Mathematics Education Issues in the Teaching of Statistics

by

Richard J. Cleary

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Cornell University
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ABSTRACT

Elementary statistics courses, especially those intended for liberal arts students, are often taught by instructors whose main field of interest and experience is mathematics. More and more attention has been focused on improving the quality of these courses by incorporating real data, using computer programs, and placing some emphasis on the communication of solutions through writing projects. In this paper we consider how these practical classroom methods in statistics correspond to the theoretical models of learning that have been under discussion in the mathematics education community. We draw some parallels between the issues in statistics education and the issues in the popular calculus reform movement by examining their common connection with ideas from the education literature.

I.) Current Issues in the Teaching of Elementary Statistics.

On many college campuses the first course in statistics is approached with dread by students and faculty alike. The students fear the unknown, and a statistics course can combine the "math anxiety" that many have with a perception of statistics as some collection of techniques to be memorized. The faculty, often mathematicians with limited experience in statistical applications, see statistics as a cookbook course and lack the training to add more interesting material and perspective. Statisticians have long recognized the dangers in letting mathematicians bear the burden of teaching the first course, as argued by D. Moore (1988).

In recent years there have been many proposals that have suggested improvements in the elementary statistics course. These efforts have been aided by the formation of a group of statisticians with support from the Sloan Foundation. The group, referred to as the Statistics and Liberal Arts Workshop (SLAW), consists of leading small college statisticians who represent the intersection of the mathematics and statistics communities. By publishing articles on statistics education in leading mathematics and statistics journals, this group has helped to call attention to the elementary course. Particularly helpful review articles are T. Moore and Roberts (1989) and T. Moore and Witmer (1990).

The articles above give teachers of the first statistics course a list of sources for improving the course. The common focus of these pieces, though, is on pedagogy and choice of topics. They are intended for teachers, and represent a teacher's point of view. Few if any consider the educational questions that have been considered for some time in the mathematics education literature.

How do students learn statistics? What are the epistemological, social, and cognitive issues? Do the ideas that have currency in mathematics education apply to statistics, or is there reason to develop a separate discipline of statistics education? While a few authors, such as Iversen (1985), have considered the role that statistics should play in a liberal arts education, the problem of how statistics is learned remains underexplored.

A close reading of the articles that include suggestions for improving statistics courses shows the authors are making assumptions about how students learn. These assumptions are sometimes explicitly stated as in Lock (1990). Here while advocating an early and in-depth introduction to applied statistics the author states that, "the best way to excite students about statistics is to demonstrate the usefulness for solving interesting problems in the real world." The idea that recognizing the utility of a subject increases the ability to understand the material is commonly expressed in the mathematics education literature in articles such as Rubin (1989). In constructivist terminology, seeing an application allows the student to create alternate representations for the concept under study.

In this paper we consider some of the practical and popular techniques under use in many elementary statistics courses. In each case we consider the underlying assumptions of the proponents of the technique, and attempt to relate those assumptions to similar ideas in the mathematics education literature. We then consider the parallels of the ideas in statistics to the current push for change in the calculus curriculum. We conclude by calling on statisticians to take advantage of research done in the field of education.

II.) The Mathematics Education Perspective on Classroom Strategies.

Theories of education and the nature of learning are never in short supply. As education becomes better understood as an academic discipline and its research methods improve, different explanations of human learning gain popularity. This is especially true in mathematics education, where the development of testing tools that try to measure the level of achievement has been a major factor in both what is taught and how it is taught. The attempt to measure mastery of certain skills has led to nearly a century in which mathematics achievement has been associated with the ability to perform well on a series of short answer drill problems.

This testing based approach to mathematics was consistent with the behaviorist school of thought in education. In simple terms, students were taught with the notion that the reward of doing well on an exam was a strong motivation to learn. This simple model of learning is still deeply imbedded in our mathematics curriculum at all levels. This is not surprising since curriculum changes will tend to lag behind changes in the beliefs about what constitutes effective teaching.

Today most experts in the field of mathematics education consider mathematics knowledge to be constructed by students. This constructivist school sees all human learning as the building of knowledge by assembling previously learned concepts into a new more powerful structure. As the learner becomes more familiar with the concept they are able to better understand it by linking it to other previously learned material. In some cases a new concept may be formed in several ways, this set of multiple representations of a single concept allows for deeper understanding as more and more previous knowledge is linked to it. Educators are developing tools for teachers to explore a student's growth of understanding. One such tool, the concept map, is discussed in detail in Novak and Gowin (1984). A thorough treatment of the philosophy behind constructivism can be found in Novak (1977).

The brief outlines of the behaviorist and constructivist schools of thought above set the stage for our discussion of the statistics curriculum. The traditional materials for the first course are a text book and set of exercises that are holdovers from a very linear, reward structured era of education. They are written and used with the understanding that new knowledge is acquired one skill at a time, and that a certain temporal ordering of topics is mandatory for comprehension. The drill and practice exercises of elementary and secondary mathematics are still evident in the problems in many statistics texts, where obviously unrealistic and uninteresting "applications" are the norm. The emphasis on computational skills rather than increased understanding of the uses of statistics to solve problems is how the problem of the first course is perceived by statisticians; but from an education perspective it appears to be a classic case of conflict between behaviorist and constructivist attitudes.

It is interesting to note that one of the motivations often expressed by statisticians, such as D. Moore (1988), for getting involved in curricular issues is that statistics as a discipline is fundamentally different in outlook than mathematics. In other words the statistician's concept of their discipline was at odds with what they saw being presented as statistics. Their clearly formed high level concept of statistics as a science could not be represented using the collection of topics in the usual first course. The act of criticizing the ordinary statistics course on such grounds shows an innate sensitivity to, if not any formal knowledge of, constructivist principles.

We now begin to consider a few of the specific suggestions for improving elementary statistics courses and consider how each fits in with the constructivist view of learning. We will attempt in each case to present the motivation for the technique as seen by the statistician and how such an activity would be interpreted by the constructivist. We specifically consider three topics: the use of real data in the classroom and on assignments, writing as a component of statistics education, and the extensive use of graphical methods in statistics.

The use of "real" data in the statistics classroom is not new, but the emphasis on data that is both real and interesting to students is welcome. Many traditional statistics texts have one or two data sets available in an appendix but the data sets are often from older classic experiments in agriculture or medicine that students do not see as important. From the constructionist point of view, they have difficulty linking the statistical methods they are learning to the application if the application itself is not a well developed concept. This difficulty is neatly summed up by Confrey (1989), in a description of problems in mathematics education when she says:

> "Mathematics is classified as formal knowledge, and hence is isolated from common experience and common sense-making. Studies of students' misconceptions in mathematics and science emphasize that students differentiate school learning from everyday experience."

Iversen (1985) carefully explains that the statistician's task is to avoid such a separation of academic and practical knowledge by choosing data and topics that encourage statistical thinking rather than methodology. Statistical thinking implies a high level concept that could easily be linked to many new situations, specific methods are more limited and difficult to connect (or easier to forget.)

Help for the mathematician trying to locate a source of real data that generates interest in students is becoming considerably easier to find. Singer and Willett (1990) give a number of suggestions. The students themselves can be excellent sources of data they are very interested in through the use of classroom surveys.

All areas of mathematics, and indeed of the college curriculum as a whole, have seen an increase in the number and variety of writing projects in recent years. Statistics is especially well-suited to writing endeavors, in no small part due to the appearance of some statistical results daily in our mass media. Many statisticians like to consider the three major tasks of the discipline as the collection of data, the organization and presentation of data, and inference from the data. All three can be addressed in writing projects of many varieties. Radke-Sharpe (1990) explains that writing provides a focus on the internalization and conceptualization of material.

Constructivist thinkers would agree that writing, if properly implemented, encourages students to explore linkages and organize and examine their key concepts. The difficulty for many teachers is their reluctance to use writing as an assessment tool when more traditional tests seem more objective. The result might be narrowly defined assignments that don't reward learning so much as the writer's ability to follow a certain form. Similar problems have been faced by teachers who choose to emphasize problem solving techniques in mathematics courses. In discussing assessment in such courses Marshall (1987) points out that while the form of the writing project (or problem solution) should be flexible, it is the instructor's responsibilty to establish what sort of concepts (or techniques) should be demonstrated.

Traditional statistics courses in mathematics departments have put a heavy emphasis on statistical inference at the expense of data collection and organization. Many college level statistics courses skip right over the already limited selection of topics in the design and reporting of experiments available in the first few chapters of many texts. Topics such as statistical graphics, and the use of graphical techniques to display data, were considered too elementary for college credit. This argument was easier to defend before the availability of computer packages that allowed easy access to a wide variety of graphics tools.

To the constructivist, depriving the student of the opportunity to use graphics takes away the possibility of establishing a set of multiple representations that would lead to deeper understanding. As a specific example, we consider the idea of the normal density function. How many representations does the statistician have for this vital concept? It is easy to list almost any number but we limit ourselves to five here for illustrative purposes. The normal density can be thought of as: a function; the "bell-shaped" graph of that function; a table that describes the area under the function; a limit of other densities; and a model for the generation of data.

The richness of the interaction of these and other ways of representing the normal density is what gives the statistician the ability to exploit its properties. How many of these representations do we present in the usual first course? Certainly almost any of the popular computer packages for a first course allow us to demonstrate all of them. We can generate data, graph densities, search tables and compare the normal to other distributions. The traditional first course might focus exclusively on calculations and deny students the chance to develop a fuller concept of normality.

It is important to understand that recognizing the validity of constructivist arguments is not enough to assure success in teaching. Even with the best of instruction and opportunity, students will need time to learn difficult concepts and put them together. If the pace of a course is hectic and rushed, the usual problems of disenchantment will occur. The long list of suggested improvements in the teaching of statistics will be of little use without teachers who can find an appropriate balance between the amount and depth of the material presented.

III.) Parallels with Calculus Reform.

While statisticians have been trying to get the attention of college mathematics departments, those departments have often been devoting their energy and resources to improving their calculus courses. The calculus reform movement has seen an increasing number of mathematicians call for changes in the way that calculus, the traditional introduction to college mathematics, is taught. The issues are both curricular, with debate over content, and pedagogical, with arguments over teaching techniques and the role of the computer in mathematics classes. Catchy phrases like "the lean and lively calculus" have been coined to try to win support among colleagues for proposed changes.

It is hard to overlook the parallels in suggestions for change in calculus and statistics. A decrease in the "cookbook" topics that are covered strictly from a computational point of view is advocated. Clever and meaningful uses of the computer are proposed. The use of real applications in calculus is encouraged for the same reasons as the use of real data in statistics. Experimental courses using all of these approaches are underway all over the country and the experience of the instructors involved is the focal point of many meetings.

Perhaps the most startling similarity in these efforts is the academic philosophy that motivates those working for changes in the curriculum. Generally mathematicians and statisticians teaching at the college level have little or no formal training in education, yet many have been able to recognize that their traditional courses are no longer adequate. Using a rather practically oriented and ad hoc collection of techniques, they have wound up suggesting the sort of methods advocated by constructivists. This is certainly a new experience for mathematicians, who are used to having their own theories exploited by other sciences rather than the other way around.

There are differences in the problems faced by calculus reformers and statisticians as they seek to make college students better able to understand the world around them. Statisticians are a minority in mathematics departments at most schools, and are not present at all at others, as detailed in Moore and Witmer (1990). It is thus harder for them to generate the sort of publicity and support that the calculus reform issues have generated. Perhaps the biggest factor in the statistician's favor is the availability of good data for their course. Even the staunchest supporters of project based calculus classes recognize the difficulty in making science and engineering problems that use calculus accessible to undergraduates. It is clear that all those interested in making college mathematics and statistics courses more valuable to students could benefit with some introduction to the constructivist theory of education.

IV.) Conclusions.

Should mathematicians teach statistics? Even if the answer is in fact, "NO!", as D. Moore (1988) suggests, the evidence shows that mathematicians are now, and will continue to be, the group teaching most college students their first statistics course. Perhaps the question that needs to be addressed is in fact, HOW should mathematicians teach statistics?

From the standpoint of course content and organization, the increasing bibliography of materials for use in the first statistics course should be adequate for any instructor's needs. Watkins (1992) presented a list of the most vital references to an audience of mathematicians at the annual meeting of the American Mathematical Society and the Mathematical Association of America in Baltimore. The soon to be released volume Statistics for the Twenty First Century, edited by Gordon and Gordon and published by the MAA, will contain a variety of useful articles. No member of a mathematics faculty will be able to plead ignorance as an excuse for a poor job in the first course.

Finally, it may be time for college mathematics and statistics faculty to recognize that the field of education may be standing ready to make a contribution to their daily endeavors. Learning about techniques like concept mapping and recognizing the many similarities in different types of education reform may allow faculty to improve courses more quickly while reducing duplication of effort. Almost every college or university has an education department available for consultation. Most scientists know colleagues who could have saved themselves a great deal of effort if they had only consulted a statistician early in their work. It could be that a few minutes of research and consultation with educators could be of similar benefit to members of the statistical community.

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