Statistical Analysis of Reported Tag-Recaptures in the Harvest from an Exploited Population

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Abstract

Tag-recapture experiments on exploited populations often utilize the information on tag recaptures reported from the annual harvest in order to estimate the vital statistics of the population. The design of the experiment consists of releasing a known number of individually identifiable tagged members into the population at the start of each year; the annual returns of tags from each batch released are separately recorded. On the assumption that survival is year-specific but not age-specific and that all tagged members present in the population are equally vulnerable to harvest regardless of the time of tagging, the "reported exploitation" rate and survival rate for each year are estimable. The fraction of a released batch of tags that are ultimately returned, and the total returns from all releases in each harvest season together constitute a sufficient summary of the data and may be transformed into estimates of annual survival rates and annual rates of "reported exploitation". The validity of this model may be tested by a connected series of contingency chi-squares.

# Statistical Analysis of Reported Tag-Recaptures in the Harvest from an Exploited Population

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## I. Introduction

Robson (1963) constructed a stochastic model and estimation formulae for a tag recapture experiment in which the population is sampled annually. At each anniversary date a sample is drawn from the population, tag-recaptures are recorded, untagged individuals are given tags, and the entire sample is returned to the population. A defect of this model as it is conventionally applied is the selectivity of the annual sample, which usually employs the same capture technique at the same geographic location each year. In the case of fish populations this annual sample is typically collected from major spawning concentrations which occur at the same sites each year and which usually represent distinct subpopulations (as evidenced from tag returns.)

Youngs (1971) adapted this model to the more appropriate and commonly occuring situation in which the population is exploited during the year and recaptures are reported from the harvest. The method of exploitation is usually non-selective within the adult population, removing the same proportion of each of the tagged and untagged subpopulations, and hence provides effectively random samples as a basis for estimating survival rates. Since harvesting eliminates the possibility of multiple recaptures, however, the stochastic model and estimation formulae are correspondingly effected, the details of which are developed here.

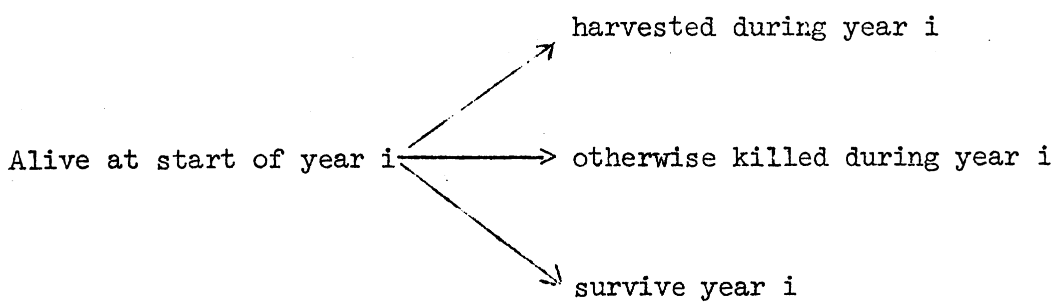
## II. A Stochastic Model for Reported Tag Recaptures.

Exploitation rates follow seasonal cycles which vary with the particular species being exploited; we therefore ignore the within-year structure and consider only the problem of modelling the annual total number of reported tag recaptures. At the i'th anniversary date $N_i$ newly marked adult individuals bearing serially numbered tags are released into the population, and in the following years the reported numbers of recaptures from this lot are $R_{i,i}, R_{i,i+1}, \ldots, R_{i,k}$ as displayed in Table 1.

Table 1. Array of reported recaptures over a k year period

| Year Tagged | Number Tagged | Year of reported recapture 1 | 2 | 3 | ... | k | Total |
|---|---|---|---|---|---|---|---|
| 1 | $N_1$ | $R_{11}$ | $R_{12}$ | $R_{13}$ | ... | $R_{1k}$ | $R_1$ |
| 2 | $N_2$ | | $R_{22}$ | $R_{23}$ | ... | $R_{2k}$ | $R_2$ |
| 3 | $N_3$ | | | $R_{33}$ | ... | $R_{3k}$ | $R_3$ |
| ⋮ | ⋮ | | | | ⋱ | ⋮ | ⋮ |
| k | $N_k$ | | | | | $R_{kk}$ | $R_k$ |
| Total | | $C_1$ | $C_2$ | $C_3$ | ... | $C_k$ | |

A tagged individual alive at the start of year i may suffer any of three fates during the course of year i :



We assume that in the adult population these mortality rates are year-specific -- i.e., depend upon the environmental conditions and exploitation pressure prevailing during the i'th year -- but are independent of the age of the individual, or the time of tagging, so that for all (tagged) individuals alive at the start of year i the following conditional probabilities obtain:

$$u_i = P \text{ (harvested during year i)}$$
$$S_i = P \text{ (survive year i)}$$
$$1 - S_i - u_i = P \text{ (otherwise killed during year i).}$$

A harvested tag-recapture may go unreported, and we assume only that for all tag-recaptures made in year i the conditional probability

$$\lambda_i = P \text{ (tag recapture reported)}$$

is independent of the year of tagging. This is virtually equivalent to assuming that tags remain permanently attached, for otherwise the probability of reporting a tagged individual which has been recaptured would decrease with the age of the tag.

With the further assumption that tagged individuals, mixed into the larger exploited population, each suffer a statistically independent fate we arrive at a likelihood function which may be expressed as a product of multinomial distributions. Letting $\{R_{ij}\}$ denote the array of random variables shown in Table 1 then for fixed numbers of tag releases $N_1, \ldots, N_k$

$$P\left(\{R_{ij}\}\right) = \prod_{i=1}^{k} \binom{N_i}{R_{i,i}, \ldots, R_{i,k}}$$

$$\left(\lambda_i u_i\right)^{R_{i,i}} \left(S_i \lambda_{i+1} u_{i+1}\right)^{R_{i,i+1}} \ldots \left(S_i S_{i+1} \ldots S_{k-1} \lambda_k u_k\right)^{R_{i,k}}$$

$$\left[1 - \lambda_i u_i - S_i \lambda_{i+1} u_{i+1} - \ldots - S_i S_{i+1} \ldots S_{k-1} \lambda_k u_k\right]^{N_i - R_i} .$$

III. Estimation formulas.

The parameters $\lambda_i$ and $u_i$ appear in the likelihood function only as the product $f_i = \lambda_i u_i$ ; hence the 2k - 1 identifiable parameters in this model are

$$f_i = P\,(\text{recapture reported in year } i \mid \text{alive at start of year } i)$$
$$\text{for } i = 1,\ldots,k$$

and

$$S_i = P\,(\text{survive year } i \mid \text{alive at start of year } i) \text{ for } i = 1,\ldots,k-1 \;.$$

Expected tag returns expressed as functions of these parameters are shown in Table 2, where it is seen that a convenient reparameterization for expressing expected values of marginal totals is given by $(\rho_1,\ldots,\rho_k,\ \xi_2,\ldots,\xi_k)$, where

$$E\left(\frac{R_i}{N_i}\right) = f_i + S_i f_{i+1} + \ldots + S_i S_{i+1} \cdots S_{k-1} f_k$$
$$\equiv \rho_i$$
$$= f_i + S_i \rho_{i+1} \quad \text{for } i = 1,\ldots,k-1 \quad (\rho_k \equiv f_k)$$

and

$$E\left(C_i\right) = (N_1 S_1 \cdots S_{i-1} + N_2 S_2 \cdots S_{i-1} + \ldots + N_i) f_i$$
$$\equiv \xi_i f_i$$
$$\xi_{i+1} = N_{i+1} + S_i \xi_i \quad \text{for } i = 1,\ldots,k-1 \; (\xi_1 \equiv N_1) \;.$$

Table 2. Array of expected tag returns

| Year Tagged | Year of Reported Recapture: 1 | 2 | 3 | ... | k | Total |
|---|---|---|---|---|---|---|
| 1 | $N_1f_1$ | $N_1S_1f_2$ | $N_1S_1S_2f_3$ | ... | $N_1S_1S_2\cdots S_{k-1}f_k$ | $N_1\rho_1$ |
| 2 | | $N_2f_2$ | $N_2S_2f_3$ | ... | $N_2S_2S_3\cdots S_{k-1}f_k$ | $N_2\rho_2$ |
| 3 | | | $N_3f_3$ | ... | $N_3S_3S_4\cdots S_{k-1}f_k$ | $N_3\rho_3$ |
| ⋮ | | | | ⋱ | ⋮ | ⋮ |
| k | | | | | $N_kf_k$ | $N_k\rho_k$ |
| Total | $\xi_1f_1$ | $\xi_2f_2$ | $\xi_3f_3$ | ... | $\xi_kf_k$ | |

This transformation of parameters is thus given by

$$S_i = \frac{\xi_{i+1} - N_{i+1}}{\xi_i} \qquad f_i = \rho_i - \rho_{i+1}\left(\frac{\xi_{i+1} - N_{i+1}}{\xi_i}\right)$$

Neyman factorization of the likelihood function reveals that the row and column totals of Table 1 form a sufficient statistic, and that a convenient transformation of this statistic is given by $(R_1,\ldots,R_k,\ T_2,\ldots,T_k)$ where

$$T_{i+1} = R_{i+1} + T_i - C_i \qquad \text{for } i = 1,\ldots,k-1 \qquad (T_1 \equiv R_1)$$

since

$$P\left(\left\{R_{ij}\right\}\right) = \prod_{i=1}^{k} \binom{N_i}{R_{ii},\ldots,R_{ik}} f_i^{C_i}(1-\rho_i)^{N_i-R_i} S_{i-1}^{T_i-R_i} \quad .$$

In terms of $T_i$ and $r_i = R_i/N_i$ the maximum likelihood estimators are then given by

$$\hat{f}_i = r_i\left(\frac{C_i}{T_i}\right) \qquad \hat{S}_i = \frac{r_i - \hat{f}_i}{r_{i+1}} \quad .$$

Consistency of these estimators as $N_i$ gets large is apparent from the relations

$$E(r_i) = \rho_i = f_i + S_i\rho_{i+1} \qquad E(C_i) = f_i\xi_i \qquad E(T_i) = \rho_i\xi_i$$

where $R_i$ is binomially distributed and $C_i$ and $T_i$ are each distributed as a convolution of binomial distributions. As shown in Section 4, for fixed $R_i$ and $T_i$ the conditional distribution of $C_i$ is also binomial,

$$P(C_i|R_i,T_i) = \binom{T_i}{C_i}\left(\frac{f_i}{\rho_i}\right)^{C_i}\left(1-\frac{f_i}{\rho_i}\right)^{T_i - C_i},$$

independent of $R_i$ and hence $\hat{f}_i$ is an unbiased estimator. Since the numerator and denominator of $\hat{S}_i$ are statistically independent we may also infer that

$$E(\hat{S}_i) = S_i E\left(\frac{\rho_{i+1}}{r_{i+1}}\right) \doteq S_i\left[1-(1-\rho_{i+1})^{N_{i+1}}\right] .$$

The approximate covariance matrix of $\left(\hat{f}_1,\ldots,\hat{f}_k, \hat{S}_1,\ldots,\hat{S}_{k-1}\right)$, obtained by conventional Taylor series approximations, is given by:

$$\sigma^2_{\hat{f}_i} \doteq \frac{f_i}{N_i\rho_i}\left[f_i(1-\rho_i) + \frac{N_i(\rho_i - f_i)}{\xi_i}\right]$$

$$\sigma^2_{\hat{S}_i} \doteq S_i^2\left[\frac{1-\rho_i}{N_i\rho_i} + \frac{1-\rho_{i+1}}{N_{i+1}\rho_{i+1}} + \frac{f_i}{\rho_i\xi_i(\rho_i - f_i)}\right]$$

$$\sigma_{\hat{f}_i\hat{S}_i} \doteq f_iS_i\left[\frac{1-\rho_i}{N_i\rho_i} - \frac{f_i}{\rho_i\xi_i}\right]$$

$$\sigma_{\hat{f}_i\hat{f}_{i+j}} \doteq -\frac{f_if_{i+j}}{\rho_i\xi_{i+1}}S_iS_{i+1}\cdots S_{i+j-1}(\rho_i - S_iS_{i+1}\cdots S_{i+j-1}\rho_{i+j})$$

$$\sigma_{\hat{f}_i\hat{S}_{i+j}} \doteq \frac{1}{\rho_{i+j+1}}\sigma_{\hat{f}_i\hat{f}_{i+j}}$$

$$\sigma_{\hat{f}_{i+j}\hat{S}_i} \doteq \begin{cases} S_i\left[\frac{1}{f_i}\sigma_{\hat{f}_i\hat{f}_{i+j}} - \frac{1}{f_{i+1}}\sigma_{\hat{f}_{i+1}\hat{f}_{i+j}}\right] & \text{for } j > 1 \\ S_i\left[\frac{1}{f_i}\sigma_{\hat{f}_i\hat{f}_{i+1}} - \frac{f_{i+1}(1-\rho_{i+1})}{N_{i+1}\rho_{i+1}}\right] & \text{for } j = 1 \end{cases}$$

$$\sigma_{\hat{S}_i\hat{S}_{i+j}} = \begin{cases} -\frac{1}{\rho_{i+j+1}}\sigma_{\hat{f}_{i+j}\hat{S}_i} & \text{for } j > 1 \\ -\frac{1}{\rho_{i+j+1}}\left[\sigma_{\hat{f}_{i+1}\hat{S}_i} + \frac{S_i(1-\rho_{i+1})}{N_{i+1}}\right] & \text{for } j = 1 \end{cases}$$

The estimated covariance matrix is obtained by substituting maximum likelihood estimates into the above formulas, noting that $\hat{\rho}_i = R_i/N_i$ and $\hat{\xi}_i = T_i/\hat{\rho}_i$ .

## IV. Conditional tests of the model.

A size $\alpha$ test of the model may be obtained by defining a critical region in the $k(k+1)/2$ dimensional sample space of $\{R_{ij}\}$ which has measure $\alpha$ with respect to the (parameter-free) conditional distribution of $\{R_{ij}\}$, given the sufficient statistic $(R_1,\ldots,R_k,\ T_2,\ldots,T_k)$. This conditional distribution has rank

$$\frac{k(k+1)}{2} - (2k-1) = \frac{(k-1)(k-2)}{2} = 1 + 2 + 3 + \ldots + (k-2)$$

and may be expressed as a product of multihypergeometric distributions of successively smaller ranks $k-2,\ k-3,\ldots,1$ . The first of these, of rank $k-2$, is obtained by noting that

$$P\left(R_{12},\ldots,R_{1k}\mid T_2 - R_2\right)$$

$$= \binom{T_2 - R_2}{R_{12},\ldots,R_{1k}} f_2^{R_{12}}(s_2f_3)^{R_{13}} \cdots (s_2s_3\cdots s_{k-1}f_k)^{R_{1k}}/\rho_2^{T_2 - R_2}$$

and

$$P\left(R_{22},\ldots,R_{2k}\mid R_2\right)$$

$$= \binom{R_2}{R_{22},\ldots,R_{2k}} f_2^{R_{22}}(s_2f_3)^{R_{23}} \cdots (s_2s_3\cdots s_{k-1}f_k)^{R_{2k}}/\rho_2^{R_2}$$

and because of statistical independence between rows in Table 1 the convolution of these two distributions is

$$P\left(C_2,R_{23}^*,\ldots,R_{2k}^*\mid T_2,R_2\right)$$

$$= \binom{T_2}{C_2,R_{23}^*,\ldots,R_{2k}^*} f_2^{C_2}(s_2f_3)^{R_{23}^*} \cdots (s_2s_3\cdots s_{k-1}f_k)^{R_{2k}^*}/\rho_2^{T_2}$$

where $R_{2j}^* = R_{1j} + R_{2j}$ . Note in particular that $C_2$ (or in general $C_i$) is conditionally binomially distributed, as mentioned in Section 3.

The multihypergeometric distribution of rank k - 2 is now obtained by conditioning on the sums $R_{2j}^*$, as well, to give

$$P\left(R_{12},\ldots,R_{1k}\mid T_2,\ R_2,\ C_2,\ R_{23}^*,\ldots,R_{2k}^*\right)$$

$$= \binom{T_2 - R_2}{R_{12},\ldots,R_{1k}}\binom{R_2}{R_{22},\ldots,R_{2k}}\bigg/\binom{T_2}{C_2,\ R_{23}^*,\ldots,R_{2k}^*} \quad .$$

Thus, a contingency chi-square test can be applied to the $2 \times (k-1)$ table:

| Year Tagged | Year of Recapture 2 | 3 | ... | k | Total |
|---|---|---|---|---|---|
| 1 | $R_{12}$ | $R_{13}$ | ... | $R_{1k}$ | $T_2 - R_2$ |
| 2 | $R_{22}$ | $R_{23}$ | | $R_{2k}$ | $R_2$ |
| Total | $C_2$ | $R^*_{23}$ | ... | $R^*_{2k}$ | $T_2$ |

Similarly,

$$P\left(R^*_{23},\ldots,R^*_{2k} \mid T_3 - R_3 = T_2 - C_2\right)$$

$$\binom{T_3 - R_3}{R^*_{23},\ldots,R^*_{2k}} f_3^{R^*_{23}} (S_3 f_4)^{R^*_{24}} \cdots (S_3 S_4 \cdots S_{k-1} f_k)^{R^*_{2k}} \Big/ \rho_3^{T_3 - R_3}$$

and, independently,

$$P\left(R_{33},\ldots,R_{3k} \mid R_3\right)$$

$$= \binom{R_3}{R_{33},\ldots,R_{3k}} f_3^{R_{33}} (S_3 f_4)^{R_{34}} \cdots (S_3 S_4 \cdots S_{k-1} f_k)^{R_{3k}} \Big/ \rho_3^{R_3}$$

so that

$$P\left(R^*_{23},\ldots,R^*_{2k} \mid T_3,\ R_3,\ C_3,\ \ T_2,\ R_2\ C_2,\ \ R^*_{34},\ldots,R^*_{3k}\right)$$

$$\binom{T_3 - R_3}{R^*_{23},\ldots,R^*_{2k}} \binom{R_3}{R_{33},\ldots,R_{3k}} \Big/ \binom{T_3}{C_3,\ R^*_{34},\ldots,R^*_{3k}}$$

where $R^*_{3j} = R^*_{2j} + R_{3j}$. The contingency chi-square test of the $2 \times (k-2)$ table:

| Year Tagged | Year of Recapture 3 | 4 | ... | k | Total |
|---|---|---|---|---|---|
| 1 or 2 | $R^*_{23}$ | $R^*_{24}$ | ... | $R^*_{2k}$ | $T_3 - R_3$ |
| 3 | $R_{33}$ | $R_{34}$ | ... | $R_{3k}$ | $R_3$ |
| Total | $C_3$ | $R^*_{34}$ | ... | $R^*_{3k}$ | $T_3$ |

is thus independent (asymptotically) of the preceeding test and the two chi-square statistics are additive. This argument continues in an obvious manner, resulting in k - 2 independent contingency chi-square statistics which may be added to give one combined test of the model with degrees of freedom $(k-1)(k-2)/2$. If marginal totals are two small for validity of the chi-square approximation then grouping will be necessary at some stages, with a corresponding reduction in degrees of freedom.

Certain specific alternatives to the model may be tested by partitioning chi-square. One alternative that is likely to arise in a fish tagging experiment is a differential mortality rate and/or vulnerability to anglers immediately after release of the tagged fish, followed by a return to normal before the next anniversary date. Tag returns in this case are expected to follow the modified pattern shown in Table 3.

Table 3. Expected tag-returns with differential rates during the first year after release

| Tags released | | Year of Recapture | | | | |
|---|---|---|---|---|---|---|
| Year | Number | 1 | 2 | 3 | ... | k |
| 1 | $N_1$ | $N_1 f_1^*$ | $N_1 S_1^* f_2$ | $N_1 S_1^* S_2 f_3$ | ... | $N_1 S_1^* S_2 \cdots S_{k-1} f_k$ |
| 2 | $N_2$ | | $N_2 f_2^*$ | $N_2 S_2^* f_3$ | ... | $N_2 S_2^* S_3 \cdots S_{k-1} f_k$ |
| 3 | $N_3$ | | | $N_3 f_3^*$ | ... | $N_3 S_3^* S_4 \cdots S_{k-1} f_k$ |
| ⋮ | ⋮ | | | | | ⋮ |
| k | $N_k$ | | | | | $N_k f_k^*$ |

A test against this alternative is obtained by partitioning each 2 × (k-i+1) table

| | | | | | |
|---|---|---|---|---|---|
| $R^*_{i-1,i}$ | $R^*_{i-1,i+1}$ | . . . | $R^*_{i-1,k}$ | $T_i - R_i$ | |
| $R_{i,i}$ | $R_{i,i+1}$ | . . . | $R_{i,k}$ | $R_i$ | $\rightarrow \chi^2_{k-i}$ |
| $C_i$ | $R^*_{i,i+1}$ | . . . | $R^*_{i,k}$ | $T_i$ | |

into two tables:

| | | | |
|---|---|---|---|
| $R^*_{i-1,i}$ | $T_i - R_i - R^*_{i-1,i}$ | $T_i - R_i$ | |
| $R_{i,i}$ | $R_i - R_{ii}$ | $R_i$ | $\rightarrow \chi^2_1$ |
| $C_i$ | $T_i - C_i$ | $T_i$ | |

| | | | | |
|---|---|---|---|---|
| $R^*_{i-1,i+1}$ | . . . | $R^*_{i-1,k}$ | $T_i - R_i - R^*_{i-1,i}$ | |
| $R_{i,i+1}$ | . . . | $R_{i,k}$ | $R_i - R_{ii}$ | $\rightarrow \chi^2_{k-i-1}$ |
| $R^*_{i,i+1}$ | . . . | $R^*_{i,k}$ | $T_i - C_i$ | |

The sum of these k - 2 single degree of freedom chi-squares then provides a test against the short term differential effects of tagging. Continued partitioning in this manner clearly provides tests against successively longer term pertubations following tagging.

## V. Numerical Example

Data from a ten-year fish tagging experiment are given in Table 4, along with the (sufficient) summary statistics and maximum likelihood estimates; approximate 95 percent confidence limits are presented with each estimate. Note from the last line of Table 4 that the estimators $\hat{f}_i$ and $\hat{S}_i$ are substantially correlated. A similarly high correlation exists between the survival estimators for adjacent years; for example,

$$\hat{\sigma}_{\hat{f}_1\hat{f}_2} = - \frac{\hat{f}_1\hat{f}_2}{\hat{\rho}_2 T_2} \hat{S}_1\hat{\rho}_2 \left(\hat{\rho}_1 - \hat{S}_1\hat{\rho}_2\right) = - .000000103$$

and

$$\hat{\sigma}_{\hat{S}_1\hat{f}_2} = \hat{S}_1 \left[ \frac{1}{\hat{f}_1} \hat{\sigma}_{\hat{f}_1\hat{f}_2} - \frac{\hat{f}_2(1-\hat{\rho}_2)}{R_2} \right] = - .000240$$

giving

$$\hat{\sigma}_{\hat{S}_1\hat{S}_2} = - \frac{1}{\hat{\rho}_3} \left[ \hat{\sigma}_{\hat{S}_1\hat{f}_2} + \frac{\hat{S}_1(1-\hat{\rho}_2)}{N_2} \right] = - .001168$$

and the estimated correlation between $\hat{S}_1$ and $\hat{S}_2$ is then

$$\frac{\hat{\sigma}_{\hat{S}_1\hat{S}_2}}{\hat{\sigma}_{\hat{S}_1}\hat{\sigma}_{\hat{S}_2}} = - \frac{.001168}{(.0594)(.0637)} = - .309$$

Table 4. Number of trout tagged during fall spawning and subsequently reported caught by anglers

| Year Tagged (i) | Number Tagged ($N_i$) | 1960-61 | 61-62 | 62-63 | 63-64 | 64-65 | 65-66 | 66-67 | 67-68 | 68-69 | 69-70 | Total ($R_i$) | $\frac{R_i}{N_i} = \hat{\rho}_i$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1960 | 1048 | 72 | 44 | 8 | 9 | 4 | 4 | 1 | 1 | 1 | 0 | 144 | .1374 |
| 1961 | 844 | | 74 | 30 | 20 | 7 | 4 | 2 | 1 | 0 | 0 | 138 | .1635 |
| 1962 | 989 | | | 54 | 48 | 13 | 23 | 5 | 4 | 2 | 0 | 149 | .1507 |
| 1963 | 971 | | | | 74 | 24 | 16 | 7 | 3 | 1 | 1 | 126 | .1298 |
| 1964 | 863 | | | | | 48 | 40 | 5 | 5 | 2 | 5 | 105 | .1217 |
| 1965 | 465 | | | | | | 31 | 10 | 6 | 3 | 2 | 52 | .1118 |
| 1966 | 845 | | | | | | | 38 | 30 | 6 | 2 | 76 | .0899 |
| 1967 | 360 | | | | | | | | 19 | 6 | 6 | 31 | .0861 |
| 1968 | 625 | | | | | | | | | 13 | 14 | 27 | .0432 |
| 1969 | 760 | | | | | | | | | | 17 | 17 | .0224 |
| Total ($T_i$) | | 72 | 210 | 241 | 275 | 229 | 185 | 143 | 106 | 64 | | | |
| Total ($C_i$) | | 72 | 118 | 92 | 151 | 96 | 118 | 68 | 69 | 34 | 47 | | |

| | 1960-61 | 61-62 | 62-63 | 63-64 | 64-65 | 65-66 | 66-67 | 67-68 | 68-69 | 69-70 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\hat{f}_i = R_i C_i / N_i \hat{T}_i$ | .0687 | .0919 | .0575 | .0710 | .0510 | .0713 | .0427 | .0561 | .0230 | .0224 |
| $\pm 1.96\hat{\sigma}_{\hat{f}_i}$ | ±.0153 | ±.0178 | ±.0126 | ±.0139 | ±.0120 | ±.0198 | ±.0118 | ±.0204 | ±.0100 | ±.0105 |
| $\hat{S}_i = (\hat{\rho}_i - \hat{f}_i)/\hat{\rho}_{i+1}$ | .420 | .476 | .718 | .483 | .632 | .450 | .550 | .696 | .905 | —— |
| $\pm 1.96\hat{\sigma}_{\hat{S}_i}$ | ±.116 | ±.125 | ±.173 | ±.133 | ±.210 | ±.173 | ±.235 | ±.392 | ±.590 | |
| $\hat{\sigma}_{\hat{f}\hat{S}} / \hat{\sigma}_{\hat{f}} \hat{\sigma}_{\hat{S}}$ | .343 | .425 | .398 | .476 | .405 | .600 | .381 | .543 | .451 | |

Likewise there is high correlation between $\hat{S}_i$ and $\hat{f}_{i+1}$, such as

$$\frac{\hat{\sigma}_{\hat{S}_1 \hat{f}_2}}{\hat{\sigma}_{\hat{S}_1} \hat{\sigma}_{\hat{f}_2}} = - \frac{.000240}{(.0594)(.00781)} = - .445 \quad .$$

The remaining correlations for non-adjacent years are negligible, such as

$$\frac{\hat{\sigma}_{\hat{S}_1 \hat{S}_3}}{\hat{\sigma}_{\hat{S}_1} \hat{\sigma}_{\hat{S}_3}} = - .0025 \quad .$$

If any further analysis were contemplated, as for example relating survival rate(S) to the effect (f) of fishing for the 9 years, then this correlated and hetergeneous error structure of the estimators would have to be taken into account.

Chi-square goodness-of-fit tests as described in Section 4 are calculated in Table 5. Some grouping was necessary in the scantier portions of the table, as in the first two years where the recaptures from 1966 to 1970 were grouped to produce a chi-square statistic with 5 degrees of freedom. In these same two rows a comparison between the 1961-62 recaptures and the 1962-70 recaptures resulted in a chi-square value of 0.279 on one degree of freedom to test against differential mortality and/or vulnerability to fishermen during the first year after release. None of the chi-square values were critical, and in total fell very near the median value.

Table 5. Contingency Chi-square tests of goodness-of-fit to the tag recapture model.

| Year Tagged | Year of Recapture 1-2 | 2-3 | 3-4 | 4-5 | 5-6 | 6-7 | 7-8 | 8-9 | 9-10 | Total | Two row Contingency chi-square | First column vs. Remaining cols. chi-square |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1960 | 44 | 8 | 9 | 4 | 4 | 1 | 1 | 1 | 0 | 72 | | |
| 1961 | 74 | 30 | 20 | 7 | 4 | 2 | 1 | 0 | 0 | 138 | 5.185 | 0.279 |
| 1960-61 | 118 | 38 | 29 | 11 | 8 | 3 | 2 | 1 | 0 | 210 | (5 d.f.) | (1 d.f.) |
| 1960-61 | | 38 | 29 | 11 | 8 | 3 | 2 | 1 | 0 | 92 | | |
| 1962 | | 54 | 48 | 13 | 23 | 5 | 4 | 2 | 0 | 149 | 3.086 | 0.390 |
| 1960-62 | | 92 | 77 | 24 | 31 | 8 | 6 | 3 | 0 | 241 | (5 d.f.) | (1 d.f.) |
| 1960-62 | | | 77 | 24 | 31 | 8 | 6 | 3 | 0 | 149 | | |
| 1963 | | | 74 | 24 | 16 | 7 | 3 | 1 | 1 | 126 | 4.219 | 0.544 |
| 1960-63 | | | 151 | 48 | 47 | 15 | 9 | 4 | 1 | 275 | (5 d.f.) | (1 d.f.) |
| 1960-63 | | | | 48 | 47 | 15 | 9 | 4 | 1 | 124 | | |
| 1964 | | | | 48 | 40 | 5 | 5 | 2 | 5 | 105 | 7.268 | 1.146 |
| 1960-64 | | | | 96 | 87 | 20 | 14 | 6 | 6 | 229 | (5.d.f.) | (1 d.f.) |
| 1960-64 | | | | | 87 | 20 | 14 | 6 | 6 | 133 | | |
| 1965 | | | | | 31 | 10 | 6 | 3 | 2 | 52 | 0.798 | 1.371 |
| 1960-65 | | | | | 118 | 30 | 20 | 9 | 8 | 185 | (4 d.f.) | (1 d.f.) |
| 1960-65 | | | | | | 30 | 20 | 9 | 8 | 67 | | |
| 1966 | | | | | | 38 | 30 | 6 | 2 | 76 | 6.601 | 0.048 |
| 1960-66 | | | | | | 68 | 50 | 15 | 10 | 143 | (3 d.f.) | (1 d.f.) |
| 1960-66 | | | | | | | 50 | 15 | 10 | 75 | | |
| 1967 | | | | | | | 19 | 6 | 6 | 31 | 0.629 | 1.078 |
| 1960-67 | | | | | | | 69 | 21 | 16 | 106 | (2 d.f.) | (1 d.f.) |
| 1960-67 | | | | | | | | 21 | 16 | 37 | | |
| 1968 | | | | | | | | 13 | 14 | 27 | 0.465 | 0.465 |
| 1960-68 | | | | | | | | 34 | 30 | 64 | (1 d.f.) | (1 d.f.) |
| | | | | | | Total chi-square | | | | | 28.25 (30 d.f.) | 4.856 (7 d.f.) |