidence Intervals
5		Replication vs. Subsampling
		Confounding in 2^k Factorial Experiments
6	2	Factorial Experiments in Complete Block Designs and Latin Square Designs
7		Representation of effects in 2^3 Factorials
8		Balanced and Partial Confounding
9-10	3	Fractional Factorials
		Split-Plot and Split-Block Experiments
11		What is a split-plot and a split-block design? (W. T. Federer)
12-14	4	Analysis of a split-plot

Texts: Snedecor and Cochran: Statistical Methods

D. R. Cox: Planning of Experiments

W. T. Federer: Statistics and Society, Chapters VI and VII.

STATISTICS AND BIOMETRY 504

REGRESSION ANALYSIS I

Lecturer: C. L. Wood

Guest Lecturer: F. B. Cady

Dates: Feb. 27 - Apr. 3

Outline:

Lecture*	Lab	Topic
1		Review of Simple Linear Regression; Matrix Algebra
2	1	Matrix Inversion; Systems of Linear Equations; Abbreviated Doolittle Method
3		Data Matrices for (i) "Designed Experiments", (ii) "Classical Regression", and (iii) "Complete Data Description".
4-5	2	Model Simplification; Partitioning Sums of Squares
6-7	3	Tests of Hypotheses: Unconditional and Conditional
8	4	Estimation of Linear Functions of Regression Coefficients
9		Classical Regression Analysis

Text: Snedecor and Cochran: Statistical Methods, Chapter 13.

* These topics will be covered in nine, seventy-five minute classes.

STATISTICS AND BIOMETRY 505

REGRESSION ANALYSIS II

Lecturer: F. B. Cady

Dates: Apr. 10 - May 8

Outline:

Lecture*	Lab	Topic
1	1	Residual Analysis
2		Interpretation of Estimated Regression Coefficients
3		Interpretation of R^2
4	2	Problems with Large Data Sets
5		Variable Selection Based on Residual Sum of Squares
6	3	Variable Selection Based on Total Mean Square Error
7	4	Biased Estimation - Ridge Regression
8		Generalized Least Squares - Transformations
9		Covariance

Text: Lecture Notes will be provided by F. B. Cady

* These topics will be covered in nine, seventy-five minute classes.

STATISTICS AND BIOMETRY 506

SAMPLING BIOLOGICAL POPULATIONS

Lecturer: D. S. Robson

Dates: Jan. 28 - Feb. 25

Outline:

Lecture*	Lab	Topic
1	1	Introduction to sample surveys
2-5		Review of socio-economic sampling and estimation procedures
6-8	2	Double sampling and associated estimation procedures
9-11	3	Variable probability sampling
12-13	4	"Tag-recapture" methods of population estimation
14		"Change-in-ratio" and "Removal" methods of population estimation

Text: Stewart: Basic Ideas of Scientific Sampling

* These topics will be covered in nine, seventy-five minute classes.

STATISTICS AND BIOMETRY 507

NON-PARAMETRIC AND DISTRIBUTION-FREE STATISTICAL METHODS

Lecturer: C. L. Wood

Dates: Apr. 9 - May 9

Outline:

Lecture	Lab	Topic
		Fundamentals
1		Introduction to Non-parametric Inference
2		Permutations and Combinations
3	1	Randomization Tests
		Non-parametric Analysis of Variance
4		Kruskal-Wallis for One-Way Layout
5		Median Tests for One-Way Layout
6	2	Tests based on Friedman Rank Sums for Two-Way Layout
7		Non-parametric Multiple Comparisons
		Tests for Dispersion
8		Ansari-Bradley and Moses' Procedures
9	3	Tests for Dispersion based on the Jackknife Statistic
		Dependence between Two Variables
10		Pitman product-moment Correlation Coefficient and Spearman Rank Order Correlation Coefficient
11		Kendall's Test for Correlation
		Goodness of Fit Tests
12	4	X^2-Goodness of Fit Tests
13		Cramer-von Mises and Kolmogorov-Smirnov Statistics
14		X^2-Tests of Independence

Text: Hollander and Wolfe: Non-parametric Statistical Inference

STATISTICS 502 FINAL EXAM Spring, 1975

Exam Value: 100 points Name _____ (last), _____ (first)

Instructions: (1) Emphasis is on concepts and not on arithmetic computations.

(2) In each problem, indicate clearly and completely the statistical procedure being used.

1. (30 points): J. W. Lambert at the University of Minnesota, in 1951, compared the effect of two row spacings (18 in. and 24 in.) on the yield of two soybean varieties (OM, B)*. The four treatments (T_1 = OM; 18 in., T_2 = B; 18 in., T_3 = OM; 24 in., T_4 = B; 24 in.) were randomly planted in 3 blocks each with 4 plots. The yield in bushels is given below:

Treatments

Block	18"		24"		Totals
	OM	B	OM	B	
1	28	23	28	25	104
2	21	22	24	21	88
3	23	24	29	20	96
Totals	72	69	81	66	288

* OM = Ottawa Mandarin; B = Blackhawk

Some useful calculations are:

$(288)^2/12 = 6{,}912.00$

$(104)^2/4 + (88)^2/4 + (96)^2/4 = 6{,}944.00$

$(72)^2/3 + (69)^2/3 + (81)^2/3 + (66)^2/3 = 6{,}954.00$

$(28)^2 + (21)^2 + \cdots + (21)^2 + (20)^2 = 7{,}010.00$

(i) Write out the appropriate model and identify each component. Indicate the assumptions that are appropriate when treatment effects are assumed to be fixed.

(ii) Set up an ANOVA table giving the sources of variation, sums of squares, degrees of freedom, and mean squares. Test whether there are differences among treatments.

(iii) Construct a meaningful set of 3 orthogonal linear contrasts of treatment means. Calculate the sums of squares for one of your contrasts.

(iv) Describe the test procedure you would use (test statistic and critical region) if all the comparisons in (iii) were to be tested and the probability of making at least one false rejection is to be less than 0.05.

2. (35 points): On February 28, 1975, the Ithaca Journal ran an article claiming that in 1974, the number of marriages/1,000 persons had dropped by 4% over that for 1973. In order to assess this claim, the following study is feasible: For each of the five geographic regions of the United States (Northeast, Southeast, Mid-west, Northwest, and Southwest), three states were randomly selected and within each state four counties were randomly selected. The number of marriages/1,000 persons was recorded and is shown below:

% Decrease in # of Marriages/1,000 persons

Region	NE			SE			MW			NW			SW		
State	Mass.	N. Y.	Vt.	Ala.	Va.	Ky.	Oh.	Mich.	Mo.	Ore.	Ida.	Wym.	Cal.	N. M.	Tex.
County 1	4	3	3	1	1	-1	3	6	3	6	2	4	7	10	8
2	2	3	5	-1	5	0	1	4	2	4	4	7	9	8	9
3	6	1	2	1	3	2	-1	6	2	4	2	5	7	12	8
4	4	1	2	-1	3	-1	1	4	5	2	4	4	9	10	11
Totals	16	8	12	0	12	0	4	20	12	16	12	20	32	40	36
	36			12			36			48			108		

Some useful calculations are:

$$(240)^2/60 = 960.00$$
$$(36)^2/12 + (12)^2/12 + (36)^2/12 + (48)^2/12 + (108)^2/12 = 1,392.00$$
$$(16)^2/4 + (8)^2/4 + \cdots + (36)^2/4 = 1,472.00$$
$$4^2 + 2^2 + 6^2 + \cdots + 8^2 + 11^2 = 1,652.00$$

(i) Write an appropriate model and identify each component.

(ii) Construct an ANOVA table, including sources of variation, sums of squares, degrees of freedom, mean squares, and expected mean squares.

(iii) Set up a 90% Confidence Interval for the variance among states within regions.

(iv) Notice that the drop in the marriage rates in Southwest U. S. is higher than for the other four regions. Does Southwestern U. S. actually differ (in the % drop in # of marriages/1,000 persons) from the other four regions?

(v) Give a point and interval estimate for μ, the overall percent decrease in marriages.

3. (20 points): A chemical experiment, investigating the effects of four levels of pressure and four levels of temperature on a process, is to be run. There are two proposals (a) and (b). Proposal (a) allocates the 4 x 4 treatment combinations at random to 16 experimental units and then 2 determinations are to be made on each experimental unit, while proposal (b) allocates two replications of the 4 x 4 treatment combinations at random to 32 experimental units and then 1 determination is to be made on each experimental unit.

   (i) Give the model equation for each proposal. Be careful with the subscripts, give the range of each subscript, and identify the terms in the model.

   (ii) Can I estimate both the experimental error and sampling error (as defined in class) from either proposal (a) or (b)? If so, explain, including any assumption(s) required. If not, explain!

   (iii) Why would an experimenter desire an estimate of both the experimental error and the sampling error? How would you design the experiment to estimate the two types of error?

   (iv) If the two replications in proposal (b) were run at two different times and the experimenter thought that a main effect of time existed, how would you design the experiment? Give the resulting model.

4. (15 points): Given below is the table of means for a two-factor experiment with two levels each of A and B:

		Factor A	
		A_1	A_2
Factor B	B_1	1	X
	B_2	Y	Z

Choose X, Y, and Z in each of the following situations such that

   (i) The mean square for A is zero;
   the mean square for B is zero;
   and the mean square for AB is zero.

   (ii) The mean square for A is non-zero;
   the mean square for B is non-zero;
   and the mean square for AB is zero.

   (iii) The mean square for A is zero;
   the mean square for B is zero;
   and the mean square for AB is non-zero.

STATISTICS 503 FINAL EXAM Spring, 1975

Exam Value: 100 points Name _____ (last), _____ (first)

1. In experiments on the effect of fire-retardant treatments applied to wood, factorial experiments of the following type are encountered. These are three different treatments and a control, and these are to be compared on material of two species. Each species is to be tested with a rough surface and with a smooth surface two days, three months, and one year after treatment. This is, formally, a 4(treatments) x 2(species) x 2(rough and smooth) x 3(times) factorial experiment. One replication is to be run and the 4-factor interaction is to be used as an estimate of the experimental error. The times were randomly selected; all other factors are fixed.

   (i) Indicate the F-statistic for testing that there are no treatment effects. Justify!

   (ii) What assumptions about the data must be made in order to conduct the test in part (i)?

2. In order to investigate the differential effects of three levels of a drug (A_1, A_2, A_3) on motor response, white rats, injected with various units of the drug, were run through a maze, one at a time, and the time required to run the course was recorded. Six rats (2 weeks old) were randomly selected from those available to the Experimental Psychology Lab for the experiment, and two were randomly assigned to each drug level. The results were summarized and reported in the following ANOVA table:

Source	SS	DF
Mean	24	1
Drug	4	2
Within	3	3
Total	31	6

Eight weeks later, the same experiment was repeated, but this time the rats used in the experiment (different rats) were ten weeks old. The results were as follows:

Source	SS	DF
Mean	6	1
Drug	4	2
Within	3	3
Total	13	6

(i) Combining the two experiments, construct an ANOVA table, including as much detail as is possible. (Do not display EMS's.) Is there a difference in average times for rats 2 weeks old as compared to rats 10 weeks old?

(ii) In the combined analysis, is it possible for SS Drugs = 0? Explain!

3. A preliminary study was conducted to investigate the moisture gradients in a field which was being considered for use in an alfalfa experiment. If certain sections of the field were found to be too wet, drainage tiles would be installed. The following sampling plan was adopted by the experimenters. The asterisks indicate positions at which soil samples would be taken. These observed values are to be analyzed to determine soil moisture.

*	*	*	*
(1.70)	(0.85)	(1.90)	(2.45)
*	*	*	*
(1.10)	(1.55)	(2.40)	(2.95)
*	*	*	*
(1.50)	(2.10)	(3.20)	(3.60)

It is known that moisture gradients in the field, if they exist, are linear from top to bottom and from left to right.

(i) Propose an analysis of this data; i.e., set up an ANOVA table. Clearly indicate how to calculate the sums of squares (do not do any calculations) and what tests are possible, and of interest.

(ii) Suppose you learn that the measurements for the top row were made in Week 1, for the middle row were made in Week 2, and for the bottom row were made in Week 3. How does this affect your analysis?

4. In certain textile investigations it is required to test a number of modifications in a process for producing a thin web of parallel fibers. One important property of the web is the number of fiber entanglements, say per mg of web, and this is measured by passing a section of web slowly over an illuminated strip, when individual entanglements can be noted and the total found. However it is difficult to define precisely what constitutes an entanglement so that, whereas one observer can get reasonably reproducible counts over a short period of time, there are liable to be large systematic differences between observers and between the same observer's counts on different days.

Suppose for definiteness that we have 6 different batches, $W_1, \cdots, W_6$, of web to be compared and 2 observers. Let us assume that the webs have been produced by six different processes. Six sections of each web are randomly selected for examination. Two experimental designs are given below:

PLAN A*:

	Order of Measurement					
	1	2	3	4	5	6
Observer 1 First period	W_4	W_1	W_2	W_5	W_6	W_3
Observer 2 First period	W_5	W_6	W_2	W_1	W_4	W_3
Observer 1 Second period	W_3	W_6	W_2	W_4	W_1	W_5
Observer 2 Second period	W_3	W_1	W_4	W_5	W_6	W_2
Observer 1 Third period	W_6	W_5	W_4	W_3	W_1	W_2
Observer 2 Third period	W_5	W_1	W_4	W_3	W_6	W_2

PLAN B:

	Order of Measurement					
	1	2	3	4	5	6
Observer 1 First period	W_3	W_6	W_1	W_4	W_2	W_5
Observer 2 First period	W_4	W_1	W_2	W_5	W_3	W_6
Observer 1 Second period	W_5	W_2	W_3	W_6	W_4	W_1
Observer 2 Second period	W_6	W_3	W_4	W_1	W_5	W_2
Observer 1 Third period	W_1	W_4	W_5	W_2	W_6	W_3
Observer 2 Third period	W_2	W_5	W_6	W_3	W_1	W_4

* The order of measurement was randomly assigned.

(i) Identify <u>each</u> design and give the ANOVA table, including sources of variation and degrees of freedom.

(ii) Under what experimental conditions would you prefer PLAN A to PLAN B and vice versa?

5. Suppose the following experiment were carried out to explore factors influencing texture of strawberries which have been frozen and rethawed. The objective is to determine freezing conditions which will produce non-mushy thawed berries.

The experimenter is interested in two factors: One factor is the speed of freezing (speeds s_1, s_2, and s_3 are used); the other factor is the concentration of sucrose in the syrup in which the berries are frozen (concentrations c_1, c_2, c_3, and c_4 are used). Both factors are fixed effects. The experimenter is primarily interested in the effect of speed of freezing and its interaction with sucrose concentration -- less interested in the effect of sucrose concentration itself.

The berries are all of one variety, but come from two localities. Localities are viewed as blocks -- it is thought that localities will <u>not</u> interact with the treatments described in the preceding paragraph.

The berries from each locality are divided into 4 portions, one portion placed in syrup of each of the 4 sucrose concentrations. Each portion is subsequently subdivided into 3 sub-portions to be frozen at the 3 freezing speeds.

Later, after thawing, 10 berries from each locality-sucrose concentration-freezing speed combination are tested with a device measuring the force required to puncture the fruit with a probe. The average reading for the 10 berries was recorded for use in the subsequent analysis. (We use 24, not 240, as the total degrees of freedom.)

Suppose the data were:

	Locality 1				Locality 2				Totals
	c_1	c_2	c_3	c_4	c_1	c_2	c_3	c_4	
s_1	13.6	15.2	16.3	14.3	14.7	16.3	15.8	16.1	122.3
s_2	16.4	15.4	17.7	15.4	15.8	19.5	19.7	18.5	138.4
s_3	16.1	17.2	20.2	16.1	17.6	19.7	21.3	16.9	145.1
Totals	46.1	47.8	54.2	45.8	48.1	55.5	56.8	51.5	405.8
	193.9				211.9				

Sucrose Concentration	c_1	c_2	c_3	c_4
Total	94.2	103.3	111.0	97.3

(i) Write out the appropriate model.

(ii) Describe the partitioning of the total SS by listing the sources of variation and the degrees of freedom associated with each source.

(iii) Then indicate how to calculate each sum of squares (i.e., plug in the numbers but don't do any multiplying or dividing).

(iv) Indicate the appropriate F ratios for testing the effects of interest.

STATISTICS 506 FINAL EXAM Spring, 1975

Name _____ (last), _____ (first)

In June of 1974 a major paper company with forest holdings throughout northeastern U. S. selected 4 forest sites to test a new insecticide against one of their major competitors, the spruce budworm. Each test site consisted of a 100-acre tract which was to be sprayed with the rapidly degradable insecticide, and the density of the budworm infestation was to be measured immediately before and immediately after spraying. Dominant (tallest) and codominant spruce trees are favored by the budworm and the midcrown of the tree is the favored target area, so the practice among forest entomologists is to count budworms per 18 inches of branch on branches deliberately selected from the midcrown of dominant and codominant trees in order to obtain an index of budworm density in a forest area.

Formally, each of the 100-acre tracts was partitioned into tenth-acre plots, of which 60 were randomly chosen for study. In each selected plot the dominant and codominant spruce trees were identified and listed in a random order. Every other tree in the list was sampled before spray by deliberately selecting the terminal 18-inch segment of four systematically chosen branches at a prescribed midcrown point of the tree. After spray the remaining half of the list was sampled in the same manner. The index of survival was taken to be the ratio of post-spray density to pre-spray density measured in this manner at each site.

To mathematize the above description, let "x of a tree" be the total number of budworms on the prescribed four 18-inch branch segments (assume that these branches are uniquely identified by some rule) at the time of the pre-spray sample, and let "y of the tree" be the post-spray number on the same branch segments of an undisturbed tree. The index of survival or any one of the four tracts may then be denoted by T_y/T_x and is estimated by $\hat{T}_y/\hat{T}_x$ where, for example,

$$\hat{T}_x = \frac{K}{k}\sum_{1}^{k} N_h \bar{x}_h ,$$

and the sampling variance of $\hat{T}_x$ is given by the formula

$$V(\hat{T}_x) = \frac{K(K-k)}{k} S^2_{T_{xh}} + \frac{K}{k}\sum_{1}^{K} N_h \frac{(N_h - n_h)}{n_h} S^2_{xh} .$$

The sampling variance of the ratio $(\hat{T}_y/\hat{T}_x)$ is approximated by

$$V\left(\frac{\hat{T}_y}{\hat{T}_x}\right) \doteq \left(\frac{T_y}{T_x}\right)^2 \left[\frac{V(\hat{T}_y)}{T_y^2} + \frac{V(\hat{T}_x)}{T_x^2} - 2\,\frac{Cov(\hat{T}_y,\hat{T}_x)}{T_yT_x}\right] .$$

When x is measured on a random sample of n_h out of N_h elements and y is measured on a random sample of m_h taken from the remaining $N_h - n_h$ elements, then the sampling covariance of $\hat{T}_y$ and $\hat{T}_x$ is given by

$$Cov(\hat{T}_y,\hat{T}_x) = \frac{K(K-k)}{k}\, S_{T_{yh}T_{xh}} - \frac{K}{k}\sum_{1}^{K} N_h S_{xyh} .$$

The following small artificial population of size K = S clusters was created to check your reading and comprehension of the above formulas:

h =	1		2		3		4		5		Total	
N_h =	4		4		4		6		8		26	
j	x_{1j}	y_{1j}	x_{2j}	y_{2j}	x_{3j}	y_{3j}	x_{4j}	y_{4j}	x_{5j}	y_{5j}		
1	0	0	0	0	0	0	0	0	2	0		
2	0	1	0	0	0	0	3	0	3	0		
3	3	0	0	1	6	0	6	0	5	1		
4	5	0	8	1	6	0	10	1	10	1		
5							10	2	11	0		
6							13	1	13	2		
7									14	0		
8									22	4		
Total	8	1	8	2	12	0	42	4	80	8	150	15
Mean	2	$\frac{1}{4}$	2	$\frac{1}{2}$	3	0	7	1	10	2	24	$3\frac{3}{4}$
Variance	6	$\frac{1}{4}$	12	$\frac{1}{3}$	12	0	24	.4	44	6/7		
Covariance	$-\frac{2}{3}$		$\frac{2}{3}$		0		3		7			

Some additional numerical results are:

$$(22)^2 + (22)^2 + (18)^2 + (12)^2 + (50)^2 = 3936$$

$$44 + 22 + 54 + 12 + 250 = 382.$$

Substitute the appropriate numbers into the preceding formulas for the case k = 2, $n_h = m_h = N_h/2$, and show enough details of your calculations so that arithmetic errors can be disregarded.

$V(\hat{T}_y) =$

$V(\hat{T}_x) =$

$Cov(\hat{T}_y, \hat{T}_x) =$

$V(\hat{T}_y/\hat{T}_x) =$

Pretending that Table 1 represents numbers of budworms before and after spray in a little test tract of 5 plots, identify one possible sample outcome (k = 2, $n_h = m_h = N_h/2$) by circling entries in Table 1, and calculate the estimate $\hat{T}_y/\hat{T}_x$ obtained from your sample:

$\hat{T}_y/\hat{T}_x =$

Note that the total number of trees measured in a sample (before spray plus after spray) is a random variable which in this example ranges from 8 to 14 with an average value of 10.4. An alternative sampling design producing a fixed sample size of 10 trees is a stratified sample in which one tree is randomly selected from each of the K = 5 plots for pre-spray measurement and a second one is selected from each plot for post-spray measurement. Calculate the variances under this sampling design. (Hint: The formulas are special cases of the general formulas given earlier.):

$V(\hat{T}_y) =$

$V(\hat{T}_x) =$

$Cov(\hat{T}_y, \hat{T}_x) =$

$V(\hat{T}_y/\hat{T}_x) =$

A third alternative design producing comparable sample size is to select one plot at random for pre-spray measurement of every tree in the plot and then select a different plot at random for post-spray measurement of every tree in the plot. Compare the variances under this design with those of the other two designs. This is another special case but you need one additional formula; if k_1 plots are randomly selected for pre-spray complete measurement and k_2 different plots are randomly selected from the remaining $K - k_1$ for post-spray complete measurement, then

$$\mathrm{Cov}\left(\frac{K}{k_1}\sum_{1}^{k_1} T_{xh}, \frac{K}{k_2}\sum_{1}^{k_2} T_{yh}\right) = -KS_{T_{xh}T_{yh}}$$

$V(\hat{T}_y) =$

$V(\hat{T}_x) =$

$\mathrm{Cov}(\hat{T}_y, \hat{T}_x) =$

$V(\hat{T}_y/\hat{T}_x) =$

No control tracts (unsprayed) were used in the experiment designed by the paper company because the time interval between pre- and post-spray samples was short enough to permit the assumption that changes in budworm density due to natural causes would be negligible. Unsprayed control tracts are commonly used in such experiments, however, when a significant time lapse occurs between the pre- and post-spray samples, so suppose each of the four test tracts was matched with a nearby unsprayed control tract of the same size, and sampled in exactly the same manner at exactly the same times. The estimate of spray-induced mortality at any one of the four sites is then given by

$$\hat{M} = 1 - \frac{(\hat{T}_y/\hat{T}_x)\ \text{spray}}{(\hat{T}_y/\hat{T}_x)\text{control}} .$$

Thus, $1 - \hat{M}$ is a ratio of two statistically uncorrelated ratios, and since we have a general formula for the sampling variance of a ratio we should be able to deduce

a formula for the sampling variance of the above estimator. Let the symbol V_s denote the sampling variance of $\hat{T}_y/\hat{T}_x$ for the sprayed tract and let V_c denote the corresponding variance for the control tract. Using the symbols V_s, V_c, $(T_y/T_x)_s$ and $(T_y/T_x)_c$, give a formula for the (approximate) sampling variance of the mortality estimator:

$V(\hat{M}) = V(1 - \hat{M}) \doteq$

Suppose that the four test sites are essentially a random sample of sites from those (very large) forest holdings of the company where the spray would potentially be applied. Mortality estimates $\hat{M}_1$, $\hat{M}_2$, $\hat{M}_3$ and $\hat{M}_4$ are available from the four sample sites and will be combined in some manner to estimate what the mortality would be for all potential sites of spraying. The numerator ratio and denominator ratio of the $\hat{M}_i$'s are now correlated <u>between</u> sites (uncorrelated <u>within</u> sites) and there is a problem in determining just how the four estimates should be combined. For the sake of simplicity suppose the average mortality is estimated by the simple average

$$\bar{M} = \tfrac{1}{4}(\hat{M}_1 + \hat{M}_2 + \hat{M}_3 + \hat{M}_4).$$

Statistical inferences to this large population (e.g., a confidence interval) will then be based upon $\bar{M}$ and

$$s^2_{\bar{M}} = \frac{1}{4(3)}\left[(\hat{M}_1 - \bar{M})^2 + (\hat{M}_2 - \bar{M})^2 + (\hat{M}_3 - \bar{M})^2 + (\hat{M}_4 - \bar{M})^2\right].$$

The point here is that estimates of <u>sampling variance</u> such as those you have just computed do <u>not</u> enter into such final calculations as $\bar{M} \pm 1.96 s_{\bar{M}}$. Explain, therefore, what utility the sampling variance formulas have in the context of this paper company's experiment: