Franz Reuleaux: Contributions to 19th C. Kinematics and Theory of Machines

Francis C. Moon

Sibley School of Mechanical and Aerospace Engineering

Cornell University, Ithaca, New York, 14850

This review surveys late 19th century kinematics and the theory of machines as seen through the contributions of the German engineering scientist, Franz Reuleaux (1829-1905), often called the "father of kinematics". Extremely famous in his time and one of the first honorary members of ASME, Reuleaux was largely forgotten in much of modern mechanics literature in English until the recent rediscovery of some of his work. In addition to his contributions to kinematics, we review Reuleaux's ideas about design synthesis, optimization and aesthetics in design, engineering education as well as his early contributions to biomechanics. A unique aspect of this review has been the use of Reuleaux's kinematic models at Cornell University and in the Deutsches Museum as a tool to rediscover lost engineering and kinematic knowledge of 19th century history of machines.

[This paper will appear in *Applied Mechanics Reviews* in early 2003; Published by ASME]

CONTENTS

Introduction

Reuleaux's Family and Early Life

Engineering Scientist and Technical Consultant

Reuleaux's Theory of Machines

Reuleaux's Kinematics

Reuleaux's Kinematics vis a vis Dynamics

Reuleaux on Symbol Notation

'Lost' Kinematic Knowledge:

 Curves of Constant Breadth

 Straight-Line Mechanisms

 Rotary Piston Machines

Visual Knowledge and Kinematic Models: The Cornell Reuleaux Collection

Early Biomechanics and Kinematics

Strength of Materials, Design and Optimization

Reuleaux's and Redtenbacher's Books

On Invention and Creativity

Engineering Education in the 19th Century

Summary

Acknowledgements

References

INTRODUCTION

Engineers are future oriented, rarely looking back on the history of their craft and science. The great age of machines spanned the improvements of the steam engine by James Watt at the end of the 18th century to the dawn of flying machines at the beginning of the 20th century. With a century now past, we have some perspective to draw lessons in a review of the age of machines through the life of one of its principal theorists, Franz Reuleaux (1829-1905), often called the 'father of modern kinematics'. (Fig 1) Reviews of twentieth century kinematics may be found in the *Applied Mechanics Review* articles of Bottema (1953) [1], and Freudenstein (1959) [2], and in a more recent survey edited by Erdman (1993) [3]. A more recent symposium on history of machines and mechanisms may be found in Ceccarelli (2000) [4]. The focus of this article is on the second half of the 19th century.

Machine mechanisms are largely hidden in modern technology and virtually absent from general mechanical engineering education, especially in North America. Yet mechanisms remain important components in many technologies including aircraft, automobile suspensions, robotic manipulators, satellites, consumer electronics and biomechanic prostheses. Central to the design of these devices is kinematics, a subject that a half-century ago had its own identity but for most students today is taught mainly as a prelude to dynamics. Today, there are a few universities that offer advanced courses in mechanism design, but for most students the kinematics of the slider crank and the four bar linkage are all they are taught. Likewise the study of machines as complete entities has become a victim of a reductionism in mechanics replaced by the study of the engineering sciences of solid and fluid mechanics, heat transfer etc. Yet the pioneers of engineering science in the 19th century had a different vision for their revolution in engineering. Franz Reuleaux was in the second generation of engineering scientists in Germany who advocated a mathematical treatment of mechanical engineering within the context of machines. His views and accomplishments in kinematics and the theory of machines is the focus of this review.

Many of our ideas about kinematics of mechanisms and multi-body systems originate in this period and stem from Reuleaux's two major books, *The Kinematics of Machinery* (1875/76) [5,6], and *The Constructor* (1861 –1893) [7,8], a machine design book which went through four editions in four languages (The complete German titles are given in the References). Reuleaux's ideas included kinematic pairs and constraints, open and closed kinematic chains, centrodes and instant centers and the use of physical models of mechanisms to capture theoretical concepts. There were earlier English, French and German texts such as Willis (1841) [9], Laboulaye (1849) [10] and Redtenbacher (1861-1866) [11-13] that influenced Reuleaux. His own books and ideas of kinematics of machines influenced many late 19th C. texts such as Kennedy (1886) [14], Burmester (1888) [15], and the early 20th C. texts of Hartmann (1913) [16], Barr and Wood (1916) [17], Durley (1907) [18], Grübler (1917) [19] and Hartenberg and Denavit (1964) [20]. What distinguished Reuleaux's work from later 20th century works, was his view that both kinematics and strength of materials should be studied in the context of machines. He was also the first to attempt to place invention, kinematic synthesis and design of the machine as a whole on a mathematical and scientific basis.

There have been a number of reviews of Reuleaux's life and work in German (eg Weihe, 1942 [21] , Mauersberger, 1988 [22], Braun, 1990 [23], Braun and Weber, 1979 [24], Severin, 2000 [25]). In English, there is an extended article by Hans Zopke (1896) [26] more than a century ago. Dimarogonas (1993) [27] gave a brief review of Reuleaux's work in a survey of the theory of machines. Recently several articles on Reuleaux's kinematics have appeared in a symposium on the history of machines. (See Ceccarelli, 2000 [4].) One of the contemporary authorities on Reuleaux is Klaus Mauersberger at the Technical University of Dresden and several of his papers are listed in the References below [22,28]. There is also a recent German dissertation on Reuleaux by Sebastian Remberger (1999, 2000) [29,30] who has also organized the papers of Reuleaux at the Archiv of the Deutsches Museum in Munich. Our own review represents a summary of a longer work on Reuleaux in progress. Several web-based articles have appeared recently in connection with the so-called *Reuleaux triangle,* which is discussed below.

In gathering materials for this review, the Author was fortunate to have access to the collections of kinematic models designed by Reuleaux at Cornell University in Ithaca New York and at the Deutsches Museum in Munich. (Fig 2 a, b) The history of these models will be described below. Many of these models are historical catalysts for pursuing threads of technical knowledge no longer in the current literature. The Author also had access to the Reuleaux papers (or *Nachlass* in German) at the Archiv of the Deutsches Museum as well as access to kinematic models collections in Dresden, Hannover, Aachen, Berlin, Karlsruhe and the Science Museum in London.

The Reuleaux oeuvre consists of over 180 publications, including four major books. In addition to technical papers, he wrote reports on the major World Exhibitions, commentaries on technical and industrial progress around the world and German translations of Robert Thurston's *The Animal as Machine and a Prime Mover* [31] and of Longfellow's *Hiawatha*. This review cannot hope to survey the vast subject of kinematics of machines and mechanisms in the 19th century. Our goal is to provide a guide and links to some of the nearly forgotten sources in the literature and artifact collections related to the history of machines.

REULEAUX’S FAMILY AND EARLY LIFE

According to Reuleaux’s family tree, (Gillispie, 1970 [32]), his father and grandfather were machine builders with roots in Liege, Belgium in the 18th century. Belgium was under the hegemony of both France and the Netherlands. There was a small German speaking area east of Liege close to Aachen (Aix la Chapelle). Franz Reuleaux was born in Eschweiler, a suburb of Aachen in 1829, the former capital of Charlemagne. Reuleaux’s French sounding name in a German family reflects the multicultural heritage of Belgium. His wife Charlotte Overbeck (1829 – 1908) was from Antwerp Belgium. (They were married in 1856.)

One of the keys to Belgium’s industrial progress was its coal and iron industry, which dates from the 16th century in the Ardenne and Meuse valleys running from the city of Mons east to Liege. In Great Britain, coal mining inspired the invention and use of the Newcomen and Watt steam engines. Belgium was the first European area to be industrialized after England in the late 18th century, perhaps because it had an active mining industry that utilized steam engines for pumping water out of deep mines. Steam engines were also used as blowing pumps for coke and iron furnaces. Thus Reuleaux’s family connection to machines had its roots in the industrial and political milieu of Belgium.

Around the beginning of the 19th century, his family moved their business about 40 kilometers west of Liege to Eschweiler, a village near Aachen, originally occupied by France, but later ceded to Prussia after the defeat of Napoleon in 1814. Reuleaux received his technical training at the Polytechnic School at Karlsruhe (1850-52), where he studied with a major machine theorist, Professor Ferdinand Redtenbacher (1809-1869) who is sometimes called the father of mechanical engineering in Germany. The program at Karlsruhe, was influenced by the French L’Ecole Polytechnique (Mauersberger, 1989 [33]). After two years at Karlsruhe, Reuleaux went to universities in Berlin and Bonn to study philosophy, logic, natural sciences and other liberal arts. After the death of his father, he returned to work in the family business. In 1856, at the age of 27, he received an invitation to become a professor of mechanical engineering at the Swiss Federal Institute in Zurich and after eight years took a position in Berlin.

ENGINEERING SCIENTIST AND TECHNICAL CONSULTANT

Franz Reuleaux began his academic career with the publication of a machine design handbook in 1854 book with a former student colleague Carl L. Moll, *Constructionslehre für den Maschinenbau* [34] or Design for Mechanical Engineering. (The original German spelling is used here which was changed in the late 19th century.) In this book Reuleaux calls himself a 'Zivilingenieur'. At the time mechanical engineering was beginning to emerge as a separate discipline. At the Polytechnic School of Zurich (ETH), in 1856, Reuleaux and Gustav Zeuner, created a new program in mechanical engineering. After the publication of his 1861 book *Der Constructeur* [7] (or The Designer) and his success at Zurich, he was called to Berlin in 1864, to develop a mechanical engineering program at the Königliche Gewerbe Akademie or Royal Industrial Academy in Berlin in 1864, serving as its Director from 1868 - 1879. This Academy merged with the architecture based Bau Akademie in 1879 to become the Königs Technischen Hochschule Berlin – Charlottenburg. Reuleaux was elected Rector of this new institution during 1890/91, one of the major technical universities in the world with more than 3000 students and 300 professors, (Zopke, 1896 [26]). He worked in this role advocating new educational programs in Germany. He also received the title of Royal Privy Councilor in the government. Reuleaux was a member of the Imperial Patent Office for eight years.

Reuleaux was not a major inventor in the mold of James Watt, nor was he an entrepreneur in the style of the Siemens brothers. He was not a pure scientist as we imagine Maxwell or Einstein to have been. He personified a new figure in the industrial age, the *engineer-scientist*; professor, kinematics theorist, head of a university, industrial consultant and confidant to capitalists, government expert and technical ambassador to the emerging global industrial world, someone like Theodore von Karman a century later. Unlike James Watt, who was an instrument maker and craftsman, Reuleaux and his fellow engineer-scientists were trained in science and mathematics, philosophy and literature as well as in "mechanical arts", influenced in part by the French 'Polytechnique' tradition with its strong mathematics and mechanics base. Unlike the craftsman-engineer who believed in trial and error, hands on education, the engineer-scientist believed that machines could be created and designed using scientific principles guided by rigorous mathematics.

Reuleaux's life spanned the period of enormous growth in travel spurred by the development of powerful steam engines that carried people across oceans and continents by steamship and railroad. He traveled to World Exhibitions in London (1862), Paris (1867), Vienna (1873), Philadelphia (1876), Sidney (1879), Melbourne (1881) and Chicago (1893), often as

German ambassador to these fairs. His professional life coincided with new communications such as overseas mail and the telegraph that linked the growing industrial world with the first internet. Records show he had regularly communicated with colleagues in North America such as Gibbs (Yale), Thurston (Stevens and Cornell) and Bovey (McGill) and many of the founders of ASME. He was one of the early honorary members of ASME (1882) along with Siemens, Eiffel, and Westinghouse.

Franz Reuleaux was one of the optimists of the machine age who believed in the power of technology to free mankind from the slavery and prejudices of peasant life, in spite of the terrible toll on the industrial worker. In his time, machines were viewed with awe and marvel. He and his generation saw the age of the machine as a continuity of progress reaching back to the Greeks and Egyptians as part of the destiny of humankind. Machines were the embodiment of man's knowledge and control over nature. He viewed the evolution in the development of the machine as an analog to the development of advanced societies in which education, crafts, manufacture and government are linked in a chain of mutual dependency for the common good (Reuleaux, 1885 [35]).

Aside from his scholarly contributions, Reuleaux was a player in the political world of the machine age. He was the German ambassador to the Centennial Exhibition in Philadelphia in 1876 as well as to other world expositions. In Philadelphia he wrote letters that were published in Berlin newspapers and appeared as a book, *Briefe aus Philadelphia* (1877), on how poor and shoddy (*billig und schlecht*) German manufactures were compared to English and American produced goods. (Reuleaux, 1877 [36]) In this book he proposed an economic design principle; when faced with competition, one should raise the quality, not lower the price. Later this principle became a hallmark of German manufacturing, (Remberger, 2000 [30]). Reuleaux was active in revamping the German patent system. He was also a consultant to the development of the Otto-Langen internal combustion engine (c 1867) as well as to the industrialists Mannesmann (c 1889) who had developed seamless pipe manufacturing. At the Chicago Columbian Exposition (1893) he created another controversy in Germany by praising American precision manufacturing methods. (See Braun and Weber, 1979 [24].)

Reuleaux believed there were scientific principles behind invention and the creation of new machines or what we call *synthesis* today. He attempted to posit principles of *design theory*, a subject that has come into vogue a century later. This belief in the primacy of scientific principles in the theory and design of machines became the hallmark of his worldwide reputation particularly in the subject of machine kinematics. However, his advocacy of scientific principles in engineering design also gained him critics, who believed he had placed too much emphasis on theory and who after his death tried to reverse the educational structure Reuleaux had helped to build in German engineering institutions.

REULEAUX'S THEORY OF MACHINES

There were many attempts to classify machines according to the tasks or type of motion they produced. Some classifications consider the machine as a whole while others are based on the notion of basic machine elements. Some attribute the deconstruction or 'dissection' of machines into basic machine elements to Leonardo da Vinci, who often mentioned in his manuscripts *elementi macchinali*, and Leonardo was thought by some to have attempted to describe the first collection of machine elements in his Codex Madrid I in 1493, a work that was unfortunately lost until 1966 (Galluzzi, 1997 [37]). More than three centuries later, Reuleaux, using topological concepts, conceived of machine elements as a kinematic chain or network of pairs of connected parts, where the motion of each part is constrained by the neighboring parts in the chain (*kinematic pairs*). This led him to represent a chain of parts in a machine by a set of symbols. Reuleaux believed that each mechanism then had a unique symbolic representation. From the idea that symbols could represent a machine mechanism, Reuleaux believed he had discovered the key to rational principles of invention and synthesis, a *language of invention.*

The importance of history to Reuleaux was two fold. First he believed in progress by *evolution of machine invention* not by genius, in which each new machine is the result of pushing the performance boundaries of the previous generation of machines. It was important to him to review 'prior art' vis a vis kinematic inventions. Quoting Alexander von Humboldt, Reuleaux despised the *'wrangling about priority'* since all ideas emerge from earlier science and technology. His second interest in history of machines was his view that machines evolved from *force-closed* kinematic pairs to more precise geometry-based kinematic pairs and linkages.

Reuleaux divided the study of machines into four categories, 1. the study of machines as complete systems, 2. the theoretical study of mainly prime movers (e. g. thermodynamics of steam and gas engines) 3. machine design (strength of materials, friction etc.) 4. the study of pure mechanisms (geometry of motion). In this context, he said, *"Kinematics is made to belong essentially not to Mechanics, as with Ampere, but to the Science of Machines,.."* (Reuleaux, 1876, p 40) His main contribution in kinematics was not so much new theorems about motions of constrained rigid bodies, but his use of constraints and geometric topology to provide tools for *kinematic synthesis*. The Machine, he said, consists of one or more mechanisms, which can be separated into kinematic chains, which in turn can be broken down into kinematic pairs or fundamental mathematical constraints. The tools of this reductionism is analysis; *"The reverse of this operation is synthesis, the placing together of the kinematic elements, chains and mechanisms, from which a machine can be built up so as to fulfill its required function. "* (Reuleaux, 1876, p 52)

His assertion that machine artifacts are the result of evolution, also led Reuleaux to recount the path of evolution to his own ideas or constructs about kinematics. These constructs came from two paths of history; one the history of mechanisms as appeared in books by Ramelli (1588) [38] and Leupold (1724) [39], while the other path had its origins in the mathematical mechanics of Euler, Chasles and Poinsot. These paths merged in France at the end of the 18th and the beginning of the 19th century with the publication of works by Monge, Hachette, Lanz and Betancourte, and Charles Laboulaye (1849, 1864) [10] who tried to use geometry to classify and analyze the motions of mechanisms and machines. (See e.g. Koetsier, (2000) [40], for a discussion of the French School of Kinematics.) Reuleaux clearly stated his intellectual debt to these forerunners of kinematics. Therefore, it is difficult to claim that Reuleaux was the first to prove this or that theorem, as there are antecedents in the works of French kinematicians, as well as in Willis and Redtenbacher. His theoretical work was a synthesis of ideas that resulted in a new interpretation of mechanisms. That his 1875 book was important was evident by the immediate and lasting success of his ideas amongst his own contemporaries and later generations in the 20th century. The growth of interest in kinematics in the 19th century was clearly enhanced by the development of the steam engine. More than two centuries past the early days of the steam engine, it is difficult to understand the challenge and excitement that this prime mover created and the role that kinematics played in its development.

The age of machines began in the coal mines of Britain with the application of the Newcomen steam engine in 1732, which was used to pump water out of deep mines. Later James Watt (1736-1819) made four major contributions to the steam engine; one was based on thermodynamics and the others were kinematic mechanisms. He invented a separate condenser for the steam, that obviated the need to reheat the piston and that greatly increased the efficiency. The second invention was a kinematic mechanism that allowed the rocker arm to produce an approximate straight-line motion. The third invention was a planetary gear mechanism to convert the oscillating motion of the piston into rotating motion of the flywheel. This allowed the steam engine to be used in factories and ships. The fourth invention of Watt was a speed governor to regulate the speed of the flywheel even when the load varies, certainly one of the milestones of control engineering. These inventions and the spread of the use of steam engines in manufacturing and transportation, spawned a plethora of kinematic and machine inventions that inspired many theoreticians such as Willis and Reuleaux and mathematicians such as Ampere, Chebychev and Sylvester to create a science that would organize this knowledge (Ferguson, 1962 [41]). In addition to his kinematic theories, Reuleaux also created a museum of 800 models of mechanisms that he hoped would codify machine elements.

Besides the development of prime mover machines, this period saw progress in mechanical instrumentation and calculators through the invention and use of mechanisms for digital adding, subtraction, multiplication and division as well as analog mechanical integration devices. (See e.g. Martin, 1992 [42].) Charles Babbage, known as the father of computer science, created designs for a mechanical computer in 1832 which he never finished. But by the end of the 19th century, there were a large group of business machine manufacturers who required ingenious kinematic mechanisms for their products. This led to the invention of many straight-line and curve following mechanisms which must have inspired the emphasis on kinematic synthesis in Reuleaux's *Kinematics of Machinery* (1876) [6].

In keeping with Reuleaux's interest in the machine as a whole, he took special interest in the class of mechanisms related to regulators and controllers for steam engines and other manufacturing machines. Thus kinematic theory and invention, which began with Watt's speed governor (circa 1760), marked the beginning of feedback control theory, a subject Reuleaux took an increasing interest.

Franz Reuleaux had an intimate connection with prime mover machines since his father and grandfather were among the early builders of steam engines on the continent. In the case of the steam engine, kinematics played almost as important a role as thermodynamics, as evidenced by the contributions of James Watt. Reuleaux was also closely involved with the development of the Otto-Langen gas or internal combustion engines. A description of Reuleaux's role in the Otto-Langen engine may be found in the book of Hardenberg (2000) [43]. Reuleaux also had detailed knowledge of the many attempts to create a rotary engine. (Wankel, 1963 [44]) In his written works however, he did not present any discussion of thermodynamic principles with regard to prime movers. His professor at Karlsruhe, Ferdinand Redtenbacher (1862) [12] had included thermodynamics in the study of machines. Reuleaux seemed to be familiar with the work of the great thermodynamicists such as Carnot and Rankine. At Zurich, he worked with a famous thermodynamicist Professor Gustav Zeuner who later taught at Dresden. There is historical evidence in the archives of J. Willard Gibbs that Reuleaux had written to Gibbs. However, in his discussion of prime mover machines, Reuleaux simply treated the gas or the fluid as a flexible link in the closed chain that characterized the machine. This lack of discussion of thermodynamic principles in his works and the lack of any discussion of dynamics were obvious failings of his general theory of machines. He simply saw the problem of kinematic synthesis or the topology of the machine as the more essential part of machine theory.

Although his work led the way toward a reductionism in kinematics of machines and machine design, Reuleaux decried the growth of specialization in mechanical engineering. *"The endless isolation of efforts must be detrimental to the whole,"* he wrote in *Kinematics of Machinery*. In spite of his crusade to teach and view machine design from a holistic vantage point, it was the narrow subject of geometric kinematics that he was remembered for in the half century after his death. However, the unity his work brought to the theory of machines was immediately recognized around 1875. His *Kinematics of Machinery* was translated almost immediately into English, French and Italian. In reading the tributes to Reuleaux's work in his time, many contemporaries hailed his theories as genius, which is difficult for most of us in mechanics to understand a century and a quarter later when machines are taken for granted and the anointed gurus of our age are in information and biological sciences.

REULEAUX'S KINEMATICS

Kinematics as a separate science was defined by Ampere in his Encyclopedia in 1830 [45]. It emerged from the late 18th century work of Monge and later through Hachette (1811) [46] in the L'Ecole Polytechnique in Paris. However, kinematic mechanisms have their origins in early civilization, certainly in the Greek and Roman empires. One of the early works on machines is that of the Roman Pollio Vitruvius (c 25 BC). Beginning in the 15th century many kinematic devices were recorded in published catalogs including those of Francesco di Georgio (1439-1501) (See Galluzzi, 1997 [37]) as well as the posthumous Codices of Leonardo da Vinci (e.g. Codex Madrid I, 1493), which were followed by others such as Besson (1569) [47], Ramelli (1588) [38], in the 16th century and later by Leupold (1724) [39] in the 18th century. These machine books however, lacked a mathematical underpinning which began to emerge in the late 18th century.

One of the problems for mathematicians vis a vis kinematics was to find a framework to categorize the plethora of inventions which appeared in the early part of the 19th century. Although there were principles of motion for free particles and rigid bodies resulting from the work of Galileo, Newton, Euler and Lagrange, how did one treat the dynamics of machines? The key to Reuleaux was the recognition that machines were constrained bodies. In the language of modern analytical dynamics, machines were subject to geometric or holonomic constraints. It is of interest to note that mechanisms occupied the interest of mathematicians such as Euler, Chebyshev, Sylvester and even Gibbs whose Ph.D. dissertation at Yale was on the shape of gear teeth. *"Take to Kinematics. It will repay you. It is more fecund than geometry; it adds a fourth dimension to space",* wrote Chebyshev to Sylvester in 1873. (Ferguson, 1962 [41])

In his *Kinematics of Machinery*, Reuleaux applied the mathematical ideas of kinematics to the evolution, invention and development of machines. This was a revolutionary idea for his time. For not only did he bring new ideas into kinematics of mechanisms, he also tied them to both the past evolution of machines as well as to the future invention of new machines. His successors in kinematics embraced his ideas about kinematic pairs and chains and all but ignored the larger ideas pertaining to history, evolution and invention of machines.

In 1864, Reuleaux developed his key idea of the Machine as a *kinematic chain* linked by pairs of geometric constraints. He said he was influenced by the theoretical work of Poinsot, although the latter wrote mainly about the motion of free rigid bodies. Although he knew geometry and differential equations, one cannot classify Reuleaux as a mathematician such as Lagrange or Gibbs. He was interested in the theory of engineering design not in mathematics per se.

Kinematics for Reuleaux was not divorced from the study of machines in general. In *Kinematics of Machinery* (1876), Reuleaux defined the nature of the machine; *"A machine is a combination of resistant bodies so arranged that by their means the mechanical forces of nature can be compelled to do work accompanied by certain determinate motions."* For Reuleaux, 'certain determinate motions' was a key element of machine invention and kinematics had a firm place in machine design. The earlier editions of *The Constructor* dealt with strength of materials and the design of machine components. But in his 4th edition, published in German in 1889 and translated into English by Suplee in 1893 [8], Reuleaux summarized his ideas of kinematics from his 1875 work. Later in 1900, shortly before his death, Reuleaux published a second volume of *Kinematics of Machinery,* which focused on specific machine components.

In summary, Reuleaux's major contributions to kinematics were:

i) the definition of a machine mechanism as a *chain* of constrained elements;
ii) the recognition that each element in this chain can be understood by looking at the geometric constraints between *kinematic pairs*;
iii) the idea that *machine evolution* has progressed from forced-closed mechanisms to more precise chains of kinematic-pairs;
iv) the search for a principle of logical *synthesis* of kinematic mechanisms and his use of a symbolic syntax to classify machine mechanisms;
v) the use of instant centers or rolling *centrodes* to represent the relative motion of two kinematic pairs of machine elements and the use of this tool for kinematic synthesis.

Regarding centrodes, Reuleaux may have been the first to provide a systematic discussion of the fact that planar relative motion between two elements in a kinematic pair can be described by the rolling of one body on another. He defined a path of instant centers or pole paths (*Polbahnen*, in German), translated at first by Kennedy as *centroid*, and later changed to *centrode*. The fact that every constrained motion of a kinematic pair is equivalent to rolling, and the idea that a machine is a chain of such kinematic pairs, led Reuleaux to redefine the machine, perhaps with tongue in cheek, as a collection of objects in which everything rolls. *"All relative motions of con-plane figures can be considered to be rolling motions, and the motions of any points on them can be determined so soon as the centroids [Polbahnen] of the figures are known."* (Reuleaux, 1876, p. 64. The bracketed term was added by the Author.) Reuleaux in a footnote (p. 590) attributes the German term Polbahn to a 'Professor Aronhold' [48]. He also cites the work of Poinsot (1834) [49] who extended the rolling idea to non-planar motions of rigid bodies. Reuleaux called what we define as screw motion, twisting; *"All relative motions of two bodies may be considered as the twisting or rolling of ruled surfaces or axiods."* (Reuleaux, 1876, [6] p. 81) Whatever Reuleaux's debt to Poinsot as regards rolling centrodes, there is no application in Poinsot to the motion of constrained bodies or mechanisms per se which is the principal contribution of Reuleaux.

In *Kinematics of Machinery* (1876) [6], Reuleaux gave a number of examples of centrodes, for example, showing beautiful details of equivalent rolling of the motion of a curved triangle in a square or rhombic shaped cavity. (Fig 3.) Further he had these curves inscribed on his kinematic models, which could be used for teaching students as physical realizations of his textbook ideas. He inscribed the centrodes for several positive return cam mechanisms on the models, in glass or brass, that beautifully illustrate the nature of dwell motions in these devices.

One of the German texts that borrowed heavily from Reuleaux was that of Professor L. Burmester of the Königlichen Technischen Hochschule, in Munich, published in 1888, under the same title as Reuleaux, *Lehrbuch der Kinematik* [15]. Burmester cited the 'original work' of Reuleaux and applied the centrode idea to the crossed double slider, a mechanism used centuries earlier to draw exact ellipse curves. Shown in Fig 4 of this review is the 'Polbahnen' for the double slider, which is exactly the same as the rolling of a planet cylinder on the inside of a hollow or sun cylinder.

Another example of Reuleaux's influence is in the 1898 American book of Professor Frederick N. Willson of Princeton Univeristy [50], whose book on theoretical graphics used *Polbahnen* or centrodes of a four bar mechnism from Reuleaux' 1876 book. (See Fig 5 in this work). Reuleaux had written to Willson praising his book, which presented the families of trochoid and cycloid curves and dozens of other beautiful curves found in mechanisms and machines in a systematic way. Willson's book contains ideas that are no longer taught in engineering, even as these curves are still used in gears and other mechanisms and illustrates some of the lost knowledge of kinematics.

The use of the kinematic chain idea of Reuleaux brought mechanism theory into analogy with electrical circuit theory. Reuleaux, with few exceptions, did not treat mechanisms with more than one circuit or one degree of freedom, where one link is active and the others are follower links. However, there are mechanisms such as differentials used in automobile transmissions which have two input links. Nor did Reuleaux develop an energy theorem for his kinematic circuit analogous to Kirchhoff's circuit law. The extension of the kinematic chain to multi-circuit mechanisms, which Reuleaux called "compound chains", was developed later in the 20th century in the form of network theory, graph theory, and screw theory. (See e.g. Davies, 1983 [51], Phillips, 1990 [52].)

Oddly, there are very few mathematical equations in Reuleaux's *Kinematics of Machinery*. However, in the 4th edition of *The Constructor* (1893), he published many formulas for use in kinematic design. In the Volume 2 of his *Lehrbuch der Kinematik* (1900) [53], he presented more details of the application of his theories to specific machine mechanisms. By 1900 however, a quarter century after his major work, there were many other kinematic textbooks which had adopted his ideas and had added new ideas of dynamic forces in mechanisms as well.

The Reuleaux "School" of kinematics which included Kennedy (1886) [14] in England and Burmester (1888) [15], Hartmann (1913) [16] and Grübler (1917) [19] in Germany, influenced the nomenclature of kinematics to this day. This can be seen in the Standards for Terminology of the International Federation for the Theory of Machines and Mechanisms (IFToMM, 1990 [54]) which lists technical terms in four languages many of which were originally defined and used by the Reuleaux "School". It is interesting to note that in the current literature one can find reference to Kennedy's three center theorem, or the Burmester point or the Grübler equation but no mention of a law or equation by Reuleaux. Yet most historical reviews credit Reuleaux for the basic fundamental ideas of kinematic analysis. As one example, in the extensive German book on the history of gear technology by Graf. Von Seherr-Thoss (1965) [55], there are over 30 references to Reuleaux's work, more than any other author.

REULEAUX'S KINEMATICS VIS A VIS DYNAMICS

Although Reuleaux's theories about machines were important contributions at the time, his theories were based largely on geometric ideas (Phoronomy) and not on dynamic principles, which were later incorporated into the theory of machines (See e.g. Hartenberg and Denavit, 1964 [20]). Nor did Reuleaux treat the problem of nonholnomic constraints. Modern texts on multi-body dynamics treat both kinematics and dynamics in a systematic way. (eg Wittenberg, 1977 [56], Craig, 1989 [57]) These dynamic theories however, view the machine as a deterministic entity whose behavior could be uniquely predicted and controlled by use of the Newton-Euler laws of motion. Reuleaux did recognize the problem of unpredictable noise in machines, especially in what he called 'force-closed' kinematic pairs, or what is now called unilateral constraints.

In the last quarter century, there have been new discoveries in nonlinear dynamics under the mantle of *"chaos theory*", (see e.g. Moon, 1992 [58]). Research has shown that many machine mechanisms can exhibit small amounts of unpredictable or chaotic dynamics due to the inevitable imperfect nature of the machine as constructed, including friction, backlash and elastic flexibility. Examples include chaotic noise in gears (Pfeiffer, 1988 [59]), and chaos in ball bearings (Mevel and Guyader, 1993 [60]). This has suggested that a modern theory of machines should admit a certain measure of chaotic noise in the behavior of mechanisms. There is some evidence that this small unpredictability may in fact be beneficial to the successful operation of the machine. The nature of unpredictability in machines was not ignored in the 19th century, especially amongst clock analysts.

Reuleaux recognized the fact that unilateral or "force-closed" constraints, were a source of *"clattering and jerking in their force-closed working."* He said, the scientific designer tries to eliminate unilateral constraints *"until all indefiniteness is removed"*. He also acknowledged the problem of determining friction forces in mechanisms and gears, which today are recognized as a major source of chaos in mechanical systems. In general however, ideas about the importance of elastic vibrations, resonance or structure-borne noise in machines did not make it into Reuleaux's work.

There are examples in his oeuvre where he exhibited interest in dynamics and control. Reuleaux (1876a) [61] published a paper titled, "Das Zentrifugalmoment: Ein Beitrag zur Dynamik" ("The centrifugal moment: a contribution to dynamics." The date is not known.) Beginning with the invention of the rotating ball speed governor of James Watt, most steam engines incorporated a controller with two spinning masses coupled to the motion of the engine valve. The design depended on the effective centrifugal force of a mass rotating about a fixed axis. Reuleaux found a general equation for the centrifugal force of extended rigid bodies rotating about an axis. He used integration methods to relate the force moment to the principal moments of inertia of the rotating body and designed an experimental apparatus to measure this moment. However, this result in dynamics did not appear in his major books.

Reuleaux was interested in the regulation of steam and gas engine motions which he discussed in the 4th edition of his *The Constructor* (1893) [8]. In these engines, slide and rotary valves were opened and closed during each machine cycle to admit steam or air-fuel mixtures or to exhaust steam or gas from the engine cylinder. In an early 20th century book by Bevan (1939) [62] he refers to a *"The Reuleaux Diagram"*, a phase diagram to describe the timing between the valves and crank motions of a steam engine. Kinematic mechanisms linked the flywheel to the valve motion and were important in engine design. Steam and gas engines were not pure geometry dependent machines since their operating speeds depended on the gas pressures generated, the flywheel, the load as well as the speed controller. It is not clear that Reuleaux completely understood that regulated machines were not pure kinematic mechanisms. In the introduction to *The Constructor* (1893), he compared valve controllers to escapements, which are true dynamic mechanisms whose motions depend on the laws of dynamics and not just on the geometry of constrained rigid bodies. He even created several so-called "power escapement" models in his famous mechanism collection, (Fig 6), which he said were analogs of machine regulators.

There is evidence that Reuleaux understood the concept of feedback control in a figure from *The Constructor* (Figures 1012, 1213, p 231) in which he tried to explain the workings of two coupled regulators of a commercial machine. This figure shows the closed loop arrangement of a Worthington Duplex Pump and has the features of a block diagram, an idea that did not appear in control theory until several decades into the 20th century.

An interesting aspect of the Moll and Reuleaux book of 1854 [34] was their statement of equilibrium as a variational method using the *"principle of virtual velocities*". Virtual velocity analysis has its origins in ancient Greek science and is similar to virtual work. It is likely that Moll and Reuleaux learned it from their professor at Karlsruhe, Ferdinand Redtenbacher. The principle is also quoted in the text of Laboulaye (1864) [10] as *de principe des vitesses virtuella*, citing the work of Ampere (1830) [30]. There is no evidence however that Reuleaux ever used this principle for dynamics problem. The dynamic counterpart of this principle was stated mathematically by Jourdain, in 1909, as a principle of 'virtual power'. Later in the 20th century, Kane and coworkers extended these methods to the dynamics of rigid bodies which has been codified in several multi-body codes. But the origins of this principle can be seen in the machine design texts of the 19th century.

REULEAUX ON SYMBOL NOTATION

In contrast to the physical sciences of mechanics and electromagnetism, where natural laws were codified with mathematical equations, in chemistry and biology attempts were made to classify the objects of these sciences with tables and abstract notation. The periodic table of elements in chemistry by Mendeleev and Myer appeared in the middle of the 19th century. Similar attempts at classification of machines were also attempted. For example Jean N. P. Hachette, in 1811 [46], constructed a table of mechanisms according to how these mechanisms change motion from say circular to linear motion or from circular to intermittent motion. Charles Babbage (1826) [63] of computer fame, created a mechanical notation using lines and arrows to show how one part of a machine drives another. Unlike Hachette, Babbage's notation tried to show relationships between different parts of the machine. However, the notation required a two dimensional tabular array for each device not unlike that in a music score. He presented an example of an hour counting mechanism for a clock that encompassed two full pages. There was a similar effort by Cambridge professor Robert Willis (1841) [9] who devoted the entire Chapter X of his book to *"mechanical notation"*. His method is similar to Babbage's in that the machine is represented by a table with entries for names of parts, numbers of gear teeth, angular velocities, and the type of motion, i.e., steady, oscillatory, or intermittent.

One of Franz Reuleaux's unique contributions to kinematics was the creation of a symbolic language with which to classify a machine, a syntax for kinematic devices which he proposed as a tool to address the problem of synthesis, a language for machine invention. In his quest for an alphabet of machine devices, Reuleaux built the world's largest collection of machine components, a dictionary of sorts of over 800 models. Using his symbolic system, along with his models, Reuleaux sought to deconstruct every machine that had been or would be invented in the future, a Genome project for the Machine Age.

The key to his classification was the recognition that every practical machine could be represented as a chain of kinematic constraints. Each constraint involved a geometric relation between adjacent parts. A piston in a steam engine, for example, is confined to slide back and forth in the cylinder. Each link on a bicycle chain is constrained to rotate about an axis relative to the adjacent link and so on. Each constraint he represented as a symbol, letters with superscripts and subscripts.

At around the same time, the Swedish biologist Linnaeus was constructing a taxonomy for plants and animals using ideas of species, genus, family, orders, etc. Some of these biological taxonomies were based on physical similarities and some on evolutionary ancestors. Initially, Reuleaux tried to classify machines based on function, such as guiding, storing, driving, and forming or *place-changing machines* versus *form-changing machines*. He abandoned the function-based approach, in favor of a syntax-based methodology using a model based on linguistics rather than biology, a model patterned after chemistry. Reuleaux visualized each machine as a chain of constrained links and the key to distinguishing one machine from another was the sequence of these different link pairs. As described above, each kinematic pair could be written as a symbol and the entire machine as a sequence of symbols. A factory is then a sequence of symbolic words or a sentence representing a complex assembly of machines.

Reuleaux introduced his symbol notation in Chapter VII of *Kinematics of Machinery* (1876) [6]. He was aware of previous attempts to classify machines. He reviewed the systems of Babbage (1826) and Willis (1841). He devoted considerable time to the 4-bar linkage and the slider crank mechanisms. Reuleaux's notation essentially maps kinematic constraint pairs onto a set of symbols

For example, Reuleaux used the symbol '**C**' to represent a cylindrically coupled or revolute kinematic pair. He used the symbol '**P**' to represent a prismatic kinematic pair, '**S**' to represent a screw pair.

Reuleaux's symbol notation has three different kinds of symbols:

Class or name symbols; [**S** screw, **P** prism, **C** cylinder, **K** cone, **G** sphere, etc.]
Form symbols; [+ full body, - open body, z teeth (Zahn), λ liquid, γ gas]
Symbols of relation; [**……** linkage**,** _______ grounded link, ||, parallel axes]

His complete symbol notation uses $\mathbf{C}^-$, $\mathbf{C}^+$, to distinguish between a cylindrical hole in an object and a solid cylinder. Thus, a cylindric joint would be represented by the pair, $\mathbf{C}^-\mathbf{C}^+$**,** namely a solid cylinder constrained by a cylindrical cavity. He then uses a compressed notation $\mathbf{C} = \mathbf{C}^-\mathbf{C}^+$**,** to represent the cylindric joint. Unlike mathematical symbols, there are no logical rules or operations between the symbols except for geometric compatibility among the constraints.

Examples of his compressed notation include:

$$(C_4^{''})^d\ ; \qquad \textbf{Four-bar linkage (link 'd' grounded)}$$

$$(C_3^{''}P^{\mathrm{T}})^d;\ \ \textbf{Slider-crank (link 'd' grounded)}$$

The first symbol indicates four cylindric or revolute (rotary) joints, all axes parallel as notated with the superscript on the letter **C**. The four links are labeled a,b,c,d. The superscript ‘d’, indicates that the d-link in grounded.

The second symbol is the compressed notation for three revolute joints with parallel axes and a prismatic joint, ie linear sliding, whose sliding motion is transverse or perpendicular to the rotary axes. (Reuleaux used an inverted Tee without serifs to indicate a perpendicular axis) His notation does not include any information of mass or moment of inertia. In this sense it is purely geometry and topology based. For example, a flywheel cannot be represented as a carrier of kinetic energy, since his notation is not based on dynamics. His method also has difficulty with, what he calls, ‘force-closed’ systems, that is, mechanisms that rely on some implicit or explicit force to maintain the kinematic constraint such as a wheel.

Using symbol notation, Reuleaux employed six ways to generate a class of mechanisms;

i) *Inversions*: changing the grounded element in the chain of kinematic pairs.

ii) *Expansion of elements*: enlarging or changing the scale of different components in the chain.

iii) *From plane to conic chains*: redefining the device from a planar one to motion on a sphere.

iv) *Reduction of kinematic chain elements*: reducing the length scale of one element to zero while maintaining the geometic constraint.

v) *Augmentation of kinematic chains*: serial linking of kinematic chains.

vi) *Generation of compound chains*: the use of more than one circuit of kinematic chains.

In his use of expansion and reduction in kinematic synthesis, Reuleaux may have been the first to make implicit use of *topology* to analyze mechanisms of machines, although the formal use of the term ‘topology’ was not used in mathematics until the 20th century. In Chapters VIII, IX, with the use of symbols, he showed how one can generate a related class of mechanisms by changing the grounded link, or the shape, size, and forcing element without changing the fundamental nature of the geometric constraints.

A classic example in topology is the identity of a coffee cup and a torus; each has one ‘hole’. By stretching the cup handle or shrinking the cup container, one can obtain a torus shaped object. In *Kinematics of Machinery* (1876), Reuleaux takes one mechanism defined by a symbol and stretches and shrinks links and joints to obtain a class of mechanisms with the same symbol or kinematic relations of the links to each other. He also showed that by taking geometric limits, e.g. shrinking some link to zero, further sets of related kinematic objects could be added to the mechanism class. Reuleaux was a member of the German Patent Board, and a practical application of his method, could determine if some proposed new mechanism was different from prior art by examining the symbol sequence of each device.

An example of generating a class of mechanisms is shown in Fig 7 a, b. The models on the left is a universal joint (Cardano and Hooke) while the one on the right represents a spherical mechanism for a rotary steam engine patented in 1836 by Taylor and Davies. The kinematic chain symbol for both is $(C_3^T C^<)^{\frac{a}{b}}$. Here the second superscript < represents an axis at an angle to the other revolute axes. The superscript ‘a’ indicates the name of the fixed link and the symbol ‘b’ the name of the driven link.

Reuleaux’s use of inversions and expansion of elements implicitly uses another set of data for the mechanism, namely the relative sizes of the links and constraint elements such as diameters of cylindrical bearings and size of the slider. For example, in the case of the slider crank, he labels each link {a,b,c,d} where the slider is ‘c’; and he labels each of the three cylindrical joints with {1,2,3} where link ‘a’, is between joints ‘1’ and ‘2’. These symbols were engraved on the links of many of his kinematic models.

An important concept in Reuleaux’s theory is use of inequality relations for machine synthesis or the idea of relative sizes of the pair and link geometries. Although this is not explicit in his text, it is clear from his writing that changing dimensionless, geometric groups can generate a family of mechanisms with the same constraint symbol. For example we could think of the symbol for the slider crank as incorporating dimensional variables (as in the modern sense of object oriented programming); i.e.,

$$C_3^{"} P^T \{\mathrm{L}_a, \mathrm{L}_b, \mathrm{L}_c, \mathrm{L}_d, \mathrm{d}_1, \mathrm{d}_2, \mathrm{d}_3, \mathrm{w}\},$$

where the ‘L’s are the lengths of the links, (L_c is the length of the slider) and the ‘d’s are the diameters of the cylindrical joints. The width of the slider is ‘w’. Reuleaux was then able to generate a family of slider crank mechanisms by changing the relative lengths as represented by inequalities. For example, the classic slider crank involves the inequalities;

$$d_1 < L_1,\ d_1 < L_4,\ \text{etc.}$$

i.e., the diameters of the cylindrical joints are less than the lengths of their neighboring links. However, Reuleaux asks the reader to imagine the mechanism with

$$d_1 > L_1 \text{ or } d_2 > L_1 + L_2 \text{ etc.},$$

and proceeds to illustrate these ‘new’ mechanisms which all have the same symbol word but have different inequality relations between the link and cylinder pair dimensions. Some of these mechanisms he reminds the reader have been invented earlier and were used in different machines, while others are academic. To further illustrate his ideas, Reuleaux made three-dimensional models to show how one can realize these novel varieties of the slider crank. In making these expansions, Reuleaux tried to ‘exhaust’ the topological possibilities of the basic slider crank kinematic chain to show “*the possibility of the machine”*. (This methodology became the basis of his theory of invention discussed below.) Two members of this family are shown in Fig 8 in which Reuleaux showed how the crank was related to the eccentric mechanism. Using this topological methodology, Reuleaux is able to enumerate 54 mechanisms from the four bar linkage and classifies them into 12 classes [6].

Although Reuleaux’s ideas about kinematic pairs and open and closed chains in mechanisms have survived in texts today, his symbol notation all but died with his passing. The mid-century text by Hartenberg and Denavit (1964) [20], attempted to summarize Reuleaux’s symbol notation but it has not entered practice. However, in the modern field of computational multi-body dynamics, graph theory symbol notation is used to represent the connection properties between bodies in a complex machine. (See e.g. Wittenburg (1977) [56].) In the field of design theory, there has also emerged new attempts to represent machines using symbols as a tool for synthesis, not unlike what Reuleaux had attempted a century earlier.

‘LOST’ KINEMATIC KNOWLEDGE: MATHEMATICAL KINEMATICS AND ROTARY ENGINES

The subject of kinematics as a formal subject is not widely taught today, especially in North America, where it is taught mainly as a preface to dynamics. Courses in system dynamics, control and mechatronics have often replaced those in kinematic mechanisms. There is an international federation of researchers who are making new advances in mechanisms and machine theory. But the vast majority of mechanical engineers trained in the last quarter century, do not have a deep knowledge of either kinematics or the wide variety of mechanisms. Although contemporary engineers have an understanding of microprocessors and root locus methods, they have ‘lost’ a certain body of knowledge in kinematics of machinery familiar to earlier generations. This ‘lost’ knowledge is embodied in many of Reuleaux’s models. Three examples are described below.

Curves of Constant Breadth

Recently, high school mathematics teachers have discovered Reuleaux’s work on ‘curves of constant breadth’ and what many call *“the Reuleaux triangle”*. Several of these references can be found on the web by searching for Reuleaux. In *Kinematics of Machinery* (1876), Reuleaux defined two classes of constraints, lower and higher pairs. A lower pair involves surfaces in contact, as in the case of a cylindric bearing. Higher pairs have line or point contacts between parts as in gear teeth. Reuleaux, in asking how many constraints are necessary to prevent a planar figure from moving, demonstrated that three point constraints may not be sufficient to prevent rotation of the object. He used as examples a curved duangle in a triangular bearing, (Fig 9), and a curved equilateral triangle in a square hole. (Fig 10) The curved triangle is an example of a *curve of constant breadth*, and some mathematics texts refer to it as the Reuleaux Triangle, although its use in cam actuated steam engine regulators can be found as early as 1830. An example may be found on a Woolf steam engine in the Science Museum in London.

He extended this idea to a whole class of curved polygons or “Reuleaux rollers” which can roll between two planes without change in the gap width, hence the term ‘curves of constant breadth or width’. Far from being mathematical curiosities, curves of constant width are used in British coins (20p, 50p coins), and as a drill to make a square hole (Smith, 1993 [64]). They were also used as positive return cams in steam engine control valves at the beginning of the 19th century. Such cams had the property of a finite dwell period without the need of any added control system.

While Reuleaux may have been the first to give a general discussion of the curved triangle, there is reference in the literature that Euler may have first presented the idea. Following Reuleaux, Burmester (1888) [15], gave a discussion of curves of constant breadth in his kinematics book. The mathematician Minkowsky (1911) [65] also worked on the problem. In the 20th century, several mathematical books and publications refer to the problem of 'Reuleaux triangles' and rollers as in Rademacher and Topletz (1957) [66], Yaglom and Boltyannskii (1961) [67], Gardner (1969) [68], and Goldberg (1948) [69] even though the subject virtually disappeared from engineering kinematics textbooks.

Straight-Line Mechanisms

Another area of 'lost' kinematic knowledge are so-called straight-line mechanisms and their more general counterparts of 'calculating' kinematic linkages. (Fig 11) It is not generally appreciated that computers and calculating machines had their origins in kinematics. An important link between kinematics and mathematics at the time was the question of the representation of *mathematical relationships using kinematic mechanisms*. Could mechanisms embody mathematical operations? An entire industry was established on this premise, inspired by the calculating machines of Leibniz and Pascal in the 17th C., and later by Babbage (1826) [63] in the 19th century. (Martin, 1992 [42]) Early in his career, Reuleaux (1862) [70] wrote a paper about one of the first calculating machines called the "Thomas calculator" in which he explained the stepped gear mechanism used in addition operations. By the middle of the 19th century, mechanical machines could perform both digital and analog mathematical operations including adding, subtraction, and multiplication as well as integration, which spawned a calculator and business machine industry that created a demand for kinematic synthesis. The highpoint of analog kinematic calculators was the 'differential analyzer' of Vanemar Bush of MIT in the 1930's and similar GE machines used during and after World War II.

James Watt was famous for inventing a four link mechanism that approximately drew a straight line for use in his steam engine, patented in 1784. The great Russian mathematician Pafnutii L'vovich Chebyshev (1821-1894) of St Petersburg University spent many years investigating the problem of the number of links necessary to draw exact mathematical curves.

There is some evidence that he had proved that a five link mechanism could not draw an exact straight line. He invented several approximate straight-line devices himself. (See e.g. Ferguson, 1962 [41].) However, it was a French engineer Charles-Nicolas Peaucellier (1864) who showed that an eight link mechanism (one link grounded) could produce an exact straight line motion on some point on one of the links. (Fig 12) According to Furguson (1962), this mechanism was later used as part of a blowing engine for ventilating the English House of Commons in 1877. Reuleaux thought that these mechanisms were so important, he designed 39 straight- line mechanisms in his model collection including those of Watt, Roberts, Evans, Chebyshev, Peaucellier, Cartwright and several of his own design. Some of these models can be seen at Cornell University, the Deutsches Museum in Munich, The University of Hannover, the Technical University of Dresden and at the Kyoto University Museum. In recent years, the Peaucellier straight-line linkage has been used in computer science to prove theorems about workspace topology in robotics. (See eg Hopcroft et al 1984 [71].) This mechanism is sometimes mentioned in advanced texts in the design of mechanisms, but for most students of mechanical engineering, it is lost knowledge.

Reuleaux also designed a double slider mechanism model to draw an exact ellipse. One of these models is in the Deutsches Museum (DM06-6214) which is called an *Ellipsenzirkel*.

Rotary Piston Machines

The most ubiquitous mechanism in the world is the slider-crank, of which perhaps a billion exist in the world's automobile engines. These internal combustion machines are based on the kinematics of translating pistons. In the 19th century however, there were many attempts to create *rotary piston engines*. The rotary turbine of Parsons made it into the 20th century, and Wankel's rotary gasoline engine appeared in the 1940's, but did not survive past the 1980's. In *Kinematics of Machinery*, Reuleaux discussed the kinematics of what he called '*chamber crank trains'* and *'chamber wheel trains'*, and included drawings of over 75 rotary engines, pumps and blowing or ventilator devices. (Fig 13) For each he cited the inventor and information on how each performed. He used his symbol notation to discuss their general classes of motions as well as the similar and dissimilar motions of these various inventions. His discussion of the rotating curved triangle in a square cavity (Fig 3) may even have inspired some rotary engine inventions.

The German inventor Felix Wankel of rotary engine fame wrote a review of the history of rotating piston machines in 1963 [44], that was translated into English in 1965. He described dozens of different rotary engine concepts and in the combined style of both Willis and Reuleaux used his own symbol classification and tabular scheme to organize this knowledge. Wankel paid great tribute to Reuleaux referring to him as *"the great dynamicist Franz Reuleaux who attempted nearly 90 years ago to bring order into the chaos of the rotary piston machine field..."* Wankel believed however that Reuleaux's symbol classification methodology was *"a little too artificial"* for the engine designer. But continuing his praise, *"Reuleaux had apparently read all he could about the unsuccessful rotary heat engines which had been proposed in the preceding 150 years," "his book included so many examples that it remained for decades the best known scientific review and collection of this type of machine."*

In general Reuleaux was skeptical as to the practical application of rotary piston devices for energy machines because of seal problems and history has validated his criticism. It is remarkable that so mathematical a treatise should include so much industrial level knowledge and advice. He cited literature and anecdotal references on machines and their performance from many countries showing his wide knowledge and communication with other engineers in machine engineering. In a world concerned with energy and the environment, Reuleaux's books and models, serve as a source of lost knowledge if there is ever a need to reexamine the rotary piston combustion engine using modern materials and control electronics.

VISUAL KNOWLEDGE AND KINEMATIC MODELS: THE CORNELL COLLECTION

The machine has had a long period of evolution stretching back before the Christian era. In the 15th century, the Tuscan artist-engineers Taccola, Francesco di Giorgio and Leonardo da Vinci created picture books based on the function of the machine. The newly discovered Codex Madrid I of Leonardo da Vinci, for example, has several hundred drawings of machine elements as well as complete machines. (See eg Reti, 1980 [72].). Another example is Ramelli's *Ingeneous Machines* (1588) [38] or Besson (1569) [47]. Reuleaux likely had access to a later 18th century compendium of Jacob Leupold, *Theatrum Machinarum Generale* of 1724 [39], which classified machines by application such as construction of forts, mills, or pumping water. The historian of engineering, Ferguson (1962, 1977, 1992) [41,73-74], has made a convincing case that the use of *visual information* was the principal mode of transfer of technical information for centuries up until the mid twentieth century, when mathematical codification of technical information became dominant. (See also Mauersberger, 1994 [75].) Examples of mechanism handbooks include Brown (1868) [76] and Knight (1874) [77]. Redtenbacher and Reuleaux's works in the 19th century represented *transition documents* combining detailed technical drawings with mathematical formulas for design. A German handbook in the early 20th century that combined hundreds of drawings and kinematic design formula is Schneider's, *Die Maschinen-Elemente* in 1903 [[78]. One of the last of these mechanism compendia is the Russian work of Artobolevsky (1975) [79] or the popular American collection of Jones (1930-51) [80]. The eclectic collections of Brown (1868) [76] and Jones (1930) [80] or Bickford (1972) [81] are typical of American approaches to mechanism books in that there is little or no theory in contrast to the European Reuleaux School of the last century.

The early, machine-age picture books contained beautiful engravings of many varieties of machines for war, pumping water, manufacture and transportation 9See e.g. White, 1822, [82]. Reuleaux's two most important books contain hundreds of drawings of machines and mechanisms. The 4th edition of *The Constructor* in 1893, boasts over 1200 illustrations. To compliment his books, Reuleaux designed and built over 800 kinematic models to illustrate his symbol machine theory. The Berlin Collection was funded by the German government. Further, he authorized the reproduction of over 350 of these models for sale to universities and museums.

Reuleaux's models were no doubt influenced by the model collection of his former professor at Karlsruhe, F.J. Redtenbacher. [See footnote 37 in Ferguson (1977) [73].] Redtenbacher had published a catalog of some eighty models (*Bewegungs Mechanismen*, 1866 [13]), including complex clock escapement mechanisms that can be found in Reuleaux's later collection. (Fig 14,15) When Reuleaux moved to Berlin he authorized a German Company, Gustav Voigt, Mechanische Werkstatt, to manufacture these models. Cornell's first President, Andrew Dickson White was ambassador to Germany in Berlin from 1879-1881 where he may have had a chance to see the Reuleaux models. In Reuleaux's letter to A.D. White in 1882, in English, he suggested that Voigt had worked for Reuleaux at the Gewerbe Akademie in Berlin. Later the Voigt company won medals at several international exhibitions for their reproductions of the Reuleaux models. Reuleaux boasted in his letter to Cornell's White that he had designed the cast iron material with an alloy to resist rust. This boast has indeed stood the test of more than a century.

Another earlier model supplier was J. Schroeder, which was founded in 1837 at the Polytechnisches Arbeits-Institute, Darmstadt, Germany. The 1884 catalog (in the Deutsches Museum Archiv) reported that the models were copied from Redtenbacher (Karlsruhe), Reuleaux (Berlin) and Moll (Riga). He listed more than a hundred kinematic models attributed to

Reuleaux. However these models were not of the same quality as the Voigt models, which were designed and closely supervised by Reuleaux himself. Some of these Schroeder catalog pages show up years later in the 1912 model catalog of the Peter Koch Modellwerk, Cologne, without any attribution to Reuleaux. It is likely that Koch had purchased or merged with Schroeder. A set of Schroeder models can be found in the Science Museum in London. However, most are in storage. Schroeder boasted that his models won medals at all the major World Exhibitions, including Paris (1867), Philadelphia (1876) (See Walker, 1878 [83]), Sidney (1879) and Melbourne (1881), where Reuleaux had been the official German ambassador or had been on the judging panels. It is likely that Reuleaux had made contact with Schroeder through these world fairs and was probably motivated to reproduce his own designs through Gustav Voigt, who had worked in Reuleaux's laboratory. Cornell has a dozen of the Schroeder models that were purchased by White in 1869 shortly after the founding of the University.

There were a number of competing model makers in Germany and France in the 19th century. However the Voigt-Reuleaux models were unique in that they were designed to be used with Reuleaux's *Kinematics of Machinery*, 1875/6. This is clear from the engravings on many of the Voigt models with letters and numbers of links and joints corresponding to figures in Reuleaux's book. The instructor was to use the models to illustrate kinematic inversions and expansion of machine elements as part of Reuleaux's theory of machine synthesis. This is clear from letters of Reuleaux to Henry Bovey, the Dean of Applied Science at McGill University. McGill had purchased a large set of Voigt models and Reuleaux implored the Dean to send someone to Berlin so that Reuleaux could show how to correctly use the models in the teaching of kinematics of machines. These letters (c. 1892) also show that Reuleaux was displeased with Cornell University because they did not have the faculty to properly use his models in teaching. The Cornell based text on kinematics by Barr and Wood (1916) for example, makes no mention of the kinematic model collection at Cornell.

A number of Voigt-Reuleaux models are of complete machines such as eight fully operating clock escapements, (similar to Fig 15), and several complex control valve mechanisms. The clock escapements have as many as fifteen moving parts, constructed from over two dozen manufactured machine elements. Many of the simpler models are clearly designed for teaching. Some are demountable so that a different link can be fixed to obtain inversions. Many have adjustments to change link angles so the user can find the optimum setting, as in models for Hooke's or universal joints. (Fig 7a) The design of these Voigt reproductions, clearly show the aesthetic machine style of Reuleaux in the shapes of the pedestals. (See Fig 16) Several drawings of similar shaped pedestals can be found in Reuleaux papers in the Deutsche Museum Archiv in Munich.

By 1907, some 368 Reuleaux models were available in the Voigt catalog and the Cornell collection was reported to number 266 items (Hartenberg and Denavit, 1964). The present inventory is 220 models from the Voigt catalog. There are also a dozen Schroeder models. Because of widespread destruction in Germany in World War II, it is believed that Cornell has the largest remaining collection of Reuleaux models. There are early references to a collection of Reuleaux models in St Petersburg, Russia but they are also presumed lost.

Several references on Reuleaux in German, mention a collection of Reuleaux models by Voigt at McGill University in Montreal as mentioned above. Copies of Reuleaux's letters to Professor Henry Bovey, Dean of Applied Science at McGill in the Deutsches Museum Archiv show that several hundred models were delivered to Montreal in the 1890's. Inquiries to McGill have failed to discover the fate or the history of this collection. There is some evidence that the models were destroyed in a disastrous fire at McGill in 1907, that consumed the Macdonald Engineering Building.

After Reuleaux's death in 1905, the Technical University of Berlin sent about 60 models to the newly founded Deutsches Museum in Munich. Records also show that Professor Wilhelm Hartmann, one of Reuleaux's students, was the curator of the remaining model collection at Berlin. It is presumed that the bulk of the model collection at Berlin was destroyed during World War II. Today, about half of the original models in the Deutsches Museum are in storage and can only be seen by appointment.

There are perhaps a dozen Reuleaux models in the Universität Hannover in the institute on computer aided design headed by Professor Braune. A large set of 19th C. kinematic models is also at the Technische Universität Dresden. (See Mauersberger, 1997 [28]) One or two of Reuleaux's students went to Dresden and helped build the collection at Dresden. About a dozen models are similar to Reuleaux's design, but were probably fabricated in Dresden. Several large modern collections of mechanisms can be found in Germany especially at Aachen, Dresden and Hannover. Professor Arthur Erdman of the University of Minnesota also has a number of modern kinematic models (See e.g. Erdman and Sandor, 1997 [84]).

Recently the Author has learned of a collection of nineteen Reuleaux/Voigt models in Japan at the Kyoto University Museum, purchased in 1890. (Shiroshita et al, 2001) [See also the web site: http://inet.museum.kyoto-u.ac.jp]

As an aside, there was a popular public exhibit of 160 kinematic mechanisms in the US in the 1930's to around 1960 at the Newark Museum, Newark, NJ. The models were designed and built by Will Clark (1943) [85], who wrote a short book on the collection. A duplicate set is in the Boston Science Museum. According to Clark, who called his collection 'his hobby', the models were copied from the illustrations in the mechanism book of Brown (1868) [76]. All the models were motorized, which attracted public interest. But with a lack of funds to maintain the collection and a de-emphasis on technology in favor of 'natural' science, the Newark Museum has relegated the collection to a warehouse. This follows the fate of other museum machine collections and indicates the sad state of interest in the science and history of machines.

Robert H. Thurston of Stevens Institute was a member of the Scientific Commission of the United States to the Vienna International Exhibition of 1873, a decade before he came to Cornell. Thurston's report (1875) [86] on the Vienna Exhibition of 1873, *Machinery and Manufactures, with an account of European Manufacturing Districts*, mentions visiting Dr Reuleaux as director of the Gewerbe Akademie in Berlin. Thurston mentioned *"the fine collection of geometrical and mechanical apparatus." "The models are lighter and neater than those usually seen in our own cases" and that "none are for sale."* However after Reuleaux exhibited 300 of his models at the 1876 Exhibition of Scientific Apparatus in London, he seems to have changed his mind about reproductions and by 1880 had engaged Voigt in the making of copies of his models.

There are documents in the Cornell University Archives that confirm that the Reuleaux Collection was acquired in 1882 or thereafter (See Scientific American 1885 [87] or Sibley J. of Engineering [88]). There is a letter in English (hand written) from Franz Reuleaux to President A.D. White dated 27th June 1882. This letter confirms that there was earlier correspondence between White and Reuleaux and that Reuleaux had supervised the shipping of the Voigt manufactured models to Ithaca. In this letter, Reuleaux also mentions his own heat treatment process to keep the cast iron models from rusting.

The minutes of the Cornell University Board of Trustees, June 14, 1882, *"Acknowledges a pledge of $8,000 from the Honorable Hiram Sibley of Rochester to secure the duplicate of the Reuleaux models in the possession of the Imperial Government of Germany."* (Hiram Sibley and Ezra Cornell both formed the Western Union Telegraph Company in 1854.)

It is likely that Reuleaux met both White and Thurston in Philadelphia at the 1876 Centennial Exhibition where they were on judging panels together. Three hundred of Reuleaux's models were also on display at the South Kensington Museum in London in September 1876 (Kennedy, 1876a,b [89-90]). The Cornell Archives on A.D. White, show that White traveled to Europe in the Fall of 1876. It is possible he may have seen the Reuleaux models at South Kensington. White was also American ambassador to Berlin from 1879-1881, and may have seen the Reuleaux models in Berlin. Later when he returned to Cornell, White wrote a paper on the German educational system and praised the technical education represented at the new Berlin Technical University where Reuleaux was professor an later rector.

There is a wonderful little book by Professor A.B.W. Kennedy of University College, London with a 19 page introduction by Robert Thurston (Kennedy 1881) [91]. The book title is *The Kinematics of Machinery: Two Lectures Relating to Reuleaux Methods.* These lectures (88 pages), were given by Kennedy at the Museum. Kennedy described Reuleaux's theory of kinematic pairs and his symbol representation of complex mechanisms. This small book illustrates the high esteem in which Reuleaux was held both in Europe and the U.S. and the relation of his theory to his models. (Kennedy later became the President of the Institute of Mechanical Engineers in Great Britain and Thurston became the first president of ASME.) Kennedy mentioned the loan to the Museum of 300 models of the Kinematic Collection of the Gewerbe Academie in Berlin, designed by Reuleaux. He also mentioned a set of models at Dresden as being essentially the same as the Berlin models.

Two years after Reuleaux's death there appeared an article in *Scientific American* about his model collection with photographs of eleven of the models. (See Gradenwitz, 1907 [92])

EARLY BIOMECHANICS AND KINEMATICS:

Robert Thurston, the first president of ASME, had published a small monograph on *The Animal as Machine and Prime Mover* (1894) [31] in which he discussed the limits of force and power of humans and animals comparing their capabilities with machines such as the steam engine. Reuleaux had had contact with Thurston, perhaps as early as 1873 when they were both at he World Exhibition in Vienna. In his earlier books, Reuleaux did not discuss the application of kinematics to biology. But Thurston had sent Reuleaux a copy of his book and Reuleaux promptly translated it into German in 1895. In the second volume of his book on kinematics, Reuleaux devoted an entire chapter to kinematics of the skeletal system and its analogy with kinematic chains in machines. (Part III of Vol 2 of *Lehrbuch der Kinematik* (1900) [53] "Kinematik in der Thiereich" or kinematics in the animal kingdom.) He analyzed the joints and linkages of several fishes (Fig 17) and crustaceans. (See e.g. Kerle and Helm, 2000 [93].) He also discussed a model for muscle actuation. In examining the anatomy of shellfish from the point of view of kinematic chains, Reuleaux described a symbol representation of the mechanisms of shellfish claws and jaws. Original drawings of his anatomical sketches may be seen in the Archiv of the Deutsches Museum. There were earlier discussions of the animal as a prime mover in Willis (1841) [9], Laboulaye (1864) [10], and Redtenbacher (1862-1864) [12] though not in the detail as in Thurston or Reuleaux's books.

Around this time Reuleaux had been in contact with a doctor of medicine, O. Thilo from Riga. Thilo later reviewed Reuleaux's chapter on animal kinematics for a journal (Thilo, 1901 [94]). After Reuleaux's death in 1905, Thilo sent the Deutsches Museum several kinematic wooden models of fish illustrating some of Reuleaux's ideas. These models were used in a display in the Museum, under the title, Kinematik in Tierreich, which was the title of the chapter in Reuleaux's book of 1900. These models (Fig 17) are now in storage in the Deutsches Museum. It is likely that Reuleaux was as much influenced by Thurston and Thilo and others as they by his work. Still it is remarkable to see the emergence of bioengineering ideas over a century ago.

By the late 19th century, mechanics, electromagnetism, optics, etc were mathematically codified to such an extent that engineers could reliably use these equations for design of machines. It was natural then that engineering scientists such as Redtenbacher, Reuleaux and Thurston would try to apply this methodology to biology not only from an intellectual point of view but also from the view of the animal as part of the technical system. One of the early biomechanics models given to the Deutsches Museum in 1910 is an arm prosthesis with mechanical fingers actuated by the upper arm muscles using kinematic linkages. This model has the name of Professor Sauerbruch, presumably from Germany. It is not known if he had any connection with the work of Reuleaux who had died in 1905.

STRENGTH OF MATERIALS, DESIGN AND OPTIMIZATION

Franz Reuleaux viewed both strength of materials and kinematics as part of machine design and deplored the specialization that was growing in mechanical engineering in the late 19th century. In his theory of machine development, he posited the idea that both stress issues and kinematics issues had pushed the boundaries of invention of new machines. Thus it is not unusual that his two major books covered both strength of materials and dynamics.

In 1854, twenty years before Reuleaux's major work on kinematics, he published a work with Carl Moll on strength of materials and machine design. There have never been claims that Reuleaux had made unique contributions to strength of materials. This subject was more mature than kinematics and dynamics of machines at the time. The theoretical work of Euler and the Bernouli family in the 18th century had laid many of the foundations of the analysis of deformable bodies and structures as they were used in machines. However, Reuleaux's first book with C. Moll, subsequently published in other editions by Reuleaux alone under the title "Der Constructeur", in 1861, was extremely popular with engineers and went through four editions. The fourth edition in 1889 under the title "Der Konstrukteur" was published in English in 1893, (translated by H. Suplee). Reuleaux's tutorial method for presenting strength of materials and machine design in this work influenced a half century of engineers. Hans Zopke (1896) [26], who studied with Reuleaux and later headed mechanical engineering at George Washington University (originally called Columbian University), credited Reuleaux with introducing the elastic limit stress as a design criteria instead of the rupture strength as was current in design at that time. Zopke also said that Reuleaux's treatment of the theory of springs was used extensively in both Europe and America.

Reuleaux's *Constructor* [7-8], (a modern translation for *Der Konstrukteur* is 'The Designer') was influenced by Redtenbacher's earlier work and introduced five aspects of machine design that were unique for that period.

(1) Moll and Reuleaux (1854) [34] attempted to state *general principles of design*. (See Dimarogonas (1993) [27] for a detailed discussion.)

(2) Like Redtenbacher, Reuleaux introduced the idea of *optimum design*, i.e., a structure with a shape in which all parts of the structure reach the elastic limit at the same time. ("Bodies of Uniform Strength", p2, The Constructor, 1893)

(3) Perhaps as a result of this emphasis on optimal shapes, he promoted the idea of an *aesthetic in machine design*; i.e., that the most pleasing form follows an optimal function.

(4) Reuleaux not only posited general principles of machine design, but also listed general rules for good design for specific machine components as in the design of bearing pedestals. Today this might be called *'best practice*' rules.

(5) He defined, more clearly than earlier works, the *modular elements* of machine construction that set the format for machine design texts to this day.

While some of these ideas appeared in other works of the period, the occurrence of all these ideas by Reuleaux in one handbook was remarkable for its time.

Examples of 'bodies of equal bending strength' appear in the first German edition of *Der Constructeur*, 1861, as well as in the English 4th edition of *The Constructor*, of 1893 in the first chapter of each book. Tables and formulas for the change in shape of cross section under gravity and other loads in bending are given. Reuleaux showed how these shapes for optimal use of mass can be derived from differential equations of bending. These were likely assembled from different sources, including Redtenbacher. By placing these tables at the beginning of his handbook Reuleaux showed that he wanted the machine designer to nurture an interest in efficient design.

Reuleaux was one of the proponents of an aesthetic in machine design, advocating a fluted cast iron column with a Greek capital in the first Otto-Langen internal combustion engine, or using a gothic arch in a machine element instead of a circular arc. (Fig 16) He was likely influenced by the close connection between civil engineering and architecture as well as his principles of optimal design. In *The Constructor* he said that by studying optimal shapes, the designer *"will be able to produce, with an artistic freedom, designs which will approach the shapes indicated by mathematical analysis."*

Another theme that appears in several places in his work, is the subject of *machine complexity*. In discussing the theory of machine development in *Kinematics of Machinery* (1876) [6], he wrote that machines evolve so as to replace force-closed chains of parts, which may exhibit rattling and shaking, with kinematic pair closure that produces more precise motion. In this he said that *"simplicity or fewness of parts does not constitute excellence in a machine, but increased exactness in the motions obtained--- at the cost of considerable multiplication of parts, -- or of links in the kinematic coupling."* On the other hand he also noted that advances in materials properties led to a decrease in the number of parts but with more complexity in their shapes. In the century since, we have seen the variety in machine topology decrease, and at the same time witnessed increases in the number of parts in machines into the hundreds and thousands. In electronic material based devices however, hundreds of circuit elements have been reduced to a single complex part or chip, conforming to Reuleaux's ideas.

One of the signs of a mature technical field is the emergence of modular design, the use of smaller systems to design more complex machines. Reuleaux delineated classes of modules in his *Kinematics of Machinery* (1875-76) as well as *The Constructor*. In the 4th edition of 1893, his Chapter XI is about *"constructive elements*". These include screws, bearings, couplings, chain drives, gears, flywheels, ratchet wheels, brake systems, valves, stuffing boxes and springs, to mention a few of his categories. This extensive list of machine subsystems does not appear in Willis (1841) [9], Rankine (1887) [95], nor in Kennedy's own kinematics book of 1886 [14]. Reuleaux clearly acknowledged the importance of understanding these subcomponents in order to do machine synthesis. There are modern claims that Leonardo da Vinci tried to catalog machine elements, however, his *Codex Madrid I* was not discovered until 1967. One of the translators of this newly found Codex, Ladislao Reti (1980) [72], has used Reuleaux's list of basic machine elements as a benchmark to compare with the variety mechanisms illustrations of Leonardo da Vinci.

REULEAUX'S AND REDTENBACHERS'S MACHINE DESIGN BOOKS

After finishing his studies with Ferdinand Redtenbacher at Karlsruhe, Reuleaux and a fellow student C. Moll wrote a handbook in 1854 on machine design based on strength of materials without any kinematics [34]. Although this book was quite popular, Redtenbacher accused his former students of plagiarizing his lecture notes. Later after Reuleaux secured a position at Zurich, he wrote a single author text called *Der Constructeur* in 1861 [7].

There is some debate among historians of machine engineering as to the relative contributions to mechanical engineering of Reuleaux and his professor Ferdinand Redtenbacher, who has been called one of the pioneers of German engineering education. Both men published influential books in machine design and both developed kinematic model collections for teaching. Redtenbacher's major work was a three volume set called *Der Maschinenbau* (1862-1865) [12] loosely 'The Mechanical Engineer'. This followed a year after Reuleaux's own *Der Constructeur* (1861) was published. Redtenbacher's first volume spanned a wide selection of topics including strength of materials, design of machine elements, friction in machine elements, kinematics of mechanisms, clock escapements, and the design of construction cranes. His second and third volumes cover hydraulics, water wheels, and turbines, locomotive design, steam engines and mining machines. The structure of the text consists of short descriptions of each topic, many formulas, some derived using differential and integral calculus, and a few line drawings at the end of each volume.

The Redtenbacher work is wider in scope than either of Reuleaux's two major works including his *Lehrbuch der Kinematik* of 1875. However, unlike his pupil, Redtenbacher did not attempt to develop a general theory of machines or mechanisms. His chapter on kinematics, classified mechanisms as to the type of motion they performed and was clearly influenced by earlier French classification schemes of Monge. Nor did Redtenbacher review the history of machines and there are not many references to the literature or machine practice. Reuleaux on the other hand, placed his 1875 *Theoretische Kinematik* in the context of the 500 year history of machines from the Renaissance to the 19th century and laced his work with many historical, scientific, technical and practical references including patents. Reuleaux made more use of geometric arguments than his predecessor and, of course, placed emphasis on the problem of kinematic synthesis. Reuleaux's books contain hundreds of beautiful drawings, in sharp contrast to many other texts of the time including Willis, Redtenbacher, Laboulaye and Kennedy. Judging from the test of history, Reuleaux's key ideas about kinematic pairs and chains as well as instant centers and centrodes, survived almost a century in the textbooks of the 20th century.

However, the influence of Redtenbacher on Reuleaux's work is unmistakable. Reuleaux seems to have borrowed sections on optimal design of constant stress beams from his former professor. Reuleaux was no doubt impressed enough with his professor's model collection at Karlsruhe to build an even larger one at Berlin. Also Redtenbacher had written a section on animal and human forces in his "*Maschinenbau*" (1862), that may have influenced both Thurston and Reuleaux to write about machines and biomechanics later decades later.

In so far as kinematics was concerned, Reuleaux was influenced more by the work of Willis (1841) in England and perhaps Babbage as regards the ideas of symbol notation for kinematic mechanisms than he was by Redtenbacher. Reuleaux did give recognition to Redtenbacher in his books but criticized his mentor for not bringing some rational order to the diversity of machines and mechanisms.

ON INVENTION AND CREATIVITY

In recent decades there has been increasing interest in artificial intelligence, synthesis and creativity, sometimes codified in mechanical sciences as design theory. (See e.g., Suh, (1990, 2001) [96-97], Dasgupta (1996) [98], Roth (2000) [99].) Reuleaux's works show many early ideas about machine invention and synthesis, machine aesthetics, design principles, modular elements as well as best practice rules for design. His first mention of design principles is in his 1854 book with Carl Moll. (Dimarogonas, 1993 [27]) Reuleaux broached the subject of synthesis in machine design and the nature of invention in the introduction to *Kinematics of Machinery* (1876) [6]. He viewed his kinematic ideas in this text as prefatory to a theory of scientific invention and design of machines. He also referred to "general laws of invention." He quoted James Watt, Isaac Newton, and Goethe on invention and creativity. He compared creative thinking to the motion of links in a machine, a process governed by logical rules. *"Essentially invention is nothing less than induction, a continually setting down and therefore analyzing of the possible solutions which present themselves by analogy. The process continues until some more or less distant goal is reached"* (*Kinematics of Machinery*, 1876, p. 52.)

He also believed that inventors and artists used similar methods of thinking. Reuleaux was quite skilled in drawing as shown in Figure 16, which was copied from his personal notebooks now in the Deutsches Museum Archiv. (See also Mauersberger, 1985 [100] for a discussion of aesthetics in machine design and Reuleaux.)

Reuleaux's symbol notation, based on kinematic constraints, could be used to generate a class of mechanisms through permutation of the symbol elements. He was the first to use topological ideas to expand a kinematic chain into all its possibilities. His methodology though did not give a rationale for choosing the “best’ or optimum design. *“The scientific abstraction only serves to show the possibility of the machine, it affords no means whatsoever of judging between ‘practical and impractical.”* (*Kinematics of Machinery*, 1876, p 54.) He showed how many disparate mechanisms are related kinematically, that is, he was able to generate a whole design genus of mechanisms and show how one inventor might be influenced by another’s invention.

Reuleaux’s contributions to design synthesis are closely related to what design theorists in contemporary kinematics call *type synthesis*, which is concerned with the form and topology of the mechanism in contrast to *dimension synthesis*, having to do with the specific sizes of the links etc. Also, he worked almost exclusively with single circuit kinematic chains with one degree of freedom. Later authors such as Grübler (1917) [19], who acknowledged his debt to Reuleaux, developed synthesis tools for multi-degree of freedom kinematic chains and what is now called *number synthesis*. (See eg Erdman and Sandor, 1997 [94])

Reuleaux used symbol notation to show how the invention of one device might naturally lead to invention of another device (Moon, 2000 [101]). In *Kinematics of Machinery* (1876) [6], Reuleaux used the example of two steam engine inventions of Simpson and Shipton (KM, p 356, 372, Plate XVII, Fig 2; Plate XXIII, Fig 1) *“A comparison of this with that of Fig 2, allows us easily to understand how one and the same inventor devised both of them for the one is simply a direct inversion of the other”*

There is also a chapter in *Kinematics of Machinery*, on “Constructive Elements of Machinery” a feature seen in later design textbooks and handbooks, listing the various categories of mechanism elements. This is an early example of what modern design theorists call *“modularization in machine design”* (See e.g., Lipson et al, 2001 [102]). Reuleaux’s discussion indicates that the designer was not to begin from scratch in producing a new machine but would combine elements from existing classes of constraint pairs and larger components. Again, he did not discuss any principles of optimal assembly of these standard machine modules as one would expect today.

Reuleaux’s general interest in the history of invention is exhibited in a book he served as editor, *The Book of Inventions* (1884) [103], a pictorial, popular book on the history of invention from the early Egyptians to the end of the 19th century

(Reuleaux, 1884). This book has recently been reissued in Germany. He did not accept the popular theory of invention as resulting from scientific discovery alone, a view that is often expressed in popular literature on technology in the United States. Nor did he believe in the discontinuous genius theory of invention, where the ‘hero’ inventor, working alone, makes an important advance that benefits humankind. He viewed both scientific discovery and technical invention as evolving from a tension between the two, sometimes within the same man;

“In inventing the steam engine, Papin was as much a physicist as a mechanician, and the same may be said of Watt when his searching genius grasped the subject.” (*Kinematics of Machinery*, 1876).

It is clear in reading Reuleaux, that he viewed the development of new machine technology as one of *evolution*, that every invention has had a close antecedent developed further by clever inventors, new scientific ideas and the pressure of marketplace competition. These ideas have appeared anew in recent books on history of technology and technical creativity. (See e.g., Bassalla (1988) [104], Brose (1998) [105] and Das Gupta (1996) [98])

ENGINEERING EDUCATION IN THE 19TH CENTURY

Franz Reuleaux practiced his profession at a time of transition in machine engineering from workshop-based design, to design based on scientific and mathematical analysis. He was trained as a ‘Civil Engineer’ when mechanical engineering as a separate entity was in its infancy. During his tenure at the Technical University in Zurich (ETH), from 1856-1864, he and Professor Gustav Zeuner created a new program in mechanical engineering called in German “machine building” or *Maschinenbau*, following Redtenbacher’s earlier work at Karlsruhe. Throughout his writings he often used the term *“engineering scientist”* to distinguish between workshop-based engineering, of the kind practiced by Boulton and Watt in the late 18th century, and the new mathematics-based rational design methodology which he used in his books. His first book with Moll (1854) was an attempt to place strength of materials on such a rational basis.

In Germany he advocated strong State support for technical education at all levels. Using his analogy of a kinematic chain, he wrote that the modern industrial society was only as strong as the individual links between craftsman, industrial worker, engineer and manufacturer and that education was a common factor in this bonding (Reuleaux, 1885 [35]). This trend in engineering education had been started earlier in Germany by Ferdinand Redtenbacher (Karlsruhe), Johann Schubert (Dresden) and Julius Weisbach (Freiberg) before Reuleaux. (Mauersberger, 1989 [33]). However using his stature in the new German state and his international reputation, Reuleaux pushed for educational reform at all levels from primary schools to engineering colleges, inspired perhaps by similar developments in North America.

As in Germany, the United States Congress saw the need for support of technical education with the passage of the post Civil War Land Grant Act in 1865 that promoted education in the "Mechanic Arts". The question for these new universities was whether mechanic arts meant shop-based training of craftsman or academic, science-based education of engineers. The transformation of engineering education in the US, from *"shop-culture"* to science-based engineering, was discussed in the book by Calvert (1967) [107]. He presented a case study of Cornell University in the period, 1878-1885. Robert Thurston had established an engineering science based program at Stevens Institute of Technology and was asked by the President of Cornell, Andrew Dickson White in 1885 to establish a similar program at Cornell, replacing a shop oriented curriculum that had been in place since Cornell's founding as a Land Grant University in 1865. Thurston took this mandate and developed mechanical engineering at Cornell into a major program in the US and often wrote about his philosophy of engineering education. Franz Reuleaux had corresponded with Thurston and had copies of Thurston's articles on this new engineering curriculum. Thurston on the other hand had visited Reuleaux in Berlin in 1873. As mentioned above, Reuleaux had likely met White and Thurston in Philadelphia in 1876. Thus we may suppose that Reuleaux and Thurston reinforced each other's ideas about engineering education.

In the 1890's, towards the end of his career, critics of Franz Reuleaux, particularly Alois Riedler, (Koetsier, 2000), became increasingly vocal about the excesses of theoretically based engineering education and the importance of engineering practice. After Reuleaux retired in 1896 and certainly after his death, the German engineering curriculum became more practice oriented while the American system continued to develop on the basis of engineering science especially in the post WWII era.

Reuleaux also influenced engineering education in Canada, through his contacts with then Dean of Applied Science Henry Bovey at McGill University who had acquired a large collection of Reuleaux kinematic models. Records suggest that McGill had tried to hire Reuleaux and even awarded him an honorary degree. Reuleaux's letter copybooks also show many contacts with early ASME leaders and members as well as American engineers and industrialists. By the late 19th century, there was a strong global network between mechanical engineers in both Europe and North America, and Franz Reuleaux was a key node in that network for Germany which no doubt influenced engineering education on both sides of the Atlantic.

Reuleaux may have influenced engineering education in Japan during their period of Westernization at the end of the 19th century. For example, Kyoto University had purchased a number of the Reuleaux/Viogt models around 1890. (Shiroshita et al, 2001 [107]) There is evidence that Reuleaux had visited India but no records of any trip to Japan however.

SUMMARY

This review has attempted to place the work of Franz Reuleaux (1829-1905) in the context of 19th century history of machines and to trace some of the origins of modern kinematics and strength of materials as applied to machines. His ideas about kinematic pairs and some work on centrodes have survived in modern kinematics teaching. His ideas on centrodes have found application in modern biomechanics for example. Some ideas related to kinematic chains have been revived in the context of modern robotics. However, the geometry based pedagogy of Reuleaux's kinematics of machines has been replaced by machine dynamics and vibrations, modern methods of multi-body dynamics, analysis of differential equations and finite element methods. Reuleaux's interest in the general area of machine synthesis and invention, especially his ideas about symbolic notation, have been echoed in the modern subject of design theory, although his original work in this field has not always been recognized.

We close with a few quotations of Reuleaux which illustrate the 19th century philosophic and romantic view of The Machine. In describing the consequences of the idea that all relative motions of machine elements can be reduced to rolling, he wrote; *"the machine becomes instinct with a life of its own through the rolling geometric forms everywhere connected with it"*,-- mechanisms carry on *"the noiseless life-work of rolling," – "they are as it were the soul of the machine ruling its utterances – the bodily motions themselves – and giving them intelligible expression. They form the geometric abstraction of the machine. "*

Finally we hope we have demonstrated that 19th century kinematic models can be a source of rediscovering lost knowledge for new applications such as Reuleaux rollers, escapements and rotary engine concepts. A web based catalog and virtual museum of these models is under construction at Cornell University and is expected to be available for access in mid 2003.

ACKNOWLEDGEMENTS

The Author was fortunate as faculty curator to have had access to the Cornell University Collection of Kinematic Models and Machines, the bulk of which contains a large number of reproductions of Reuleaux models. He was also privileged to have been given access to model collections at the Deutsches Museum in Munich, The Science Museum in London, Universität Hannover, Technische Universität Dresden, Universität Karlsruhe, and the Hauch Physike Cabinet in Sorø Denmark.

The Author is also grateful to the Humboldt Stiftung of Germany for financial support to visit the Reuleaux archives and collections of the Deutsches Museum in Munich. The work in Munich would not have been possible without the support from my friend and colleague Professor Franz Kollmann formerly with the Technische Universität Darmstadt. Of particular help at the Deutsches Museum were Dr. Wilhelm Füssl, head of the Archiv, Karl Allwang, Curator for Mechanical Engineering, Manfred Spachtholz, Museum engineer and Mr. Sebastian Remberger.

Also of help were Professor Reinhard Braune of the Institut für Getriebetechnik, Universität Hannover, Professor Friedrich Pfeiffer of the Technische Universität München, Professor Karl-Heinz Modler, and Dr Klaus Mauersberger, Technische Universität Dresden, Professor Werner Goldsmith of the University of California Berkeley, Michael Wright of the Science Museum in London, Professor D. Severin, of the Technische Universität Berlin, Dr S. Shiroshita, Kyoto University Museum, Jørgen F. Andersen of Sorø Academy, Denmark and Gerald H.F. Seiflow of Leicester England for information on the Reuleaux family [108]. The Author also had support from the Cornell Library staff, especially John Saylor, head of the Engineering Library and Elaine Engst, University Archivist. Finally thanks are due Elizabeth E. Moon for transcriptions of the Reuleaux letter copy books in the Archiv of the Deutsches Museum.

REFERENCES

[45] Ampere A-M (1830), Essai sur la Philosophie des Sciences

[48] Aronhold SH (1872), Grundzüge der kinematische Geomtrie, *Verhandlungen des Vereins zur Beförderung des Gewerbefleisses in Preussen* **29**, 1872, 129-155.

[79] Artobolevsky II (1975, 1979) *Mechanisms in Modern Engineering Design,* in 2 Volumes (2288 mechanisms), Mir Publishers, Moscow, 1975.

[63] Babbage C (1826), On a Method of Expressing by Signs The Action of Machinery, *Phil Trans Roy Soc*, **116**, 250-265, in *The Works of Charles Babbage*, Ed M Campbell-Kelly, Vol 3, William Pickering, London, 1989.

[17] Barr JH and Wood EH (1916), *Kinematics of Machinery*, 2nd Ed, J Wiley & Sons, NY.

[104] Bassala G (1988), *The Evolution of Technology*, Cambridge University Press.

[47] Besson J (1569), *Theatre des Instruments*

[62] Bevan T (1939, 1956), *The Theory of Machines*, Longmans, Green and Co, London 3rd Ed, 1956, 147-149. 1st Ed, 1939.

[81] Bickford JH (1972), *Mechanisms for Intermittent Motion* Industrial Press Inc, NY.

[1] Bottema O (1953), Recent Works in Kinematics, *Applied Mechanics Reviews*, **6**, April, 169-170.

[23] Braun HJ (1990), Franz Reuleaux, in *Berlinische Lebensbilder*, Band 6 Techniker, W Treue and W König, Ed, Colloquium Verlag, Berlin, 279-292. [Mentions correspondence between Reuleaux and Thurston of Cornell.]

[24] Braun, HJ and Weber W (1979), Ingenieurwissenschaft und Gesellschaftspolitik: Das Wirken von Franz Reuleaux [Engineering Science and Corporate Politics: The Work of Franz Reuleaux.] by in *Technik und Gesellschaft*, **1**, 285-300. [Contains 42 references. This article discusses Reuleaux's activities in the politics of engineering.]

[105] Brose ED (1998), *Technology and Science in the Industrializing Nations*, *1500-1914*, Humanities Press, NJ.

[15] Burmester L (1888), *Lehrbuch der Kinematik*, Leipzig.

[76] Brown HT (1868), Editor, *Five Hundred and Seven Mechanical Movements*, Brown, Coombs & Co, NY.

[106] Calvert M (1967), *History of Mechanical Engineering*, J Hopkins Press.

[4] Ceccarelli M (2000), Ed, *International Symposium on History of Machines and Mechanisms, Proceedings HMM 2000*, Kluwer Academic Publ, Dordrecht.

[85] Clark WM (1943), *A Manual of Mechanical Movements*, Garden City Publishing Co, Garden City, NY.

[57] Craig JJ (1989), *Introduction to Robotics*, 2nd Ed, Addison Wesley, Reading, MA..

[98] Dasgupta S (1996), *Technology and Creativity*, Oxford University Press, NY.

[51] Davies, T.H., "Couplings, Coupling Networks and Their Graphs" *Mech. Mach. Theory* **30** (7), 991-1012.

[27] Dimarogonas AD (1993), The Origins of The Theory of Machines and Mechanisms Section 1.2 in *Modern Kinematics,* A.G. Erdman, Ed, J Wiley & Sons, NY.

[18] Durley RJ (1907), *Kinematics of Machines*, J Wiley & Sons.

[3] Erdman AG (1993), Ed, *Modern Kinematics; Developments in the last Forty Years*, J Wiley & Sons.

[84] Erdman AG and Sandor GN (1997), *Mechanism Design: Analysis and Synthesis*, Third Edition, Prentice Hall, Upper Saddle River, NJ.

[74] Ferguson ES (1992), *Engineering and the Mind's Eye*, MIT Press, Cambridge, Mass.

[73] Ferguson ES (1977), The Mind's Eye: Nonverbal Thought in Technology, *Science*, **197**, No 4300, 827-836.

[41] Ferguson, ES (1962), Kinematics of Mechanisms from the Time of Watt, *United States National Museum Bulletin*, **228**, Smithsonian Institution, Washington, DC paper 27, 185-230.

[2] Freudenstein F (1959), Trends in Kinematics of Mechanisms, *Applied Mechanics Reviews*, September, **12**, 587-590.

[37] Galluzzi (1997)

[68] Gardner M (1969), *The Unexpected Hanging and Other Mathematical Diversions*, A Touchstone Book Simon & Schuster, Inc., NY Chapter Eighteen; "Curves of Constant Width" 212-221 and 254.

[32] Gillispie CC (1970), Editor, *Dictionary of Scientific Biography*, Charles Scribners Sons, NY, Vol XI [See Reuleaux, Franz, by Otto Mayr, 383-385.]

[69] Goldberg M (1948), Circular-Arc Rotors in Regular Polygons, *The American Mathematical Monthly* **55**(7).

[92] Gradenwitz A (1908), The Mutual Relations of Geometry and Mechanics and Prof Reuleaux's Mechanical Movements, *Scientific American*, March 21, 204-205.

[55] Graf von Seherr-Thoss C (1965), *Die Entwicklung Der Zahnrad-Technik,* Springer-Verlag, Berlin.

[19] Grübler M (1917), *Getriebelehre*, Verlag von Julius Springer, Berlin.

[43] Hardenberg HH (2000), *The Middle Ages of the Internal Combustion Engine 1794-1886*, Society of Automotive Engineers, Warrendale, Pa.

[20] Hartenberg RS and Denavit J (1964), *Kinematic Synthesis of Linkages*, McGraw-Hill Book Co, New York, p 75.

[46] Hachette Jean NP (1811), *Traité Elementaire des Machines*, Paris.

[16] Hartmann W (1913), *Die Maschinengetriebe*, Stuttgart.

[71] Hopcroft J, Joseph D, Whitesides S, (1984), Movement problems for 2-Dimensional Linkages, *SIAM J Comput* **13** (3), August.

[54] IFToMM (1990), Terminology for the Theory of Machines and Mechanisms, *Mechanism and Machine Theory*, **26** (5), Pergamon Press, Oxford.

[80] Jones FD (1930-1951), *Ingeneous Mechanisms for Designers and Inventors,* 1st Edition, Industrial Press, New York, 3 volumes, edited by H.L. Horton.

[14] Kennedy ABW (1886), *The Mechanics of Machinery*, MacMillan and Co, London.

[91] Kennedy ABW (1881), *The Kinematics of Machinery: Two Lectures Relating to Reuleaux Methods.* Delivered at the South Kensington Museum with an Introduction by Professor RH Thurston, AM, CE Reprinted from Van Nostrand's Magazine. D Van Nostrand, Publisher, New York.

[90] Kennedy ABW, (1876a), Book Review, *The Kinematics of Machinery; Outlines of a Theory of Machines*, F Reuleaux, translated by ABW Kennedy, *Engineering*, **22**, p 197, London.

[89] Kennedy ABW, (1876b), The Berlin Kinematic Models, *Engineering*, **22**, 239-240, London.

[93] Kerle M and Helm M (2000), Animal Kinematics – A Review of a Study of Franz Reuleaux About Restained Animal Motions in, *International Symposium on History of Machines and Mechanisms, Proceedings HMM 2000,* Editor, M Ceccarelli, Kluwer Academic Publ, Dordrecht, 181-186.

[77] Knight EH (1874-1876), *Knight's American Mechanical Dictionary*, 3 volumes, New York.

[40] Koetsier T (2000), Mechanism and Machine Science: Its History and Its Identity in, *International Symposium on History of Machines and Mechanisms, Proceedings HMM 2000*, Editor, M Ceccarelli, Kluwer Academic Publ, Dordrecht, 5-24.

[10] Laboulaye C (1849, 1864), *Traite de Cinematique ou Theorie des Mechanismes*, Paris, 1849, 2nd edition 1864.

[39] Leupold J (1724), *Theatrum Machinarum*, Leipzig.

[102] Lipson H, Pollack JB, and Suh NP (2001), On the Evolution of Modularity, Cornell University Report. June 2001.

[41] Martin E (1992), *The Calculating Machines* (Die Rechnemaschinen), MIT Press.

[100] Mauersberger, K (1985), Suche nach dem Attribut der Technik; Zum Maschinenbaustil des 19 Jahrhunderts, (On mechanical engineering style in the 19th century), *Form + Zweck,* DDR, (Form and Function), **1**/1985, 15-20.

[22] Mauersberger K (1988), Franz Reuleaux – Begründer der Kinematik, *Feingerätetechnik*, 37, Berlin, 38 – 41.

[33] Mauersberger K (1989), The Development of German Engineering Education in the Nineteenth Century – a Comparison with Great Britain and France, *European Historiography of Technology*, Ed Dan Ch Christensen.

[75] Mauersberger K (1994), Visuelles Denken und nichtverbales Wissen in Mascinenbau (Visual thinking and nonverbal knowledge in mechanical engineering) *Beitreage zur Geschichte von Technik und Techniker Bildung*, Folge, Hochschule für Technik, Wirtschaft und Kultur, Leipzig (FH), 3 – 27.

[28] Mauersberger K (1997), Die Getriebemodellsammlung der Technischen Universität Dresden (The mechanism model collection of the Technical University Dresden), *Wissenschaftliche Zeitschrift der Technischen Universität* Dresden **46** Heft 3.

[65] Minkowski H (1911), Über die Korper konstanter Breite, (Of curves of constant breadth) *Gesammelte Abhandlungen* Bd II, Leipzig, S 277-279, p 67.

[60] Mevel B, Guyader JL (1993), Routes to Chaos in Ball Bearings, *J Sound and Vibration*, **162**(3), 471-487.

[34] Moll CL and Reuleaux F (1854), *Constructionslehre für den Maschinenbau,* (Design for Mechanical Engineering) Druck und Verlag von Friedrich Vieweg und Sohn, Braunschweig.

[58] Moon FC (1992), *Chaotic and Fractal Dynamics*, J Wiley & Sons, New York.

[101] Moon FC (2000), The Language of Invention, *The Bookpress*, **10** (1), Ithaca, NY

[59] Pfeiffer F (1988), Seltsame Attraktoren in Zahnradgetrieben *Ingenieur Archiv*, **58,** 113-125.

[52] Phillips J (1990), *Freedom in Machinery, Screw Theory Exemplified*, Vol 1,2, Cambridge University Press, Cambridge.

[49] Poinsot M (1834), *Outlines of A New Theory of Rotatory Motion*, Translation from the French by Charles Whitley, Cambridge.

[66] Rademacher H and Toeplitz O (1957), *The Enjoyment of Mathematics*, Princeton University Press, Princeton, NJ, 163-77, 203.

[38] Ramelli Agostino (1588) *Le Diverse et Artificiose Machine*, Paris. [Reprinted by Dover Publ, New York.]

[95] Rankine WJM (1887), *A Manual of Machinery and Millwork*, Edition 6, London.

[11] Redtenbacher F (1861), *Resultate für den Maschinenbau.*, Mannheim, Verlag von F. Bassermann.

[12] Redtenbacher F (1862-65), *Der Maschinenbau*, 3 Volumes.

[13] Redtenbacher F (1866), *Die Bewegungs Mechanismen.* Verlagsbuchhandlung von F. Bassermann, Mannheim.

[29] Remberger S (1999), Franz Reuleaux: Ansichten und Selbsverstandnis eines Ingenieurs in der deutschen Gesellschaft des spatten 19 Jahrhunderts, Magisterarbeit, Ludwig-Maximilians-Universität, München.

[30] Remberger S (2000), Billig und schlecht – Franz Reuleaux zu den Weltausstellungen in Philadelphia und Chicago, *Kultur & Technik*, **3**, July – September, pp 42-45.

[72] Reti L (1980), Editor, *The Unknown Leonardo*, see Chapter, The Engineer, by L Reti, McGraw Hill.

[7] Reuleaux, F (1861), *Der Constructeur : Ein Handbuch zum Gebrauch beim Maschinen-Entwerfen*, Verlag von Friedrich Vieweg und Sohn, Braunschweig (1st Edition; in German).

[70] Reuleaux F (1862), *Die Thomasche Rechenmaschine* Civilingenieur, Band VIII, Heft 3.

[5] Reuleaux F (1875), *Theoretische Kinematik; Grundzüge einer Theorie des Maschinenwesens*, Verlag Vieweg & Sohn Braunschweig.

[6] Reuleaux F (1876), *Kinematics of Machinery; Outlines of a Theory of Machines*, ABW Kennedy, Transl, MacMillan and Co, London.

[61] Reuleaux F (1876a), Das Zentrifugalmoment, Ein Beitrag zur Dynamik, *Verhandlungen des Verein zur Beforderung des Gewerbefleisses*, **55** (Sitzungsberichte), 50-88.

[36] Reuleaux F (1877), *Briefe Aus Philadelphia*, Braunschweig.

[103] Reuleaux F (1884), Editor, *Buch der Erfindungen*; *Gewerbe und Industrien.* [The Book of Inventions.], reprinted by Bechtermünz Verlag, 1998.

[35] Reuleaux F (1885), The Influence of the Technical Sciences upon General Culture, Translated from the German by W. Kunhardt. *School of Mines Quarterly*, Vol VII, No 1, October. [Translation of "Cultur und Technik" in *Zeitschrift des Vereins deutscher Ingenieure*, 1885, S 24.]

[8] Reuleaux F (1893), *The Constructor; A Handbook of Machine Design,* 4th Edition, H.H. Suplee, Transl 1892, Philadelphia.

[53] Reuleaux F (1900), *Lehrbuch der Kinematik; Zweiter Band. Die Praktischen Beziehungen Kinematik zu Geometrie und Mechanik,* Verlag von F. Vieweg und Sohn, Braunschweig.

[99] Roth B (2000), The Search for the Fundamental Principles of Mechanism Design, in, *International Symposium on History of Machines and Mechanisms, Proceedings HMM 2000*, Editor, M Ceccarelli, Kluwer Academic Publ, Dordrecht, 187-195.

[78] Schneider (1903) *Die Maschinen -Elemente*

[88] *Scientific American*, October 17, 1885, Vol. LIII, No. 16 [New Series]. {The cover shows a montage of pictures of the Sibley College, Cornell University. The upper left corner shows cabinets with Reuleaux kinematic models and the footnote says this is the "Sibley College Museum of Mechanisms."}

[25] Severin D (2000), Franz Reuleaux, der Wissenschaftler und Ingenieur, *Von der Bauakademie zur Technischen Universität Berlin: Geschichte und Zukunft, 1799-1999*, Earnst & Son (Wiley), 300-304.

[107] Shiroshita S, Kumamoto H, Nishihara O, and Jing D (2001) Constructing a Virtual Museum of Machine Mechanism Models Imported from Germany During Japanese Westernization for Higher Education: 3D Animations Based on Kinematics and Dynamics, Presented at Museums and the Web 2001. [http://www.archimuse.com/mw2001/papers/Shiroshita.html]

[87] *The Sibley Journal of Engineering*, June 1894, Vol. VIII, (9), published by the Students of Sibley College, Cornell University. {On page 382, article notes that Mr. Hiram Sibley had given money for the Reuleaux collection.}

[108] Seiflow GHF (1999), The Seiflow Archives, Web site with Reuleaux-Seiflow family tree. [http://freespace.virgin.net/patrick.seiflow/Archives.htm]

[64] Smith S (1993), Drilling square holes. *Mathematics Teacher* **86** (Oct), 579-583.

[96] Suh NP (1990), *The Principles of Design*, Oxford Press, New York.

[97] Suh NP (2001), *Axiomatic Design*, Oxford Press, New York.

[94] Thilo O (1901), F. Reuleaux, Kinematik in Tierreiche, Sonderabdruck aus dem “Biologischen Centralblatt” Bd.XXI Nr 16. 15. August, 514-528.

[86] Thurston RH (1873), *Machinery and Manufacturers with an Account of European Manufacturing Districts*, report on the Vienna Exhibition published by The US Department of State.

[31] Thurston RH (1894), *The Animal as Machine and a Prime Mover*, J Wiley.

[83] Walker FA (1878), Editor, *International Exhibition 1876*, *United States Centennial Commissioner Reports and Awards*, Group XXVIII, JB Lippincott & Co, Philadelphia, 1878. {This volume shows that President AD White of Cornell University was a judge in the Group on Education and Science and that the German Commissioner, F Reuleaux, assisted in the judging, p 203, p vi.

[44] Wankel F (1963), *Einteilung der Rotations-kolbenmaschnen*, Deutsche Verlags-Anstalt, Stuttgart. [Published in English as; *Rotary Piston Machines*].

[21] Weihe C, (1942) Franz Reuleaux und die Grundlagen seiner Kinematik, Deutsches Museum, Abhandlungen und Berichte, VDI-Verlag Gmbh, Berlin, 14 Jahrgang, Heft 4, 1942.{C Wiehe was Reuleaux's last student.}

[82] White J, *A New Century of Inventions*, Manchester, 1822.

[9] Willis R (1841), *Principles of Mechanisms*, London.

[50] Willson FN (1898), *Theoretical and Practical Graphics.*

[56] Wittenburg (1977), *Dynamics of Systems of Rigid Bodies*, BG Teubner, Stuttgart.

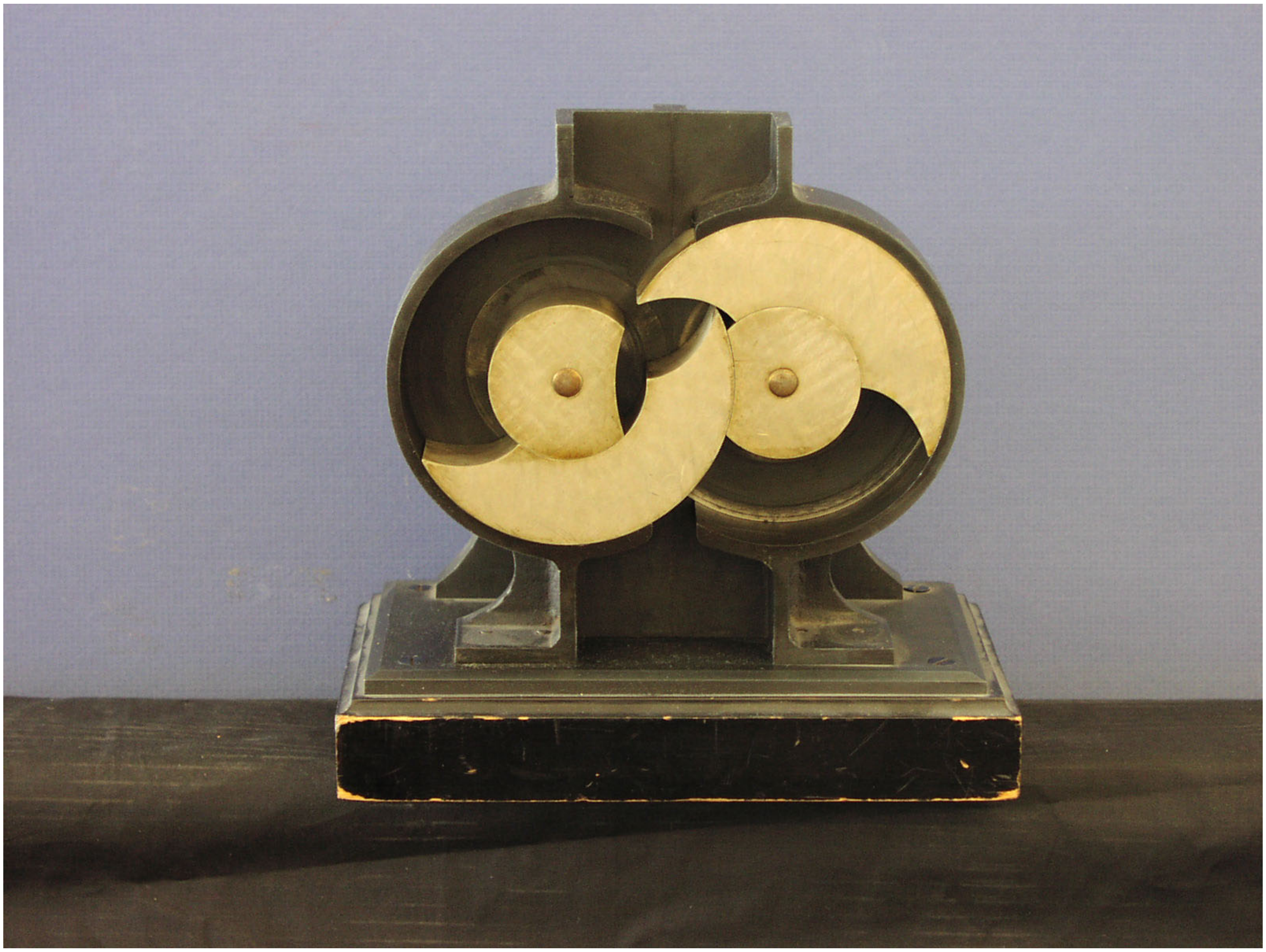
[67] Yaglom IM and Boltyanskii VG (1961), *Convex Figures*, Holt, Rinehart and Winston, NY, Chapters 7, 8.

[26] Zopke H (1896-97), Professor Franz Reuleaux, A Biographical Sketch, *Cassiers Magazine*, New York.

LIST OF FIGURE CAPTIONS

1. Engraving of Franz Reuleaux (1829 – 1905).

2 a) Rotary engine mechanism: Model No. I7, Voigt Catalog Cornell Reuleaux Collection.



b) Rolling hyperboloid friction wheel mechanism. Model in the Deutsches Museum, Munich.

3. Locus of instant centers or centrodes of pure rolling, for a curved triangle PQR rotating in a square cavity ABCD. Here the motion is equivalent to the curved triangle 1’2’3 rolling on the curved rhombus 1234. Taken from Reuleaux, *Kinematics of Machinery* (1876), Fig 113.

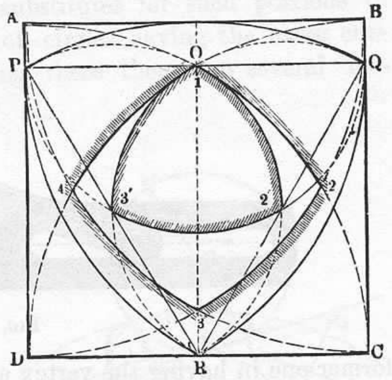
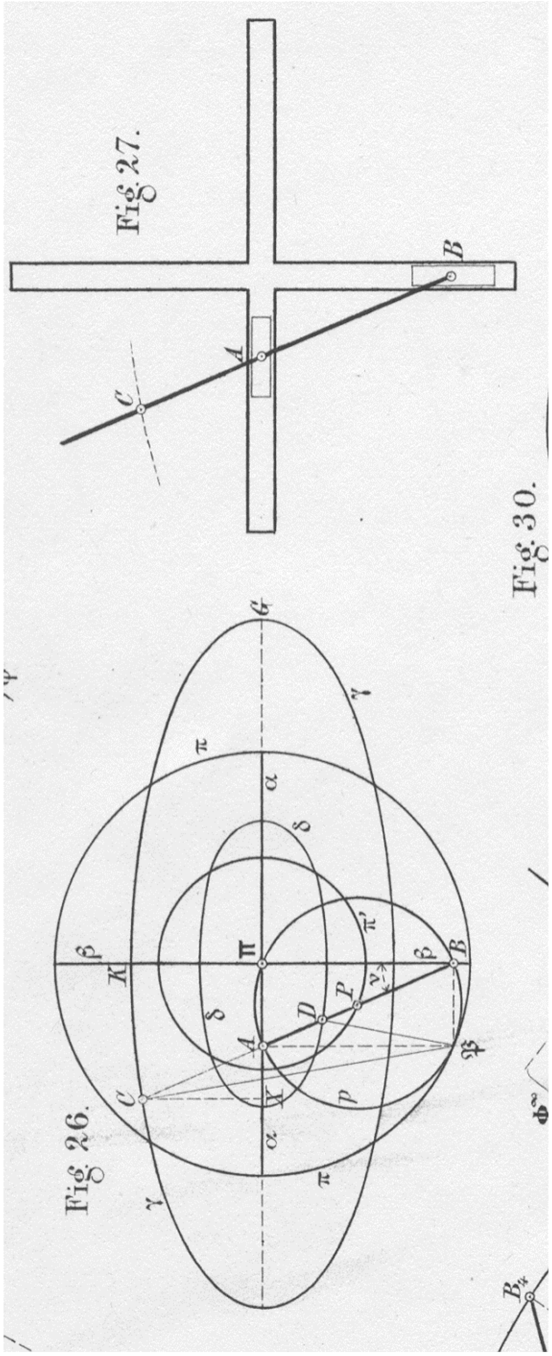


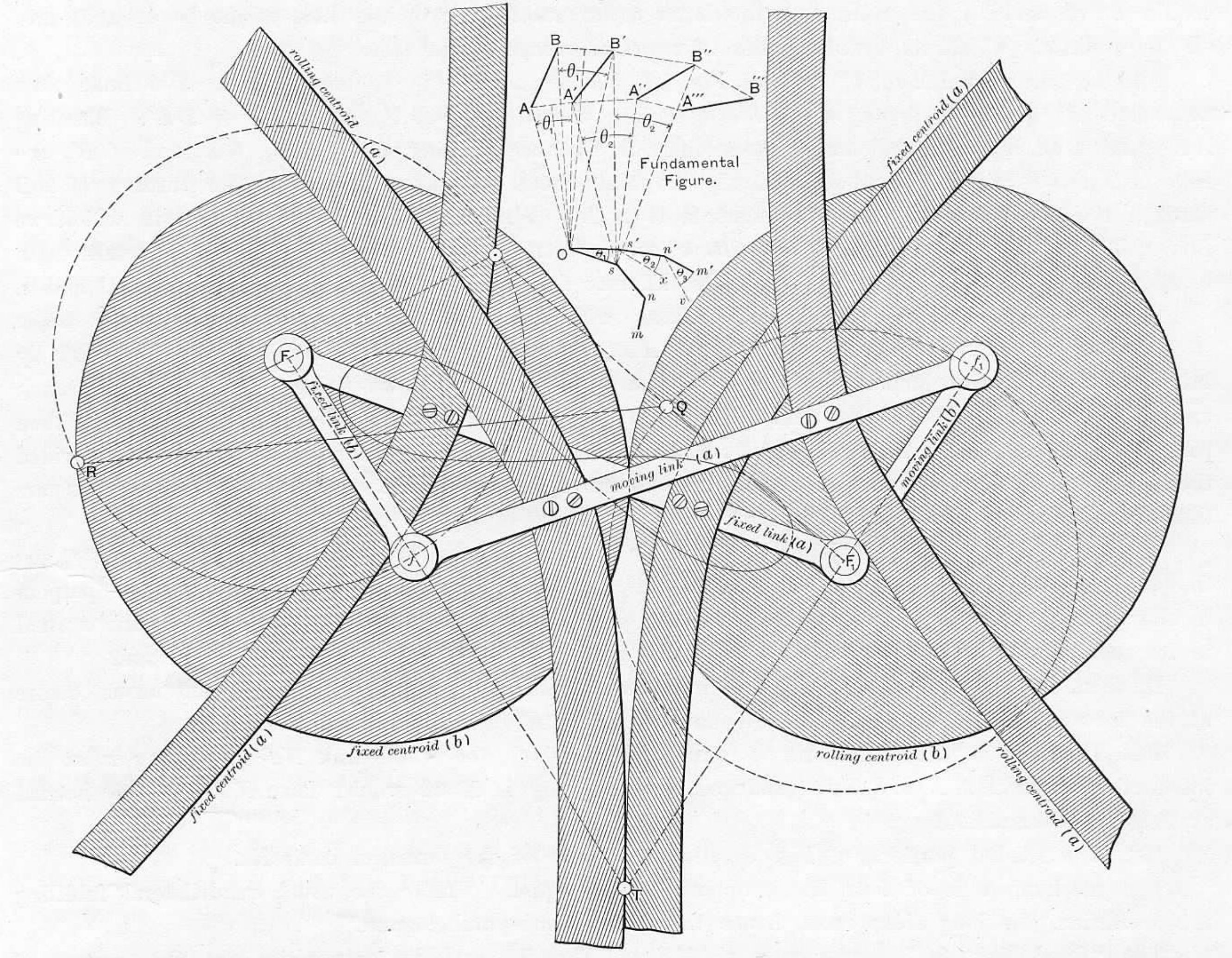
FIG. 113.

4. Centrodes for pure rolling of a link constrained by a crossed double slider prismatic joint. (From Burmester, 1888.)



5. (up) Centrodes for pure rolling for links of a four bar mechanism. (From Wilson, 1998, after Reuleaux, 1876)

Fig. 99.



5. (down) Centrodes for pure rolling for links of a four bar mechanism. (From Wilson, 1998, after Reuleaux, 1876)

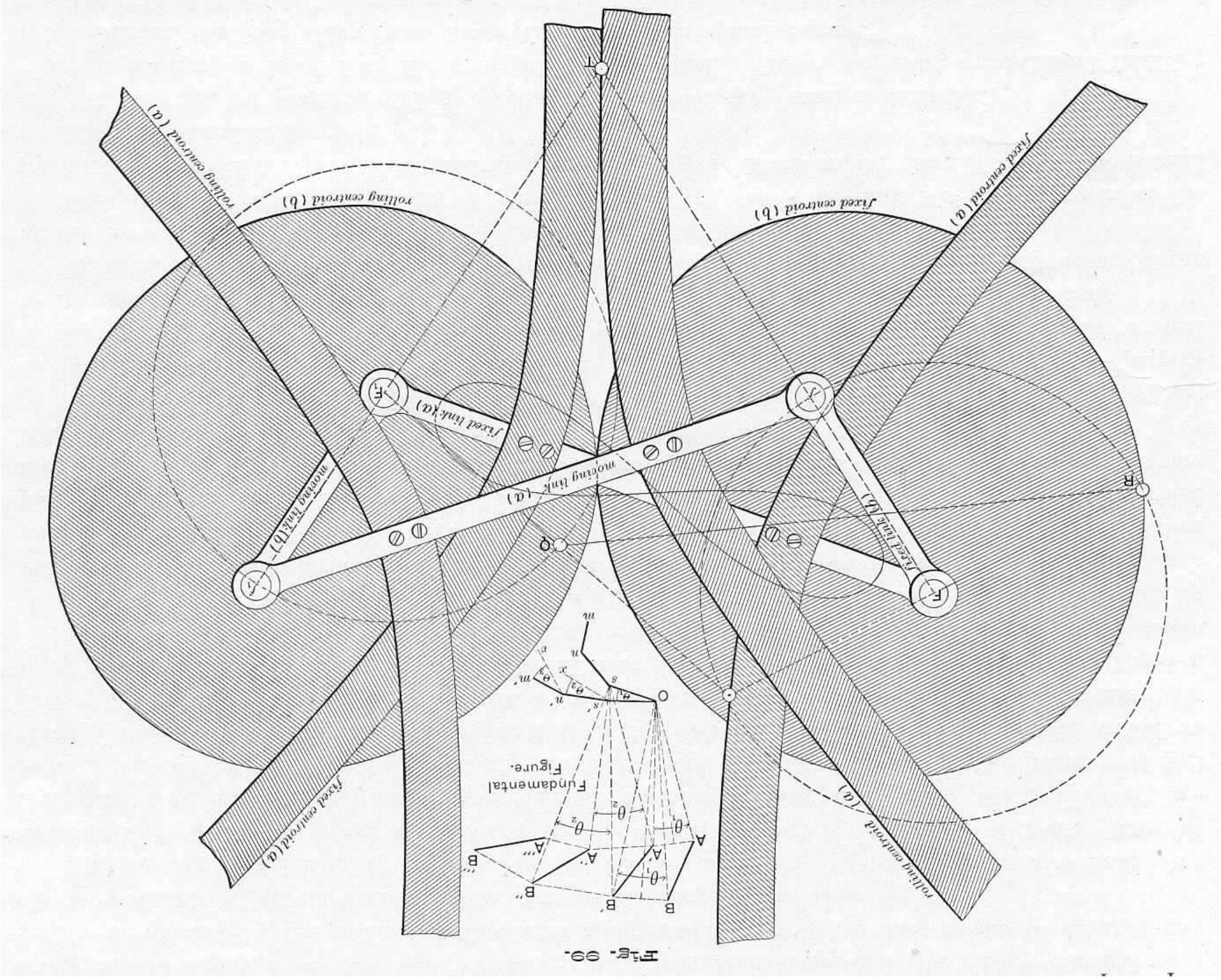
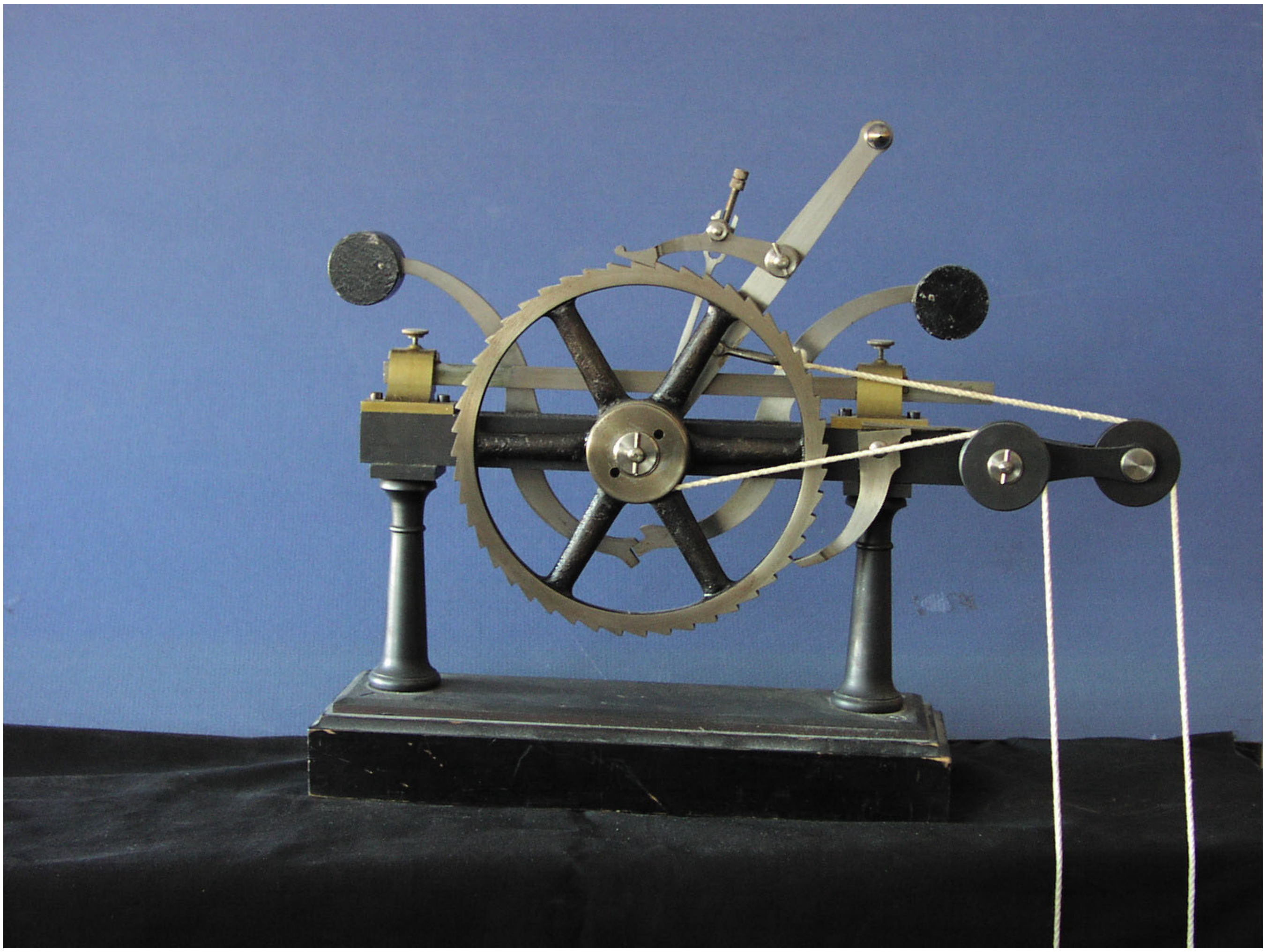
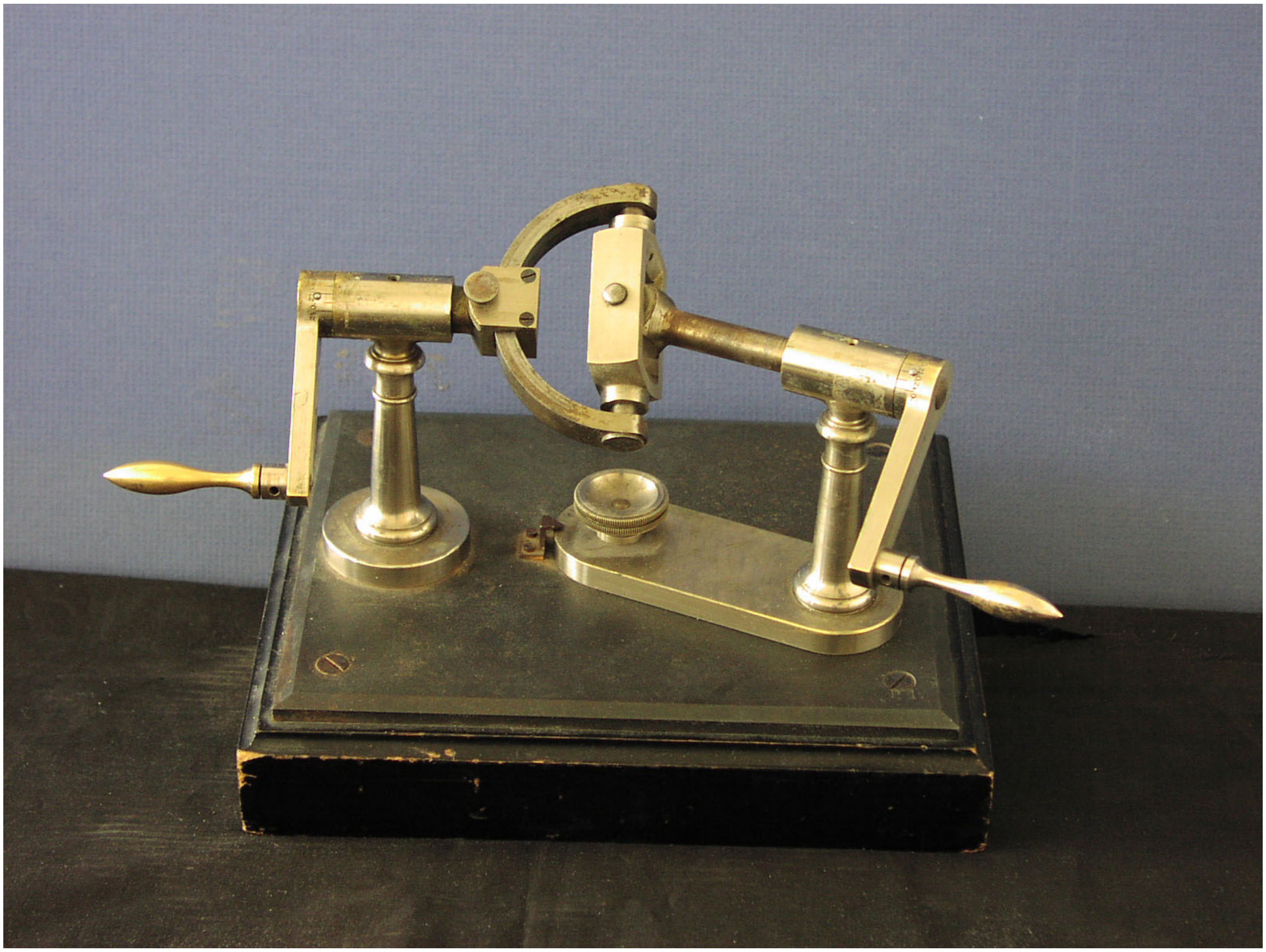


Fig. 99.

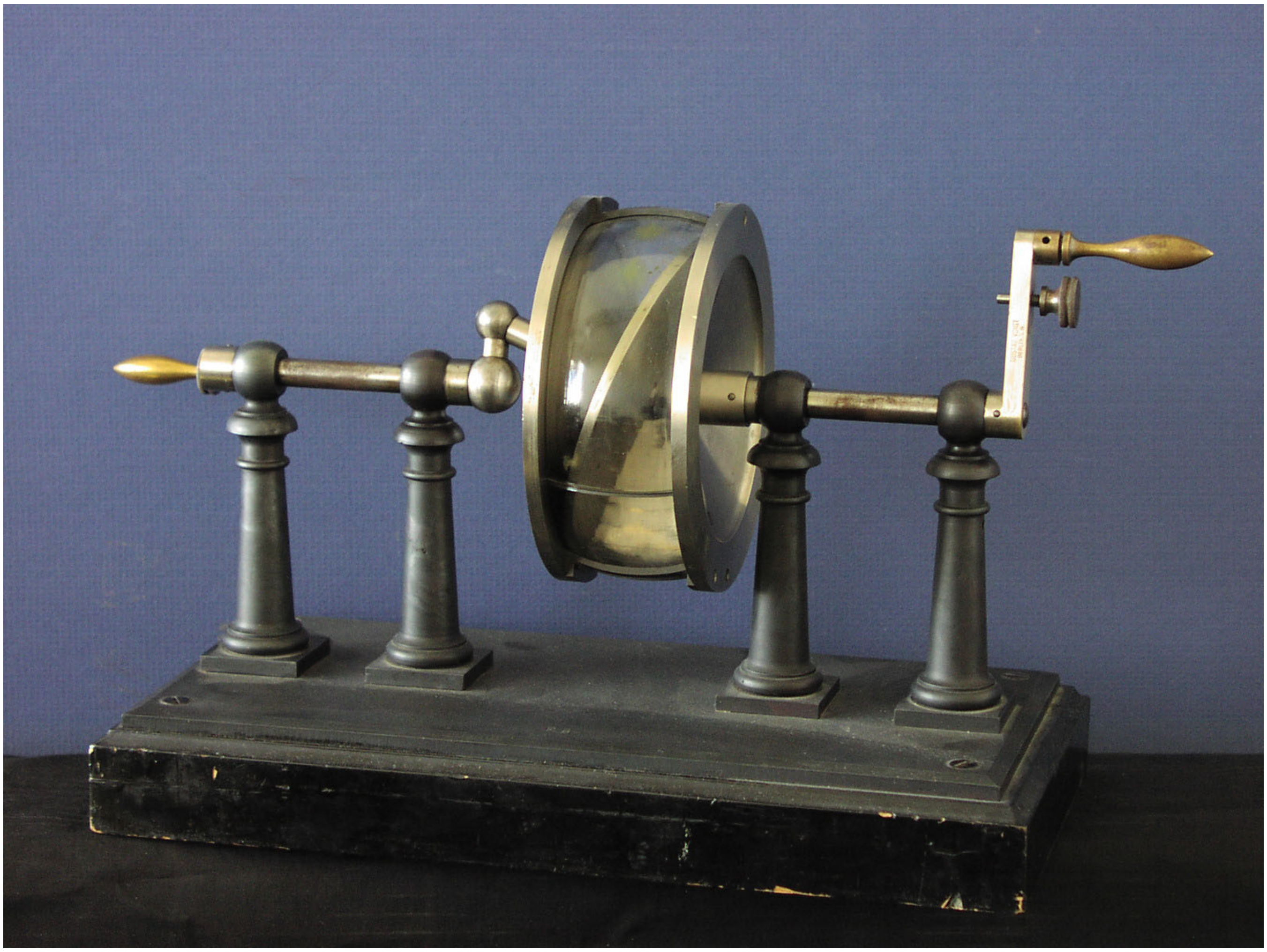
6. Reuleaux model of a ‘power escapement’. Model N24, Voigt Catalog, Cornell Reuleaux Collection.



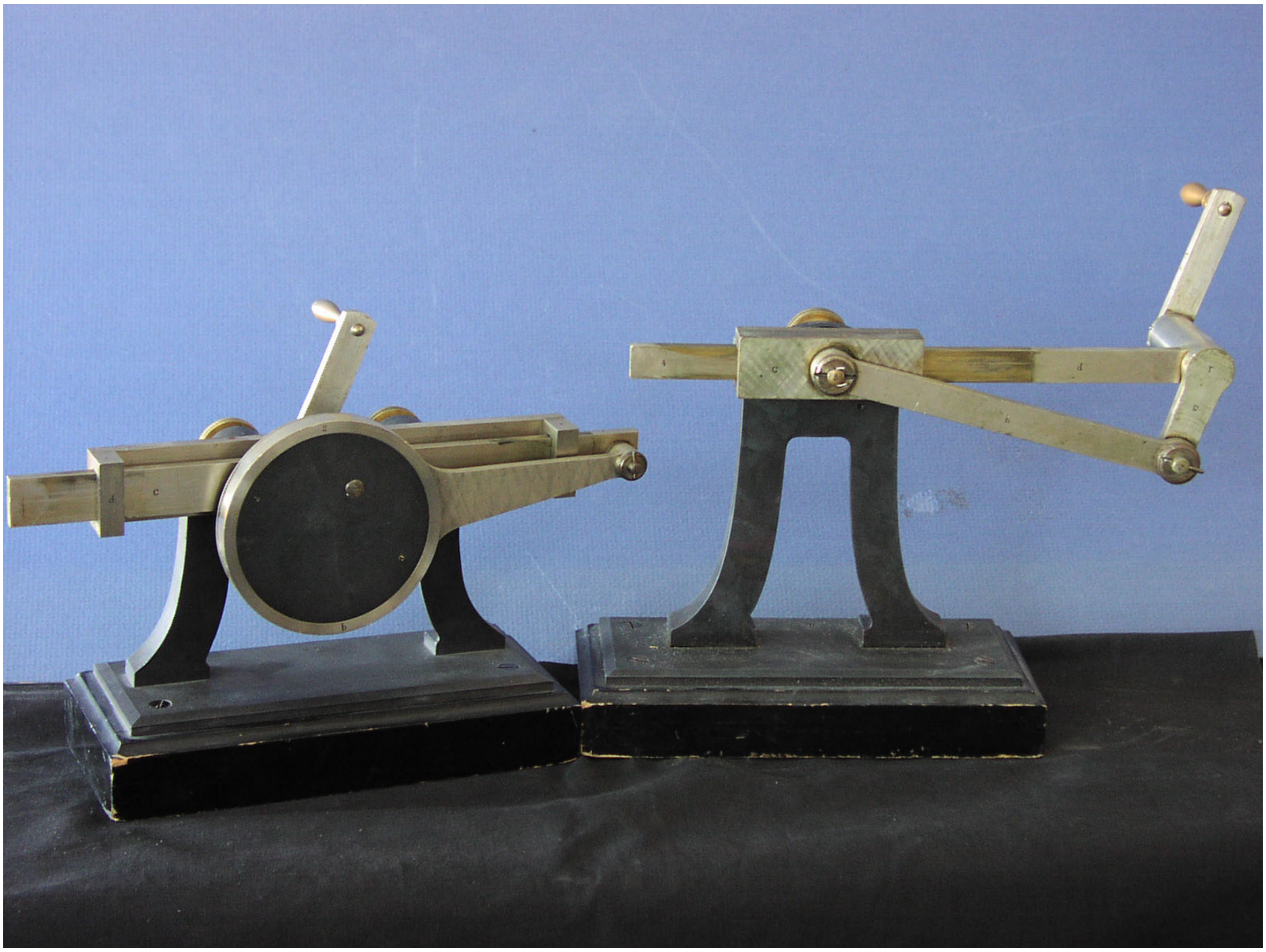
7 a) Universal joint (Hooke or Cardano). Model No P1, Voigt Catalog, Cornell Reuleaux Collection.



b) Spherical engine mechanism of Taylor and Davies, 1840. Model No F6, Voigt Catalog, Cornell Reuleaux Collection.



8. Two members of the slider crank family with the same kinematic chain symbol. Models No E2, C2, Voigt Catalog, Cornell Reuleaux Collection.



9. Example of “higher” pairs. Sketch of motion of a ‘duangle’ rotating in a equilateral triangle. From *Kinematics of Machinery* (1876), Figure 121b.

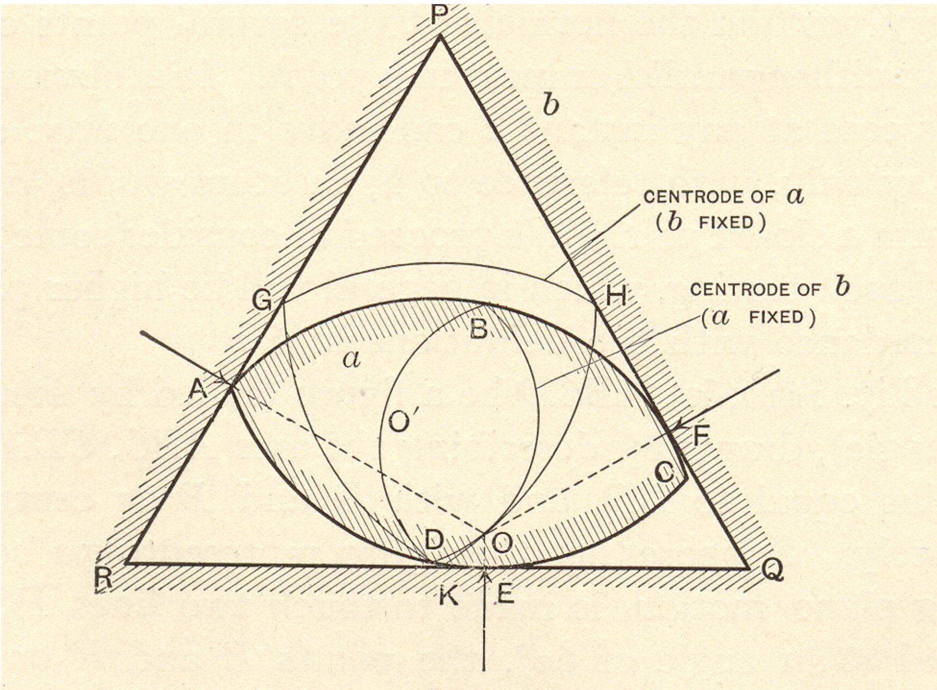
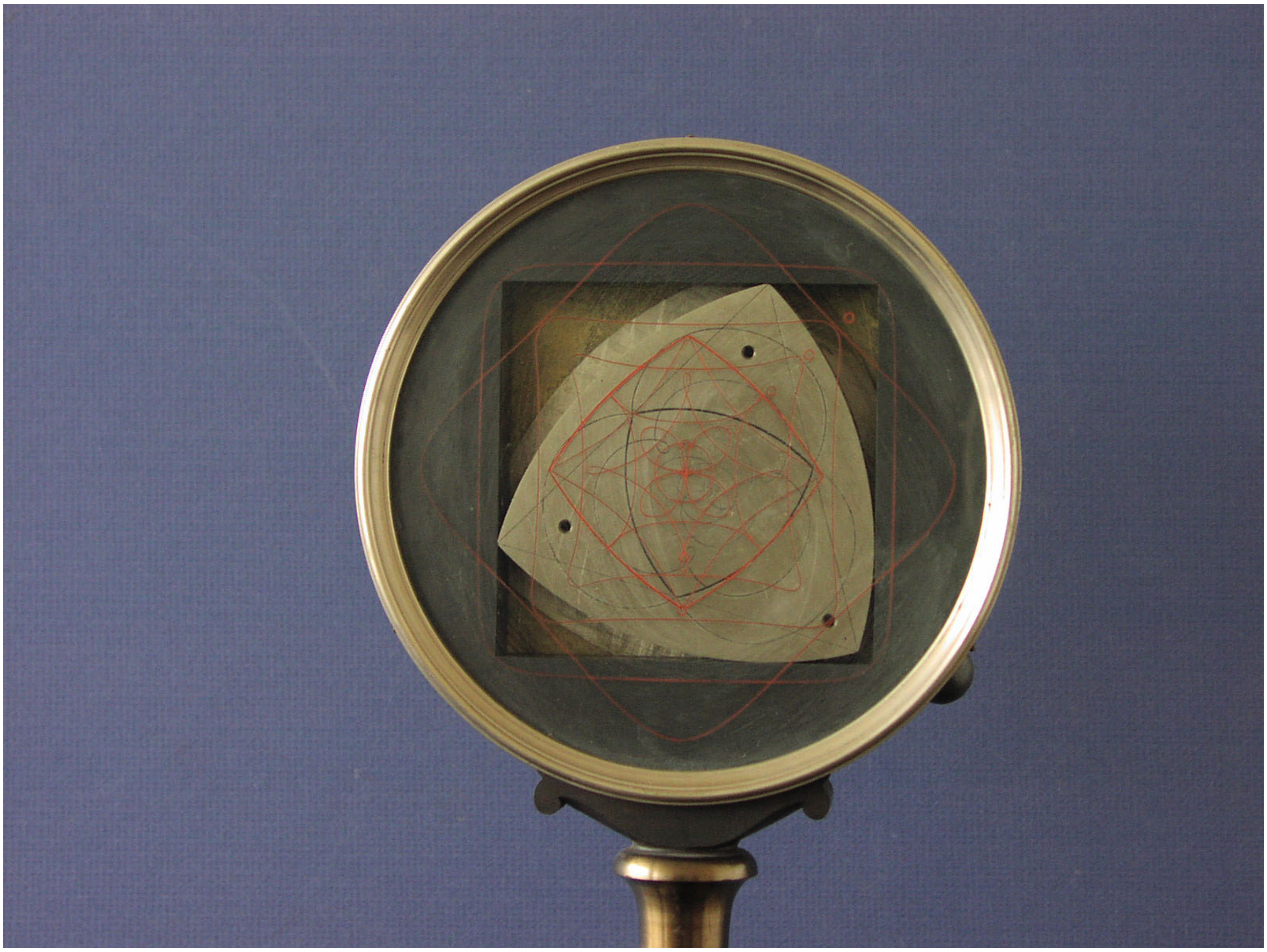
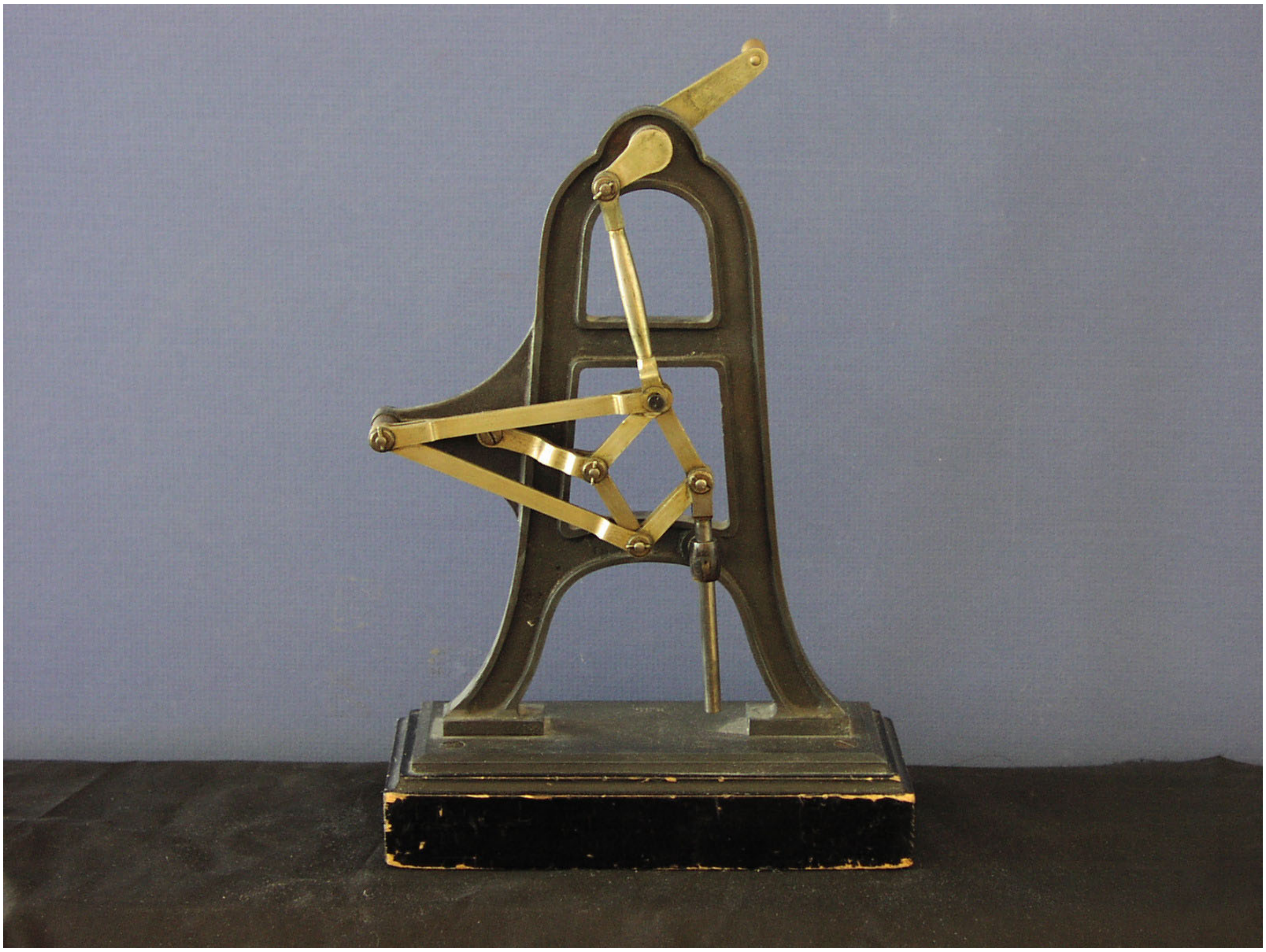


FIG. 121*b*.

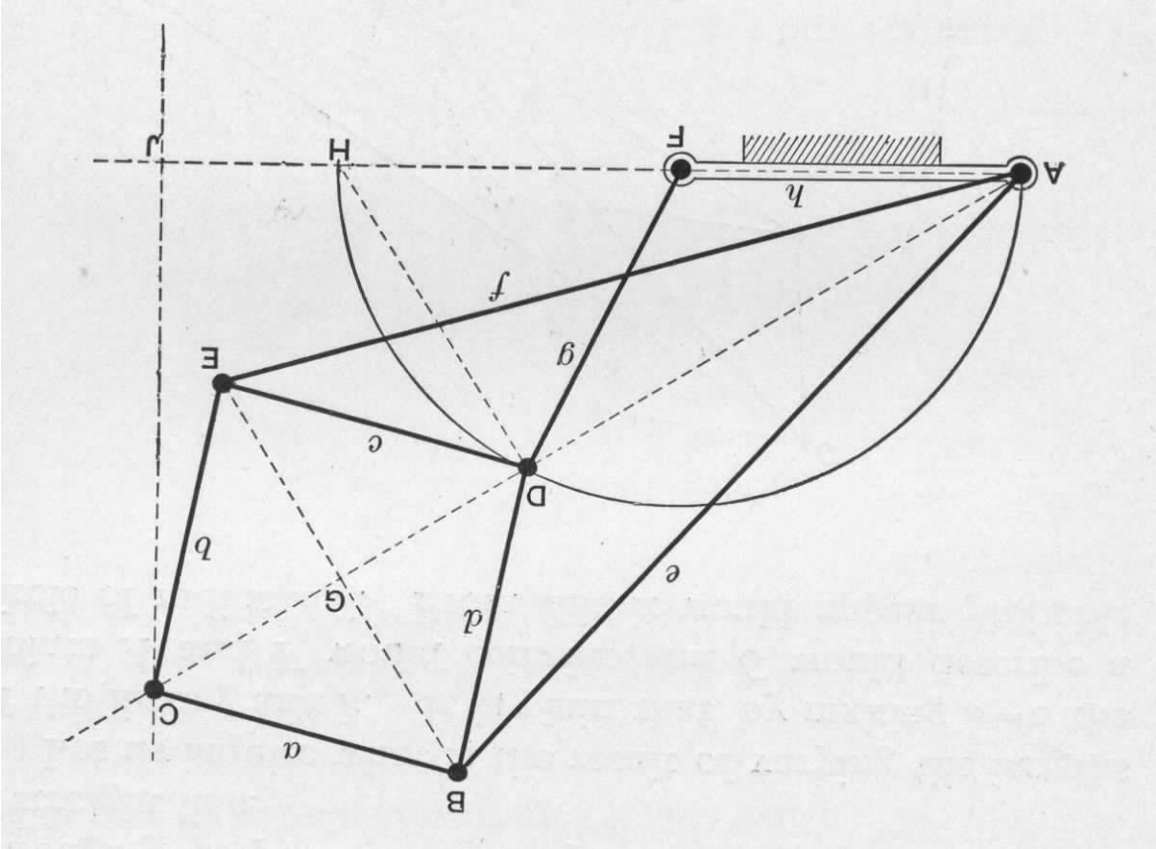
10. Reuleaux “triangle” rotating in a square bearing. Cornell Collection; Model B2, Voigt Catalog, Cornell Reuleaux Collection.



11. Peaucellier Straight Line Mechanism, c 1876. Model S35, Voigt Catalog, Cornell Reuleaux Collection.



12. Sketch of straight-line motion for a Peaucellier mechanism.



13. Chamber wheel train for a steam engine or pump. Model I4 Voigt Catalog, Cornell Reuleaux Collection.

.

14. Drawing of a cylinder escapement by F Redtenbacher, from *Bewegungs Mechanismen*

Fig. 1.

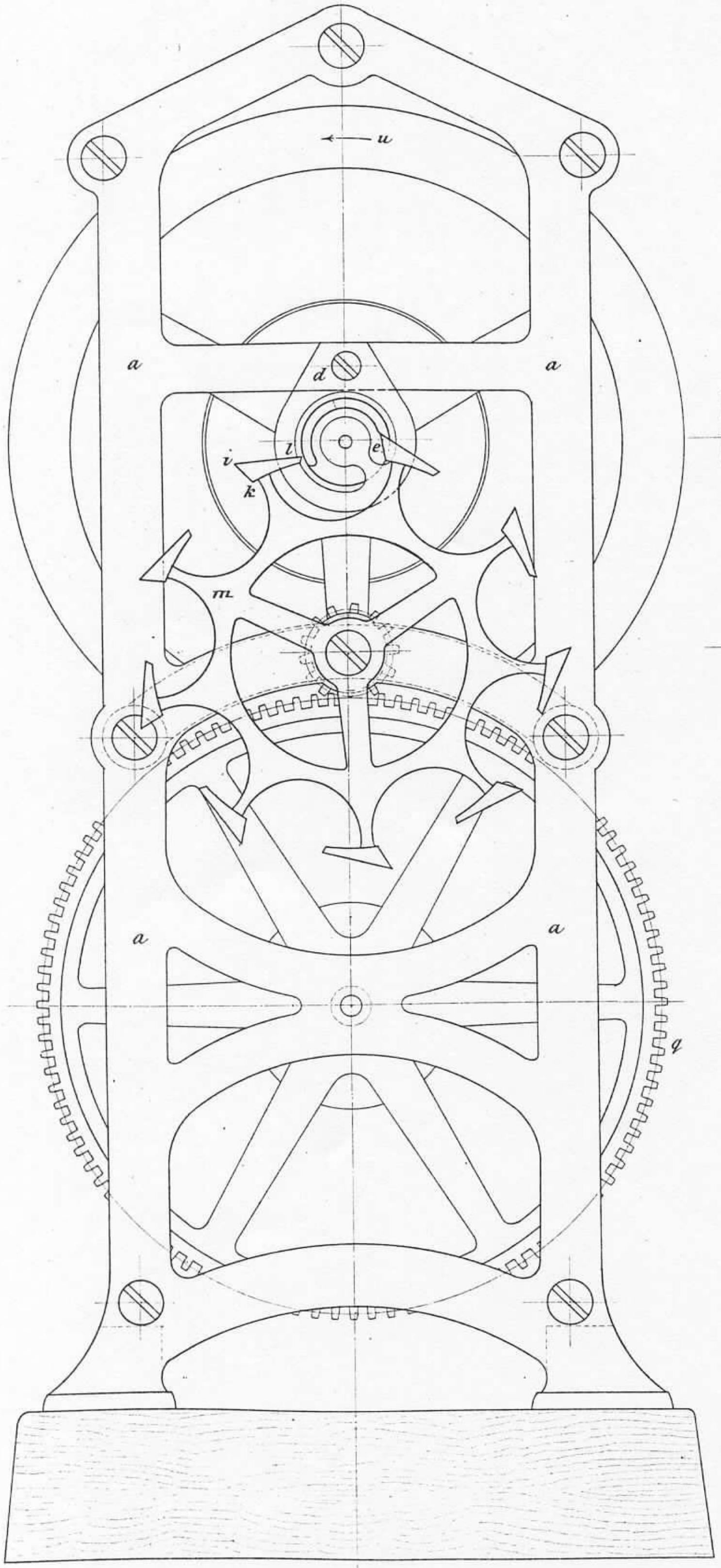
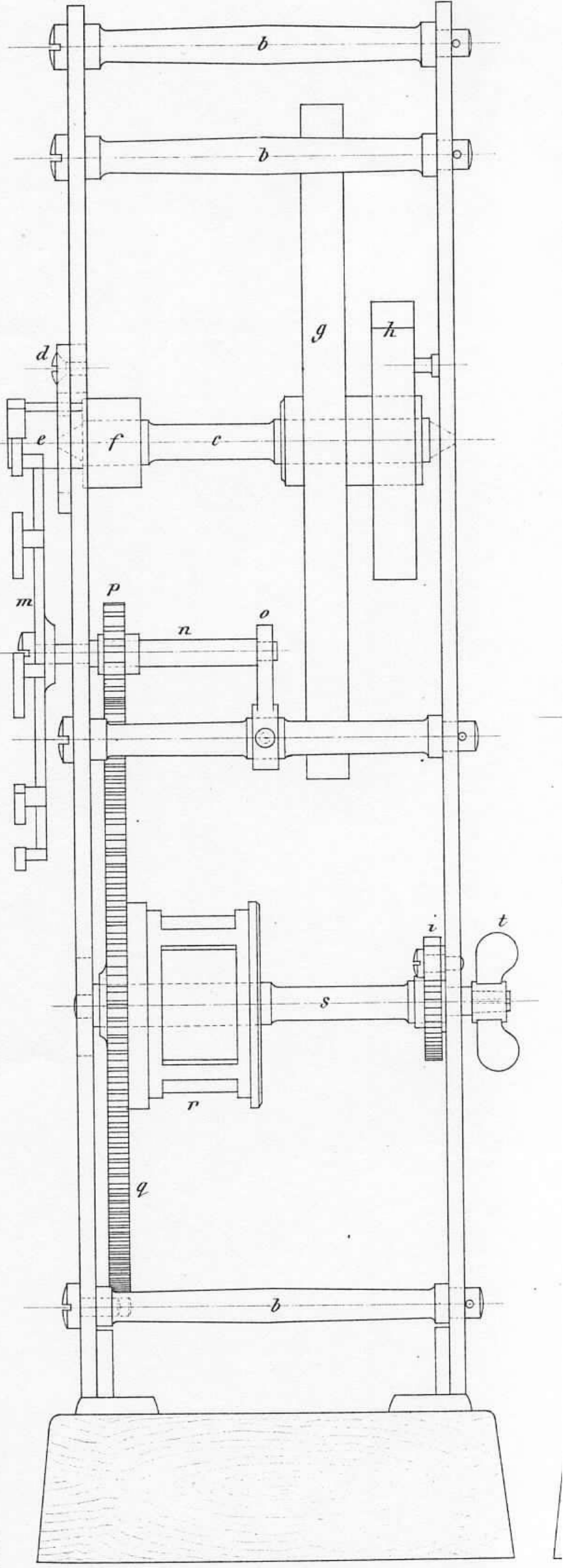
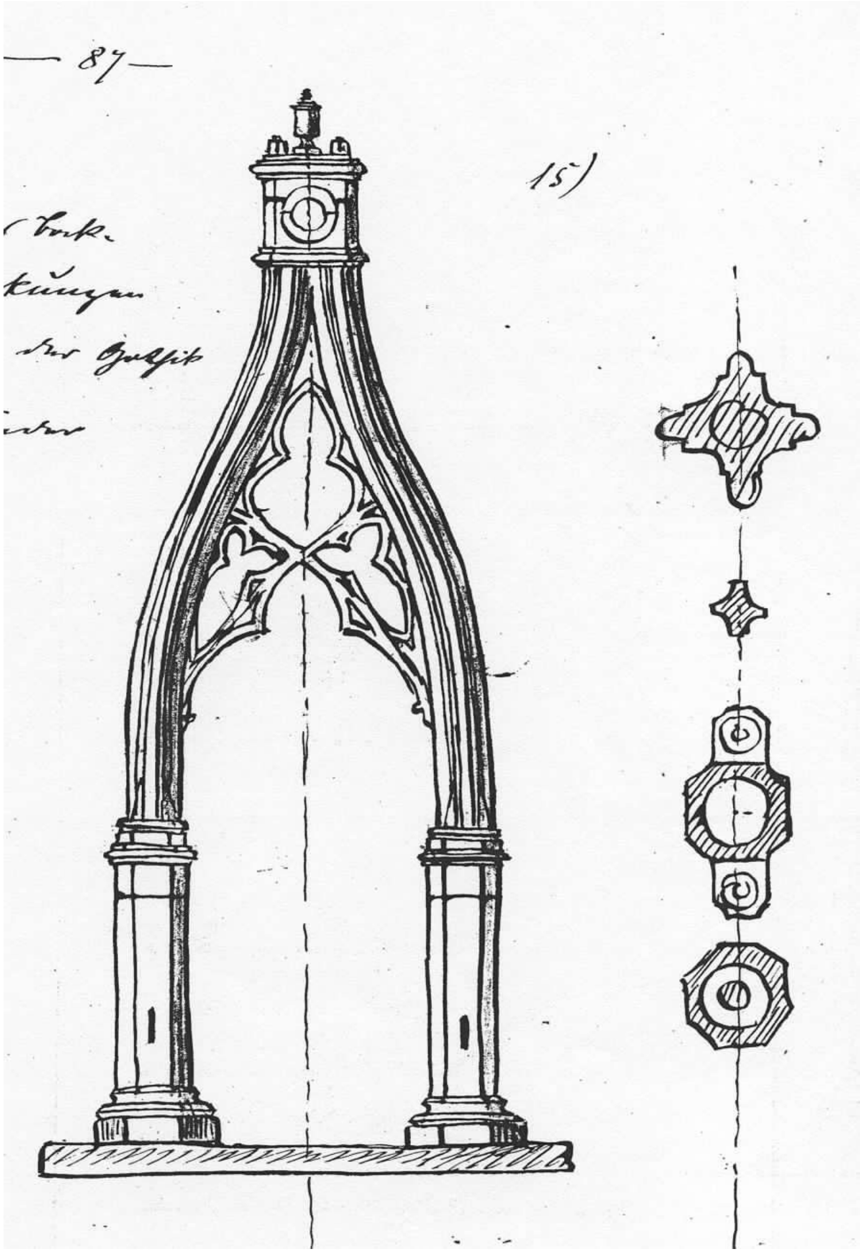


Fig. 2.



15. Cylinder escapement. Model X2, Voigt Catalog, Cornell Reuleaux Collection.

16. Drawing from Franz Reuleaux’s sketch book. Archiv of the Deutsches Museum, Munich.



17. Equivalent kinematic linkage for a fish jaw, after F. Reuleaux, (1900), Model in the Deutsches Museum by Thilo (1901)